Phenomenology of the QCD phase transition

A unified equation of state from the cluster virial expansion
within the generalized Beth-Uhlenbeck approach

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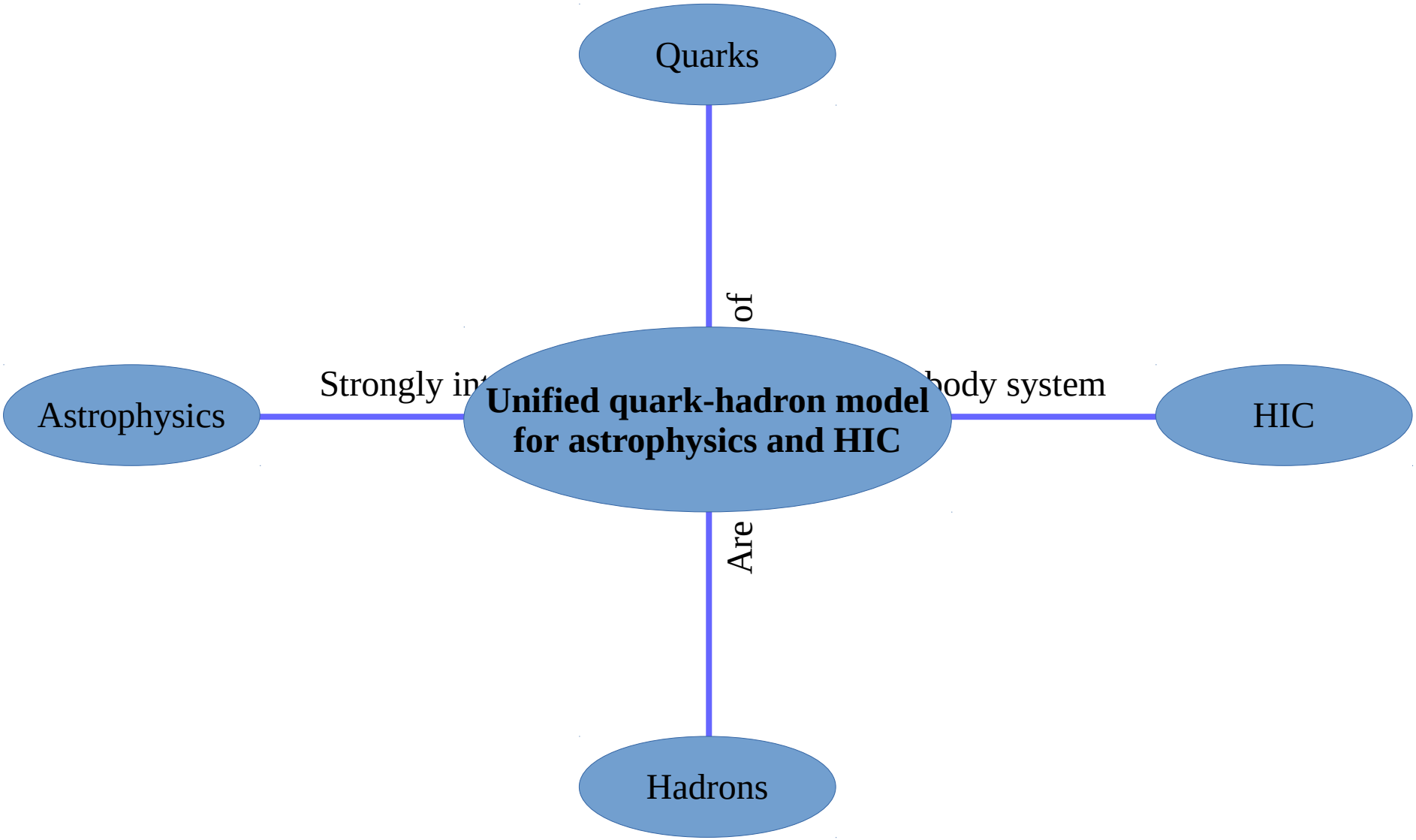
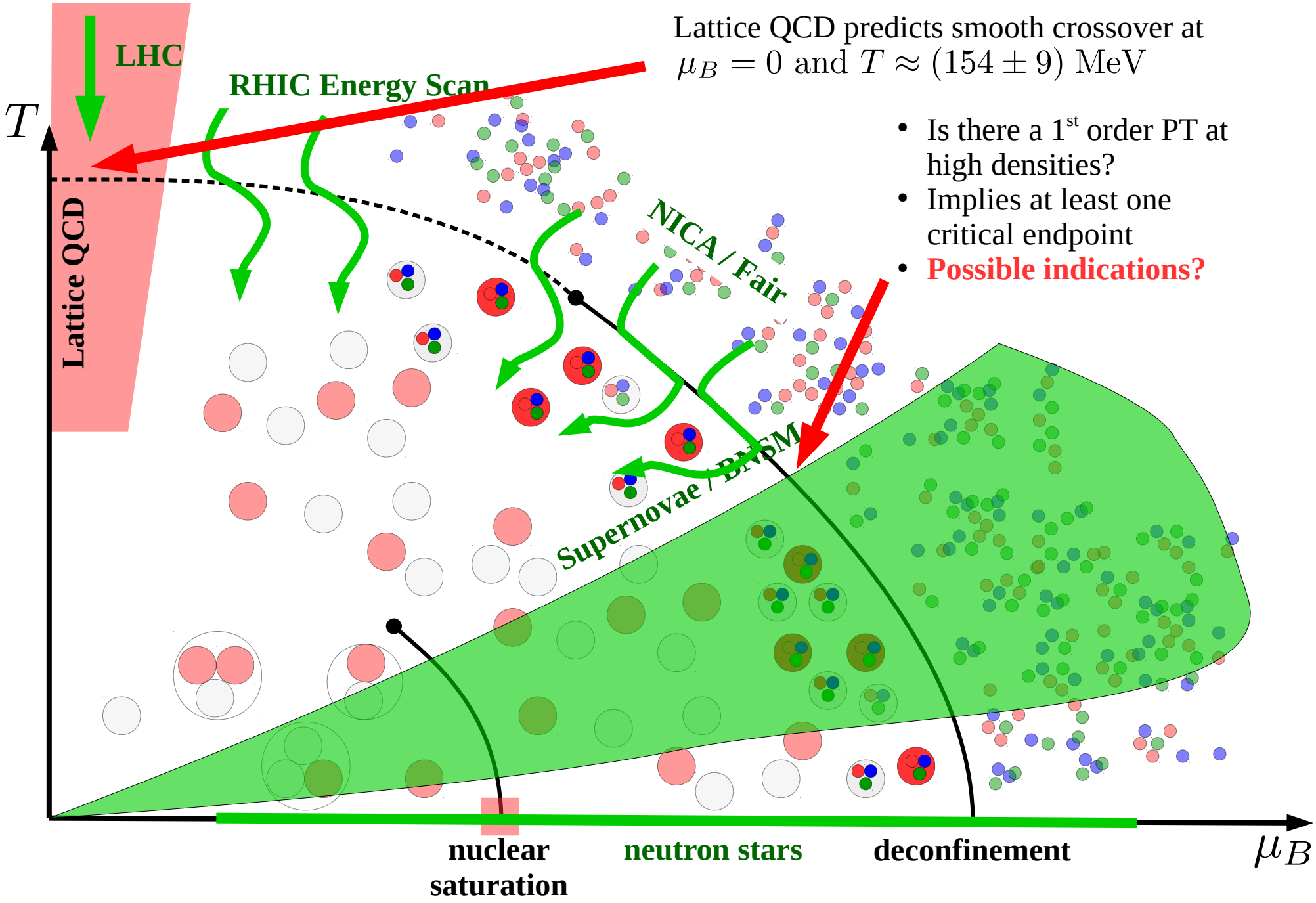
Jena, 14th of August 2019

Uniwersytet Wrocławski

Unified equation of state



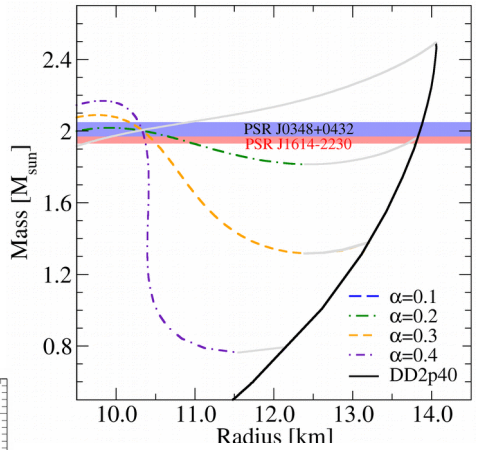
Possibility of 1st order PT at high densities

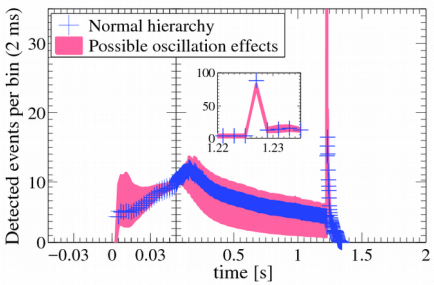


Outline

Possible signals of 1st – order phase transitions.

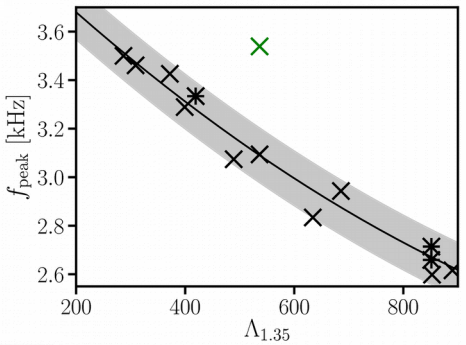
Neutron star configurations

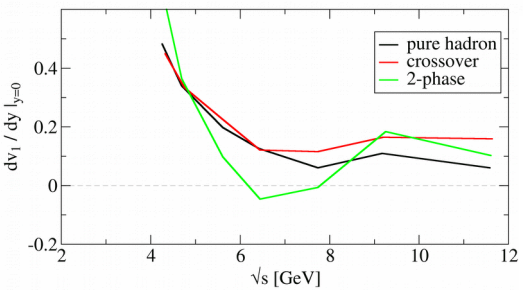




Supernova explosions of 50Ms stars

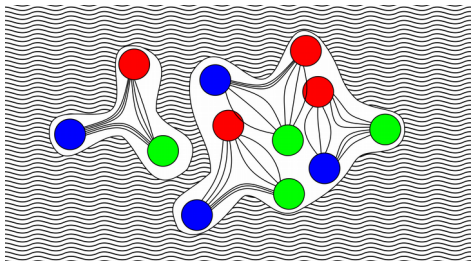
Binary neutron star mergers





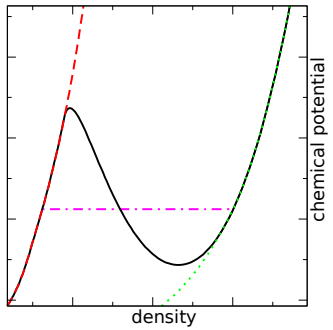
Heavy-Ion Collisions

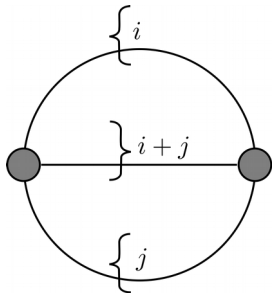
Unified description of the equation of state.



density functional theory

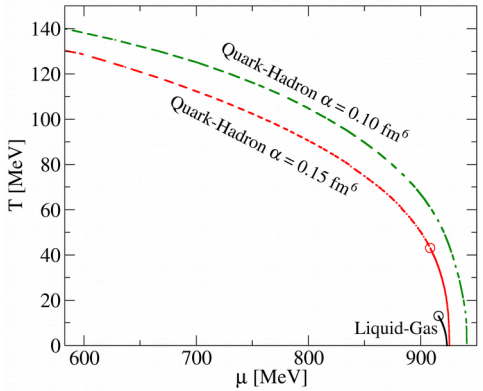
construction of phase transitions





Phi-derivable formalism

current status



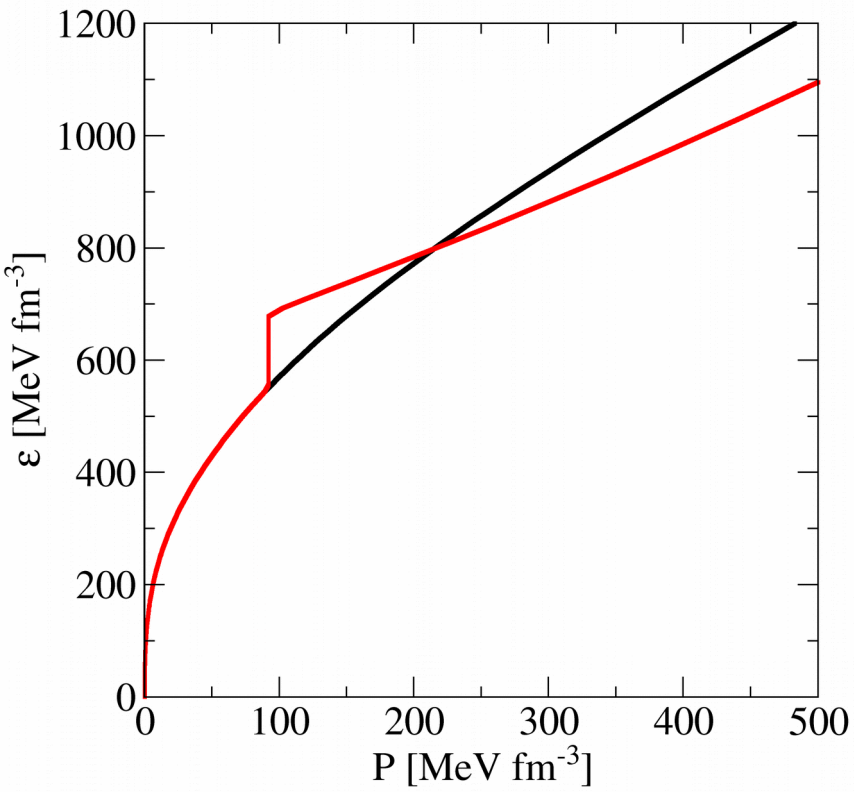
1st order PT – Neutron stars

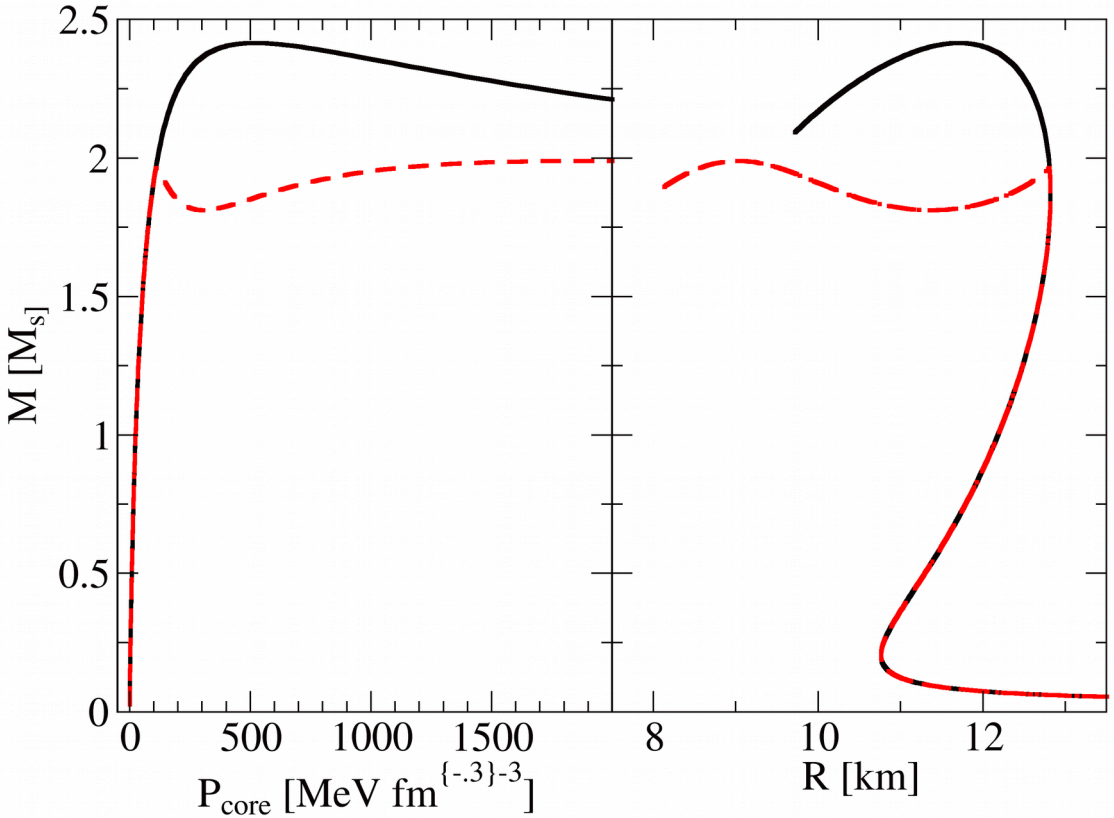
Tolman-Oppenheimer-Volkoff equations

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm(r)\epsilon(r)}{r^2}\frac{\left[1+P(r)/\epsilon(r)\right]\left[1+4\pi r^3 P(r)/m(r)\right]}{1-2Gm(r)/r} \qquad \frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi\epsilon(r)r^2$$

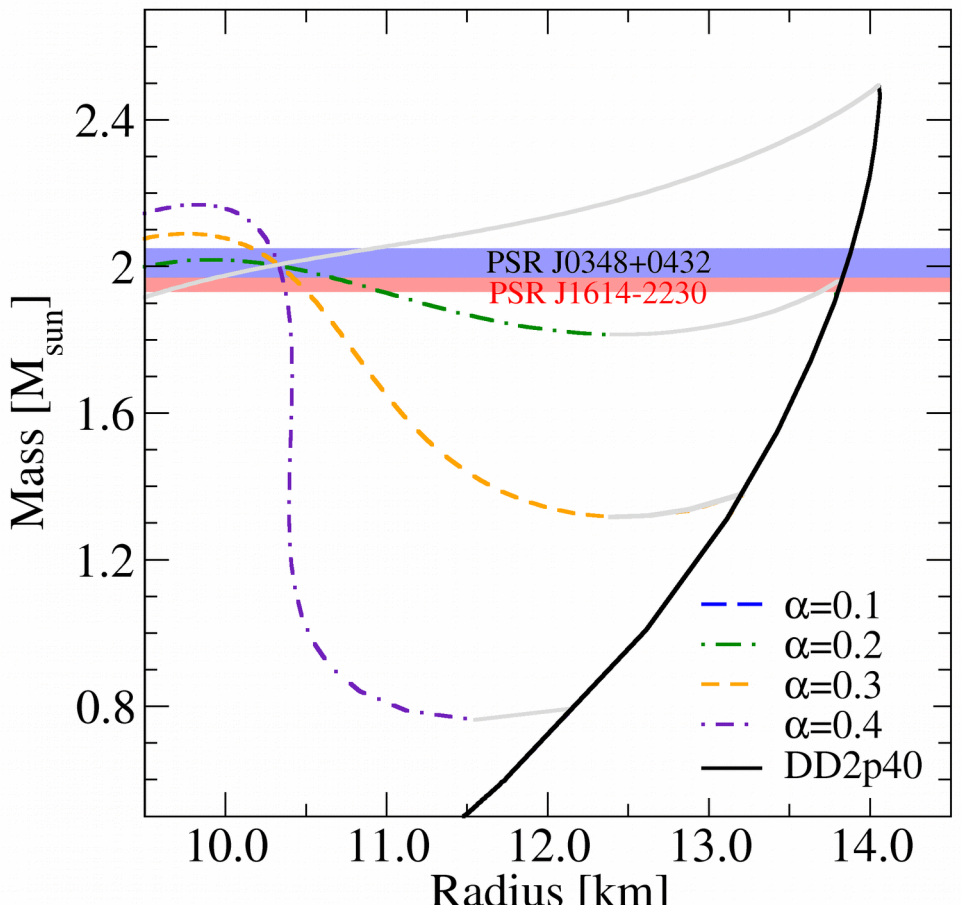
Needs an equation of state and the boundary condition of core density/pressure/energy density

$$\epsilon = \epsilon(P) \qquad n_0,\ P_0,\ \epsilon_0$$

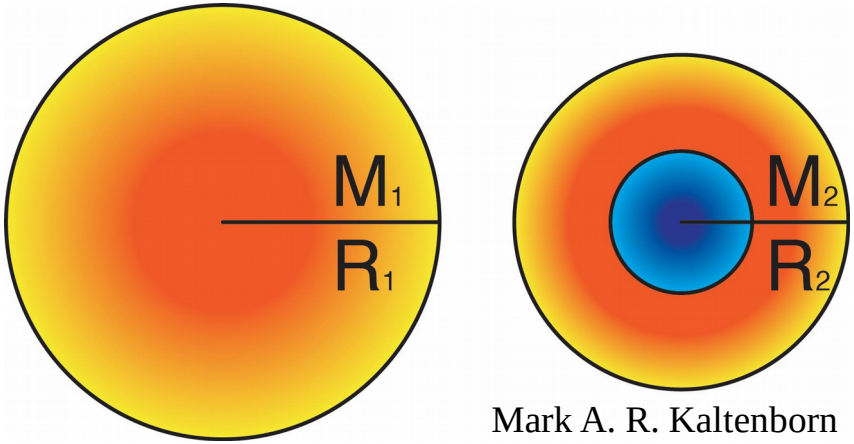




1st order PT – Neutron stars



- Star configurations with same masses, but different radii



Mark A. R. Kaltenborn

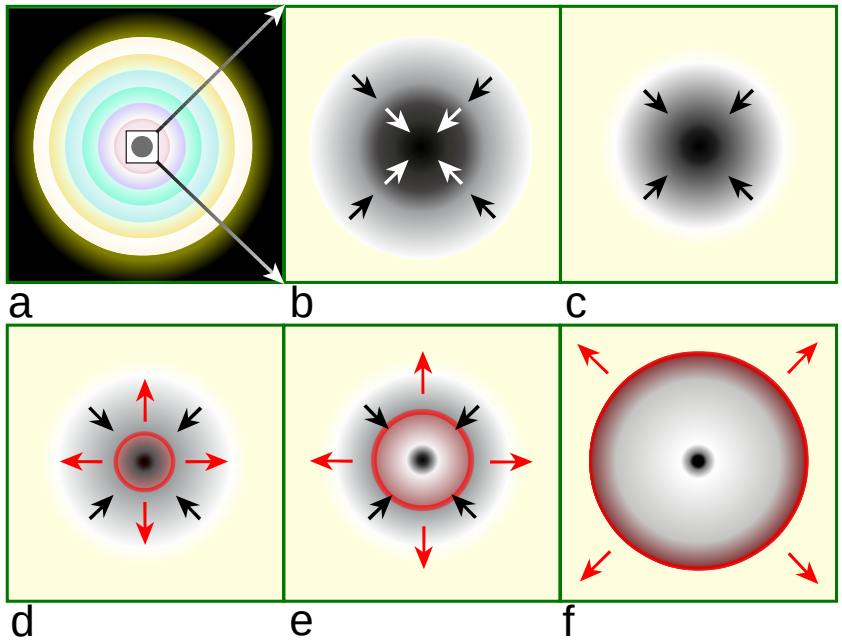
- **New class of EOS, that features high mass twins**
- NASA NICER mission: radii measurements ~ 0.5 km
- Existence of twins implies 1st order phase-transition and hence a critical point

M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

Core-collapse supernova explosions

Massive stars (~ 8 Ms)

- Sequential burning stages of light elements
- Onion structure with iron core (1.4 Ms)
- Gravitational collapse
- Bounce shock through stiffness of EOS
- Mainly neutrino heating drives shock wave

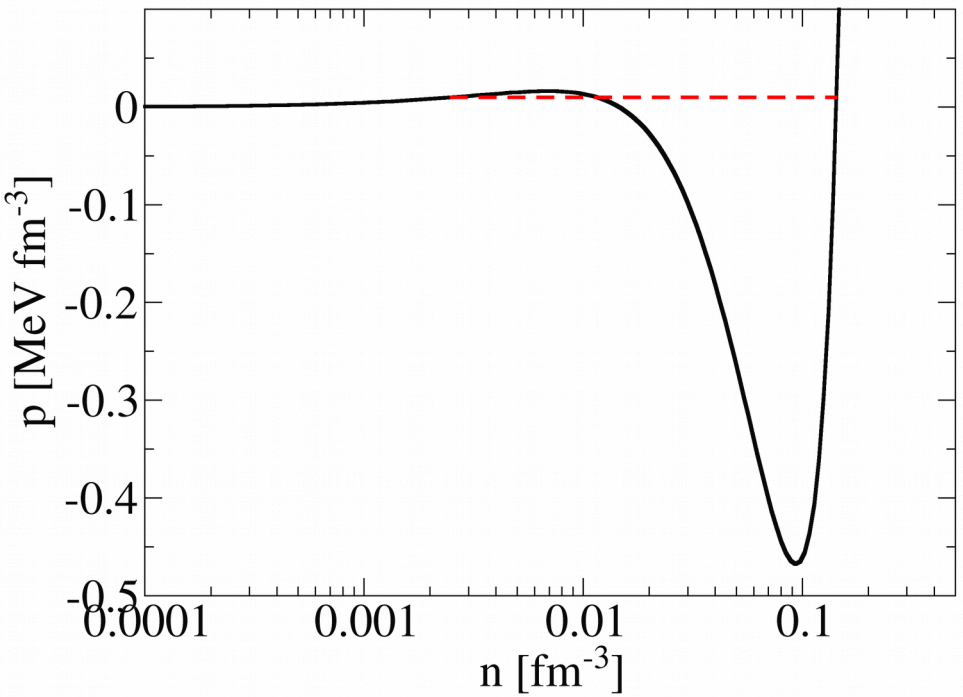


Super-massive stars (~ 50 Ms)

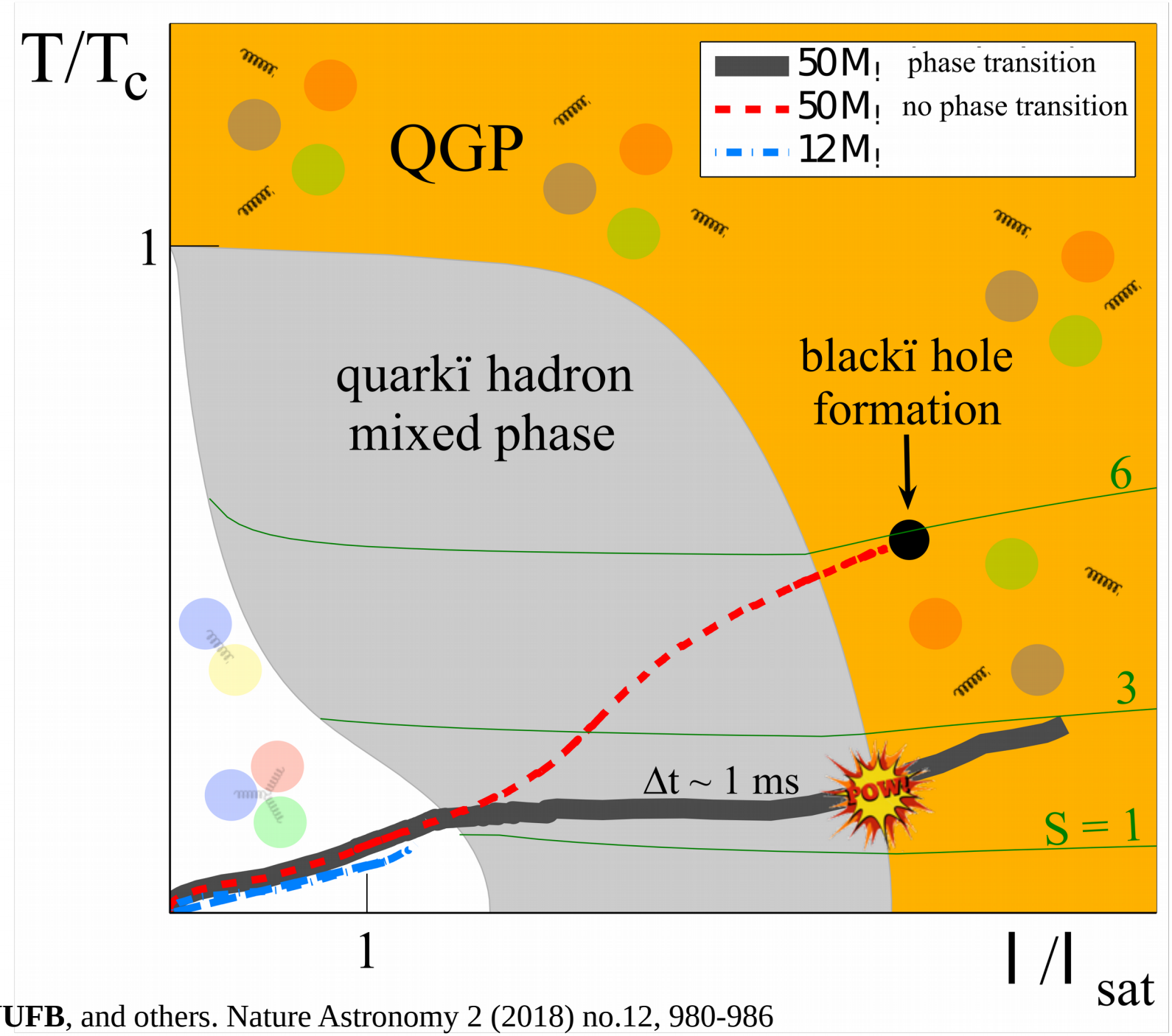
- Can not be explained by canonical models
- Have observational evidences
- One of biggest uncertainties:

high density EOS

how about a quark-hadron PT?

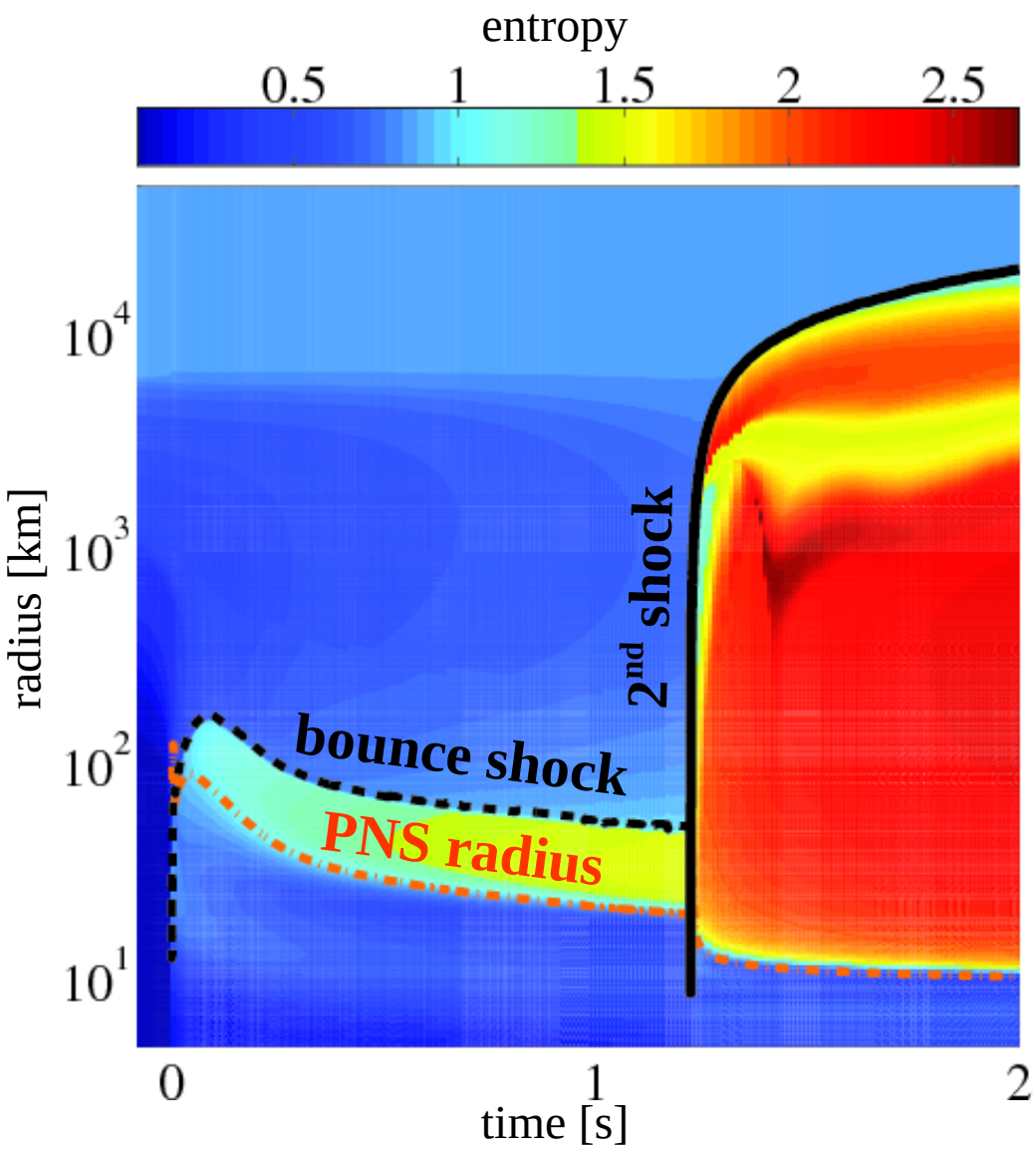


1st order PT – Supernovae of massive stars

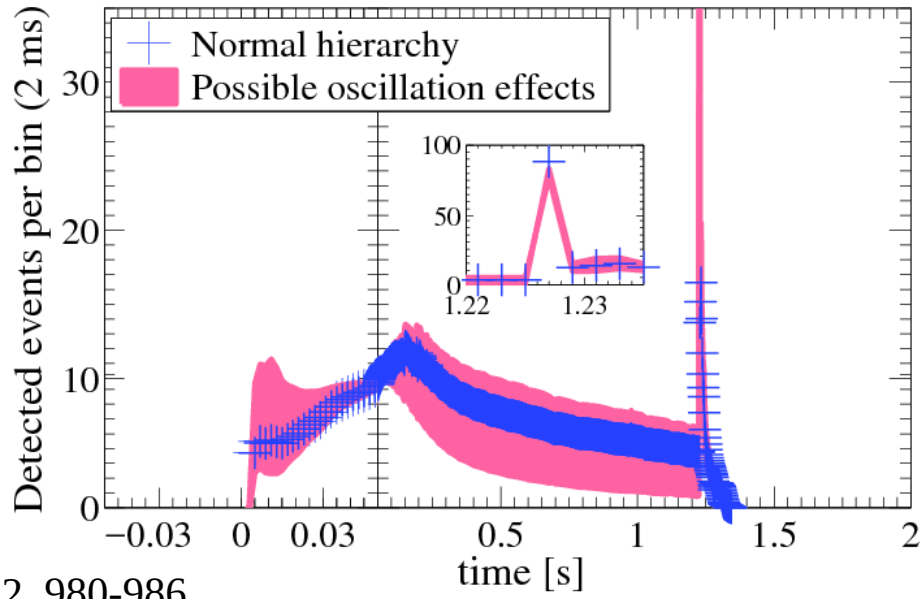


T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1st order PT – Supernovae of massive stars

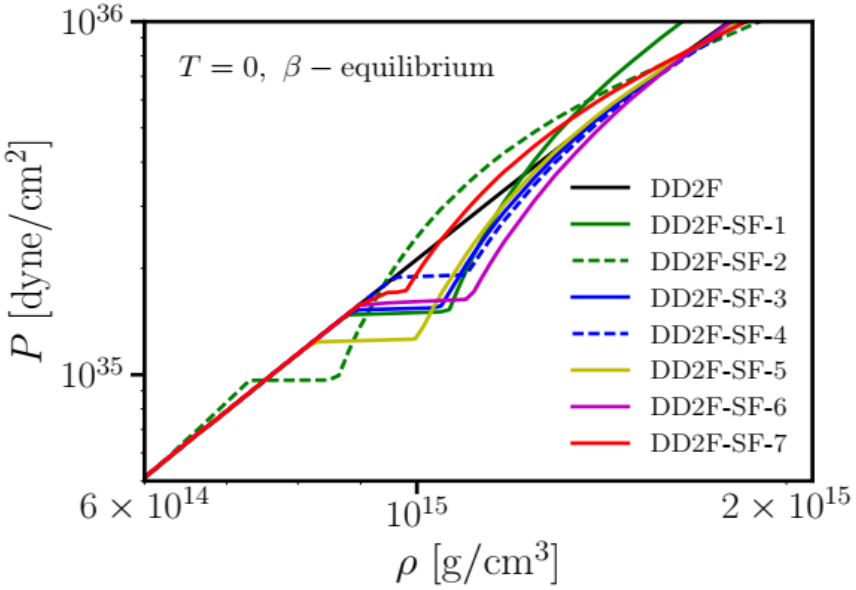


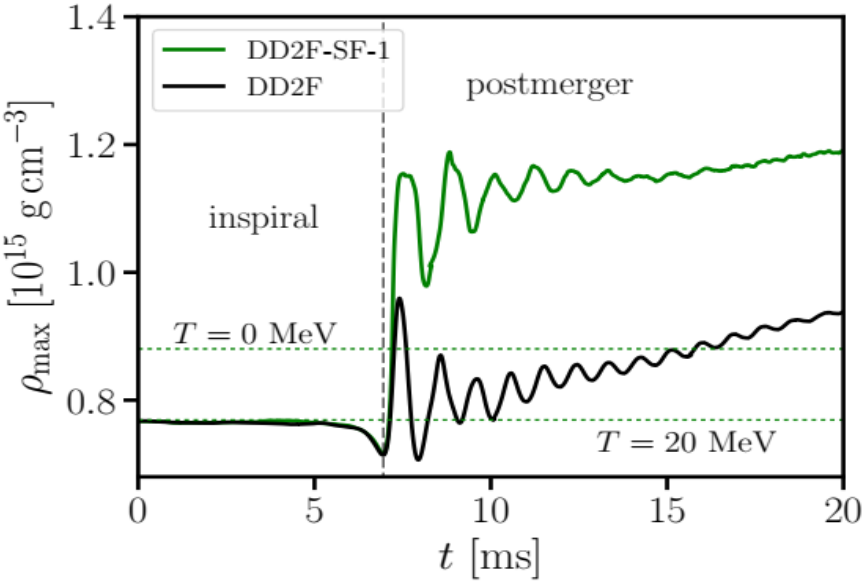
- EoS constructed under consideration of all constraints which are important in astrophysics
- Phase transition releases latent heat to explode "very" massive stars
- Remnant: $2M_\odot$ neutron stars (with quark core) at birth
- Neutrino signal measurable
- Energetic explosion, but almost no nickel

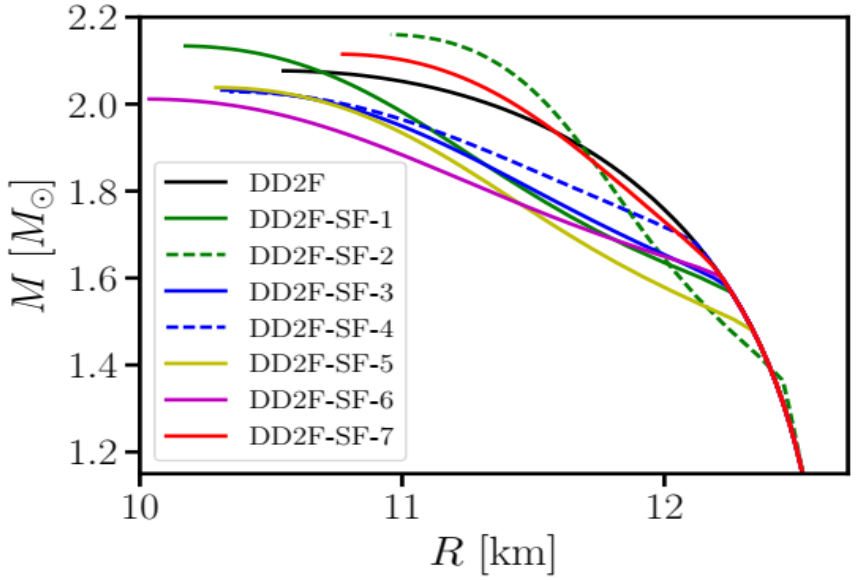


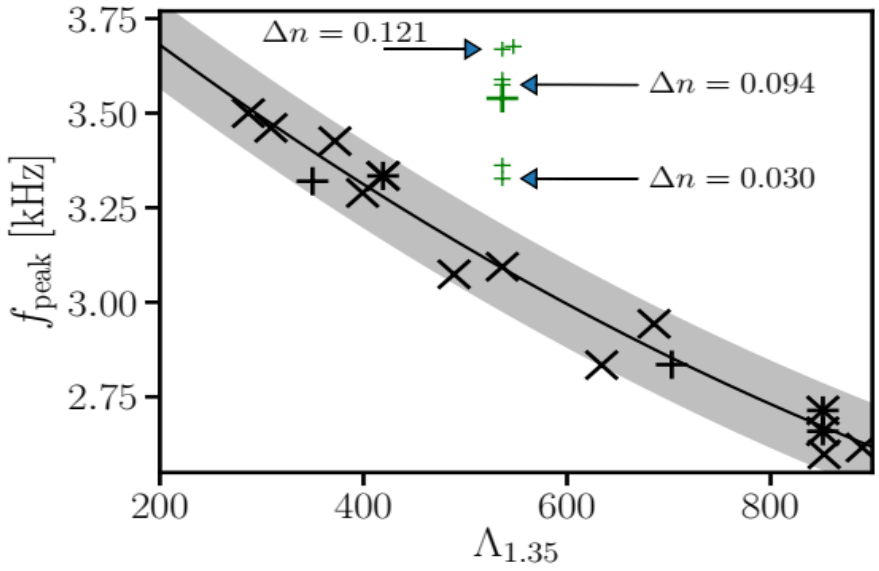
T. Fischer, **NUFB**, and others. Nature Astronomy 2 (2018) no.12, 980-986

1st order PT - Neutron star merger





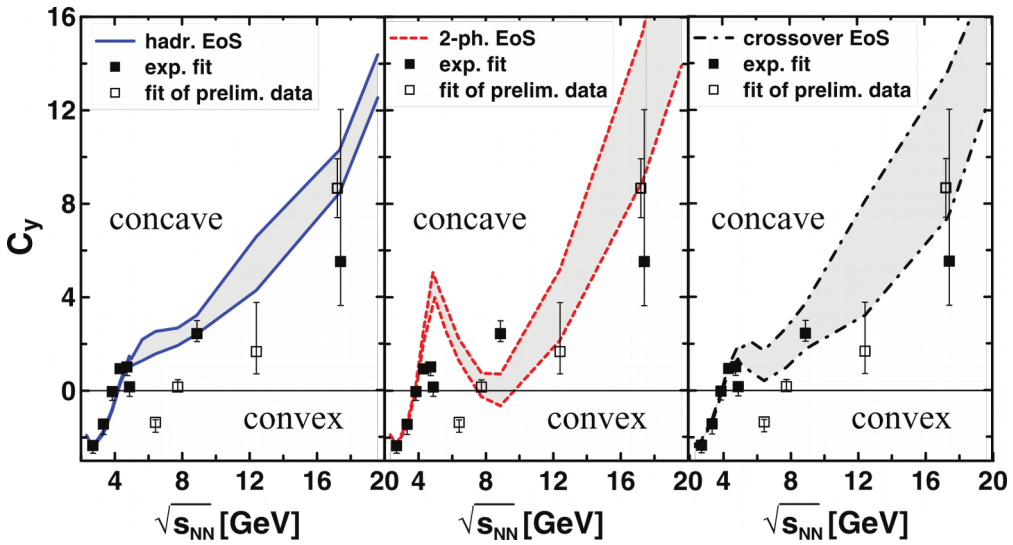




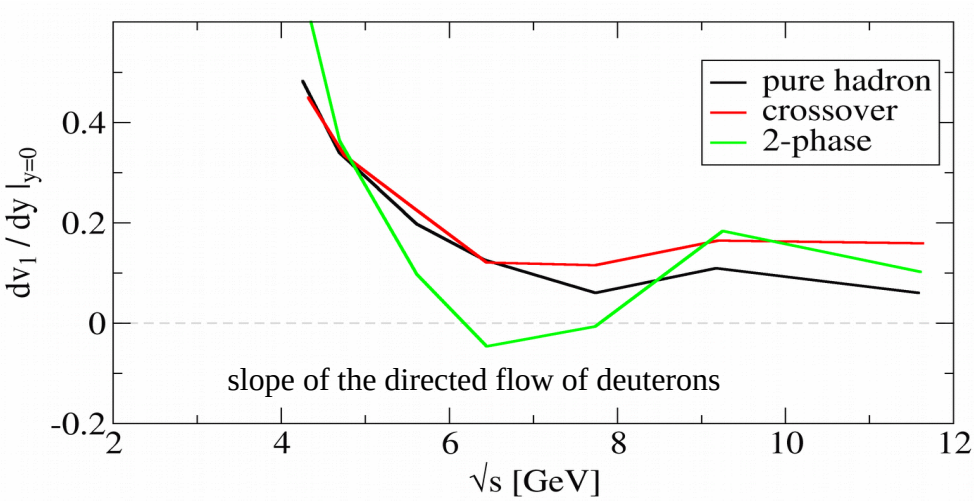
A. Bauswein, **NUFB**, and others, Phys.Rev.Lett. 122 (2019) no.6, 061102

1st order PT – Heavy Ion Collisions

strong signal (wiggle) in the baryon stopping signal [1]



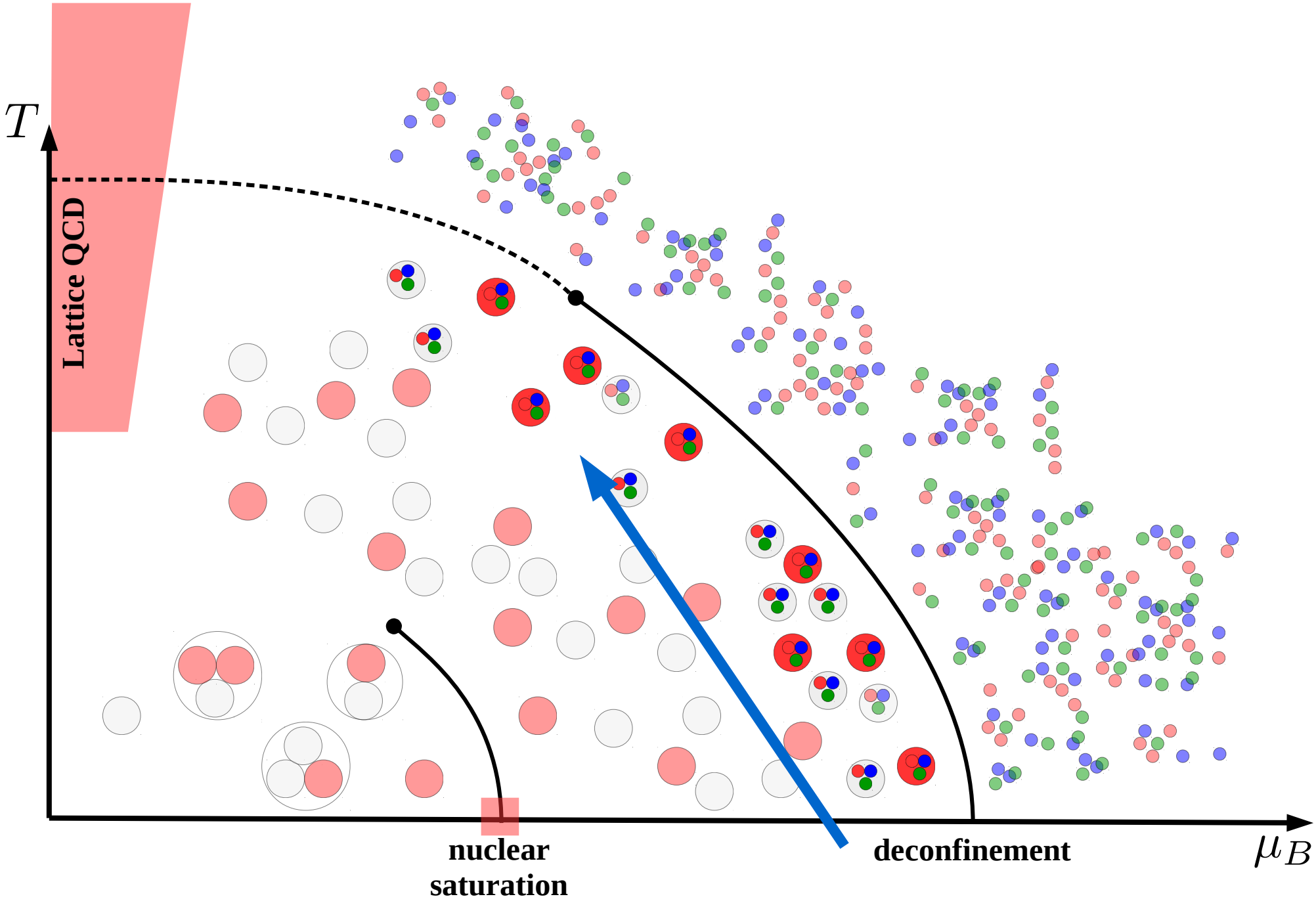
Anti-flow of clusters occur [2]



- Application of the SFM to HIC is ongoing work

[1] Yu. B. Ivanov, PRC 87, 064904 (2013)
[2] **NUFB**, P. Batyuk, D. Blaschke, and others, Eur.Phys.J. A52 (2016) no.8, 244

Model for everything?

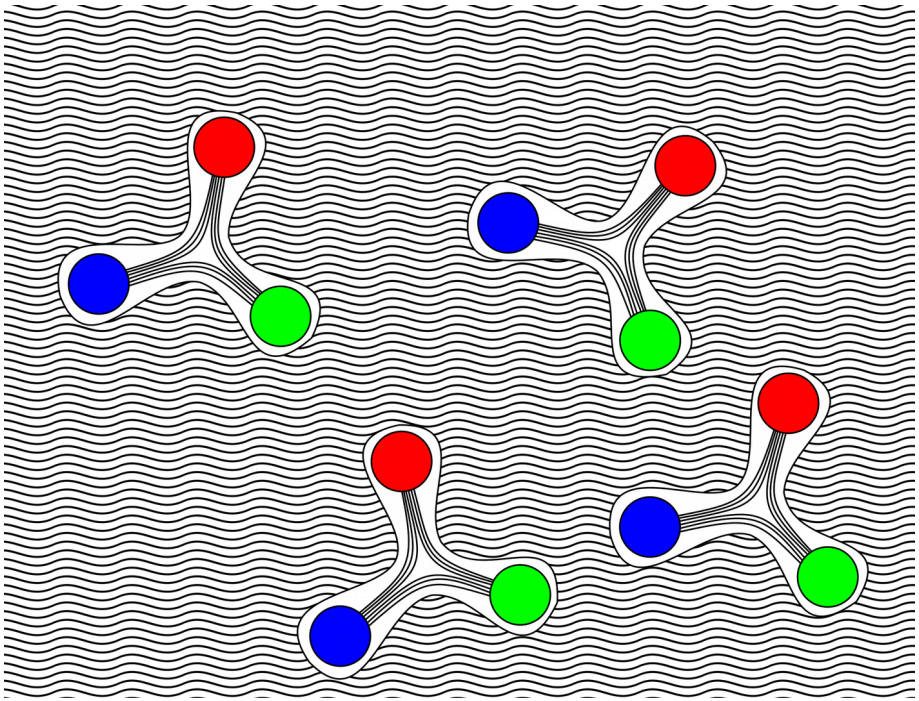


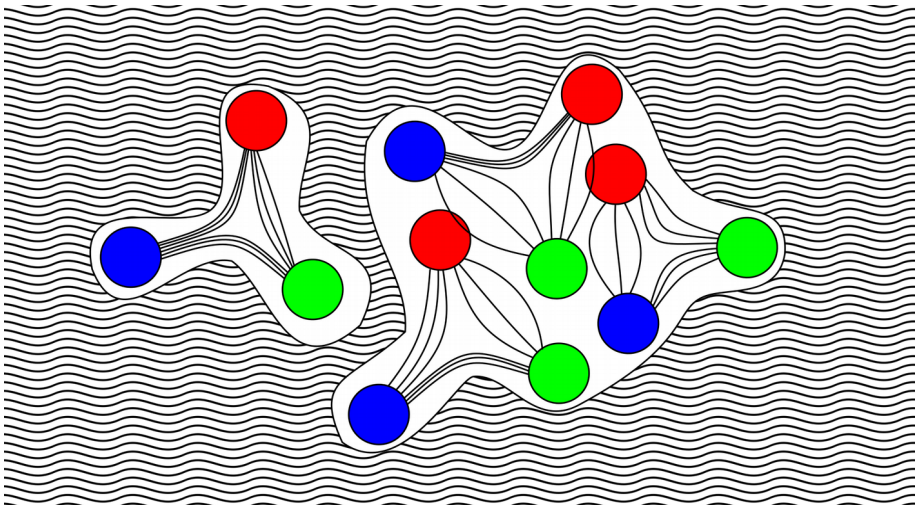
Density functional approach: Stringflip model

Low density

- Color field lines compressed by dual Meissner effect
- String-potential

$$V(r) = \sigma r \sim n^{-1/3}$$





High density

- Dual superconducting vacuum occupied by hadrons
- Pressure on field lines reduced
- Effective string-tension reduced

$$\sigma = \Phi\sigma_0$$

$$U^{\mathrm{SF}}(n_{\mathrm{S}}, n_{\mathrm{V}}) = D(n_{\mathrm{V}}) n_{\mathrm{S}}^{2/3}$$

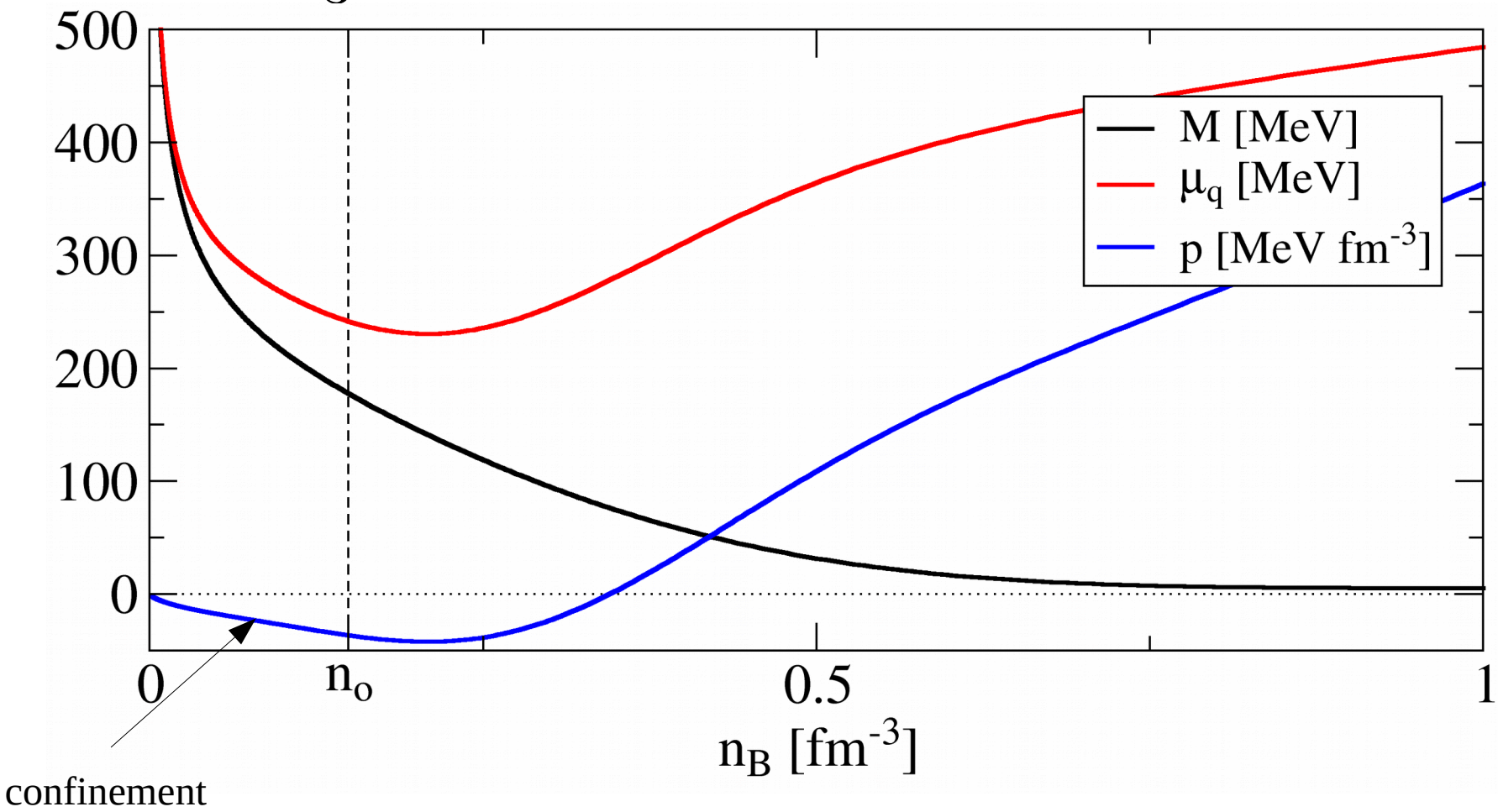
G. Ropke, et. al., Phys.Rev. D34 (1986) 3499-3513
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

Stringflip model – effective mass

Mean-field model

$$M_i = m_i + \frac{2}{3} D \cdot (n^s)^{-1/3}$$

$$D = D_0 e^{-\alpha (n - n_0)^2}$$



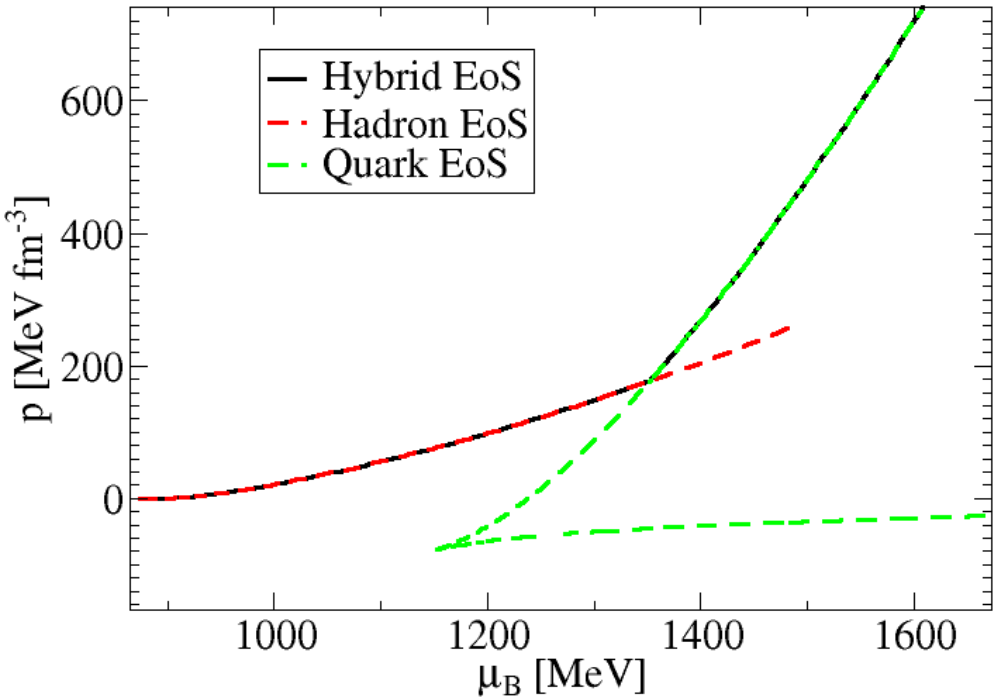
M. Kaltenborn, **NUFB**, D. Blaschke, PRD 96, 056024 (2017)

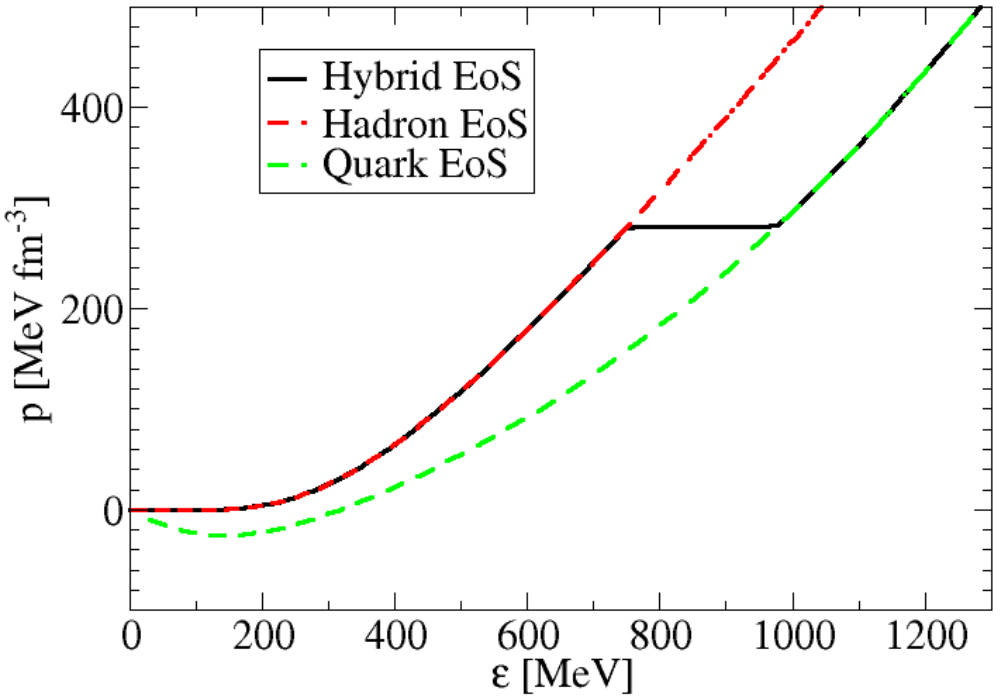
2-phase approach

old

- Two independent models for hadrons and quarks
- Match while fulfilling Gibbs condition for thermal, mechanical and chemical phase equilibrium

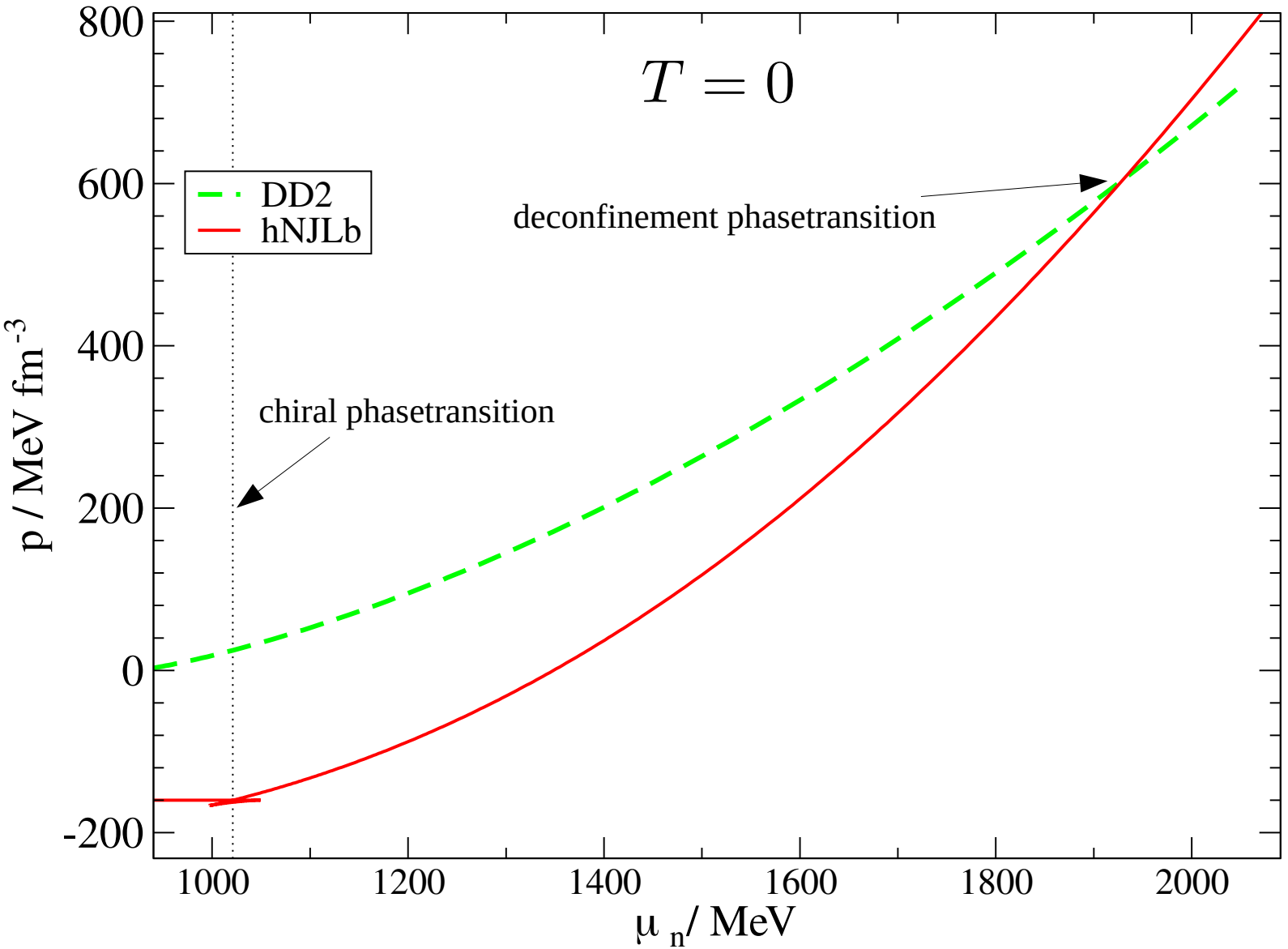
$$T^{\mathrm{H}} = T^{\mathrm{Q}} \quad p^{\mathrm{H}} = p^{\mathrm{Q}} \quad \mu^{\mathrm{H}} = \mu^{\mathrm{Q}}$$





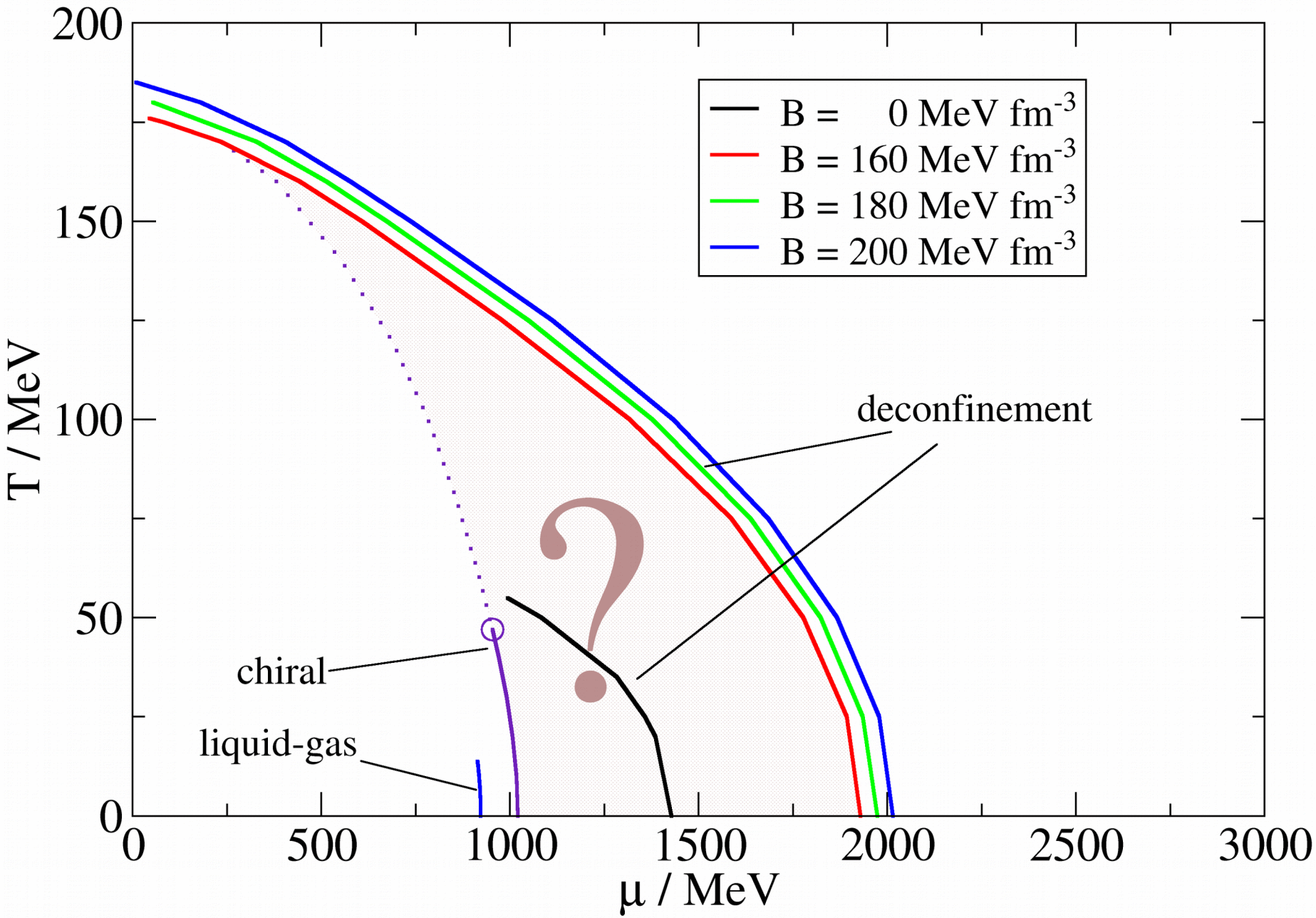
2-phase approach (with NJL)

old



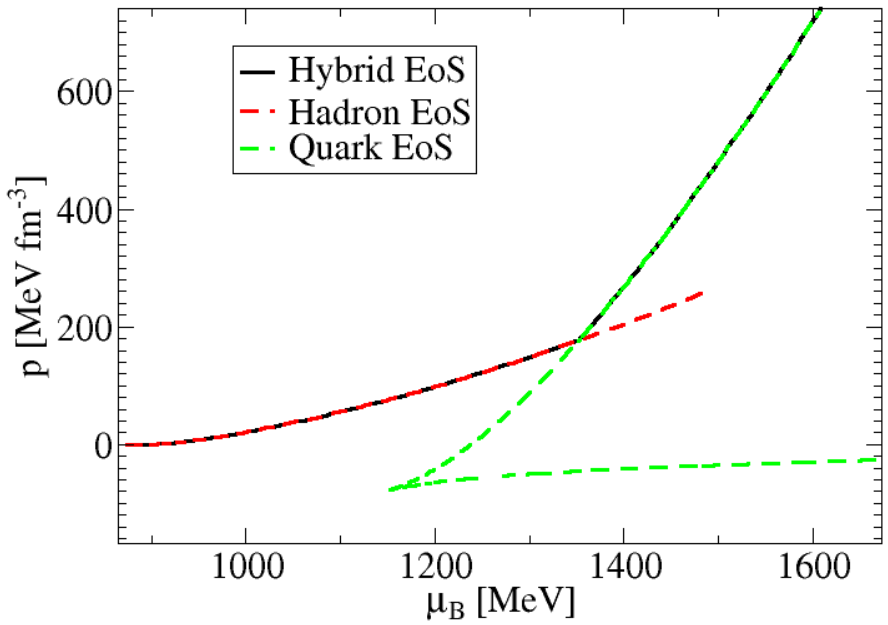
NUFB, D. Blaschke, 2016 J. Phys.: Conf. Ser. 668 012042

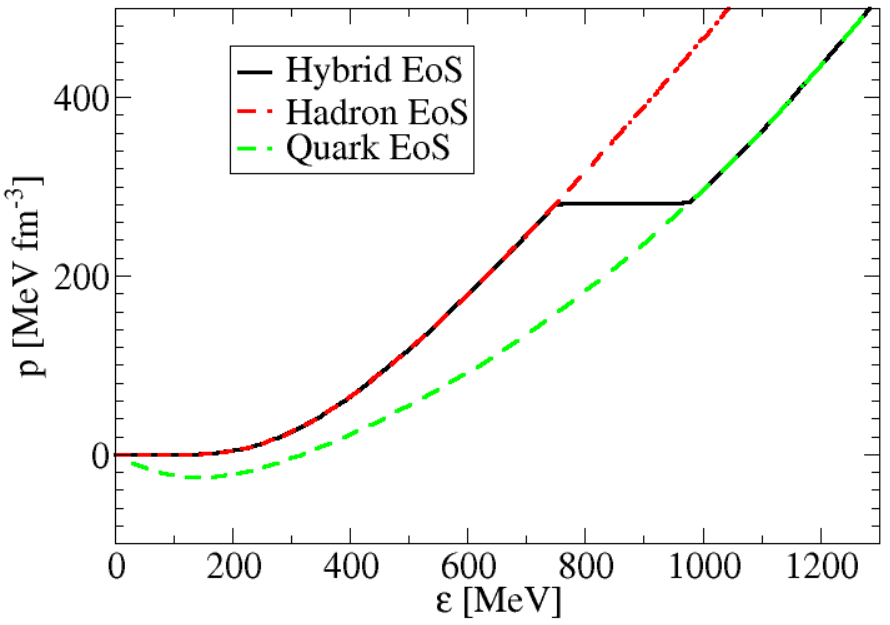
2-phase construction with NJL

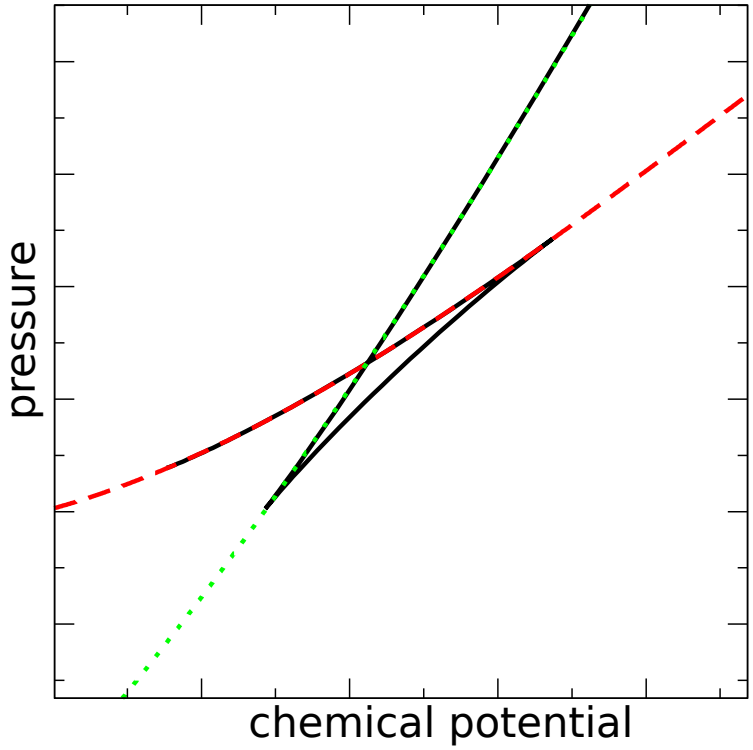


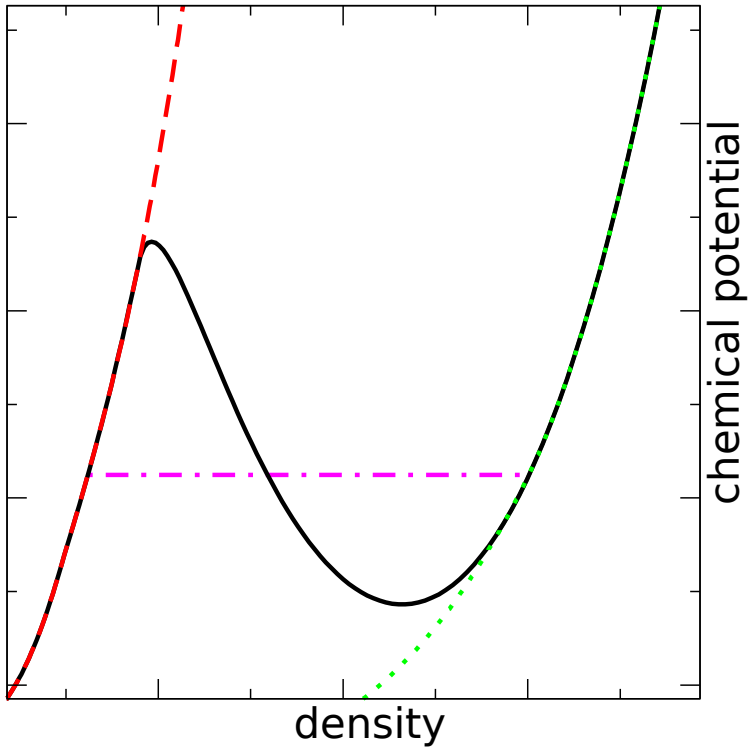
NUFB, D. Blaschke, 2016 J. Phys.: Conf. Ser. 668 012042

Two-phase approach vs van der Waals wiggle









Cluster expansion

Generating functional formalism by Baym and Kadanoff [1,2]

$$\Omega = -\mathrm{Tr}\ \ln(-G_1^{-1}) - \mathrm{Tr}\Sigma_1 G_1 + \Phi \qquad \text{With} \qquad \Sigma_1(1,1') = \frac{\delta\Phi}{\delta G_1(1,1')}.$$

Can be generalized for a consistent cluster expansion[3]

$$\Omega = \sum_{l=1}^{A} \Omega_l = \sum_{l=1}^{A} \left\{ c_l \left[\mathrm{Tr}\ln\left(-G_l^{-1}\right) + \mathrm{Tr}\left(\Sigma_l\ G_l\right)\right] + \sum_{\substack{i,j \\ i+j=l}} \Phi[G_i, G_j, G_{i+j}] \right\}$$

with

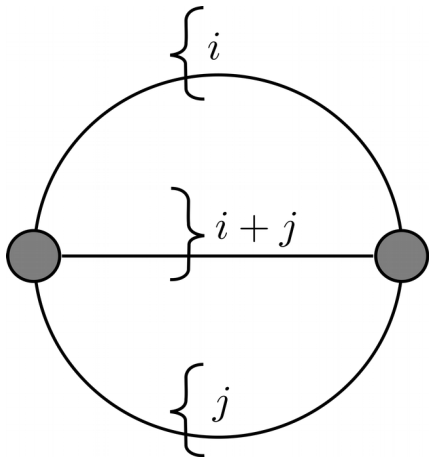
$$\Sigma_A(1\ldots A, 1'\ldots A', z_A) = \frac{\delta\Phi}{\delta G_A(1\ldots A, 1'\ldots A', z_A)}$$

Always sustains full Dyson equation and thermodynamic stability

$$G_A^{-1} = {G_A^0}^{-1} - \Sigma_A^{-1} \qquad \frac{\partial\Omega}{\partial G_A} = 0$$

Reduction on generalized sunset diagrams is recommended

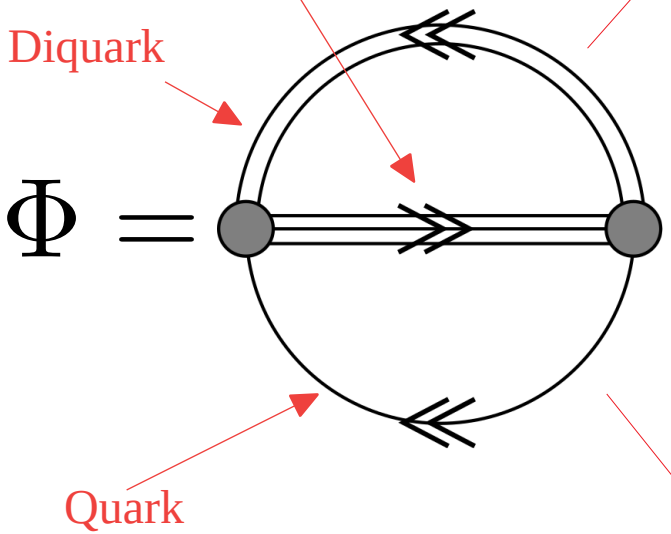
$$\Phi[G_i, G_j, G_{i+j}] =$$



[1]Baym, G.; Kadanoff, L.P. Phys. Rev. 1961, 124, 287–299.
[2]Baym, G. Phys. Rev. 1962, 127, 1391–1401.
[3]**NUFB**, and others, Universe 2018, 4(6), 67

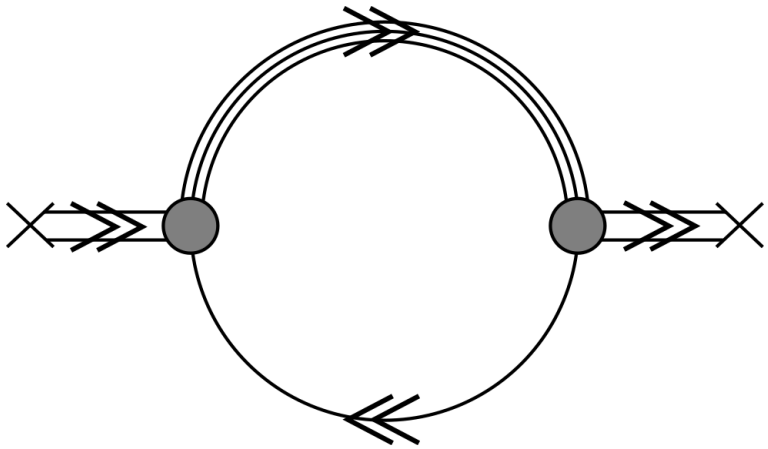
Self energy

$$\Sigma_A = \frac{\delta\Phi}{\delta G_A}$$

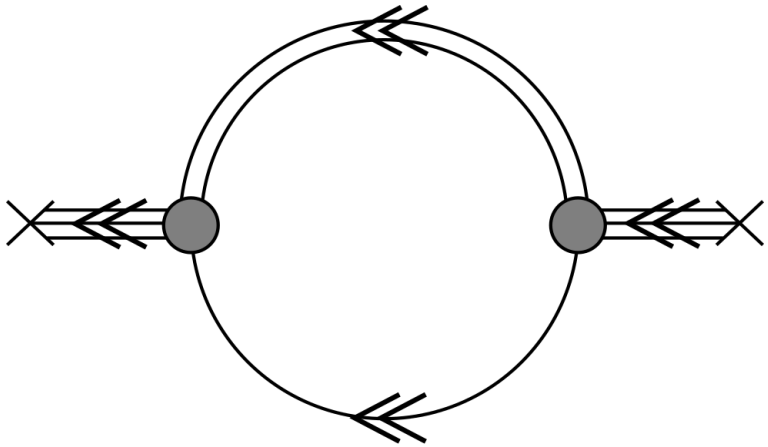


$$\Phi =$$

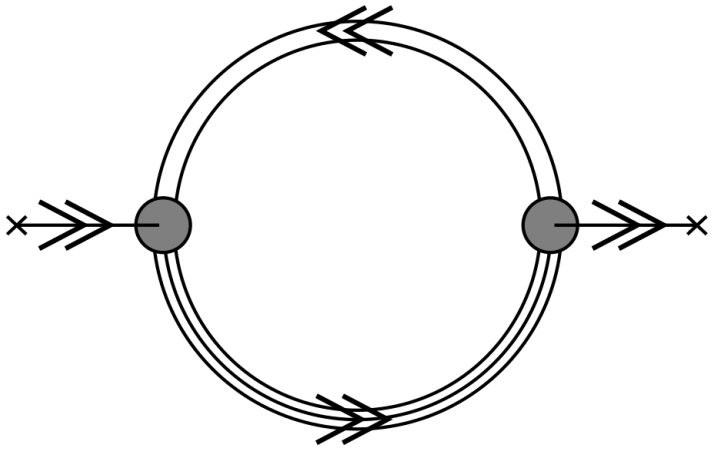
$$\Sigma_{\mathrm{D}} = \frac{\delta\Phi}{\delta G_{\mathrm{D}}} =$$



$$\Sigma_{\mathrm{B}} = \frac{\delta\Phi}{\delta G_{\mathrm{B}}} =$$

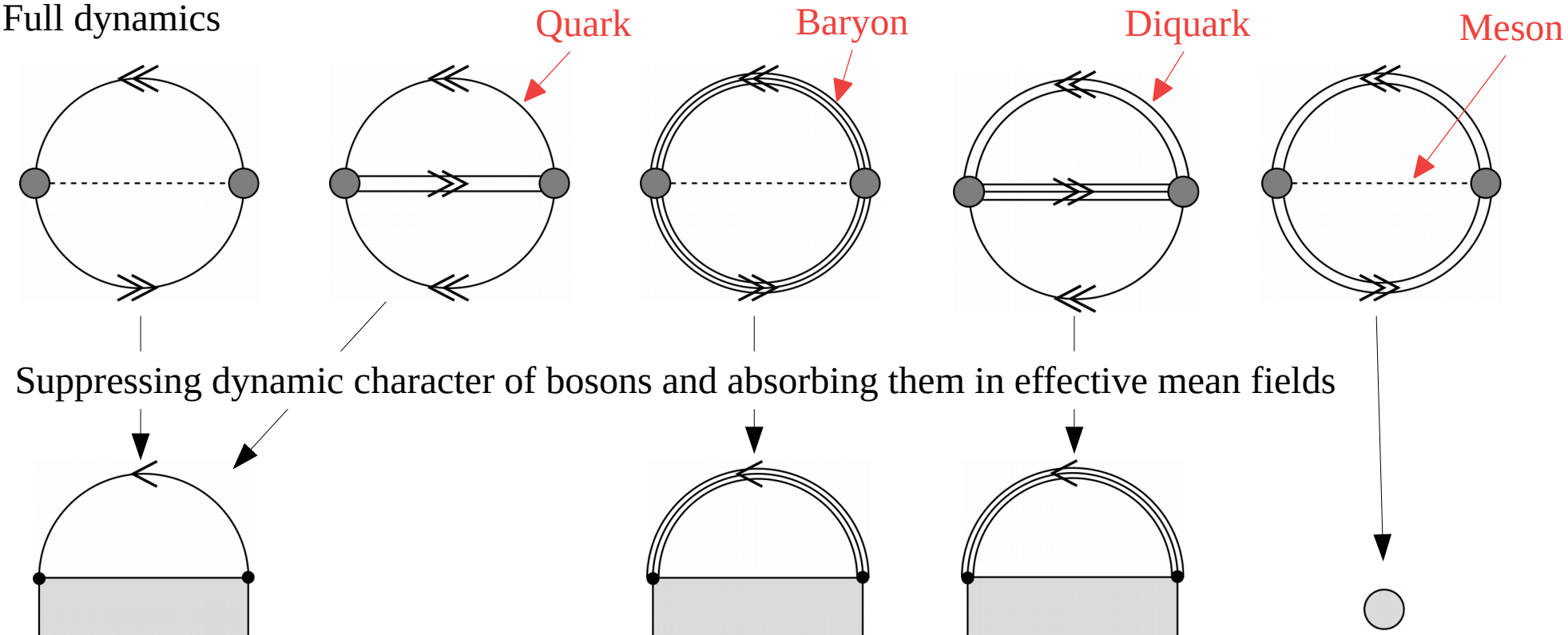


$$\Sigma_{\mathrm{Q}} = \frac{\delta\Phi}{\delta G_{\mathrm{Q}}} =$$



The Quark-Diquark-Meson-Baryon Model

Full dynamics



Suppressing dynamic character of bosons and absorbing them in effective mean fields

quark loop

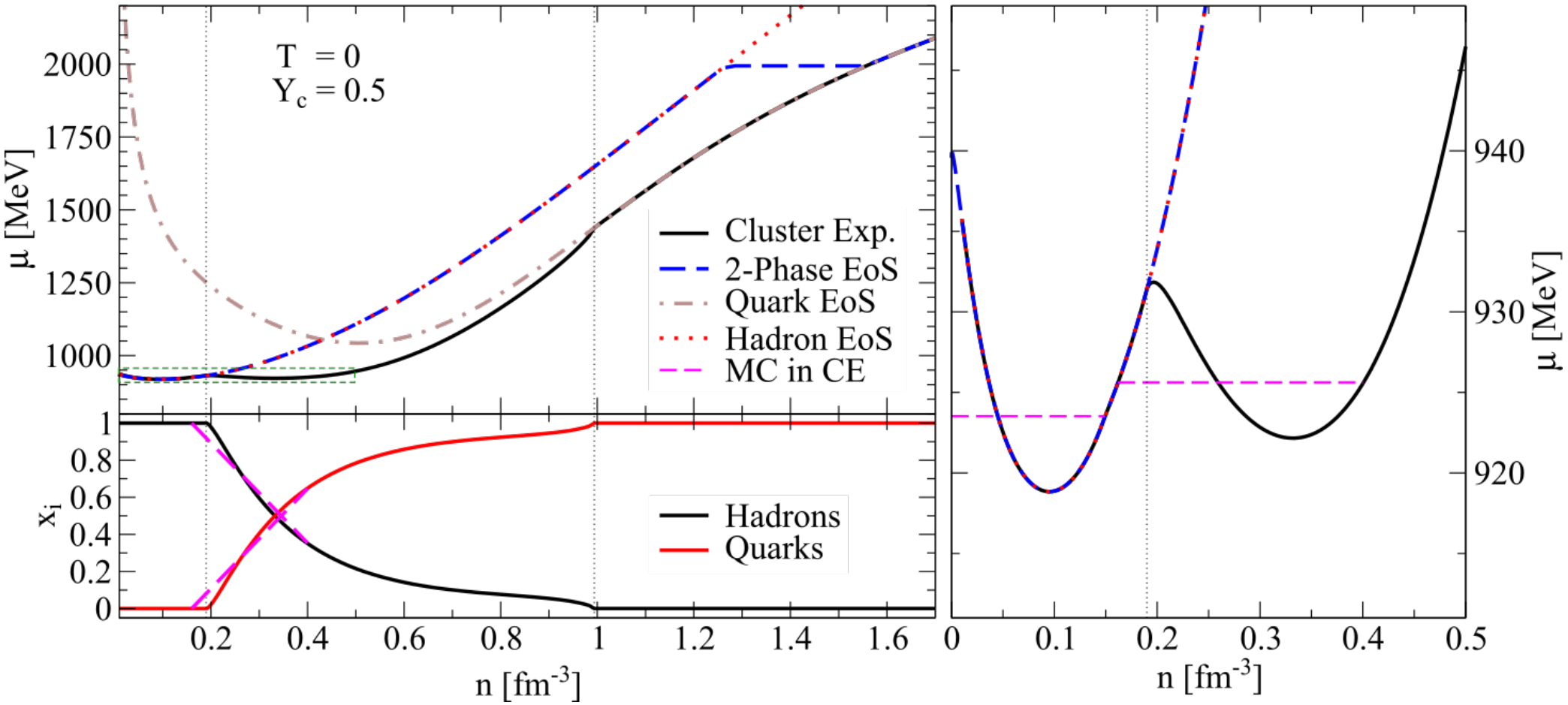
baryon loop

quark-baryon interaction

→ Real self energies → quasi particles

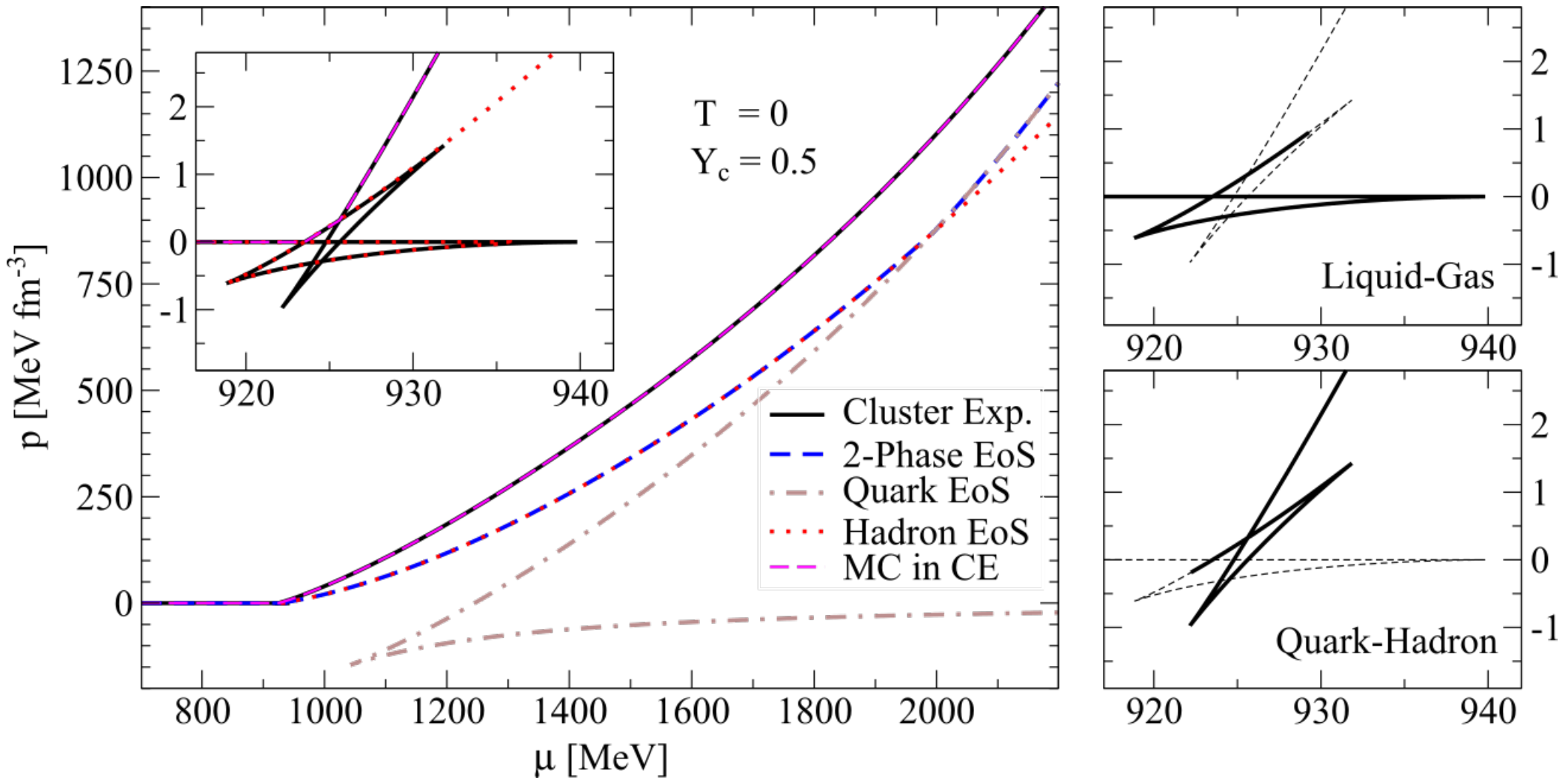
$$\delta := \arctan \frac{\mathrm{Im}\,\Sigma}{\mathrm{Re}\,\Sigma} = n\pi$$

Cluster-expansion of Quarks



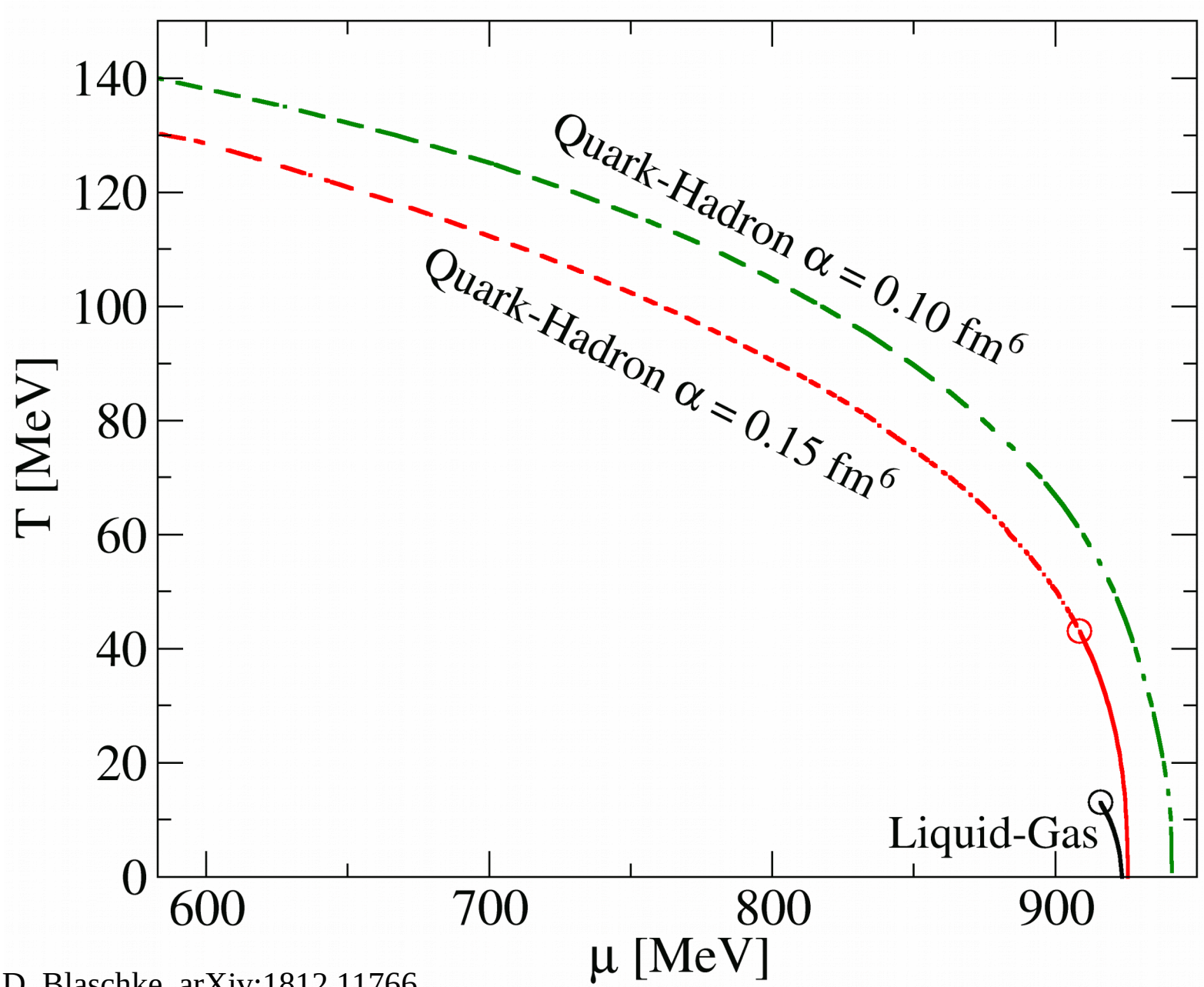
NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion of Quarks



NUFB, D. Blaschke, arXiv:1812.11766

Cluster-expansion



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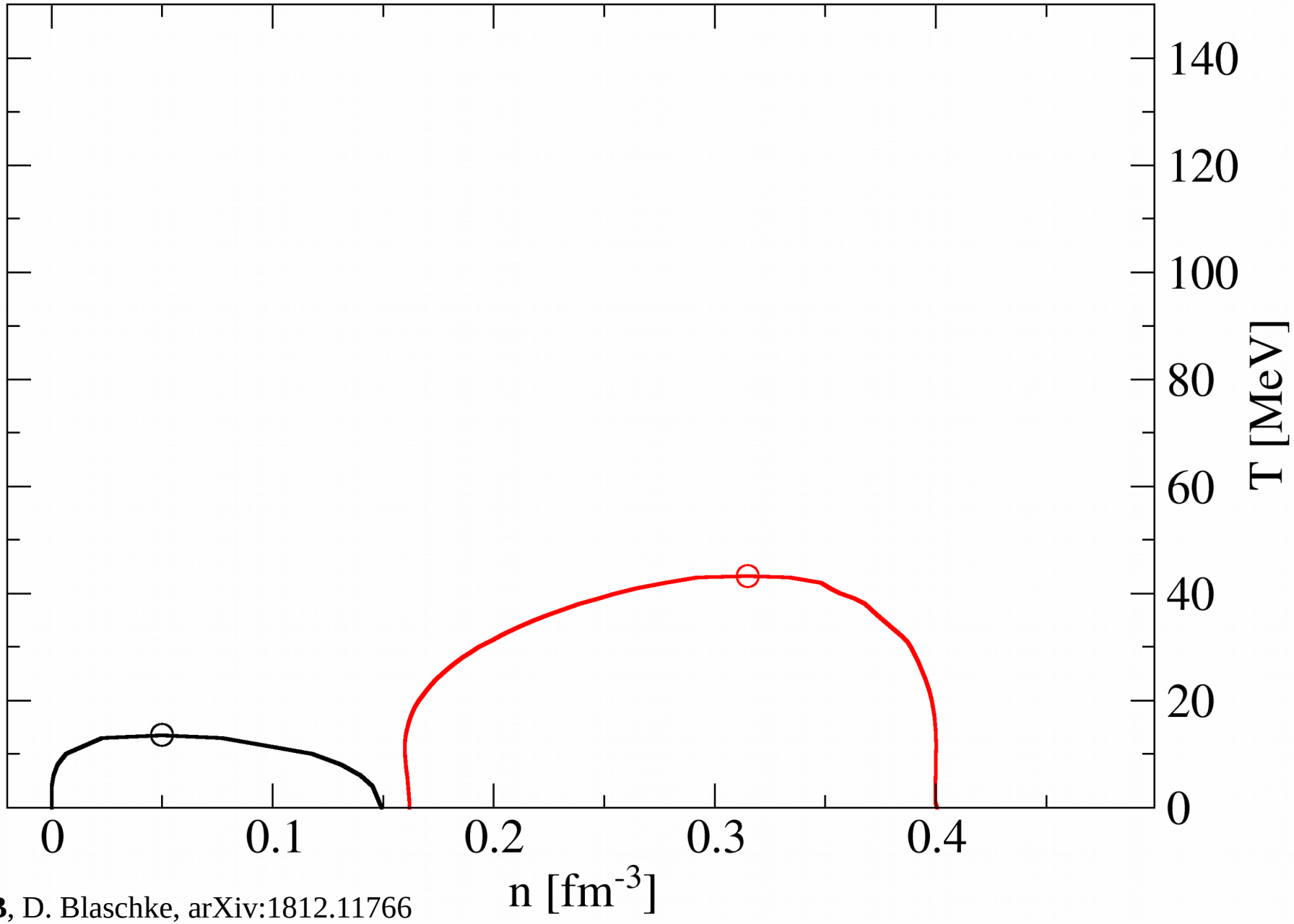
Cluster-expansion

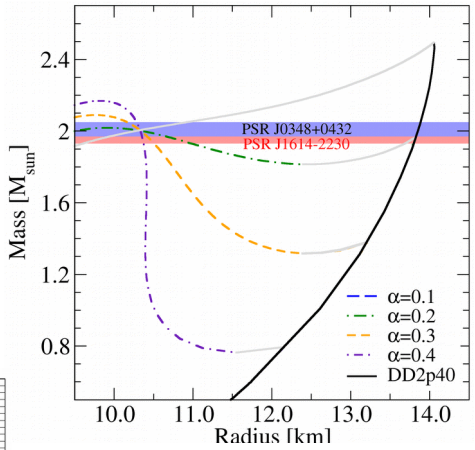


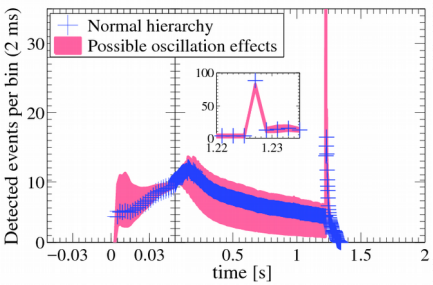
NUFB, D. Blaschke, arXiv:1812.11766

~~Outline~~ Summary

Possible signals of 1st – order phase transitions.

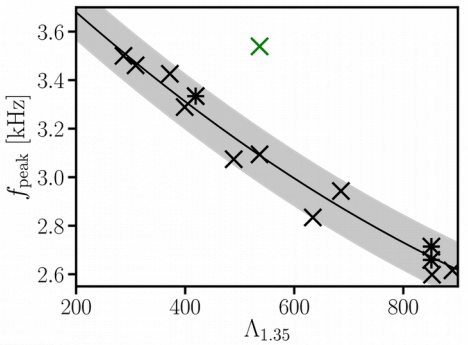
Neutron star configurations

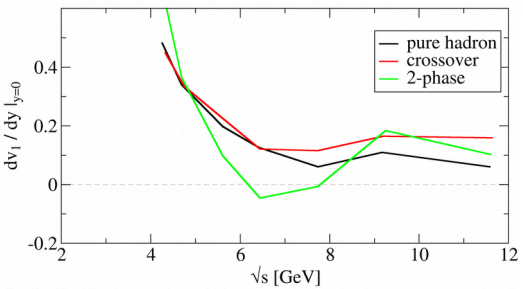




Supernova explosions of 50Ms stars

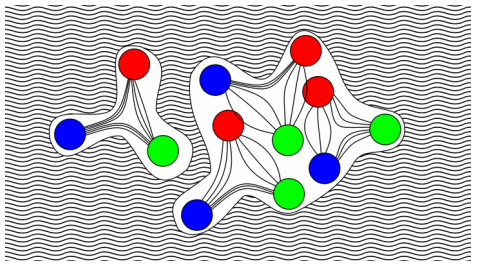
Binary neutron star mergers





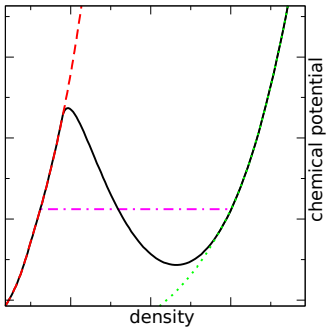
Heavy-Ion Collisions

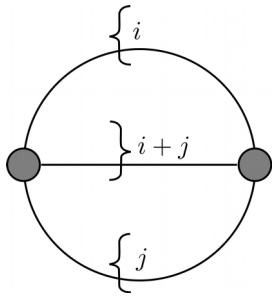
Unified description of the equation of state.



density functional theory

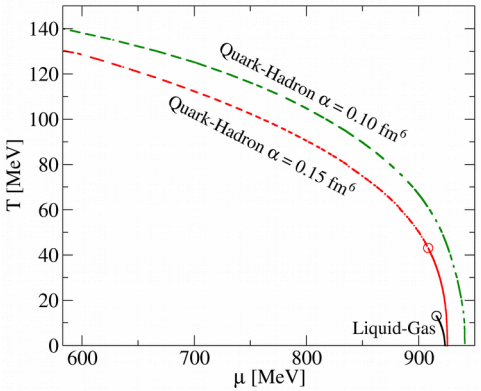
construction of phase transitions





Phi-derivable formalism

current status



Last Slide

Conclusions

- Possible scenarios are explored in which a 1st order phase transition is detectable in
  - neutron star configurations
  - neutrino signals of supernova explosions
  - Gravitational wave signal of binary neutron star mergers
  - Flow data of heavy-ion collision experiments
- Astrophysical objects and HIC collisions are based on the same physics of strongly interacting many-particle systems
- Hadrons are bound states of quarks and should be treated as such
- A cluster virial expansion within the Beth-Uhlenbeck formalism can be derived from the PHI-derivable approach
- Initial reduction to mean field already results in a consistent description of Quark-Hadron phase transition

Outlook

- Density functional with chiral physics
- Reproduction of Lattice results
- Continuum contributions and substructure effects
- Cluster mean field

Thank you!

Collaboration

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