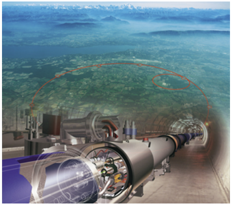
Quantenfeldtheorie aus der Vogelperspektive

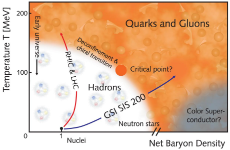
Holger Gies

Friedrich-Schiller-Universität Jena

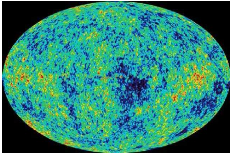
QFT



[particle physics @ CERN]

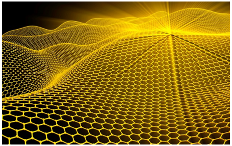


[QCD phase diagram]

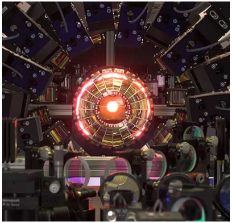


[CMB]

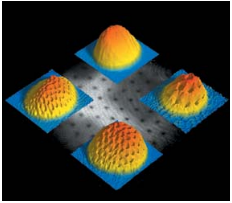
QFT



[Strongly correlated electrons]



[strong fields @ JENA]



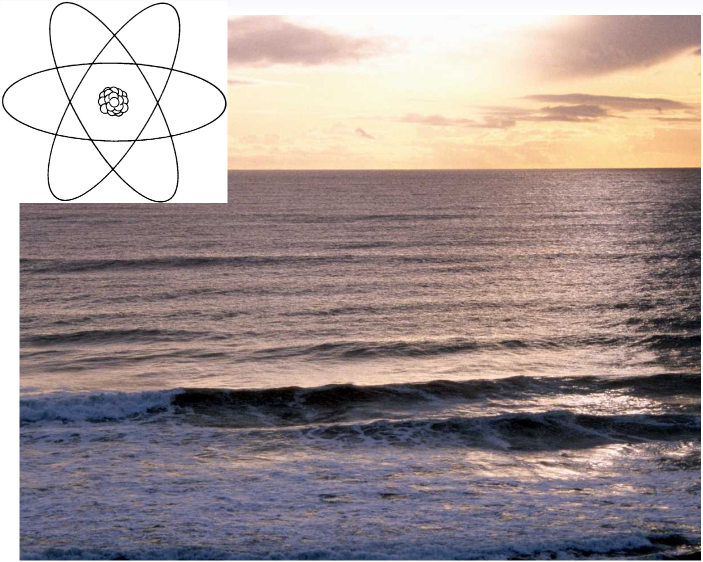
[ultracold atom gases]

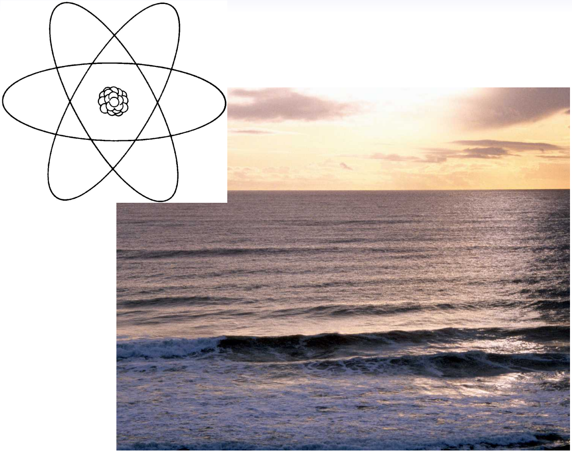
QFT

Quantum Field Theory

Quantum Field

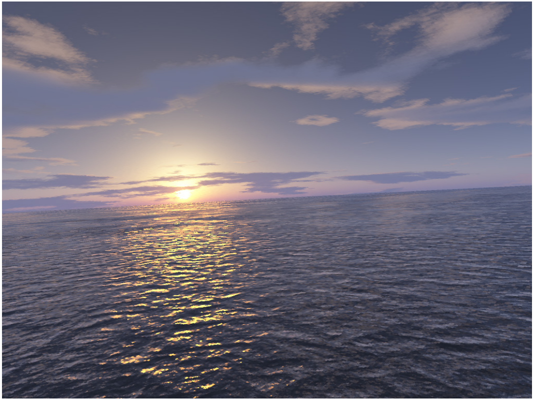
Field





CAVE: $P = |\Psi|^2$ $E^2 = (mc^2)^2 + \mathbf{p}^2 c^2$

CAVE: $$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$ (Heisenberg)



CAVE: $$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$ (Heisenberg)

Prerequisites:

Quantum mechanics:

$$i\frac{\partial}{\partial t}\Psi = H\Psi, \qquad P = |\Psi|^2$$

Special theory of relativity:

$$E^2 = m^2 + \mathbf{p}^2$$

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Quantum mechanics:

$$i\frac{\partial}{\partial t}\Psi = H\Psi, \qquad P = |\Psi|^2$$

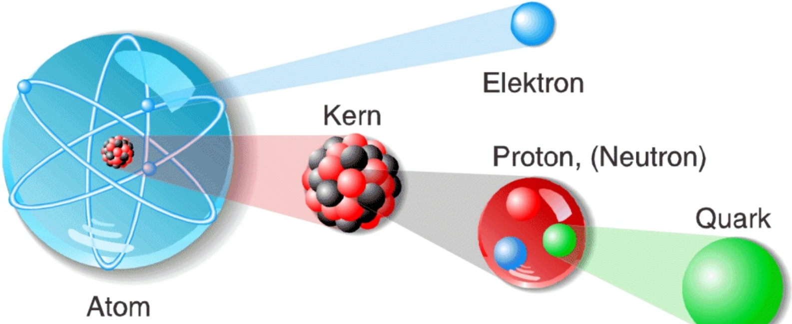
Special theory of relativity:

$$E^2 = m^2 + \mathbf{p}^2$$

Of course: $\hbar = 1 = c$

Building blocks of Nature

matter:

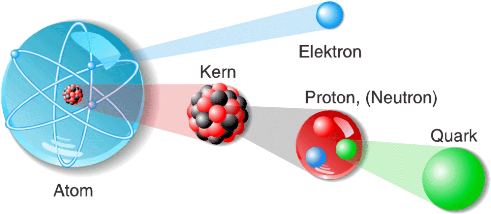


u(p) and d(own) quarks:

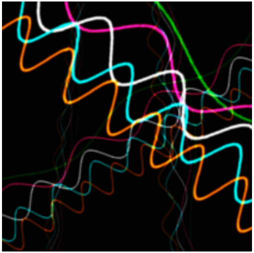
proton: u u d
neutron: u d d

Building blocks of Nature

matter:



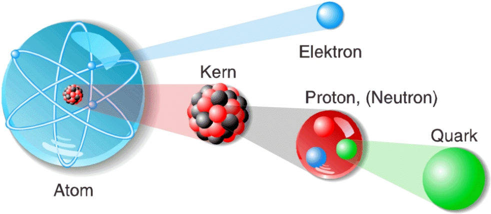
force carriers:



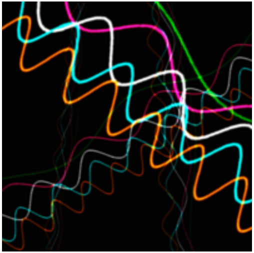
Photon γ

Building blocks of Nature

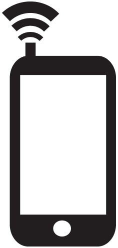
matter:



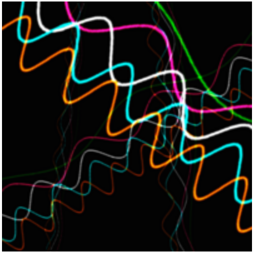
force carriers:



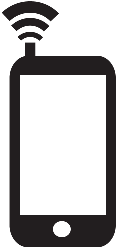
Photon γ



Building blocks of Nature

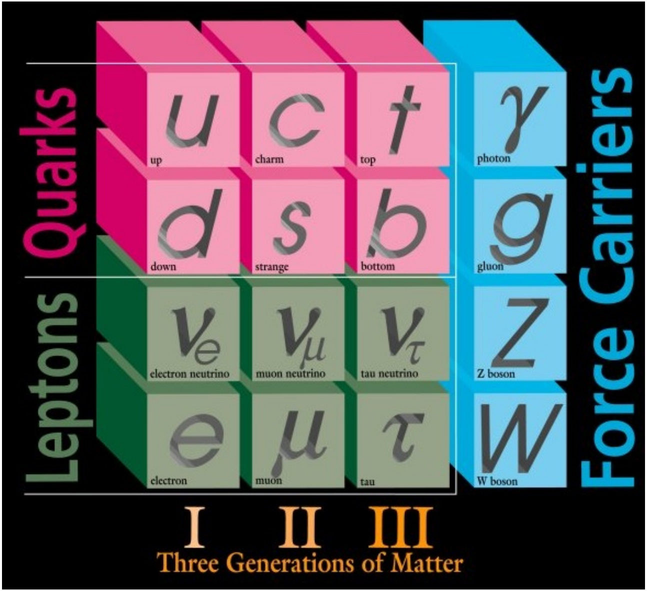


Photon γ



10 000 000 000 000 000 000 000 photons/sec

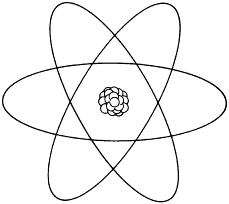
Building blocks of Nature



Why quantum field theory?

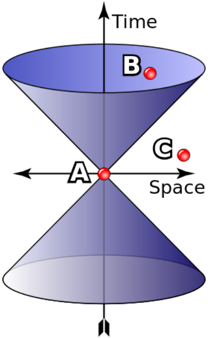
Quantum mechanics:
necessary if

$$S[x_{\text{cl}}] \sim \hbar \sim 10^{-34}\text{Js}$$



special relativity:
necessary if

$$v \lesssim c \sim 3 \times 10^{8} m/s$$



[WIKIPEDIA]

Why quantum field theory?

For particle physics:

relativistic quantum mechanics should be sufficient …?

Why quantum field theory?

For particle physics:

relativistic quantum mechanics should be sufficient …?

A simple (counter-) example:
QM:

$$i\frac{\partial}{\partial t}\Psi = H\Psi$$

SRT:

$$E^2 = m^2 + \mathbf{p}^2, \qquad \text{choose}: \ E \to H = \sqrt{\mathbf{p}^2 + m^2}$$

use replacement rule: $\mathbf{p} \to -i\boldsymbol{\nabla}$

Why quantum field theory?

$\Longrightarrow$ relativistic QM equation:

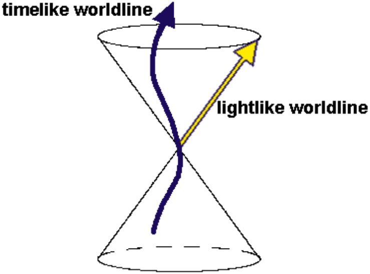
$$i\frac{\partial}{\partial t}\Psi(\mathbf{x},t) = \sqrt{-\boldsymbol{\nabla}^2 + m^2}\,\Psi(\mathbf{x},t)$$

admits plane wave solutions:

$$\Psi_{\mathbf{p}}(\mathbf{x},t) = e^{-iEt+i\mathbf{p}\cdot\mathbf{x}}$$

$\Longrightarrow$ free relativistic quantum particle!

Why quantum field theory?



Causality check:
localized point source

$$\Psi(\mathbf{x}, t = 0) = \delta^{(3)}(\mathbf{x})$$

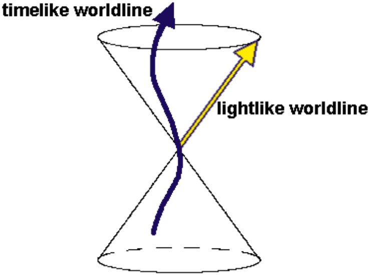
causality requires

$$\Psi(\mathbf{x}, t) = 0 \text{ for } \mathbf{x}^2 > t^2$$

Consider Green's function

$$\begin{aligned} G(\mathbf{x}, t) &= \langle \mathbf{x} | e^{-iHt} | \mathbf{0} \rangle \\ &= \int \frac{d^3p}{(2\pi)^3} e^{-i\sqrt{\mathbf{p}^2+m^2}t+i\mathbf{p}\cdot\mathbf{x}} \end{aligned}$$

Why quantum field theory?



Beyond the light cone:

$$G(\mathbf{x}, t) \sim \exp\left(-m\sqrt{\mathbf{x}^2 - t^2}\right) \neq 0$$

for $\mathbf{x}^2 > t^2$

finite probability that particle is detected outside the light cone

$\Longrightarrow$ causality can be violated

Note: probability decays exponentially with range

$$\lambda \sim \frac{1}{m} \qquad \text{(Compton wave length)}$$

…similar problems with Klein-Gordon or Dirac equation

Basics of quantum field theory

Idea: start with classical causal field theory …and quantize!

e.g.: classical electrodynamics (in vacuum)

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \qquad \boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

$$\boldsymbol{\nabla} \cdot \mathbf{E} = 0 \qquad \boldsymbol{\nabla} \times \mathbf{B} = \frac{\partial}{\partial t}\mathbf{E}$$

Quantization: try to understand Maxwell's equations as QM Heisenberg equations of motion:

$$\text{a la} \quad \frac{d}{dt}\mathbf{x} = -i[\mathbf{x}, H], \quad \frac{d}{dt}\mathbf{p} = -i[\mathbf{p}, H]$$

Basics of quantum field theory

Dictionary:

e.g., harmonic oscillator

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$$

canonical variables: q, p

Q-EoM:

$$\dot{x} = -i[x, H], \dot{p} = -i[p, H]$$

$$\Longrightarrow \ddot{q} + \omega^2 q = 0$$

fundamental commutators:

$$[q, p] = \quad i,$$

$$[q, q] = \quad 0, \qquad [p, p] = \quad 0$$

Basics of quantum field theory

Dictionary:

EM field energy → Hamiltonian

$$H = \frac{1}{2}\int_x \mathbf{E}^2 + \mathbf{B}^2$$

canonical variables: $\mathbf{A}$, $\mathbf{E}$

(magnetic field: $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$)

Q-EoM:

$$\dot{\mathbf{A}}_{\mathbf{x}} = -i[\mathbf{A}_{\mathbf{x}}, H], \dot{\mathbf{E}}_{\mathbf{x}} = -i[\mathbf{E}_{\mathbf{x}}, H]$$

Maxwell's eq. (Coulomb-Weyl gauge)

fundamental commutators:

$$\begin{aligned}[\mathbf{A}_{\mathbf{x}}, \mathbf{E}_{\mathbf{y}}] &= \quad i\delta_{\mathrm{T}}^{(3)}(\mathbf{x} - \mathbf{y}), \\ [\mathbf{A}_{\mathbf{x}}, \mathbf{A}_{\mathbf{y}}] &= \quad 0, \ [\mathbf{E}_{\mathbf{x}}, \mathbf{E}_{\mathbf{y}}] = 0\end{aligned}$$

e.g., harmonic oscillator

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2$$

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Q-EoM:

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Basics of quantum field theory

Dictionary:

canonical variables: q, p

ladder operators:

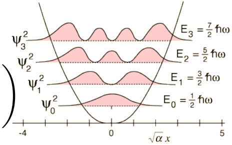
$$q = \frac{1}{\sqrt{2\omega}}(a+a^\dagger),\ p = -i\sqrt{\frac{\omega}{2}}(a-a^\dagger)$$

quantized states

$$|n\rangle,\ \text{e.g.}\ |1\rangle = a^\dagger|0\rangle$$

energy levels:

$$E_n = \omega\left(n + \frac{1}{2}\right)$$



Basics of quantum field theory

Dictionary:

canonical variables: **A**, **E**

creation/annihilation operators:

$$\mathbf{A}_{\mathbf{x}} = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}}^{\dagger}\right)$$

photon states

$$|n_{\mathbf{p}_1}, n_{\mathbf{p}_2}, \ldots\rangle, \text{ e.g. } |1_{\mathbf{p}}\rangle = a_{\mathbf{p}}^{\dagger}|0\rangle$$

energy levels:

$$E = \int_{\mathbf{p}} \omega_{\mathbf{p}} \left(n_{\mathbf{p}} + \frac{1}{2}\right)$$

canonical variables: q, p

ladder operators:

$$q = \frac{1}{\sqrt{2\omega}}(a+a^{\dagger}),\ p = -i\sqrt{\frac{\omega}{2}}(a-a^{\dagger})$$

quantized states

$$|n\rangle, \text{ e.g. } |1\rangle = a^{\dagger}|0\rangle$$

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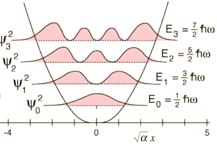
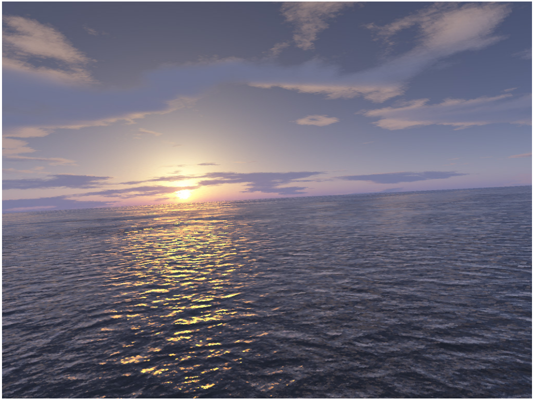


Quantized EM field:

$$\mathbf{A}_{\mathbf{x}} = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}}^{\dagger} \right)$$



zero-point fluctuations:

$$E_0 = \frac{1}{2}\int_{\mathbf{p}} \omega_{\mathbf{p}} \qquad \rightarrow \text{Part II}$$

Summary I

- “Unification” of quantum mechanics and special relativity?

  …causality (+ other) problems in relativistic QM
- consistent “unification” possible:

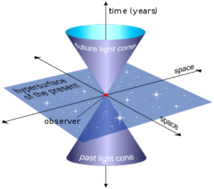
  …by quantizing relativistic field theories
- $\Longrightarrow$ field operators

$$\mathbf{A_x} = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}}^{\dagger} \right)$$

…can create and annihilate excitations = particles

- Causality: built in!

$$[\mathbf{A_x}, \mathbf{A_y}] = 0$$

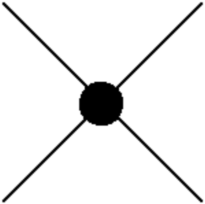


Interacting quantum fields

▷ general field operator $\phi(\mathbf{x})$:

$$\phi_x = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} a^\dagger_{\mathbf{p}} \right)$$

creates/annihilates particles



▷ general interactions, e.g., $\sim g\,\phi^4$

$\Longrightarrow$ interactions can create / annihilate particles

$\Longrightarrow$ QFT $\hat{=}$ many-body theory

$\Longrightarrow$ QFT generalizes QM to $\leq$ infinitely many degrees of freedom

Interacting quantum fields

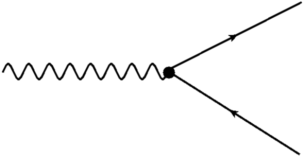
- ▷ field operator that carries a conserved charge: (e.g. electron e^-)

$$\psi_x = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} a_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} b^\dagger_{\mathbf{p}} \right)$$

annihilates particle / creates anti-particle

- ▷ e.g. photon ↔ electron-positron:

$$\sim ie\bar{\psi}(x)\gamma_\mu A_\mu(x)\psi(x)$$



- ▷ conjugate field operator

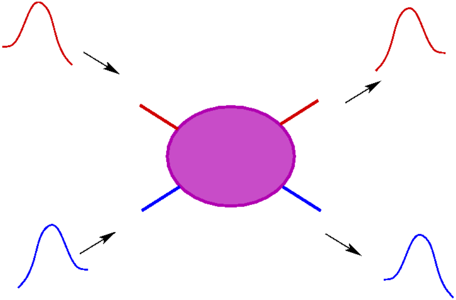
$$\bar{\psi}_x = \int_{\mathbf{p}} \frac{1}{\sqrt{2\omega_{\mathbf{p}}}} \left(e^{i\mathbf{p}\cdot\mathbf{x}} b_{\mathbf{p}} + e^{-i\mathbf{p}\cdot\mathbf{x}} a^\dagger_{\mathbf{p}} \right)$$

QFT requires the existence of anti-particles / holes

Interacting quantum fields

- ▷ physics can be extracted from correlators:

$$G^{(4)}(x_1, x_2, x_3, x_4) \sim$$



Interacting quantum fields

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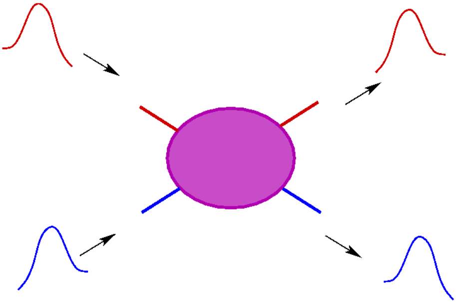
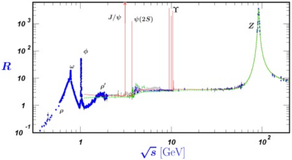
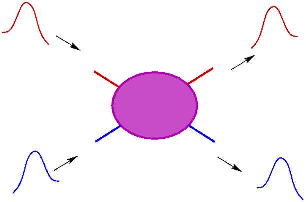


- QFT “recipes”:

$$G^{(n)}(x_1, x_2, x_3, \ldots, x_n) \qquad \rightarrow \qquad \frac{d\sigma}{d\Omega}$$



Interacting quantum fields



- Master Formula:

$$G^{(n)}(x_1, \ldots, x_n) = \frac{\langle 0 | \mathcal{T}\left(\phi(x_1) \ldots \phi(x_n) e^{-iH_{\text{int}}}\right) | 0 \rangle}{\langle 0 | e^{-iH_{\text{int}}} | 0 \rangle}$$

- e.g.,

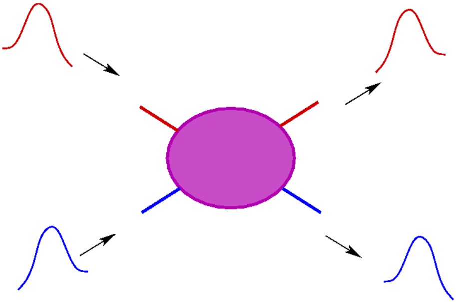
$$H_{\text{int}} = g \int d^4x \phi^4(x), \qquad H_{\text{int}}^{\text{QED}} = ie \int d^4x \bar{\psi}(x) \gamma_\mu A_\mu \psi(x), \ldots$$

different representation, cf. talks by A. Wipf, A. Sternbeck

Interacting quantum fields

▷ Master Formula: $\sim$ constitutional freedom of quantum fields

`Everything which is not forbidden is allowed`

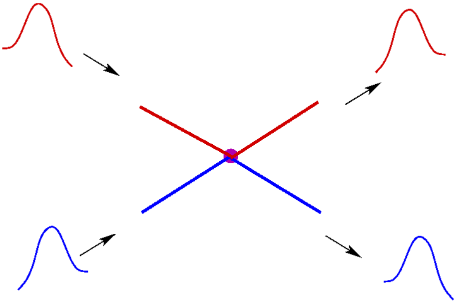


Perturbative expansion:

Interacting quantum fields

▷ Master Formula: ∼ constitutional freedom of quantum fields

`Everything which is not forbidden is allowed`



Perturbative expansion:

Feynman diagrams

(Feynman'48)

Interacting quantum fields

▷ Master Formula: ∼ constitutional freedom of quantum fields

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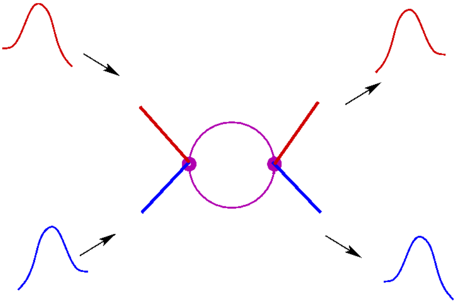


Perturbative expansion:

Feynman diagrams

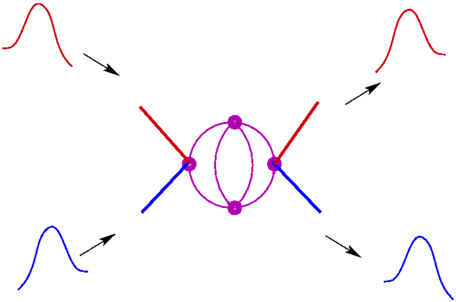
(Feynman'48)

each diagram
∼ # of integrals

Interacting quantum fields

▷ Master Formula: ∼ constitutional freedom of quantum fields

`Everything which is not forbidden is allowed`



Perturbative expansion:

Feynman diagrams

(Feynman'48)

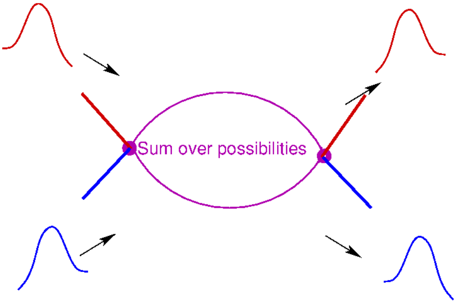
each diagram
∼ # of integrals

of increasing complexity

Interacting quantum fields

▷ Master Formula: $\sim$ constitutional freedom of quantum fields

`Everything which is not forbidden is allowed`



Perturbative expansion:

Feynman diagrams

(Feynman'48)

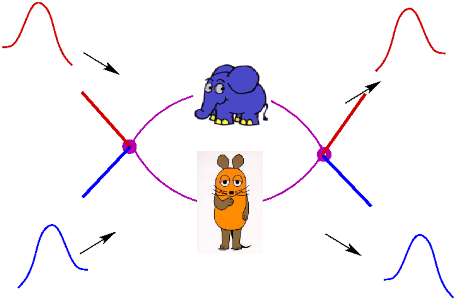
Sum of all possibilities

compatible with
conservation laws

Interacting quantum fields

▷ Master Formula: ∼ constitutional freedom of quantum fields

`Everything which is not forbidden is allowed`



Perturbative expansion:

Feynman diagrams

(Feynman'48)

really ALL possibilities

compatible with
conservation laws

Summary II

- physical observables from correlation functions $G^{(n)}(x_1, x_2, \ldots, x_n)$

- QFT: Master formula for $G^{(n)}$

  …+ recipes for computing observables

- evaluation of Master formula is difficult

  …exact solutions known only for simple systems

- perturbative weak coupling expansion

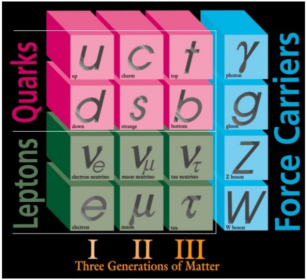
  → Feynman diagrams
  …sum over possibilities/fluctuations

- nonperturbative methods:

  cf. talks by M. Ammon, G. Bergner, A. Sternbeck, A. Wipf

For instance: particle physics

- ▷ Standard Model:



- ▷ interactions:

“under the spell of the gauge principle”
('t Hooft)

$$H_{\text{int}} = \cdots + ie\, \bar{e}\gamma_\mu A_\mu e \quad \ldots \quad + ig\, \bar{u}\gamma_\mu \tau^a G^a_\mu u \quad \ldots$$

$$\cdots + \frac{ig_{\text{w}}}{2\sqrt{2}}\, W^-_\mu [(\bar{e}\gamma^\mu(1+\gamma^5)\nu) + (\bar{d}C^\dagger_{u\bar{d}}\gamma^\mu(1+\gamma^5)u)] \ldots$$

Theory of Everything

Theory of (almost) Everything (observed so far)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a \\
& + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu \\
& -\frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 \\
& -\beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) \\
& - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) \\
& - A_\nu (W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu \\
& + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) \\
& + g^2 s_w c_w [A_\mu Z^0_\nu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] \\
& - \frac{1}{8} g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - gMW^+_\mu W^-_\mu H \\
& - \frac{1}{2} g \frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig[W^+_\mu (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W^-_\mu (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] \\
& + \frac{1}{2} g[W^+_\mu (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W^-_\mu (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)] + \frac{1}{2} g \frac{1}{c_w} (Z^0_\mu (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) \\
& - ig \frac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) + igs_w M A_\mu (W^+_\mu \phi^- - W^-_\mu \phi^+) - ig \frac{1 - 2c_w^2}{2c_w} Z^0_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+)
\end{aligned}
$$

Theory of (almost) Everything (observed so far)

$$+igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^-[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \frac{1}{4}g^2\frac{1}{c_w^2}Z_\mu^0 Z_\mu^0[H^2 + (\phi^0)^2$$

$$+2(2s_w^2 - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z_\mu^0\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z_\mu^0 H(W_\mu^+\phi^- - W_\mu^-\phi^+)$$

$$+\frac{1}{2}g^2 s_w A_\mu\phi^0(W_\mu^+\phi^- + W_\mu^-\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W_\mu^+\phi^- - W_\mu^-\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z_\mu^0 A_\mu\phi^+\phi^-$$

$$-g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda - \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda$$

$$+igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)] + \frac{ig}{4c_w}Z_\mu^0[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) +$$

$$(\bar{u}_j^\lambda\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)] + \frac{ig}{2\sqrt{2}}W_\mu^+[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(1 + \gamma^5)C_{\lambda\kappa}d_j^\kappa)]$$

$$+\frac{ig}{2\sqrt{2}}W_\mu^-[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger\gamma^\mu(1 + \gamma^5)u_j^\lambda)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)]$$

$$-\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1 - \gamma^5)d_j^\kappa) + m_u^\lambda(\bar{u}_j^\lambda C_{\lambda\kappa}(1 + \gamma^5)d_j^\kappa]$$

$$+\frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 + \gamma^5)u_j^\kappa) - m_u^\kappa(\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger(1 - \gamma^5)u_j^\kappa)] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}_j^\lambda d_j^\lambda)$$

$$+\frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \frac{M^2}{c_w^2})X^0$$

$$+\bar{Y}\partial^2 Y + igc_w W_\mu^+(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W_\mu^+(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W_\mu^-(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+)$$

Theory of (almost) Everything (observed so far)

$$+igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-)$$

$$-\frac{1}{2} gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1 - 2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-]$$

$$+igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2} igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

$$+\kappa \sqrt{-g}(R - 2\Lambda)$$

Theory of (almost) Everything (observed so far)

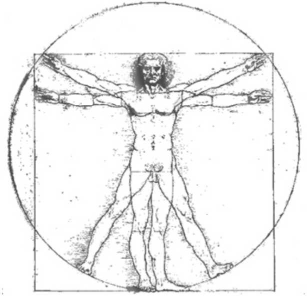
$$+igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-)$$

$$-\frac{1}{2} gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1 - 2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-]$$

$$+igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2} igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

$$+\kappa \sqrt{-g}(R - 2\Lambda)$$

$\longrightarrow$



Theory of (almost) Everything (observed so far)

$$+igs_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + igs_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-)$$

$$-\frac{1}{2} gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM[\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-]$$

$$+igMs_w[\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + \frac{1}{2} igM[\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]$$

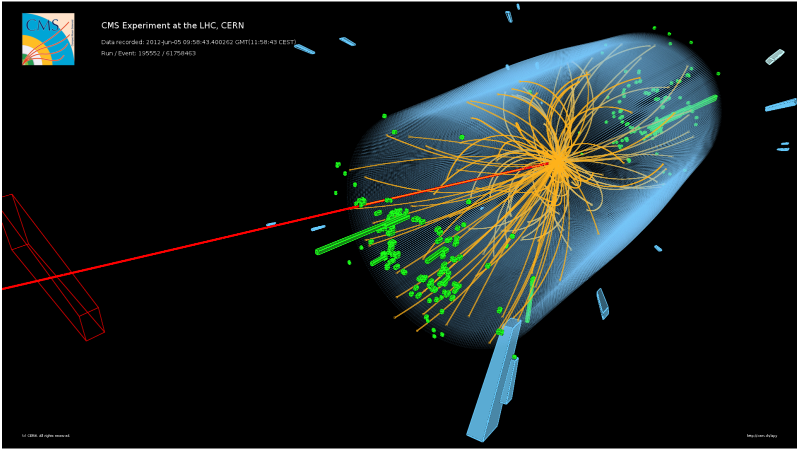
$$+\kappa \sqrt{-g}(R - 2\Lambda)$$

$$+\,?$$

Particle physics experiments

Particle physics experiments

Particle physics experiments

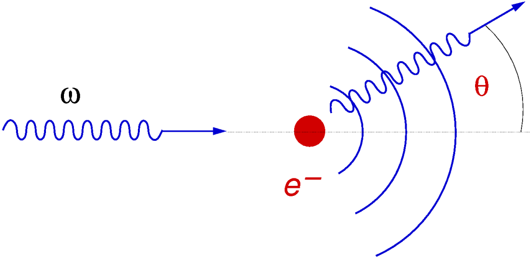


Particle physics experiments

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3

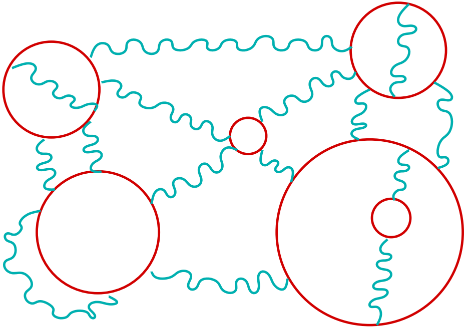
Example: Quantum electrodynamics

- Thomson scattering ($\omega \to 0$)



$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}(1+\cos^2\theta), \quad \alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

Ubiquitous Quantum Fluctuations

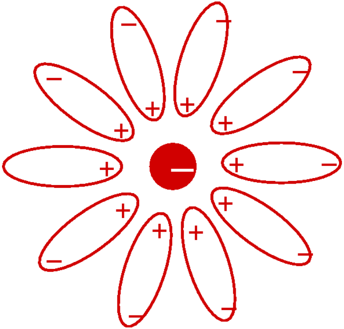


$$E_0 = \frac{1}{2}\int_p \hbar\,\omega_p$$

Example: Quantum electrodynamics

Example: Quantum electrodynamics

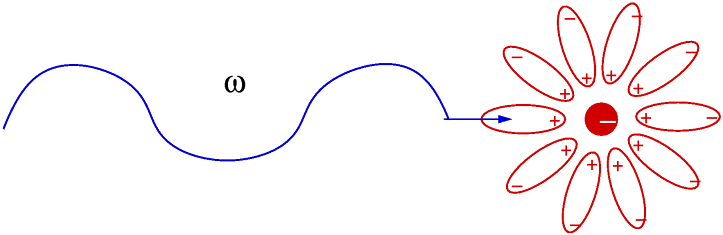
▷ vacuum polarization



⟹ screening of charges

Example: Quantum electrodynamics

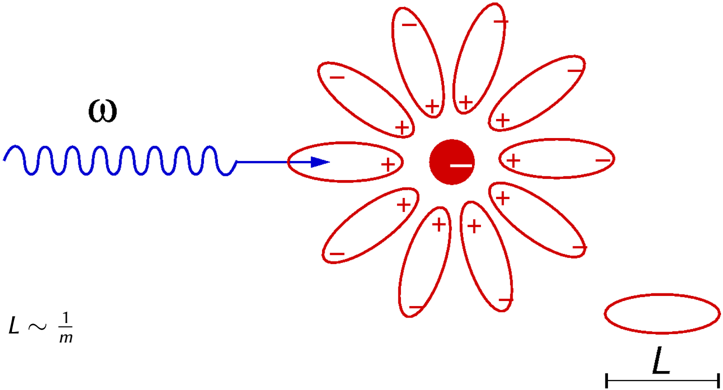
- Thomson scattering ($\omega \to 0$)



…sees averaged coupling $\alpha \simeq \frac{1}{137}$

Example: Quantum electrodynamics

- short wavelength photon ($\omega \gg m \simeq 511\text{keV}$)

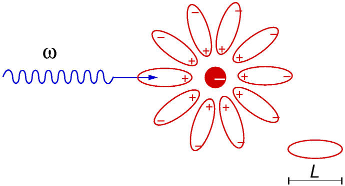


$L \sim \frac{1}{m}$

…sees less screened coupling $\alpha \nearrow$

Example: Quantum electrodynamics

- ▷ short wavelength photon ($\omega \gg m \simeq 511\text{keV}$)



- ▷ α at LEP (m_Z):

(EIDELMANN,JEGERLEHNER'95)

$$\alpha \simeq \frac{1}{128.9}$$

…mainly hadronic corrections

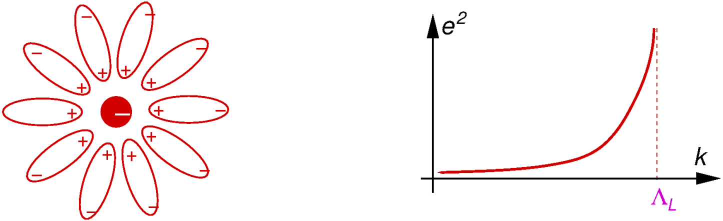
$\Longrightarrow$ measured values of couplings/parameters can depend on the scale:

$\Longrightarrow$ Running Couplings

Fundamental QED?

▷ Extrapolating perturbative running (Landau'55)

(Gell-Mann, Low'54)



Landau pole singularity

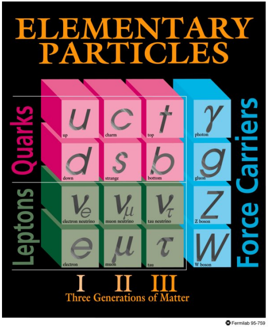
▷ world's best tested theory! But ill-defined?

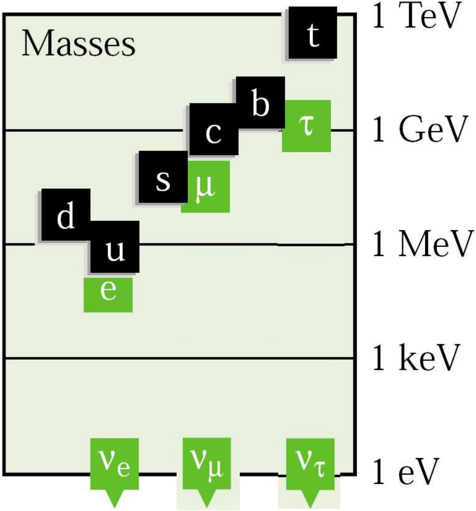
▷ evidence for scale of maximum high-energy extension: (Gockeler et al.'98; HG,Jaeckel'04)

$$\Lambda_L \simeq 10^{272}\ \mathrm{GeV}$$

$\Longrightarrow$ QFTs can predict their own failure!

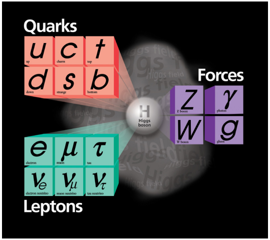
Standard model particles: pre-LHC

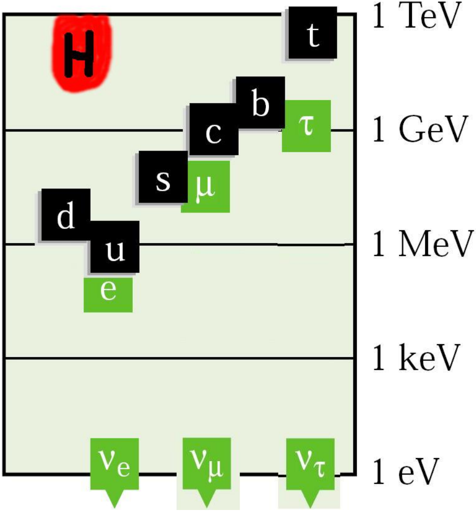




Standard model

Consistent description requires Higgs field ?

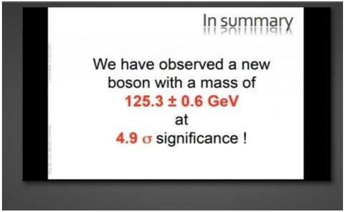




(Anderson'62; Brout,Englert'64; Higgs'64; Guralnik,Hagen,Kibble'64)

Search for the Higgs boson

$\triangleright$ 4 Jul. 2012
ATLAS & CMS
@CERN



In summary

We have observed a new boson with a mass of
125.3 ± 0.6 GeV
at
4.9 σ significance !

$\triangleright$ 14 Mar 2013, CERN press release:

“…the new particle is looking more and more like a Higgs boson …”

$$\text{CMS'12}: \quad 125.3 \pm 0.4(stat) \pm 0.5(sys) GeV,$$

$$\text{ATLAS'12}: \quad 126.0 \pm 0.4(stat) \pm 0.4(sys) GeV$$

Theory of (almost) Everything (observed so far)

$$\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a \\
& + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu \\
& - \frac{1}{2}\partial_\mu H \partial_\mu H - \frac{1}{2} m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2} M \phi^0 \phi^0 \\
& - \beta_h [\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \frac{2M^4}{g^2}\alpha_h - i g c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu)] \\
& \cdots
\end{aligned}$$

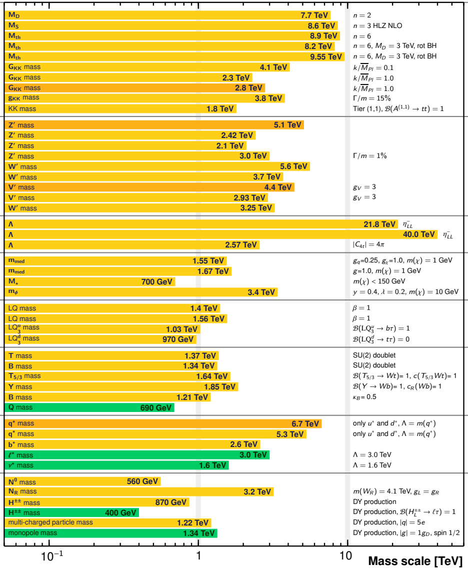
Standard model works (surprisingly) well so far …

ATLAS Exotics Searches* - 95% CL Upper Exclusion Limits

Status: March 2019

ATLAS Preliminary

$\int\mathcal{L}\,dt = (3.2 - 139)\ \mathrm{fb}^{-1}$ $\sqrt{s} = 8,\ 13$ TeV



	Model	ℓ, γ	Jets†	E_T^{miss}	$\int\mathcal{L}\,dt[\mathrm{fb}^{-1}]$	Limit		Reference
Extra dimensions	ADD $G_{KK} + g/q$	0 e, μ	1 – 4 j	Yes	36.1	M_D 7.7 TeV	$n = 2$	1711.03301
	ADD non-resonant $\gamma\gamma$	2 γ	–	–	36.7	M_S 8.6 TeV	$n = 3$ HLZ NLO	1707.04147
	ADD QBH	–	2 j	–	37.0	M_{th} 8.9 TeV	$n = 6$	1703.09127
	ADD BH high $\sum p_T$	≥ 1 e, μ	≥ 2 j	–	3.2	M_{th} 8.2 TeV	$n = 6$, $M_D = 3$ TeV, rot BH	1606.02265
	ADD BH multijet	–	≥ 3 j	–	3.6	M_{th} 9.55 TeV	$n = 6$, $M_D = 3$ TeV, rot BH	1512.02586
	RS1 $G_{KK} \to \gamma\gamma$	2 γ	–	–	36.7	G_{KK} mass 4.1 TeV	$k/\overline{M}_{Pl} = 0.1$	1707.04147
	Bulk RS $G_{KK} \to WW/ZZ$	multi-channel			36.1	G_{KK} mass 2.3 TeV	$k/\overline{M}_{Pl} = 1.0$	1808.02380
	Bulk RS $G_{KK} \to WW/ZZ \to qqqq$	0 e, μ	2 J	–	139	G_{KK} mass 2.8 TeV	$k/\overline{M}_{Pl} = 1.0$	ATLAS-CONF-2019-003
	Bulk RS $g_{KK} \to tt$	1 e, μ	≥ 1 b, ≥ 1J/2j	Yes	36.1	g_{KK} mass 3.8 TeV	$\Gamma/m = 15\%$	1804.10823
	2UED / RPP	1 e, μ	≥ 2 b, ≥ 3 j	Yes	36.1	KK mass 1.8 TeV	Tier (1,1), $\mathcal{B}(A^{(1,1)} \to tt) = 1$	1803.09678
Gauge bosons	SSM $Z' \to \ell\ell$	2 e, μ	–	–	139	Z' mass 5.1 TeV		1903.06248
	SSM $Z' \to \tau\tau$	2 τ	–	–	36.1	Z' mass 2.42 TeV		1709.07242
	Leptophobic $Z' \to bb$	–	2 b	–	36.1	Z' mass 2.1 TeV		1805.09299
	Leptophobic $Z' \to tt$	1 e, μ	≥ 1 b, ≥ 1J/2j	Yes	36.1	Z' mass 3.0 TeV	$\Gamma/m = 1\%$	1804.10823
	SSM $W' \to \ell\nu$	1 e, μ	–	Yes	79.8	W' mass 5.6 TeV		ATLAS-CONF-2018-017
	SSM $W' \to \tau\nu$	1 τ	–	Yes	36.1	W' mass 3.7 TeV		1801.06992
	HVT $V' \to WV \to qqqq$ model B	0 e, μ	2 J	–	139	V' mass 4.4 TeV	$g_V = 3$	ATLAS-CONF-2019-003
	HVT $V' \to WH/ZH$ model B	multi-channel			36.1	V' mass 2.93 TeV	$g_V = 3$	1712.06518
	LRSM $W'_R \to tb$	multi-channel			36.1	W' mass 3.25 TeV		1807.10473
CI	CI $qqqq$	–	2 j	–	37.0	Λ 21.8 TeV	η^-_{LL}	1703.09127
	CI $\ell\ell qq$	2 e, μ	–	–	36.1	Λ 40.0 TeV	η^-_{LL}	1707.02424
	CI $tttt$	$\geq$1 e,μ	$\geq$1 b, $\geq$1 j	Yes	36.1	Λ 2.57 TeV	$\lvert C_{4t}\rvert = 4\pi$	1811.02305
DM	Axial-vector mediator (Dirac DM)	0 e, μ	1 – 4 j	Yes	36.1	m_{med} 1.55 TeV	g_q=0.25, g_χ=1.0, $m(\chi) = 1$ GeV	1711.03301
	Colored scalar mediator (Dirac DM)	0 e, μ	1 – 4 j	Yes	36.1	m_{med} 1.67 TeV	g=1.0, $m(\chi) = 1$ GeV	1711.03301
	$VV\chi\chi$ EFT (Dirac DM)	0 e, μ	1 J, ≤ 1 j	Yes	3.2	M_* 700 GeV	$m(\chi) < 150$ GeV	1608.02372
	Scalar reson. $\phi \to t\chi$ (Dirac DM)	0-1 e, μ	1 b, 0-1 J	Yes	36.1	m_ϕ 3.4 TeV	$y = 0.4$, $\lambda = 0.2$, $m(\chi) = 10$ GeV	1812.09743
LQ	Scalar LQ 1st gen	1,2 e	≥ 2 j	Yes	36.1	LQ mass 1.4 TeV	$\beta = 1$	1902.00377
	Scalar LQ 2nd gen	1,2 μ	≥ 2 j	Yes	36.1	LQ mass 1.56 TeV	$\beta = 1$	1902.00377
	Scalar LQ 3rd gen	2 τ	2 b	–	36.1	LQ^u_3 mass 1.03 TeV	$\mathcal{B}(\mathrm{LQ}^u_3 \to b\tau) = 1$	1902.08103
	Scalar LQ 3rd gen	0-1 e, μ	2 b	Yes	36.1	LQ^d_3 mass 970 GeV	$\mathcal{B}(\mathrm{LQ}^d_3 \to t\tau) = 0$	1902.08103
Heavy quarks	VLQ $TT \to Ht/Zt/Wb + X$	multi-channel			36.1	T mass 1.37 TeV	SU(2) doublet	1808.02343
	VLQ $BB \to Wt/Zb + X$	multi-channel			36.1	B mass 1.34 TeV	SU(2) doublet	1808.02343
	VLQ $T_{5/3}T_{5/3} \vert T_{5/3} \to Wt + X$	2(SS)/$\geq$3 e,μ	$\geq$1 b, $\geq$1 j	Yes	36.1	$T_{5/3}$ mass 1.64 TeV	$\mathcal{B}(T_{5/3} \to Wt)$= 1, $c(T_{5/3}Wt)$= 1	1807.11883
	VLQ $Y \to Wb + X$	1 e, μ	≥ 1 b, ≥ 1j	Yes	36.1	Y mass 1.85 TeV	$\mathcal{B}(Y \to Wb)$= 1, $c_R(Wb)$= 1	1812.07343
	VLQ $B \to Hb + X$	0 e,μ, 2 γ	≥ 1 b, ≥ 1j	Yes	79.8	B mass 1.21 TeV	κ_B= 0.5	ATLAS-CONF-2018-024
	VLQ $QQ \to WqWq$	1 e, μ	≥ 4 j	Yes	20.3	Q mass 690 GeV		1509.04261
Excited fermions	Excited quark $q^* \to qg$	–	2 j	–	139	q^* mass 6.7 TeV	only u^* and d^*, $\Lambda = m(q^*)$	ATLAS-CONF-2019-007
	Excited quark $q^* \to q\gamma$	1 γ	1 j	–	36.7	q^* mass 5.3 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1709.10440
	Excited quark $b^* \to bg$	–	1 b, 1 j	–	36.1	b^* mass 2.6 TeV		1805.09299
	Excited lepton ℓ^*	3 e, μ	–	–	20.3	ℓ^* mass 3.0 TeV	$\Lambda = 3.0$ TeV	1411.2921
	Excited lepton ν^*	3 e, μ, τ	–	–	20.3	ν^* mass 1.6 TeV	$\Lambda = 1.6$ TeV	1411.2921
Other	Type III Seesaw	1 e, μ	≥ 2 j	Yes	79.8	N^0 mass 560 GeV		ATLAS-CONF-2018-020
	LRSM Majorana ν	2 μ	2 j	–	36.1	N_R mass 3.2 TeV	$m(W_R) = 4.1$ TeV, $g_L = g_R$	1809.11105
	Higgs triplet $H^{\pm\pm} \to \ell\ell$	2,3,4 e, μ (SS)	–	–	36.1	$H^{\pm\pm}$ mass 870 GeV	DY production	1710.09748
	Higgs triplet $H^{\pm\pm} \to \ell\tau$	3 e, μ, τ	–	–	20.3	$H^{\pm\pm}$ mass 400 GeV	DY production, $\mathcal{B}(H^{\pm\pm}_L \to \ell\tau) = 1$	1411.2921
	Multi-charged particles	–	–	–	36.1	multi-charged particle mass 1.22 TeV	DY production, $\lvert q\rvert = 5e$	1812.03673
	Magnetic monopoles	–	–	–	7.0	monopole mass 1.34 TeV	DY production, $\lvert g\rvert = 1g_D$, spin 1/2	1509.08059

🟩 $\sqrt{s}$ = 8 TeV

🟨 $\sqrt{s}$ = 13 TeV partial data

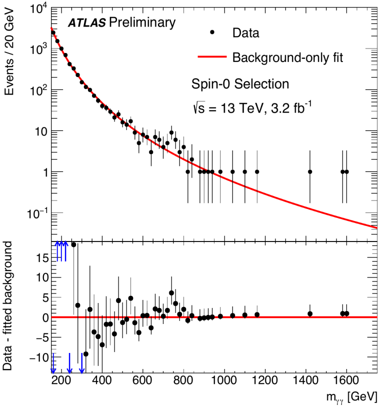
🟧 $\sqrt{s}$ = 13 TeV full data

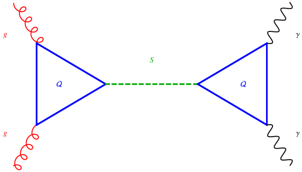
*Only a selection of the available mass limits on new states or phenomena is shown.

†Small-radius (large-radius) jets are denoted by the letter j (J).

LHC “Anomalies”

- 2016: 750 GeV resonance in diphoton channel?





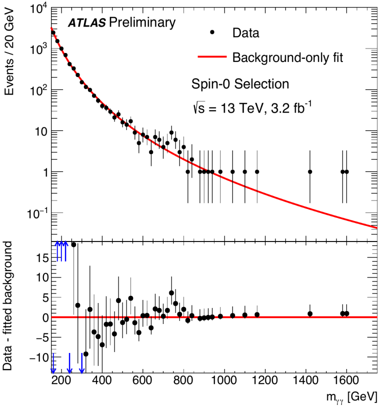
Maybe the main …

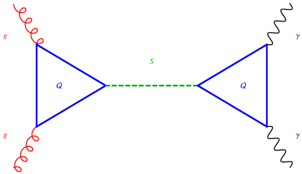
$$\left\{ \begin{array}{c} \text{discovery} \\ \text{statistical fluke} \end{array} \right\}$$

…in the last decades.

LHC “Anomalies”

- ▷ 2016: 750 GeV resonance in diphoton channel?





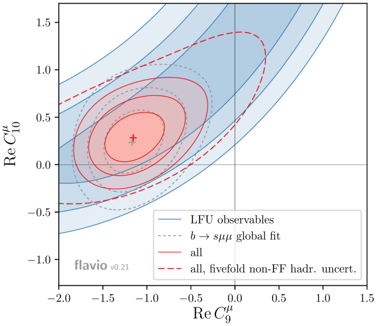
Maybe the main …

{ 2017: statistical fluke }

…in the last decades.

LHC “Anomalies”

- ▷ 2017: Lepton Flavor Universality violation?



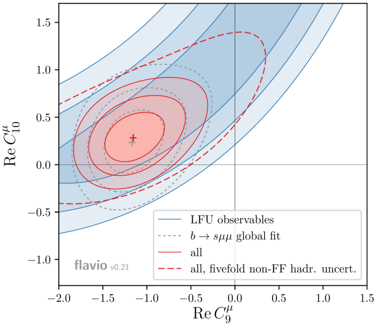
B meson decay

$\to K^{(*)}\mu\mu/ee$

2017: Discovery or Fluke?

LHC "Anomalies"

- 2017: Lepton Flavor Universality violation?



B meson decay

$\to K^{(*)}\mu\mu/ee$

2019:
consistent with SM at 2.5σ

still worthwhile to watch

Summary III

- To(a)E: Quantum Field Theory + Standard Model

…tested by experiment to high precision
(a): neutrino masses, indirect evidence for dark matter

- QFT: running couplings, masses, parameters are scale-dependent

…renormalization theory

- QFT: can be fundamental or predict its own failure

…may predict scale of “New Physics”

- recent discovery of the Higgs particle

…SM at special point ?
→ my talk on Thursday

Classification of QFTs

▷ assumptions: Lorentz invariance, local interactions, $D = 3 + 1$
(perturbative) renormalizability ($\hat{=}$ predictivity)

spin	H_{int}	particle physics
0	ϕ^4	Higgs scattering
1/2	$\bar{\psi}\psi\phi$	fermion-Higgs interactions, fermion masses
1/2	$\bar{\psi}\gamma_\mu A_\mu \psi$	gauge interactions
1	$f^{abc} A^a_\mu A^b_\nu \partial_\mu A^c_\nu$ $f^{abc} f^{ade} A^a_\mu A^b_\nu A^d_\mu A^e_\nu$	gluon self-interactions
3/2	–	–
2	–	graviton ?

Classification of QFTs

▷ assumptions: Lorentz invariance, local interactions, (perturbative) renormalizability (≙ predictivity)

$D > 3 + 1$

spin	H_{int}	particle physics
0	_	–
1/2	_	–
1/2	_	–
1	_	–
3/2	_	–
≥ 2	_	–

Summary IV

- all types of local, Lorentz-invariant, “(perturbatively) renormalizable” interactions realized in Nature

…success of “renormalization theory”

- Einstein gravity (ART) doesn’t fit!

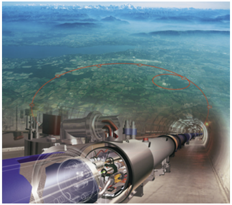
$$\text{options: give up} \begin{cases} \text{locality} \\ \text{Lorentz invariance} \\ \text{weak coupling approach} \\ \text{QFT} \end{cases}$$

- $D = 3 + 1$ (where “good” QFT’s can exist) $\equiv$ $D = 3 + 1$ of Nature

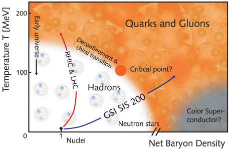
…?

Conclusion

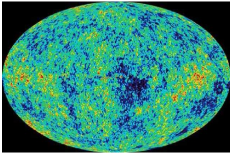
QFT



[particle physics @ CERN]

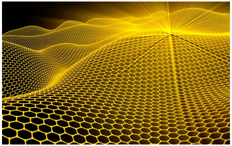


[QCD phase diagram]

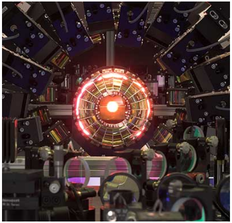


[CMB]

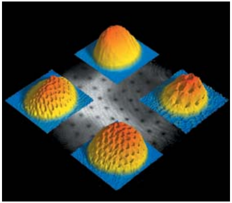
Q F T



[Strongly correlated electrons]



[strong fields @ JENA]



[ultracold atom gases]

CAVEAT: …aus der Vogelperspektive

Verlockend ist der äußre Schein
der Weise dringet tiefer ein.

(Wilhelm Busch)