Felder, Teilchen, Masse und das Higgs-Boson

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Wie schwer bin ich?



[SUEDDEUTSCHE.DE]

NB: gravitational mass = inertial mass¹

(NEWTON 1686; EULER 1750)



¹equivalence principle \rightarrow general relativity

Towards the fundamental building blocks



Masses of elementary particles

e.g., hydrogen atom







Masses of elementary particles

e.g., hydrogen atom

(c = 1)



 $m_{\rm e} = 510998 {\rm eV} \simeq 511 {\rm keV}$

 $1\text{eV}\simeq 1.783\cdot 10^{-36}\text{kg}$

 $m_{\rm p} = 938272013 \mathrm{eV} \simeq 938 \mathrm{MeV}$

Masses of elementary particles



 \simeq 70 kg protons

\simeq 35 g electrons

Energy - mass equivalence

(Einstein 1905)

$E = m c^2$

Energy - mass equivalence

(Einstein 1905)

$$E = \sqrt{(\mathbf{p}c)^2 + (m c^2)^2}$$

Energy – momentum – *m*ass – relation

(dispersion relation)

Analogue: linear chain

▷ dispersion relation of a linear chain:

$$\omega = \sqrt{(\mathbf{p}\mathbf{v})^2 + \Omega^2}$$

 $\triangleright \Omega$: restoring force



Particle \leftrightarrow Wave ...?

▷ quantum field theory:

particles = (quantized) excitations of fields

Particle \leftrightarrow Wave ...? Field!

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Particle \leftrightarrow Wave ...? Field!

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particles = (quantized) excitations of fields



 $E = \sqrt{(\mathbf{p}c)^2 + (mc^2)^2}$

Particles as field excitations

 \triangleright particle mass \sim "restoring force" of quantum field amplitudes

$$E = \sqrt{(\mathbf{p}c)^2 + (m\,c^2)^2}$$



 \triangleright energy-momentum dispersion relation ($\hbar = c = 1$)

$$E^2 = \mathbf{p}^2 + m^2$$

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 \triangleright corresponding wave equation ($E = p^0$, metric $g_{\mu\nu} = (-, +, +, +)$):

$$0 = (-E^2 + \mathbf{p}^2 + m^2)\phi(E, \mathbf{p}) = (p_\mu p^\mu + m^2)\phi(p)$$

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 \triangleright in coordinate space:

$$0 = (-\partial^2 + m^2)\phi(x), \qquad \partial^2 = \frac{\partial}{\partial x_{\mu}}\frac{\partial}{\partial x^{\mu}}$$

Klein-Gordon equation

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Klein-Gordon equation

▷ corresponding Langrangian:

$$\mathcal{L}=rac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-rac{1}{2}m^2\phi^2, \qquad ext{cf.} \quad L=T-U=rac{1}{2}\dot{x}^2-\left(rac{1}{2}\omega^2x^2
ight)$$

 \triangleright in coordinate space:

$$0 = (-\partial^2 + m^2)\phi(x), \qquad \partial^2 = rac{\partial}{\partial x_\mu} rac{\partial}{\partial x^\mu}$$

Klein-Gordon equation

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ight)$$

 \implies mass appears as prefactor of quadratic term in Lagrangian

penalizes field amplitude excitations

Summary I

mass

- gravitational mass ("weight"): source of the graviational field
- inertial mass
- mass: parameter in relativistic energy momentum dispersion relation

• quantum field theory:

- particles \sim (quantized) field excitations
- mass: cf. restoring force for field amplitude
- mass of ordinary matter: 0.05% electrons, 99.95% protons, neutrons

Q?:

Where does the mass of protons, neutrons and electrons come from?

Exercise: Mass of the Photon





> Maxwell Lagrangian:

$$\mathcal{L} = \frac{1}{2} \left(\mathbf{E}^2 - \mathbf{B}^2 \right) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} \left[(\partial_{\mu} A_{\nu})^2 - (\partial_{\mu} A_{\mu})^2 \right]$$

 \vartriangleright use gauge freedom: e.g., Lorenz gauge, $\partial_{\mu}A_{\mu}=$ 0:

$$\mathcal{L}=rac{1}{2}(\partial_{\mu}A_{
u})^{2}$$

 \implies No mass term: $m_{\gamma} = 0$ (gauge symmetry!)

exp.: $m_{\gamma} \leq 10^{-18} \, eV$

(PDG'09)

electric



[LMTM.DE]

gauge field $A_{\mu} = (A_0, \mathbf{A})$

magnetic field strength

 $\mathbf{B} = \nabla \times \mathbf{A}$



[DE.WIKIPEDIA.ORG]



&

electric

$$\mathbf{E} = -\nabla A_0 - \frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}$$

magnetic field strength

 $\mathbf{B} = \nabla \times \mathbf{A}$

gauge transformation $A_{\mu}
ightarrow A_{\mu} + \partial_{\mu} \lambda$





 $\implies \mathbf{E} \rightarrow \mathbf{E}, \qquad \mathbf{B} \rightarrow \mathbf{B}$ gauge invariant!

mass \sim restoring force



theory:

mass term

$$\mathcal{S}_{m_{\gamma}} = \int \mathcal{L}_{m_{\gamma}} = rac{1}{2} \int m_{\gamma}^2 A_{\mu} A^{\mu}$$

$$S_{m_{\gamma}}[A + \partial_{\mu}\lambda] \neq S_{m_{\gamma}}[A]$$

violates symmetry

Is the photon massless?

▷ if gauge invariance is a symmetry of Nature, then

$$m_\gamma=0$$

Consequences of gauge symmetry

▷ electromagnetic field excitations have the least possible restoring force

▷ long range force:

[NETPHYSIK.DE]

also couplings to charges must obey gauge symmetry



Photon mass from symmetry breaking

> Meissner-Ochsenfeld effect in superconductors (type I)





[PICASAWEB.GOOGLE.COM]

[COMMONS.WIKIMEDIA.ORG]

Photon mass from symmetry breaking

 $T > T_c$ ▷ Cooper pairs $T < T_c$

 \implies vacuum condensate from Cooper pairs:

$$\left< \bigcirc \right> = \frac{1}{\sqrt{2}} v$$

(BARDEEN, COOPER, SCHRIEFFER 1957)

 \triangleright photon field: A_{μ} , Cooper pair field: ϕ ; minimal coupling:

$$\partial_{\mu}\phi \rightarrow D_{\mu}[A]\phi = (\partial_{\mu} - i(2e)A_{\mu})\phi$$

gauge covariant derivative

Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_{\mu}[A]\phi)^{*}(D^{\mu}[A]\phi) - V(\phi^{*}\phi), \qquad V = \frac{1}{2}m_{\phi}^{2}|\phi|^{2} + \dots$$



 $egin{aligned} m_{\phi}^2 &> 0 \ o \mbox{massive Cooper pair} \ o \mbox{massless photon} \end{aligned}$

 $m_{\phi}^2 \sim (T - T_c) < 0$ \rightarrow massive Cooper field excitation \rightarrow massive photon

Photon mass from symmetry breaking

vacuum condensate from Cooper pairs:

$$\left< \bigcirc \right> \equiv \left< \phi \right> = \frac{1}{\sqrt{2}} v$$



photon mass below T_c :

$$\mathcal{L} = \dots (D_{\mu}[A]\phi)^* (D^{\mu}[A]\phi) \dots = \dots - (2e)^2 A_{\mu} A^{\mu} |\phi|^2 \dots \to \dots \frac{1}{2} (2e)^2 v^2 A_{\mu} A^{\mu}$$
 $m_{\gamma} = 2ev$

▷ London penetration depth:

$$\lambda \sim \frac{1}{m_{\gamma}}, \qquad \mathbf{B} \sim \exp(-x/\lambda)$$

short range

Summary II

- photons: exchange bosons of electromagnetic interactions gauge invariance!
- symmetry keeps photon massless

long range interactions

• exception: condensates (e.g., superconductor) "break"(hide) symmetry $m_{\gamma} > 0 \implies \text{short range interaction}$

Q?:

Are all masses generated from symmetry breaking?

Where does the proton mass come from?

> scattering experiments: proton is a bound state of 3 quarks



BUT: $m_q \simeq 5 \text{ MeV}$; $3 \times 5 \text{ MeV} \neq 938 \text{ MeV} \dots$?

Quark masses and chiral symmetry

▷ theory of quark interactions:

quantum chromodynamics (QCD)

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{q} i \not\!\!{D} [A] q + m_{\rm q} \bar{q} q$$

 \triangleright chirality: $q = q_L + q_R$

$$\mathcal{L}_{ ext{QCD}} = -rac{1}{4}F^a_{\mu
u}F^{a\mu
u} + ar{q}_{ ext{L}}ioldsymbol{p}[A]q_{ ext{L}} + ar{q}_{ ext{R}}ioldsymbol{p}[A]q_{ ext{R}} + m_{ ext{q}}(ar{q}_{ ext{L}}q_{ ext{R}} + ar{q}_{ ext{R}}q_{ ext{L}})$$

▷ if $m_q = 0$, QCD has a chiral symmetry:

QCD: strong interaction



strong

force
QCD: strong interaction



Proton

strong

force



Quantum chromodynamics



Quantum chromodynamics



Quantum chromodynamics



Quantum Chromodynamics



Quantum Chromodynamics





Quantum Chromodynamics

QCD: quarks, gluons cond-mat: electron states cold gases: atoms





QCD: mesons, baryons cond-mat: Cooper-pairs, longrange order cold gases: BEC, superfluidity

Mass generation from chiral symmetry breaking (sketch!)

▷ dynamics of quarks and gluons generates "effective" interactions:

e.g., 4 quark int.
$$\sim \bar{q}_{L}q_{R}\bar{q}_{R}q_{L} \sim$$

quark bilinears form bosons at strong coupling:

e.g.,
$$\phi \sim \bar{q}_{L}q_{R}, \ \phi^{\dagger} \sim \bar{q}_{R}q_{L}$$

> chiral "quark condensates" form at small momentum scales:

$$\langle \phi
angle = rac{1}{\sqrt{2}} v, \qquad ar{q}_{\mathsf{L}} q_{\mathsf{R}} ar{q}_{\mathsf{R}} q_{\mathsf{L}} \sim v (ar{q}_{\mathsf{L}} q_{\mathsf{R}} + ar{q}_{\mathsf{R}} q_{\mathsf{L}})$$

Breaking of chiral symmetry

▷ strong interaction leads to quark bound states:

$$v\sim \langle ar{q}q
angle = \left\langle igodot
ight
angle
ight
angle$$

▷ "constituent" quark mass:





> symmetry breaking through chiral vacuum condensates

Breaking of chiral symmetry

▷ strong interaction leads to quark bound states:

$$v\sim \langle ar{q}q
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angle$$

▷ "constituent" quark mass:

 $m_{
m constit.} \sim v$





Nobel-Preis 2008:

Yoichiro Nambu

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Mass of the proton

pprox 3 $\, imes\,$ 5 MeV



Mass of the proton

\approx 3 \times 5 MeV + QCD vacuum condensates



Human mass



70 kg human

35 g electrons

1,050 kg quarks

68,915 kg vacuum condensates

Is there a T_c for protons?

\triangleright *T*_c: melting point of condensates?



[GSI.DE]

 $ightarrow T_{\rm c} \simeq 156 \, {
m MeV} = 1\,810\,000\,000\,000 \, {
m Kelvin}$

 $ightarrow T > T_c$: quark-gluon-plasma (chirally symmetric)

QCD phase transition

 \triangleright in the early universe ($\sim 1 \, \mu s$ after the Big Bang):



[MAP.GSFC.NASA.GOV/]

QCD phase transition

in heavy ion collisions (RHIC@BNL, LHC@CERN seit 2010, FAIR@GSI >2023)



[BNL.GOV]

Summary III

- proton/neutron mass (and thus the largest part of all visible matter in the universe) are generated by chiral symmetry breaking
- (approximate) chiral symmetry and its breaking are properties of QCD

theory of the strong interactions

- restoration of chiral symmetry is investigated in the laboratory heavy ion collisions
- quark-gluon plasma: state of matter in the early universe

Q?:

Where does m_e , m_q , ... come from?

The renormalization group (RG)





are parameters of the action:

$$S = \int \mathcal{L} = S(\alpha_i, m_i, \dots)$$

 \implies idea: promote action *S* to a scale-dependent quantity integrate fluctuations scale by scale

$$S
ightarrow \Gamma_k$$
 1PI effective action

(...Gell-Mann,Low;Bogoliubov,Shirkhov;Callan;Symanzik;Kadanoff;Wegner,Houghton;<u>Wilson;</u>Polchinski;Wetterich, ...)



 \triangleright e.g., Wetterich equation:

$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_k R_k$$



▶ RG trajectory:



 \triangleright e.g., Wetterich equation:

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$$\partial_k \Gamma_k = \frac{1}{2} \operatorname{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_k R_k$$

> RG trajectory: $\Gamma_{k \to 0} = \Gamma$





Example: RG flow towards chiral symmetry breaking in QCD

 \triangleright Approach to chiral symmetry breaking in SU(3) ($N_{\rm f} = 1$) QCD:

 \triangleright effective potential for $\phi \sim \langle \bar{q}_{R} q_{L} \rangle$:



⁽without $U_A(1)$ anomaly)

Example: RG flow towards chiral symmetry breaking in QCD

 \triangleright Approach to chiral symmetry breaking in SU(3) ($N_f = 1$) QCD:



▷ IR prediction: "constituent quark mass"

 $m_{\rm f} \simeq 250 {\rm MeV}$

(without $U_A(1)$ anomaly)

RG flow in strongly coupled systems

QCD: quarks, gluons cond-mat: electron states cold gases: atoms





QCD: mesons, baryons cond-mat: Cooper-pairs, longrange order cold gases: BEC, superfluidity

RG flow in strongly coupled systems





RG flow in strongly coupled systems



"Wolkenbruch" Hans-Werner Sahm

Summary IV

• The Renormalization Group: flexible method to investigate strongly-interacting field theories

Where do m_e , m_q , ... come from?





35 g electrons

1,050 kg quarks

68,915 kg vacuum condensates

Building blocks of the universe



 $\triangleright \text{ BUT: } m_{\text{e}}, m_{\text{q}}, \dots m_{\text{W}}, m_{\text{Z}} \neq 0 \text{ inconsistent!}$ problems: electroweak gauge symmetry / renormalizability

Building blocks of the universe

(ENGLERT, BROUT, HIGGS, GURALNIK, HAGEN, KIBBLE 1964)



ABEHGHKt'H mechanism

(Anderson,Brout,Englert,Higgs,Guralnik,Hagen,Kibble,t'Hooft)

"spontaneously broken" (hidden) symmetry generates masses

 $\langle H \rangle \sim v \simeq 246 \text{GeV}$ "Fermi scale"

- W, Z masses a la superconductivity: $(D_{\mu}[W]H)^{\dagger}(D^{\mu}[W]H) \sim g'^2 v^2 W_{\mu} W^{\mu}$
- $m_{\rm e}, m_{\rm q}, \ldots$ from Yukawa couplings:

$$y_e \bar{\psi}^a_L H^a e_R + \ldots \sim y_e v \, \bar{e}_L e_R$$



ABEHGHKt'H mechanism

(Anderson, Brout, Englert, Higgs, Guralnik, Hagen, Kibble, t'Hooft)

ightarrow all masses \sim Fermi scale:

 $m_{W,Z,e,\mu,\tau,u,d,s,c,b,t} \sim V$

mass values parametrized by (Yukawa) couplings:

e.g., $m_e = y_e v, m_t = y_t v, ...$

 \triangleright origin of hierarchy ...?

$$y_e \simeq 2.1 \times 10^{-6}, \ldots, y_t \simeq 0.7$$



The Higgs Boson

(HIGGS 1964)

▷ Higgs boson: excitation on top of condensate

 $H(x) \sim v + h(x)$





 \triangleright Higgs self-coupling λH^4 induces mass:

$$m_h = \sqrt{\lambda} v$$

Search for the Higgs boson

▷ 4 Jul. 2012ATLAS & CMS@CERN



▷ 14 Mar 2013, CERN press release:

"...the new particle is looking more and more like a Higgs boson ..."

CMS& ATLAS'15 : $125.09 \pm 0.21(stat) \pm 0.11(sys)GeV$,

 \triangleright numbers matter: \implies




Numbers matter

standard model

Construction and the state of t

$\frac{\text{best before:}}{\Lambda = M_{\text{Planck}}}?$

Validity range of the standard model

- ⊳ Λ:
 - UV cutoff

SM as effective theory

- scale of maximum UV extension
- scale of new physics: $\Lambda_{NP} \leq \Lambda$
- ▷ typical guess: Planck scale

$$\Lambda \stackrel{?}{=} 10^{19} \text{GeV}$$

















Effective potential from functional RG



Higgs mass bounds for the Standard Model

 \triangleright upper triviality/unitarity bound (~ strong coupling in UV)



 lower "stability" bound
(~ non-interacting/unstable in UV)

lower "consistency" bound (~ stable but more involved UV theory)

(KRIVE,LINDE'76; MAIANI,PARISI,PETRONZIO'78;...HG, SONDENHEIMER'13,'14)

 \implies if $\Lambda = 10^{19}$ GeV, 128 GeV $\lesssim m_h \lesssim 150$ GeV

Experiment: $m_h \simeq 125 \text{GeV}$!?!

Perturbative lower Bound of Higgs boson mass





NNLO calculation (Degrassi et al.'12)

▷ meta-stability:

tunneling time > age of universe



Lower Bound of Higgs boson mass

▷ "Near critical" standard model: (BUTTAZZO ET AL.'13: UPDATE V4)







Lower Bound of Higgs boson mass



"consistency" bound relaxes tension

"Best-before date" of the Standard Model



▷ Near-Critical Standard Model?

(BUTTAZZO ET AL.'13; HG, GNEITING, SONDENHEIMER'14)

Summary V

• mass parameters $m_{W,Z,e,\mu,\tau,u,d,s,c,b,t}$ from ABEHGHKt'H mechanism

mass hierarchy parametrized by (Yukawa couplings)

New massive excitation

Higgs boson (discovered in 2012)

• *m_h*, *m_t* closely related to "best-before date" of standard model

"Near-criticality" of standard model

Wie schwer bin ich?



70 kg Mensch

35 g Elektronen (Higgs)

1,050 kg Quarks (Higgs)

68,915 kg Vakuumkondensate (QCD)

Wie schwer bin ich?



... viel Physik hinter einer Zahl ...