III. QFT in Curved Space-time

Semiclassical Einstein Eqs.

- Semiclassical theory of Quantum Gravity

$$R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) - \Lambda\, g_{\mu\nu} = 8\pi G \; \langle \psi \mid \hat{T}_{\mu\nu}(g) \mid \psi \rangle$$

matter fields are quantized (in some appropriate state $\mid \psi\rangle$) but gravitational field is kept classical

- This is valid in the limit that the length and time scales of the physical processes $\gg$ Planck length and time

$$\left(G\hbar/c^3\right)^{1/2} \sim 10^{-33} cm \quad \text{and} \quad \left(G\hbar/c^5\right)^{1/2} \sim 10^{-44} s$$

- In practise, solve the semiclasssical eqs. *perturbatively*:

$$R_{\mu\nu}(g^{(c)}) - \frac{1}{2} g^{(c)}_{\mu\nu} R(g^{(c)}) - \Lambda\, \underset{\substack{\uparrow \\ \textit{quantum-corrected}\text{ metric}}}{g^{(c)}_{\mu\nu}} = 8\pi G \; \langle \psi \mid \hat{T}_{\mu\nu}(\underset{\substack{\uparrow \\ \textit{background}\text{ metric}}}{g}) \mid \psi \rangle$$

Renormalization

- However, $\langle\psi \mid \hat{T}_{\mu\nu} \mid \psi\rangle$ suffers from ultraviolet divergences since it's quadratic in $\hat{\phi}$, which is an operator-valued *distribution*

- So we need to renormalize: $\langle\psi \mid \hat{T}_{\mu\nu} \mid \psi\rangle \to \langle\psi \mid \hat{T}_{\mu\nu} \mid \psi\rangle_{ren}$

RSET

- Energy now creates s-t curvature via Einstein eqs., so we cannot renormalize by simply subtracting infinities like we did in flat s-t

- First point-split $\hat{\phi}^2(x) \to \hat{\phi}(x)\hat{\phi}(x')$, then remove the divergences in a physically meaningful way and afterwards take the coincidence limit:

$$\langle\psi \mid \hat{T}_{\mu\nu}(x) \mid \psi\rangle_{ren} = \lim_{x' \to x} \Big(\langle\psi \mid \hat{T}_{\mu\nu}(x,x') \mid \psi\rangle - T^{div}_{\mu\nu}(x,x') \Big)$$

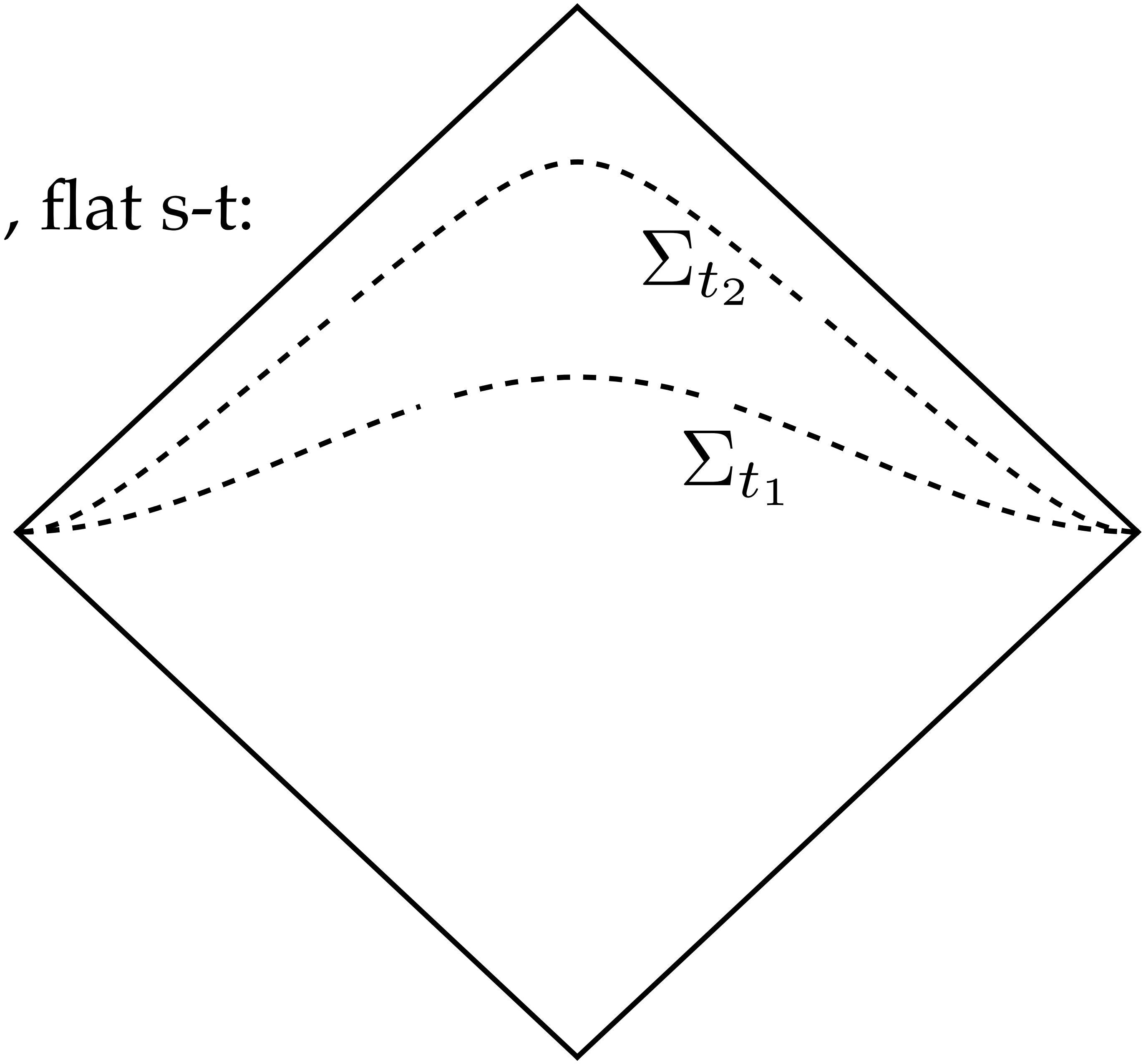
it's conserved and is causal

only depends on the geometry $g_{\mu\nu}$

Scalar Field

- Consider a globally hyperbolic s-t $(M, g_{\mu\nu})$ -> Cauchy surfaces Σ_t labelled by a parameter t

E.g., flat s-t:



- Free scalar field ϕ satisfies the Klein-Gordon eq.: $\Box\phi - m^2\phi = 0$

$$\underset{\displaystyle g^{\mu\nu}\nabla_\mu\nabla_\nu}{\underset{\shortparallel}{\Box}}\phi - m^2\phi = 0$$

Scalar Product

$$(\phi_1, \phi_2) \equiv i \int_{\Sigma_t} d\Sigma^\mu \left(\phi_1^* \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1^*\right)$$

- defines a scalar product (and an inner prod. when restricted to "pos. freq." slns.)

- is independent of the Cauchy surface if ϕ_i are slns. of the Klein-Gordon eq.

Quantization Procedure: like in Flat S-t

(1) Choose a set $\{u_i(x), u_i^*(x), \forall i\}$ of slns. of field eq. that is complete and orthonormal:

$$\left(u_i, u_j\right) = \delta_{ij} = -\left(u_i^*, u_j^*\right), \quad \left(u_i, u_j^*\right) = 0$$

↑ it may be a Dirac- δ

(2) Then any sln. of the field eq. may be expanded as

$$\phi(x) = \sum_i a_i u_i(x) + a_i^* u_i^*(x), \quad a_i = (u_i, \phi), \quad a_i^* = -(u_i^*, \phi)$$

(3) Quantize by promoting to ops. $a_i \to \hat{a}_i, \quad a_i^* \to \hat{a}_i^\dagger, \quad \phi \to \hat{\phi}$

and impose comm. rlns. $\left[\hat{a}_i, \hat{a}_j^\dagger\right] = \hat{\mathbb{I}}\, \delta_{ij}$

(4) Define a vacuum by $\hat{a}_i\,|0\rangle = 0, \forall i$ excited states by $\left(\hat{a}_i^\dagger\right)^n |\,0\rangle$ etc

Choice of vacuum depends (via $\hat{a}_i$) on choice of complete set of slns. u_i

IV. Black Holes

(a) Hawking Radiation, etc

Eternal Schwarzschild Space-time

- The unique sln. of Einstein eqs. in vacuum describing the spacetime outside a spherically-symmetric body of mass M is Schwarzschild :

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

Event horizon at $r = 2M$

Curvature singularity at $r = 0$

- Killing vectors: slike ∂_φ (axisymmetry) and tlike ∂_t (stationarity)

Null Coordinates

- Null coords.: $u \equiv t - r_*, \ v \equiv t + r_*$

$$r_* \equiv r + 2M \ln \left| \frac{r}{2M} - 1 \right| \in (-\infty, \infty) \quad \text{for} \quad r \in (2M, \infty)$$

Radially in/outgoing null geodesics are at u,v=const.

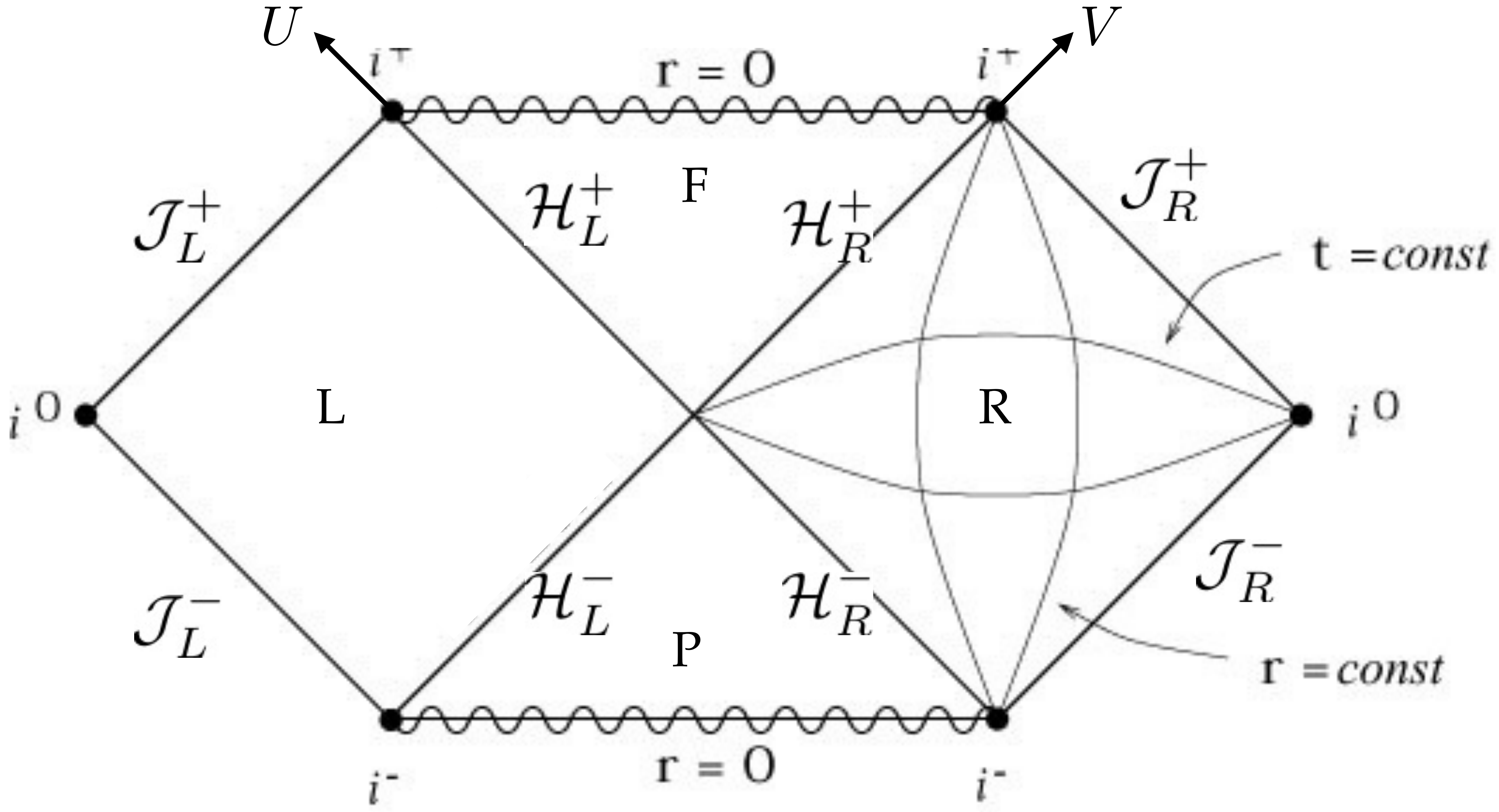
- Kruskal coords. to cover beyond EH: $U \equiv -e^{-\kappa u}, \ V \equiv e^{\kappa v}$

$\kappa \equiv \dfrac{1}{4M}$ surface gravity=force done at infinity to hold unit mass above EH

Similar transformations to cover whole s-t $U, \ V \in (-\infty, \infty)$

- *Compactify* s-t via $\bar{U} \equiv \arctan U, \ \bar{V} \equiv \arctan V \in (-\pi/2, \pi/2)$

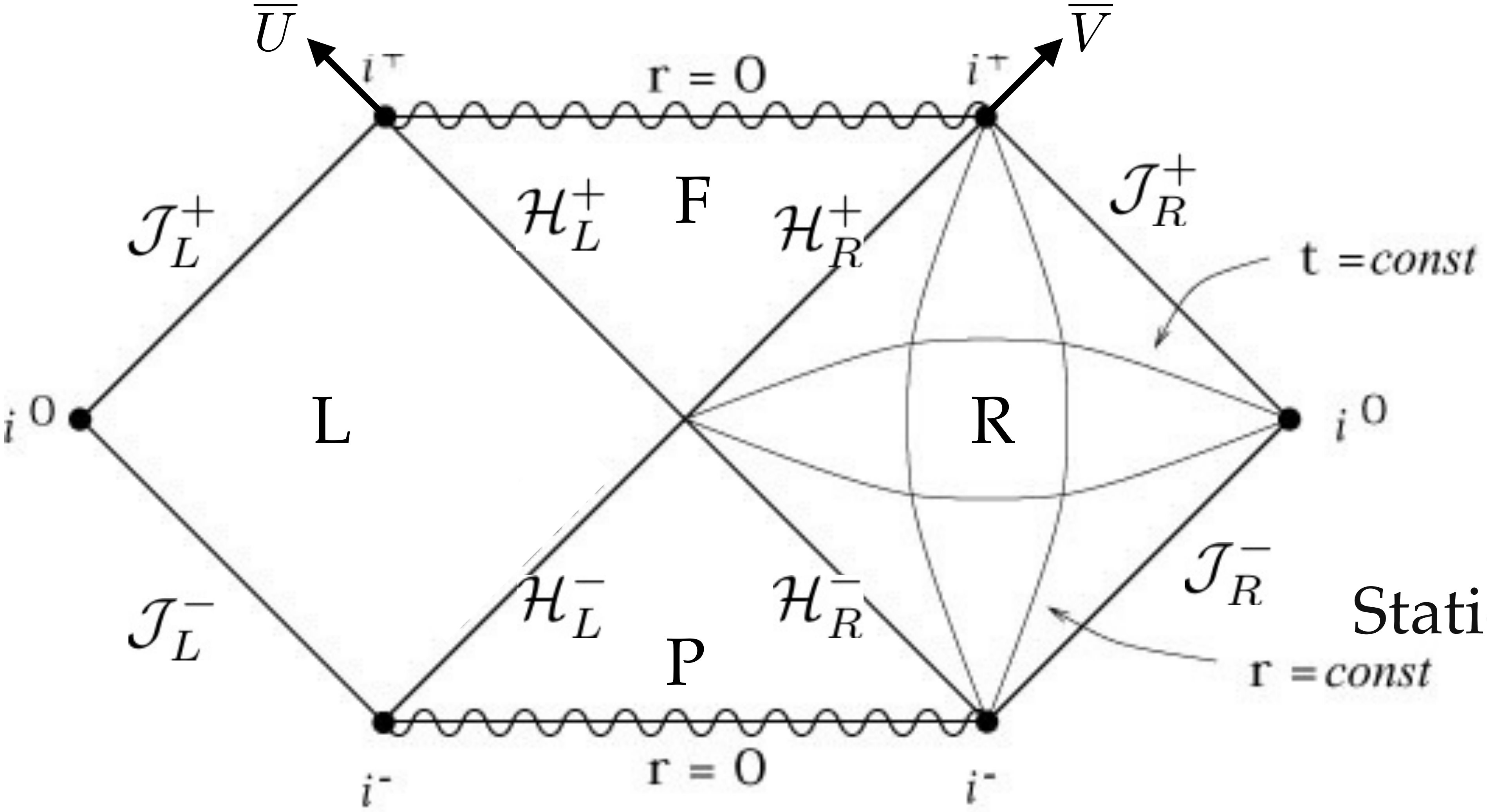
Penrose Diagram



Maximally extended Schwarzschild s-t

F: black hole, P: white hole

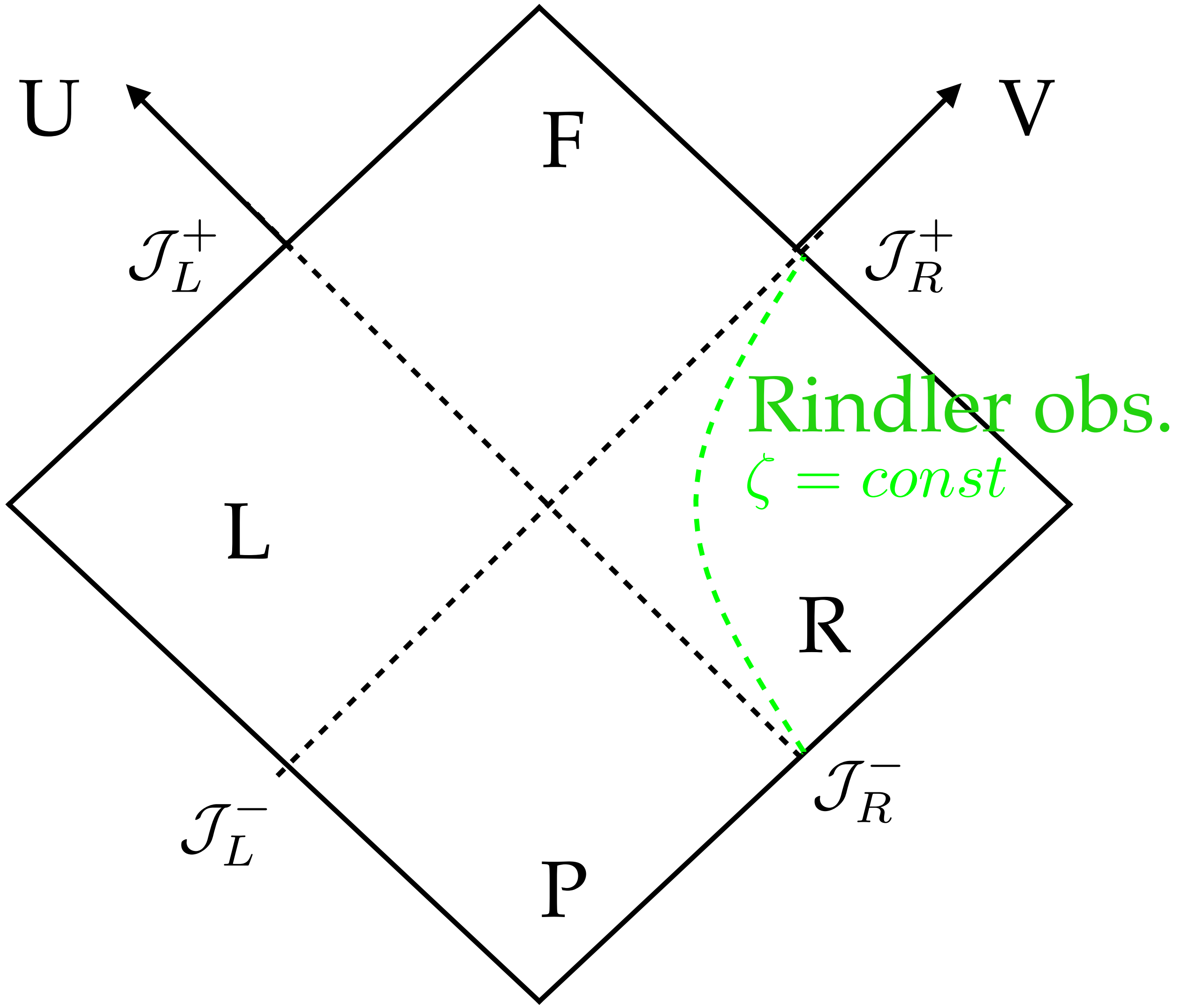
U is an affine parameter along $\mathcal{H}_R^-$



← Schwarzschild

Static obs. (which is accelerated)

flat/Rindler →



Schw.		Flat s-t
Static obs.	⟷	Rindler obs.
Free-fall obs.	⟷	Inertial obs.

Scalar Field

Klein-Gordon eq., $\Box\phi = 0$, in Schwarzschild background separates by variables:

Gral. sln.: $$\phi(x) = \int_{-\infty}^{\infty} d\omega \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_\Lambda \phi_\Lambda(x)$$

$$\Lambda \equiv \{\ell, m, \omega\}$$

Field modes: $$\phi_\Lambda(x) = N_{\ell\omega} R_{\ell\omega}(r) Y_{\ell m}(\theta, \varphi) e^{-i\omega t}$$

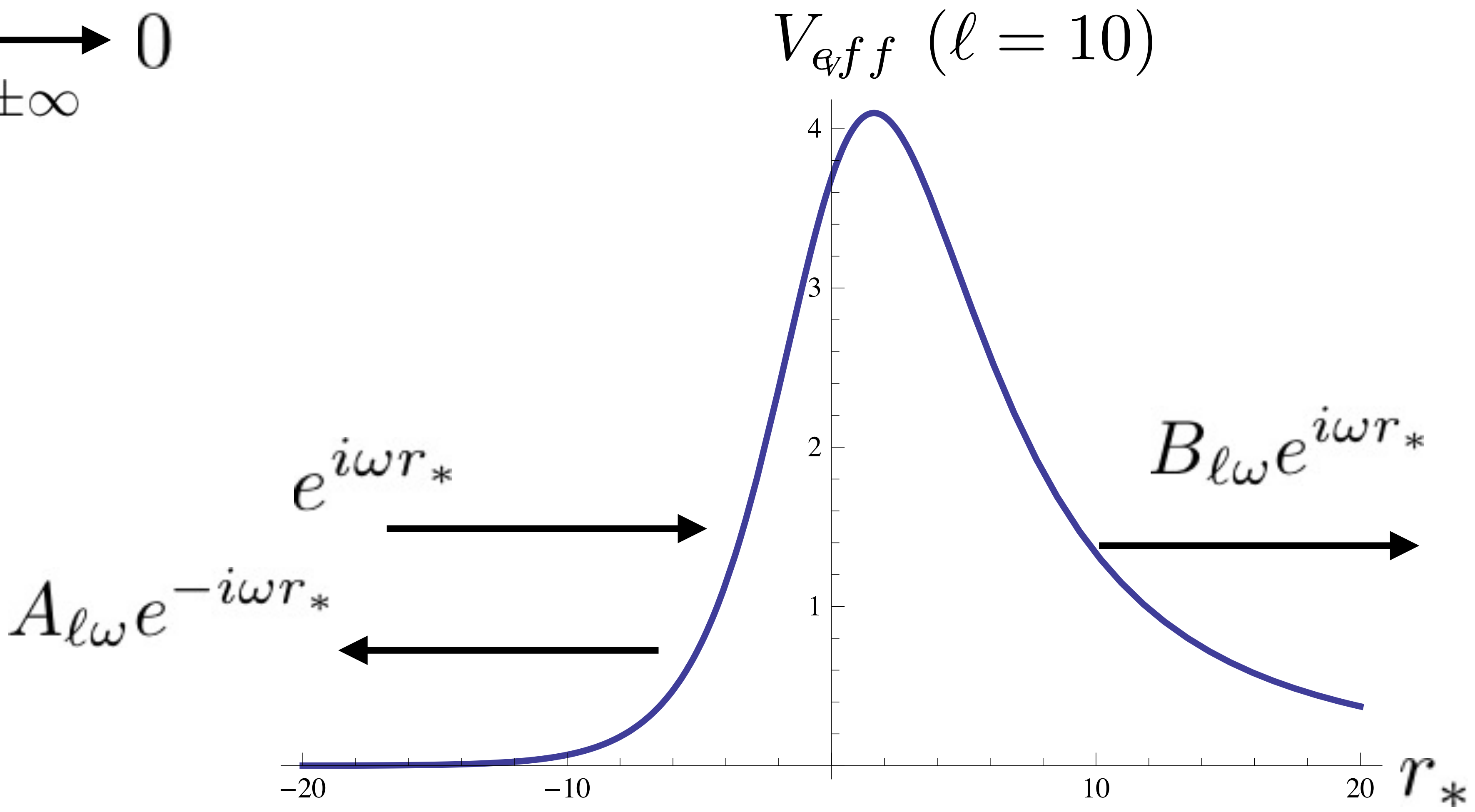
normalization const.

spherical harmonics

Radial eq.: 2nd order linear ODE -> choose 2 lin. indep. slns.

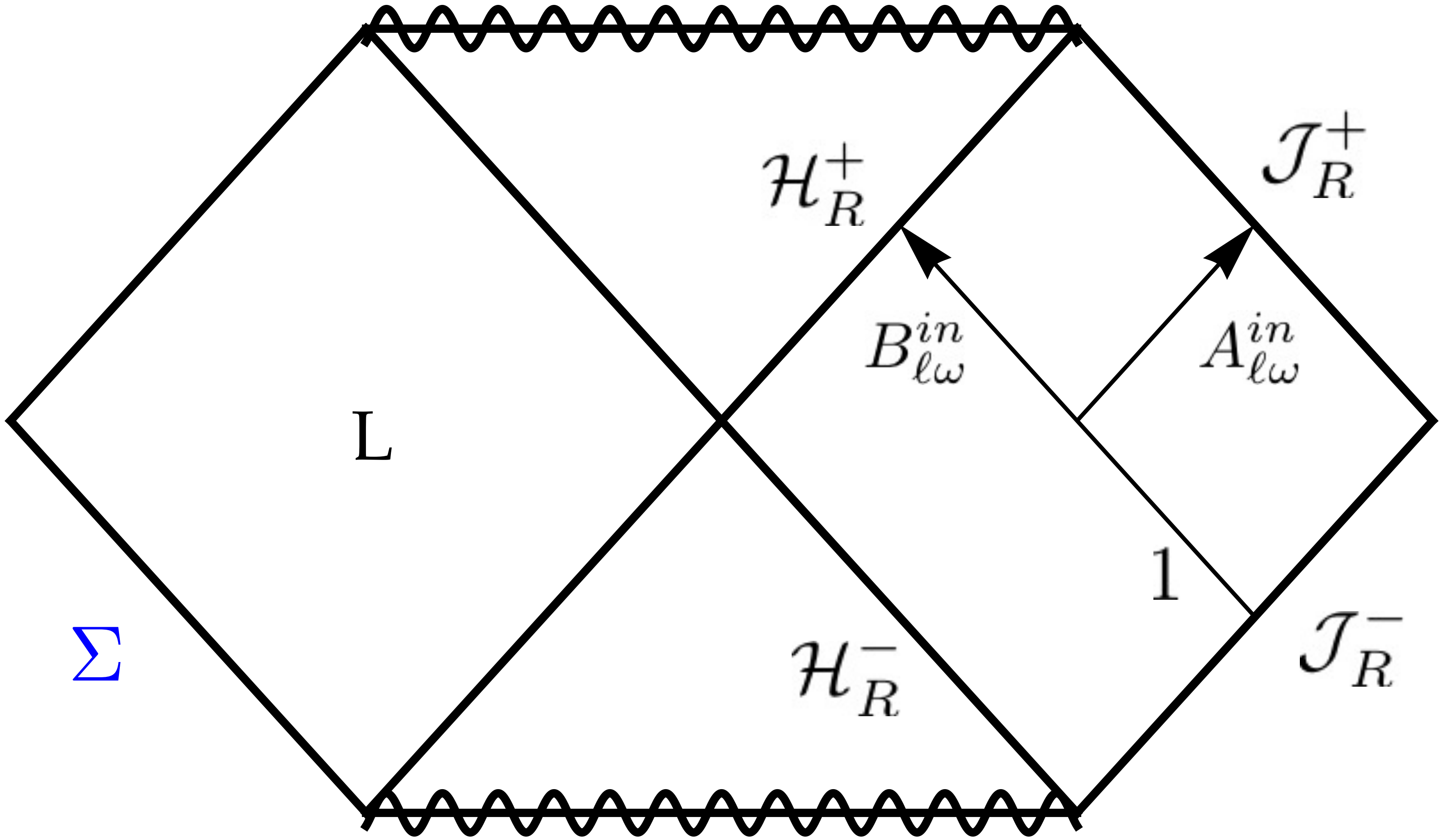
$$\left\{\frac{d^2}{dr_*^2}+\omega^2-\underbrace{\left(1-\frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2}+\frac{2M}{r^3}\right]}_{V_{eff}}\right\}R_{\ell\omega}=0$$

$$V_{eff}\xrightarrow[r_*\to\pm\infty]{}0$$



Field Modes

‘in,R’ modes



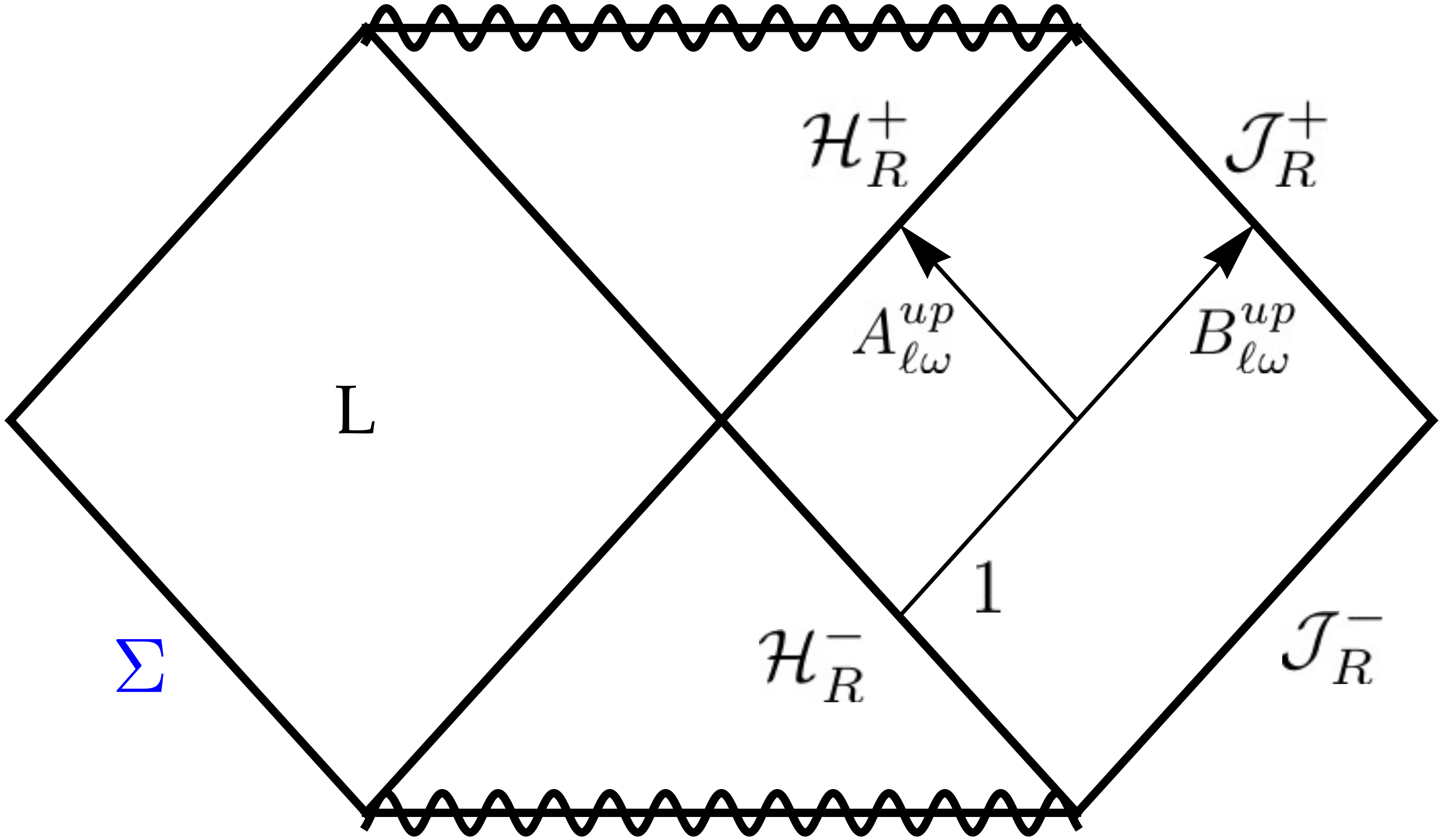
Minkowski modes in flat s-t, so correspond to Minkowski vac. in $\mathcal{J}_R^-$

$$\phi_\Lambda^{in,R}(x) = 0$$

$$\forall x \in L$$

$$\phi_\Lambda^{in,R}(x) \sim \frac{Y_{\ell m}(\Omega)}{r} \begin{cases} 0, & x \sim \mathcal{H}_R^- \\ e^{-i\omega v}, & x \sim \mathcal{J}_R^- \\ A^{in}_{\ell\omega} e^{-i\omega u}, & x \sim \mathcal{J}_R^+ \\ B^{in}_{\ell\omega} e^{-i\omega v}, & x \sim \mathcal{H}_R^+ \end{cases}$$

‘up,R’ modes



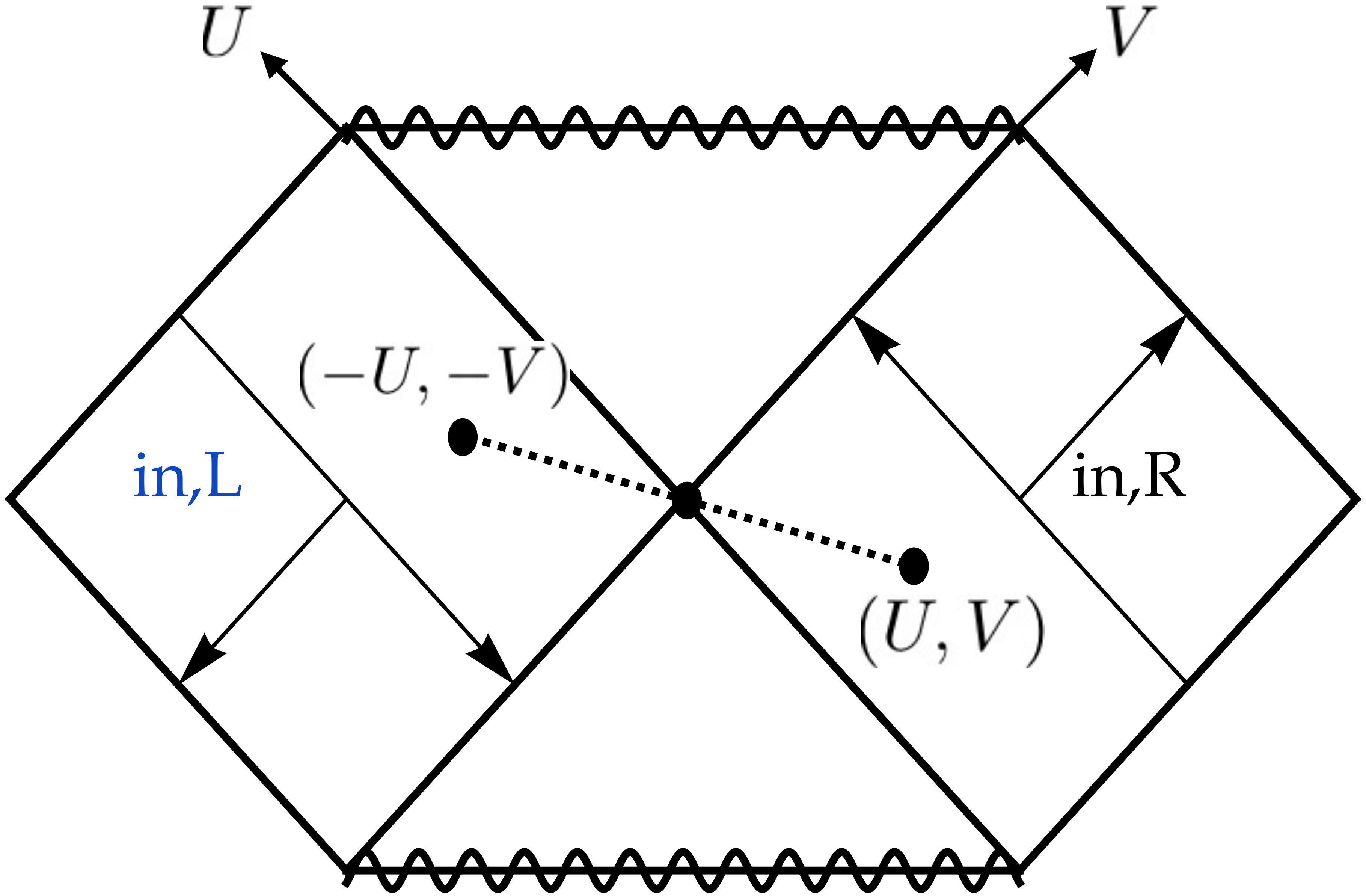
$$\phi_\Lambda^{up,R}(x) = 0$$

$$\forall x \in L$$

$$\phi_\Lambda^{up,R}(x) \sim \frac{Y_{\ell m}(\Omega)}{r} \begin{cases} e^{-i\omega u}, & x \sim \mathcal{H}_R^- \\ 0, & x \sim \mathcal{J}_R^- \\ B_{\ell\omega}^{up} e^{-i\omega u}, & x \sim \mathcal{J}_R^+ \\ A_{\ell\omega}^{up} e^{-i\omega v}, & x \sim \mathcal{H}_R^+ \end{cases}$$

‘in & up,L’ modes

$$\phi_\Lambda^{in/up,L}(x) = 0,\ \forall x \in R$$



$$\phi_\Lambda^{in/up,L}(U, V, \Omega) = \phi_\Lambda^{in/up,R^*}(-U, -V, \Omega)$$

Boulware State

- On the max. extended Schwarzschild s-t, $\left\{\phi_{\Lambda}^{in/up,L/R} \ \& \text{ c.c.}, \ \ \underset{\substack{\uparrow \\ \omega > 0}}{\forall\Lambda}\right\}$ is:

  - orthonormal: $\left(\phi_{\Lambda}^{in/up,L/R}, \phi_{\Lambda'}^{in/up,L/R}\right) = \delta_{\Lambda,\Lambda'}$

  - complete

$$\hat{\phi}(x) = \sum_{\ell,m} \int_0^\infty d\omega \left\{\hat{a}_{\Lambda}^{in,L}\phi_{\Lambda}^{in,L} + \hat{a}_{\Lambda}^{up,L}\phi_{\Lambda}^{up,L} + \hat{a}_{\Lambda}^{in,R}\phi_{\Lambda}^{in,R} + \hat{a}_{\Lambda}^{up,R}\phi_{\Lambda}^{up,R} + h.c.\right\}$$

- Boulware state is defined via

$$\hat{a}_{\Lambda}^{in/up,L} \mid B\rangle = 0 = \hat{a}_{\Lambda}^{in/up,R} \mid B\rangle, \quad \forall\Lambda$$

Boulware state is defined using modes that are pos. freq. wrt ∂_t

$$\partial_t \phi_\Lambda^{in/up,L/R} = -i\,\omega\,\phi_\Lambda^{in/up,L/R}, \quad \omega > 0$$

$\Longrightarrow$ it's seen as empty by static obs. (ie, at $r, \theta, \varphi = const$)

Analogy:

Schw. $\qquad$ Flat s-t

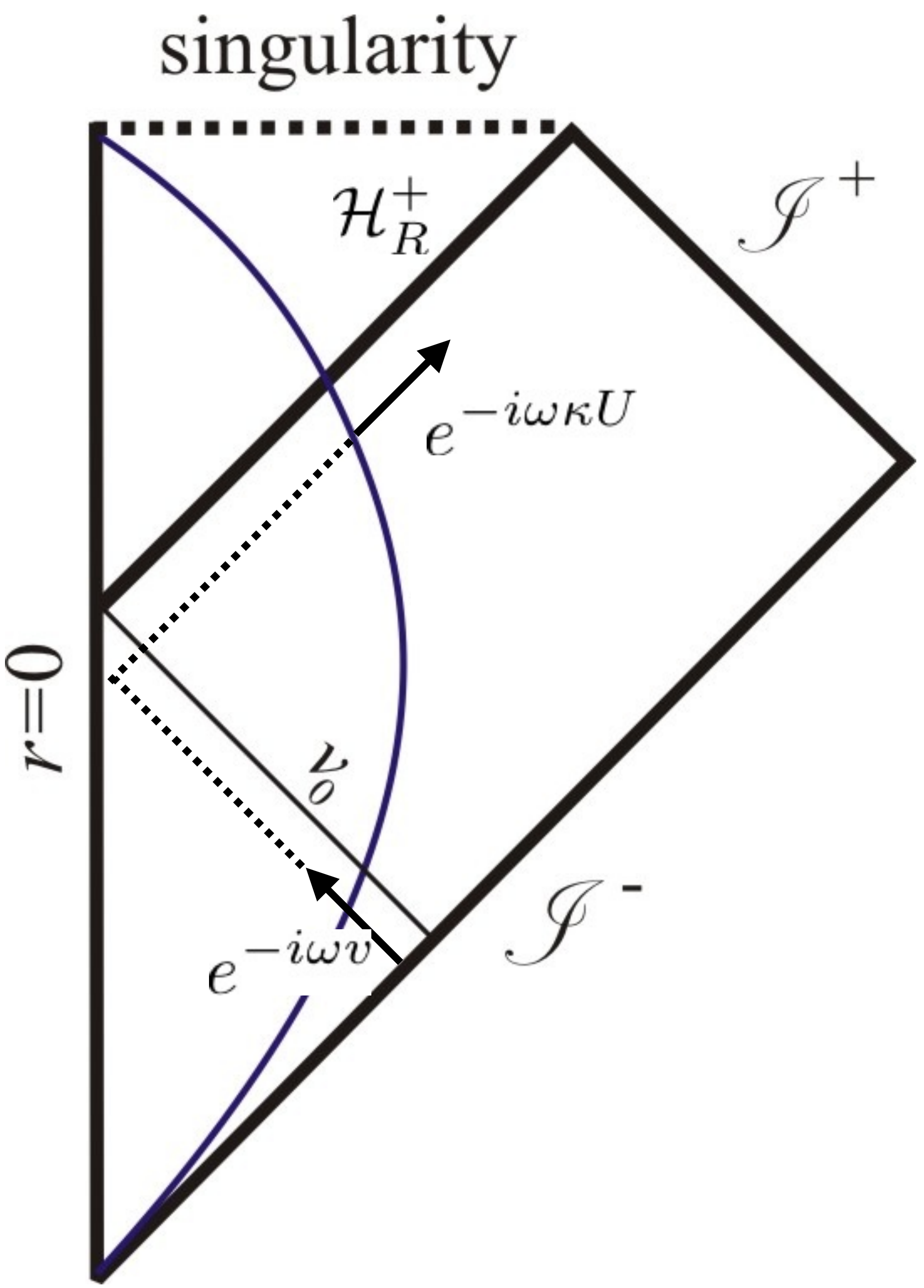
Static obs. $\leftrightsquigarrow$ Rindler obs.

defined using $\{t, r_*\}$, which diverges on $\leftrightsquigarrow$ defined using $\{\tau, \xi\}$, which diverges on horizons

$| B\rangle \leftrightsquigarrow | R\rangle$ (RSET irregular on horizons)

Gravitationally-collapsing Star

Astrophysical BH's are not eternal, they are formed from gravitational collapse of a star



Hawking'75 showed that modes

$\phi_\Lambda^{in,R} \sim e^{-i\omega v}$ at $\mathcal{J}_R^-$

behave like $e^{-i\omega\kappa U}$

ie, they are pos. freq. modes wrt ∂_U, as they leave the star [He showed this leads to the emission of quantum radiation by the BH, as we next see]

Unruh State

- We model BH evaporation in *eternal BH s-t* by using modes that are pos. freq. wrt ∂_U on $\mathcal{H}_R^-$ instead of using $\phi_\Lambda^{up,R}$
- Eg, we could directly use $e^{-i\omega\kappa U}$ but in order to obtain Bogolyubov coeffs. Unruh'76 instead used the modes:

$$\phi_\Lambda^{UP} \equiv \frac{e^{\pi\omega/(2\kappa)}}{\sqrt{2\sinh\left(\pi\omega/\kappa\right)}}\left[\phi_\Lambda^{up,R} + e^{-\pi\omega/\kappa}\phi_\Lambda^{up,L^*}\right]$$

Similar rln. as between Rindler & Minkowski modes in flat s-t!

Remember:

$$u_{\bar{k}}^R \equiv \frac{e^{\pi\bar{\omega}/(2\alpha)}}{\sqrt{2\sinh\left(\pi\bar{\omega}/\alpha\right)}}\left[\bar{\phi}_{\bar{k}}^R + e^{-\pi\bar{\omega}/\alpha}\bar{\phi}_{-\bar{k}}^{L^*}\right]$$

they define $|M\rangle$

they define $|R\rangle$

- Similarly define $\phi_\Lambda^{\overline{UP}}$ modes with $L \leftrightarrow R$ in ϕ_Λ^{UP}

- On the maximally extended Schwarzschild s-t, $\left\{\phi_\Lambda^{in,L/R}, \phi_\Lambda^{UP/\overline{UP}} \text{ \& c.c.}, \ \forall\Lambda\right\}$ form a set that is:

  - orthonormal
  - complete:

$$\hat{\phi}(x) = \sum_{\ell,m} \int_0^\infty d\omega \left\{ \hat{a}_\Lambda^{in,L} \phi_\Lambda^{in,L} + \hat{a}_\Lambda^{UP} \phi_\Lambda^{UP} + \hat{a}_\Lambda^{in,R} \phi_\Lambda^{in,R} + \hat{a}_\Lambda^{\overline{UP}} \phi_\Lambda^{\overline{UP}} + h.c. \right\}$$

- Unruh state is defined via

$$\hat{a}_\Lambda^{in,R/L} \mid U\rangle = \hat{a}_\Lambda^{UP} \mid U\rangle = \hat{a}_\Lambda^{\overline{UP}} \mid U\rangle = 0, \quad \forall\Lambda$$

Hartle-Hawking State

- We can also define the IN version of the UP modes as:

ϕ_Λ^{IN} & $\phi_\Lambda^{\overline{IN}}$ defined like ϕ_Λ^{UP} & $\phi_\Lambda^{\overline{UP}}$ with $\phi_\Lambda^{up,R/L} \to \phi_\Lambda^{in,R/L}$

- On the maximally extended Schwarzschild s-t, $\left\{\phi_\Lambda^{UP/IN/\overline{IN}/\overline{UP}} \text{ \& c.c. } \forall\Lambda\right\}$ form a set that is:

  - orthonormal

  - complete:

$$\hat{\phi}(x) = \sum_{\ell,m}\int_0^\infty d\omega \left\{\hat{a}_\Lambda^{IN}\phi_\Lambda^{IN} + \hat{a}_\Lambda^{UP}\phi_\Lambda^{UP} + \hat{a}_\Lambda^{\overline{IN}}\phi_\Lambda^{\overline{IN}} + \hat{a}_\Lambda^{\overline{UP}}\phi_\Lambda^{\overline{UP}} + h.c.\right\}$$

- Hartle-Hawking state is defined via

$$\hat{a}_\Lambda^{IN} \mid H\rangle = \hat{a}_\Lambda^{UP} \mid H\rangle = \hat{a}_\Lambda^{\overline{IN}} \mid H\rangle = \hat{a}_\Lambda^{\overline{UP}} \mid H\rangle = 0, \quad \forall\Lambda$$

- It can be shown that, $\forall\omega$, ϕ_Λ^{UP} & $\phi_\Lambda^{\overline{UP}}$ are pos. freq. wrt ∂_U on $\mathcal{H}_R^-$

  ϕ_Λ^{IN} & $\phi_\Lambda^{\overline{IN}}$ are pos. freq. wrt ∂_V on $\mathcal{H}_R^+$

- Hartle-Hawking is defined using modes that are pos. freq. wrt affine parameter U on $\mathcal{H}_R^-$ and affine parameter V on $\mathcal{H}_R^+$ $\Longrightarrow$ it's seen as empty by free-falling obs. on $\mathcal{H}_R^\pm$

Analogy:

Schw.		Flat s-t
free-falling obs. (on $\mathcal{H}_R^\pm$)	$\leftrightsquigarrow$	inertial obs.
defined using $\{U, V\}$, which is regular everywhere	$\leftrightsquigarrow$	defined using $\{t, y\}$, which is regular everywhere
UP - up and IN - in modes relationship	$\leftrightsquigarrow$	Rindler&Minkowski modes relationship
$\vert H\rangle$	$\leftrightsquigarrow$	$\vert M\rangle$ (RSET regular everywhere)

Properties of Boulware State

- RSET $\langle B \mid \hat{T}^{\mu}{}_{\nu} \mid B \rangle_{ren}$

  - goes to zero like $O\left(\frac{M^2}{r^6}\right)$ as $r \to \infty$

  - diverges on $\mathcal{H}_R^{\pm}$ in a free-falling frame (as expected)

- Boulware state:

  - is the equivalent of the Rindler state in flat s-t

  - models a cold star (no horizon and empty at infinity)

Properties of Unruh State

- RSET $\langle U | \hat{T}_\mu{}^\nu | U \rangle_{ren}$

- diverges at $\mathcal{H}^-$ (but this surface doesn't exist for astrophysical BH)

- is regular at $\mathcal{H}^+$

- has a flux of thermal Hawking radiation at $\mathcal{I}^+$:

$$\sim \frac{L}{4\pi r^2} \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad r \to \infty \qquad\qquad \mu, \nu = \{t, r, \theta, \varphi\}$$

Luminosity: $L = \frac{1}{2\pi} \int_0^\infty d\omega \frac{\omega \left| B^{in}_{\ell\omega} \right|^2}{e^{\omega / T_H} - 1}$

Hawking temperature: $T_H = \frac{\kappa}{2\pi} = \frac{1}{8\pi M}$

Unruh state models a BH that is evaporating via the emission of Hawking radiation

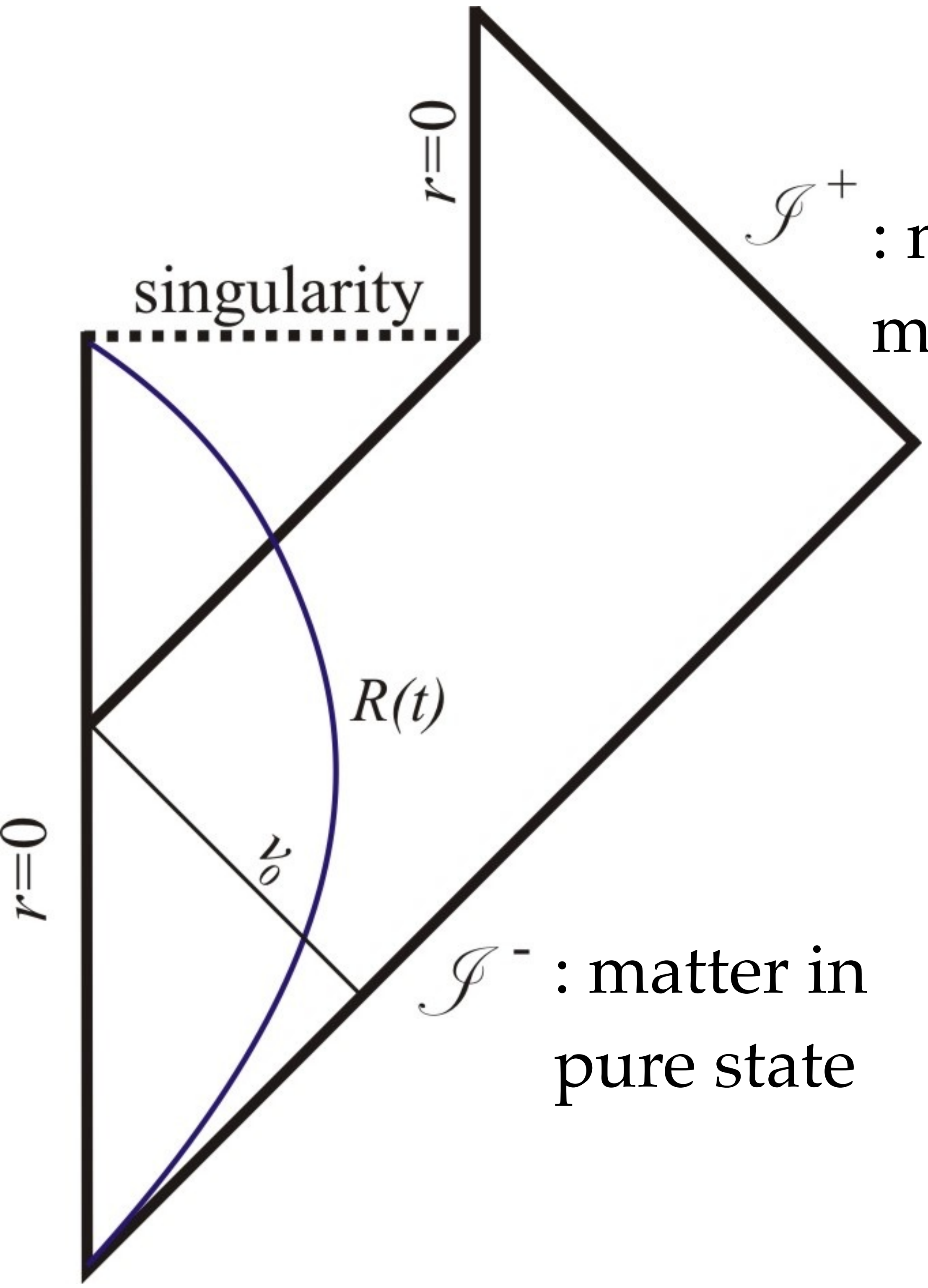
$$T_H \approx 10^{-7} \frac{M_\odot}{M} K \ll T_{CMB}$$

Stefan-Boltzman law: $-\frac{dM}{dt} \sim \text{area} \cdot T_H^4$

Lifetime of BH with $M = M_\odot$ is $\sim 10^{54} \cdot (\text{age of the Universe})$

BH evaporation poses the BH information paradox:

even if initial state of the matter is pure, the final state of Hawking radiation is thermal - in a mixed state



$\mathscr{I}^+$: matter in mixed state

$\mathscr{I}^-$: matter in pure state

$$\text{pure} \to \text{mixed}$$

$$|\psi\rangle\langle\psi| \to \rho$$

$$\mathrm{Tr}\left(\rho^2\right) < 1$$

But, according to Quantum Physics, the evolution of a physical system is unitary: if the initial state of a system is pure, then it cannot evolve into a mixed state

Properties of Hartle-Hawking State

- RSET $\langle H \mid \hat{T}_{\mu}{}^{\nu} \mid H \rangle_{ren}$

  - has a bath of thermal radiation at $\mathcal{J}_R^{\pm}$:

$$\frac{1}{2\pi^2}\int_0^\infty d\omega \frac{\omega^3}{e^{2\pi\omega/\kappa}-1}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 \end{pmatrix}, \quad r \to \infty$$

$$\mu, \nu = \{t, r, \theta, \varphi\}$$

  - is regular at $\mathcal{H}_R^{\pm}$ (and everywhere else, as expected)

  - is invariant under symmetries of BH s-t

- Hartle-Hawking state:

  - is the equivalent of the Minkowski state in flat s-t

  - models a BH in thermal (unstable) equilibrium with its own radiation, so it’s the relevant state for the laws of BH mechanics

Laws of Black Hole Mechanics

[Bardeen, Carter, Hawking 1973]

- 0th law (temp. T=const. in syst. in thermal equilibrium)

BH: κ is const. throughout EH of stationary BH

- 1st law ($\delta E = T\delta S + P\delta V$)

BH:

$$\underset{\substack{\uparrow\\ \text{mass}}}{\delta M} = \frac{\kappa}{8\pi}\underset{\substack{\uparrow\\ \text{area}}}{\delta A} + \underset{\substack{\uparrow\\ \text{ang.vel.}}}{\Omega_+}\underset{\substack{\uparrow\\ \text{ang.mom.}}}{\delta J}$$

$$T \leftrightarrow \kappa/(2\pi)$$

$$S \leftrightarrow A/4$$

- 2nd law ($\delta S \geq 0$ for isolated syst.)

BH (Hawking's rea th.): $\delta A \geq 0$ (if an -"null"- energy cond. is satisfied)

- 3rd law ($T = 0$ not achievable in finite series of processes)

BH: $\kappa = 0$ not achievable in finite time (if an energy cond. is satisfied)

IV. Black Holes

(b) Rotation, etc

Kerr Spacetime

Astrophysical BHs: Kerr BH with angular momentum $J =: a\,M$ and mass M

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt^2 - a\sin^2\theta d\varphi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left[\left(r^2+a^2\right)d\varphi - adt\right]^2$$

$$\Delta \equiv (r-r_+)(r-r_-) \qquad \Sigma \equiv r^2 + a^2\cos^2\theta$$

It has:

- an event horizon at radius $r = r_+ \equiv M + \sqrt{M^2 - a^2}$

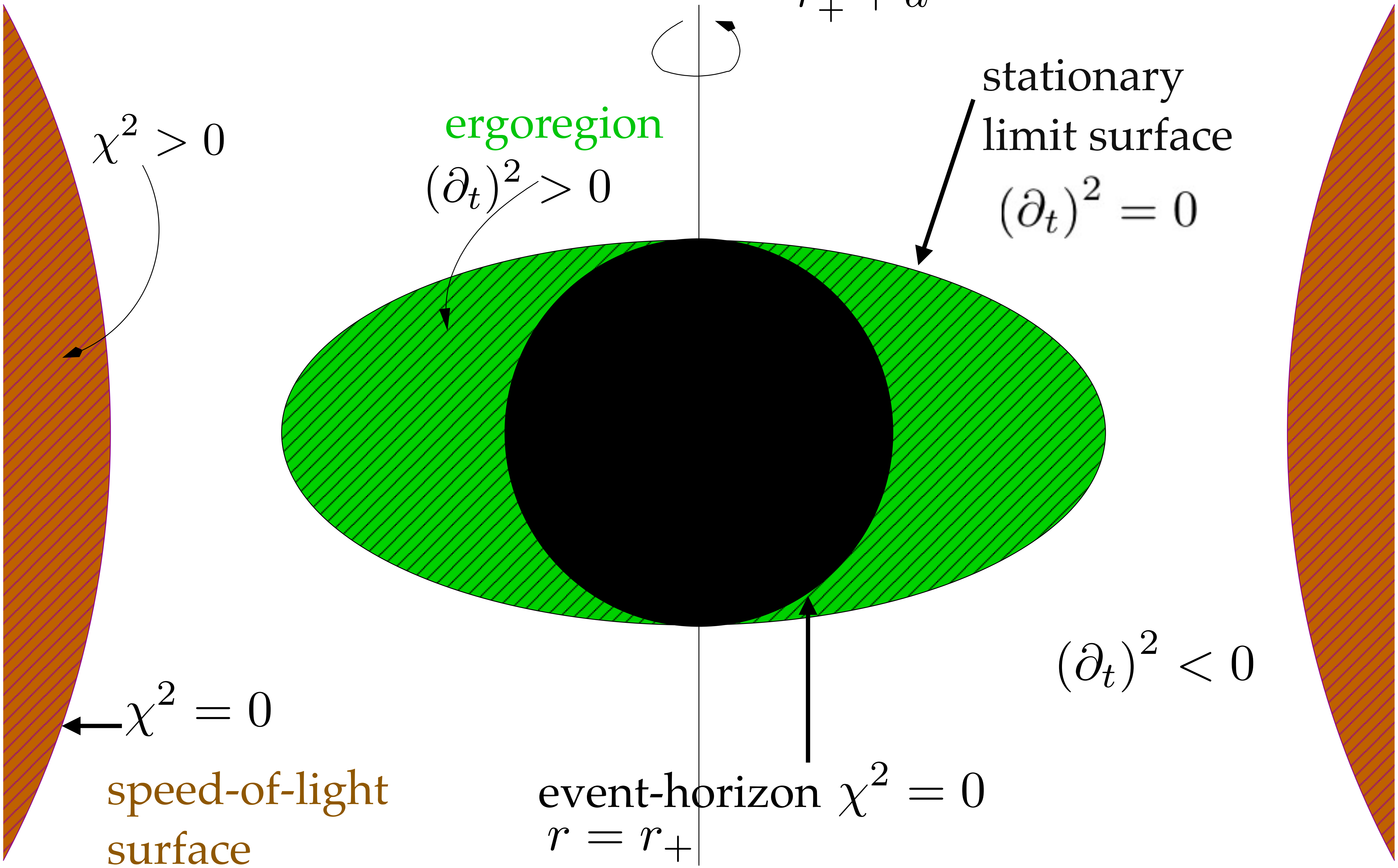
↓

maximally-rotating (extremal) is for $a = M$

- an *inner* (Cauchy horizon) at $r = r_- \equiv M - \sqrt{M^2 - a^2} \in [0, r_+]$
- a curvature singularity at $r = 0$
- two symmetries: stationarity (∂_t) and axi-symmetry (∂_φ)

$$\chi \equiv \partial_t + \Omega \partial_\varphi$$

$$\Omega_+ \equiv \frac{a}{r_+^2 + a^2}$$: angular velocity



stationary limit surface

$(\partial_t)^2 = 0$

ergoregion

$(\partial_t)^2 > 0$

$\chi^2 > 0$

$(\partial_t)^2 < 0$

$\chi^2 = 0$

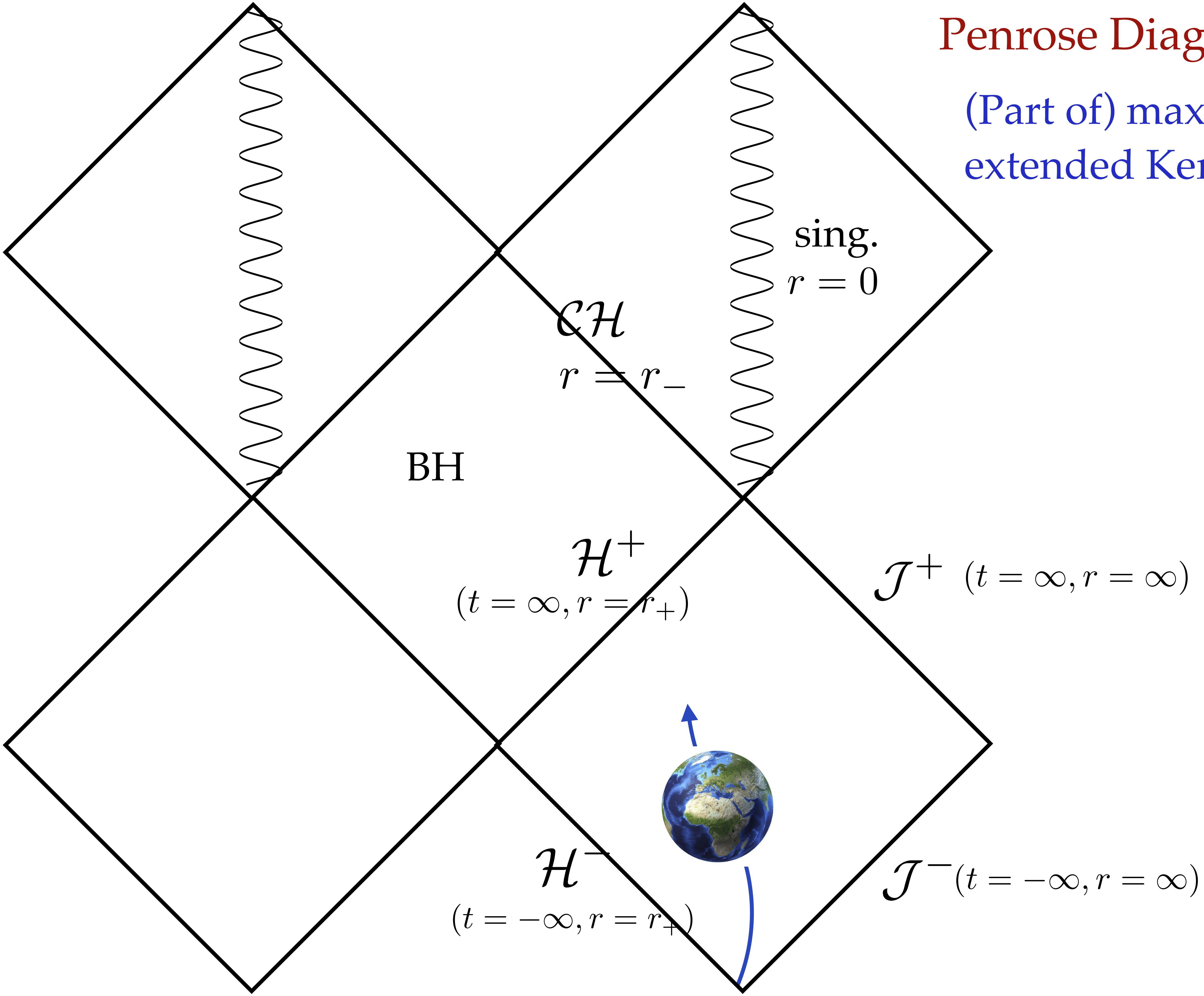
speed-of-light surface

event-horizon $\chi^2 = 0$

$r = r_+$

Penrose Diagram

(Part of) max. extended Kerr s-t



Bosons Fields

Field eq.: $\text{``}\Box\text{''}\,\phi = 0$ separates by variables

Mode slns.: $\phi_\Lambda = R_\Lambda(r) S_\Lambda(\theta) e^{im\varphi - i\omega t}$

$$\Lambda \equiv \{\ell, m, \omega\}$$

$$-\Lambda \equiv \{\ell, -m, -\omega\}$$

They satisfy similar ODEs
as in Schwarzschild

$\phi_\Lambda^{in,L/R}$ have positive norm $\forall \omega > 0$

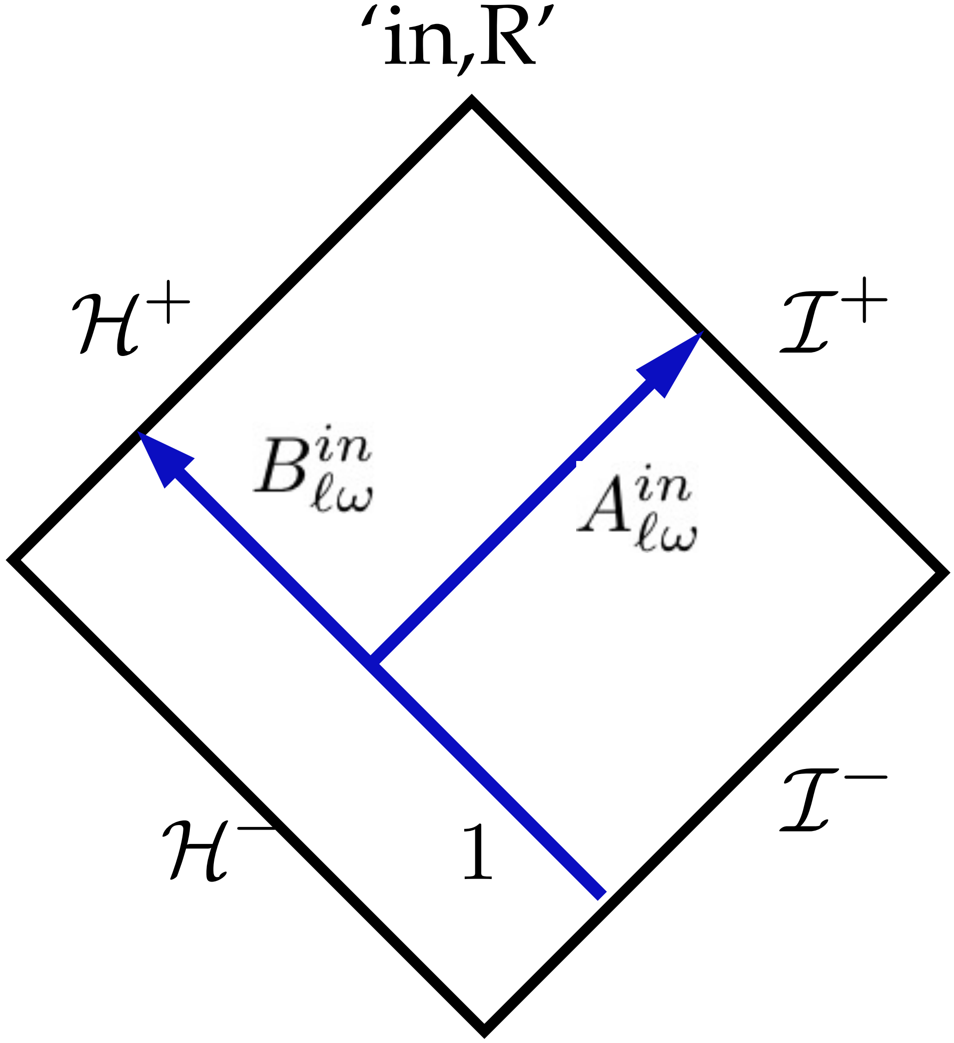
$\phi_\Lambda^{up,L/R}$ have positive norm $\forall \tilde{\omega} \equiv \omega - m\Omega_+ > 0$

1st law of BH mechanics:

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_+ \delta J$$

Send wave in with $\frac{\delta M}{\delta J} = \frac{\omega}{m}$ at infinity

$$\longrightarrow \quad \frac{\tilde{\omega}}{\omega}\delta M = \frac{\kappa}{8\pi}\delta A$$



Area th.: $\delta A \geq 0$ if null-energy cond. is satisfied (ie, $T_{\mu\nu}k^\mu k^\nu \geq 0$

for any future-directed null vector k)

$\Rightarrow \quad \delta M \leq 0$ for $\omega\tilde{\omega} < 0$ $\Rightarrow$ such a wave extracts *rotational* energy from the BH (superradiance)

$$\left|A^{in}_{\ell\omega}\right| > \left|B^{in}_{\ell\omega}\right|$$

Superradiance is due to the existence of the ergosphere

Quantum States for Bosons

- Boulware $\hat{a}_{\Lambda}^{in/up,L} \mid B\rangle = 0 = \hat{a}_{\Lambda}^{in/up,R} \mid B\rangle$

Properties:

- irregular at $\mathcal{H}_R^{\pm}$, regular elsewhere outside BH

- empty at $\mathcal{H}_R^{-}$ and $\mathcal{I}_R^{-}$

- has Unruh-Starobinsky radiation at $\mathcal{I}_R^{+}$ due to *superradiant* modes

NB: there is no state that's empty at $\mathcal{I}_R^{\pm}$

- Hartle-Hawking

- Remember: in *Schwarzschild*, this state was regular everywhere, satisfied the symmetries of the s-t, contained a thermal bath at infinity, modelled a BH in thermal eq. with its own radiation (so it's the relevant one for laws of BHs and AdS/CFT correspondence)

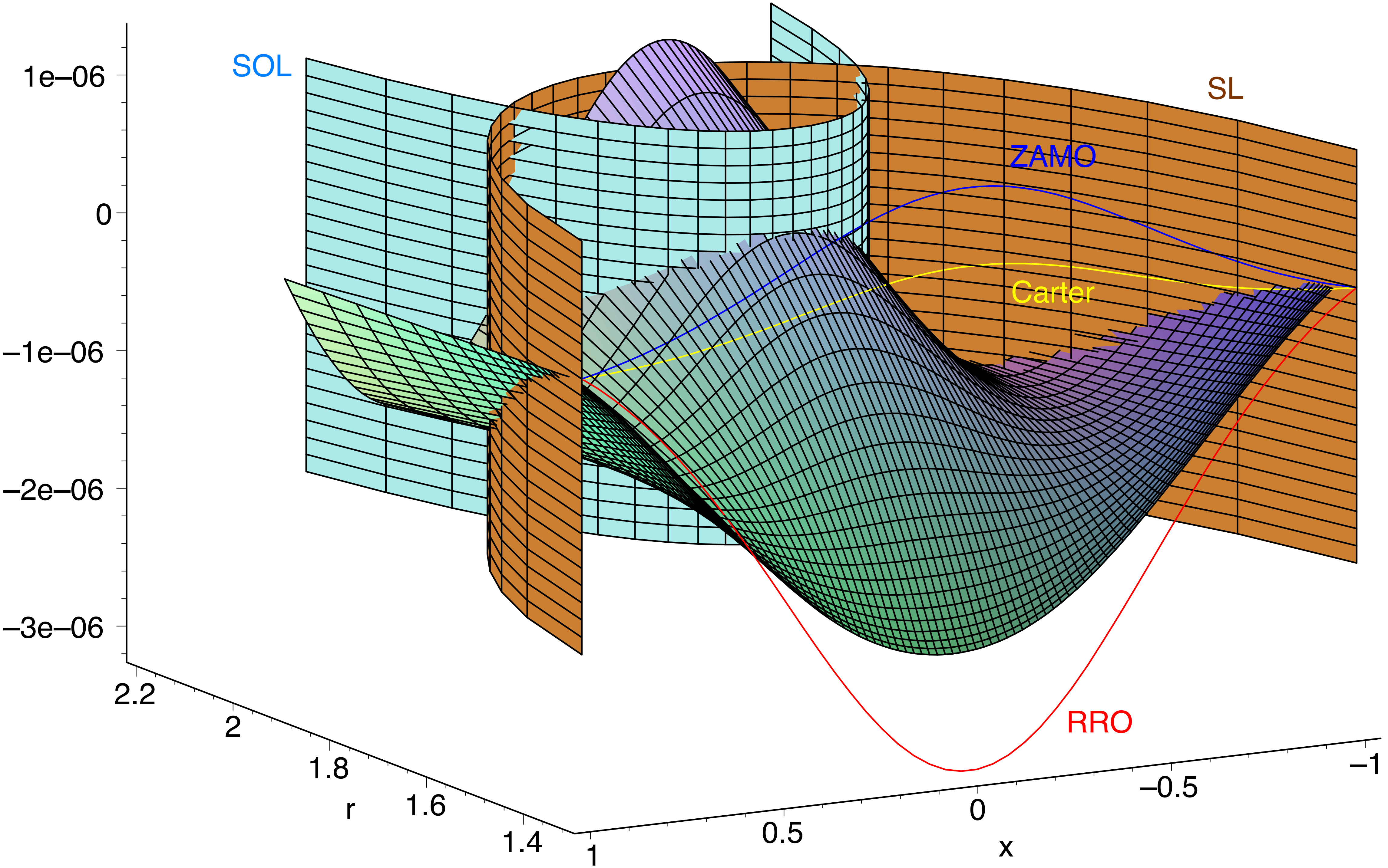
- No such state exists for bosons in Kerr, due to superradiance [Kay & Wald'91; Ottewill&Winstanley'00]

- Candelas, Chrzanowski & Howard'81 constructed a H-H-like state where modes are thermalized wrt their 'natural energy' (ie, 'in' modes wrt ω and 'up' modes wrt $\tilde{\omega}$)

CCH is regular everywhere but does not satisfy symmetries of space-time [Casals & Ottewill'05]

$$\Delta \left\langle \hat{T}_{t_+\varphi_+} \right\rangle^{CCH-B}$$

$a = 0.95M$



- Unruh $\hat{a}_{\Lambda}^{in,R/L} \mid U\rangle = \hat{a}_{\Lambda}^{UP} \mid U\rangle = \hat{a}_{\Lambda}^{\overline{UP}} \mid U\rangle = 0$

Properties in Kerr similar to those in Schwarzschild:

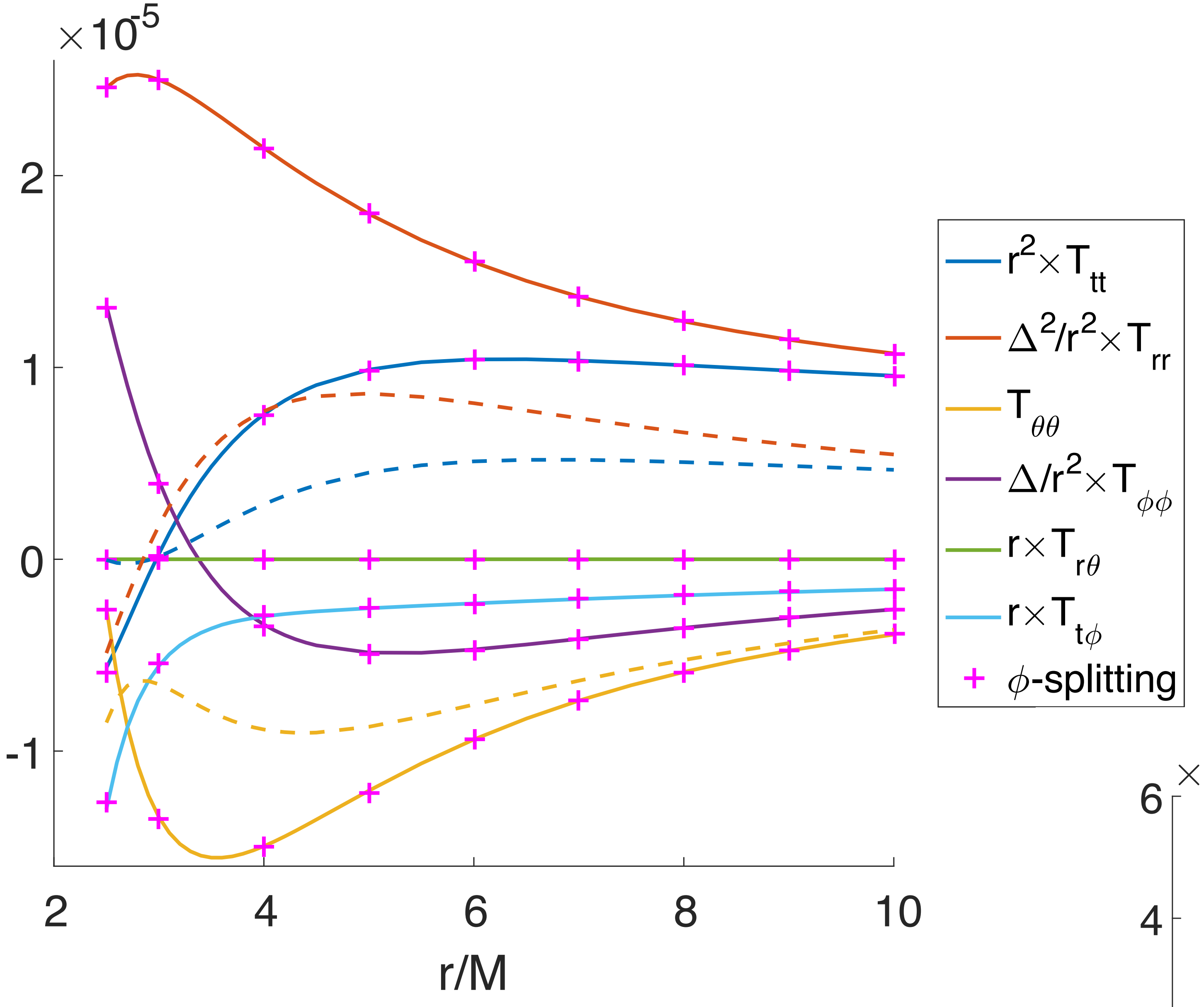
- irregular at $\mathcal{H}_R^-$ but regular elsewhere outside BH

- empty at $\mathcal{I}_R^-$

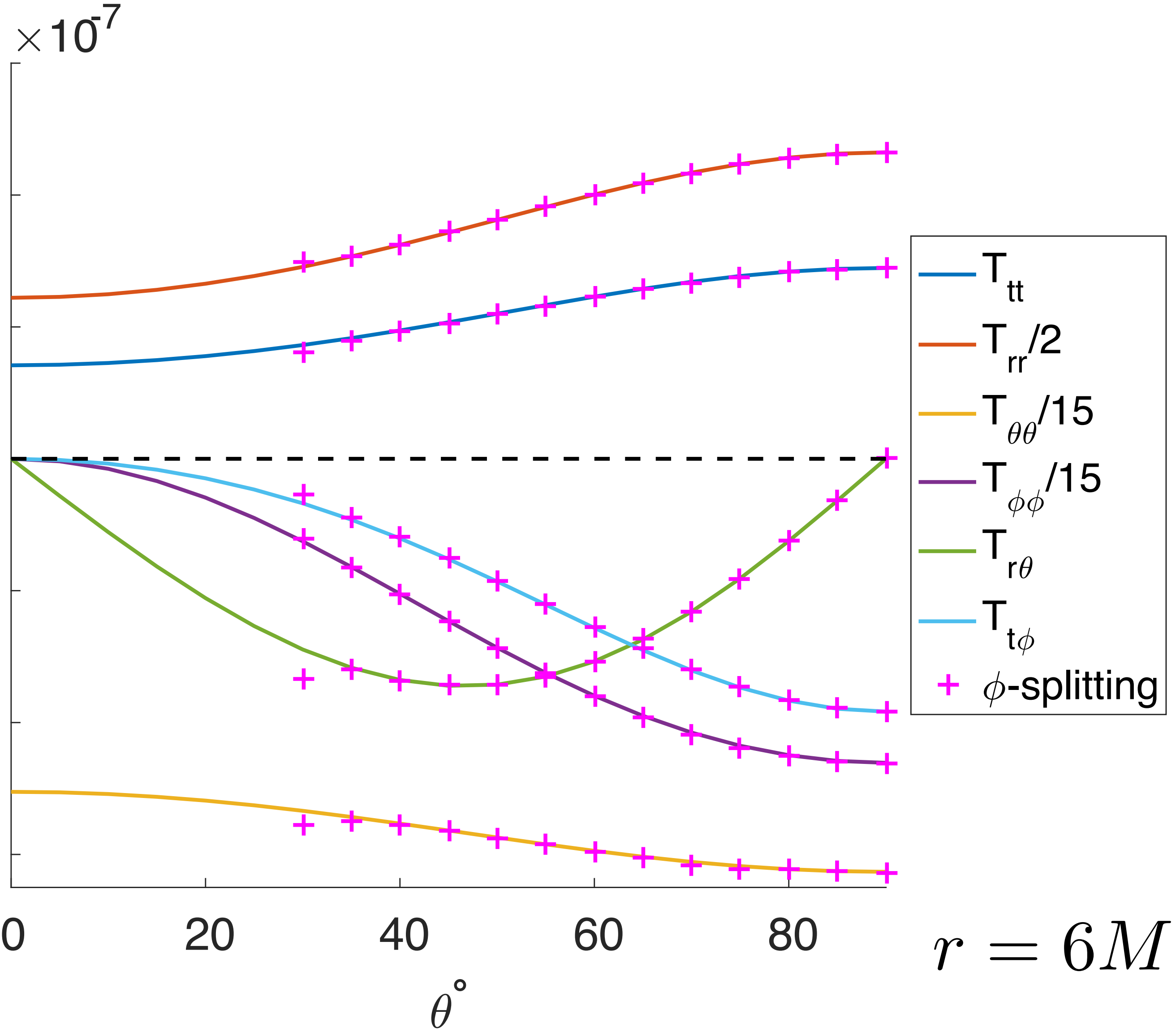
- Hawking radiation at $\mathcal{I}_R^+$ at temperature $T_H = \dfrac{\kappa}{2\pi}$

$\langle \hat{T}_{\mu\nu} \rangle^U_{ren}$ [Levi, Eilon, Ori & Meent'16]

$a = 0.7M$



$\theta = 0, \frac{\pi}{2}$



$r = 6M$

CFT for Fermions

- Dirac field eq. $\gamma^{\mu}\left(\partial_{\mu}-\Gamma_{\mu}\right)\phi=0$

Dirac matrices

$$\{\gamma^{\mu},\gamma^{\nu}\}=2g^{\mu\nu}$$

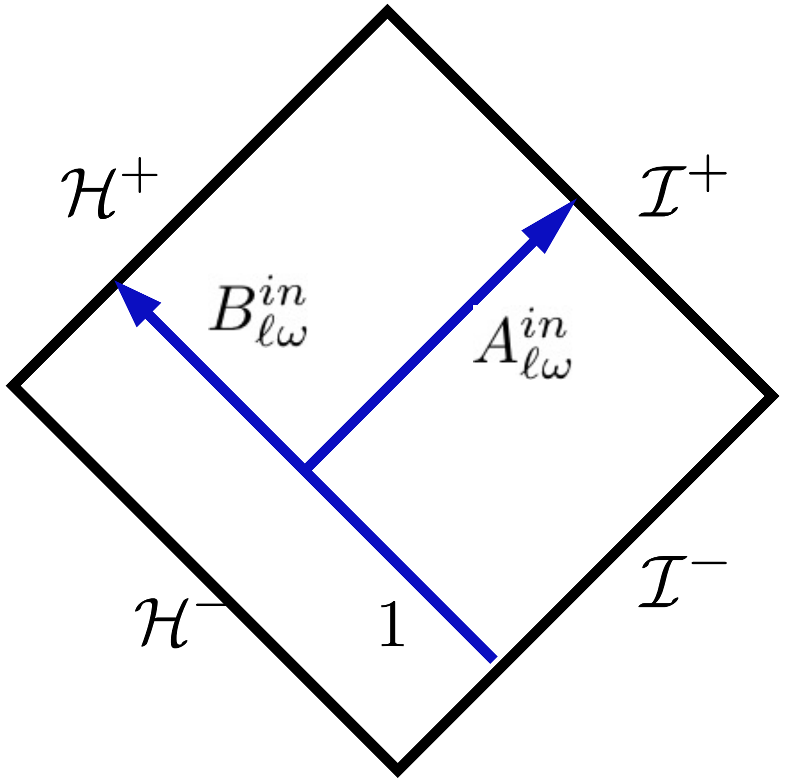
Dirac 4-spinors

Spinor connection matrices

$$\partial_{\nu}\gamma^{\mu}+\Gamma^{\mu}_{\nu\kappa}\gamma^{\kappa}-\Gamma_{\nu}\gamma^{\mu}+\gamma^{\mu}\Gamma_{\nu}=0$$

The Dirac field eq. in Kerr separates by variables and decouples for the different spinor components

- No superradiance $\left|A^{in}_{\ell\omega}\right|\leq\left|B^{in}_{\ell\omega}\right|$



Classical fermions do not satisfy the weak-energy condition -> area th. does not apply

QFT for Fermions

- Suppose $\{\phi_\Lambda, \phi_{-\Lambda}, \forall\Lambda\}$ is a complete set of slns., then

$$\hat{\phi} = \sum_\Lambda \hat{a}_\Lambda \phi_\Lambda + \hat{b}^\dagger_\Lambda \phi_{-\Lambda}$$

- $J^\mu = \left(\phi_1^\dagger \tilde{\gamma}^0\right) \gamma^\mu \phi_2$ is a conserved current $\nabla_\mu J^\mu = 0$

which is used to construct an inner product: $(\phi_1, \phi_2) = \int_{t=const} J^t dS$

- If $(\phi_{\pm\Lambda}, \phi_{\pm\Lambda'}) = \delta_{\Lambda,\Lambda'}$ *all* positive norm wrt inner product

then anticommutation rlns. $\left\{\hat{a}_\Lambda, \hat{a}^\dagger_{\Lambda'}\right\} = \left\{\hat{b}_\Lambda, \hat{b}^\dagger_{\Lambda'}\right\} = \delta_{\Lambda,\Lambda'}$

follow from $\left\{\hat{\phi}(\vec{x}, t), \hat{\Pi}(\vec{x}', t)\right\} = \hat{\mathbb{I}}\, i\, \delta^{(3)}(\vec{x} - \vec{x}')$

States in Kerr - Fermions

[Casals, Dolan, Nolan, Ottewill, Winstanley'12]

- Boulware: similar as for bosons. Eg, it has Unruh-Starobinsky radiation (even if fermions have no classical superradiance) at $\mathcal{I}_R^+$

  However, one may define:

$$\hat{a}_{\Lambda}^{in} |B\rangle = 0 \quad \forall\omega > 0$$

$$\hat{a}_{\Lambda}^{up} |B\rangle = 0 \quad \forall\omega > 0 \;\; \text{(for bosons: } \forall\tilde{\omega} > 0\text{)}$$

Expected to be empty at $\mathcal{I}^{\pm}$

Boulware for Fermions

$a \approx 0.91M$

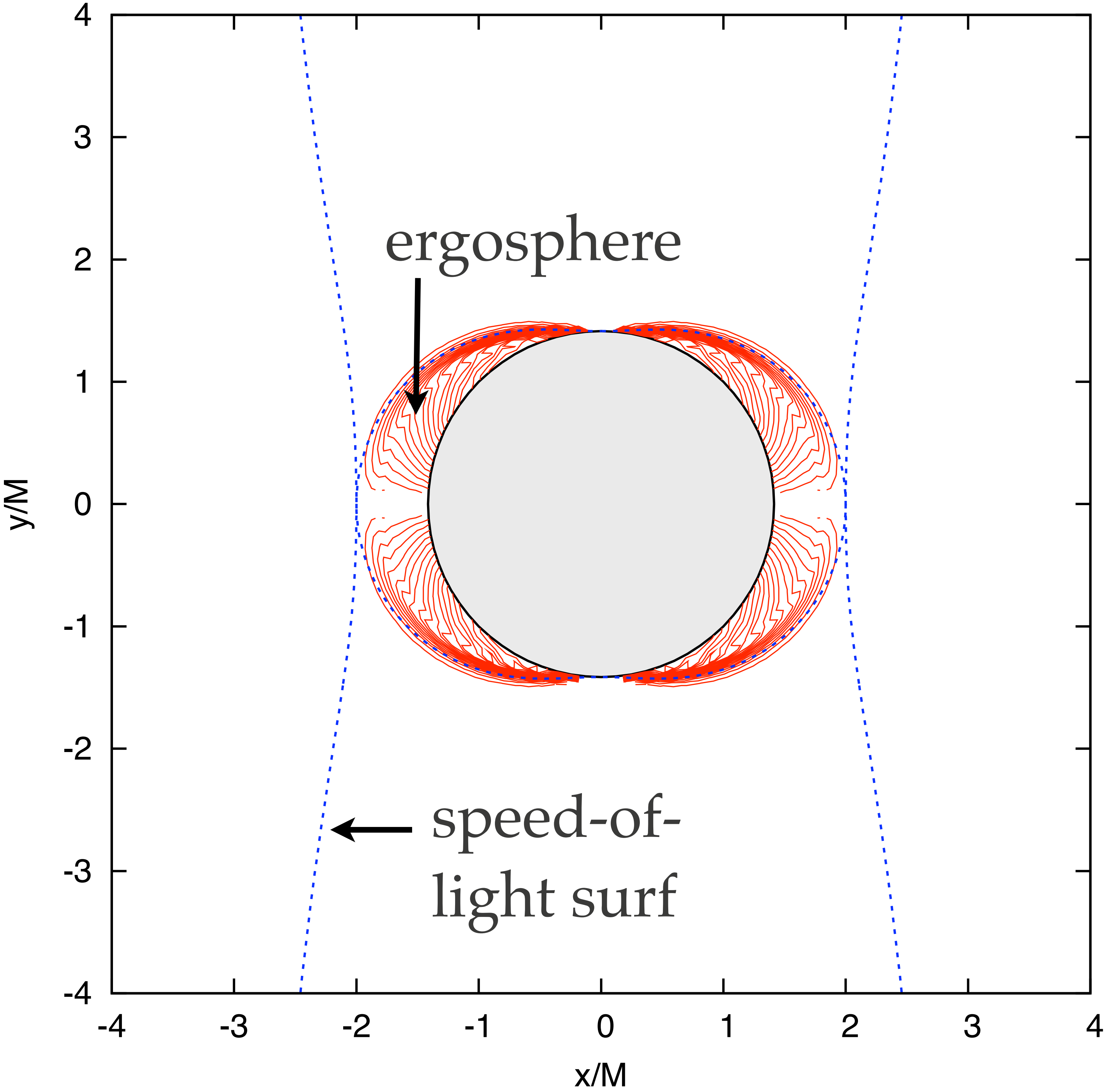
Ratio test in ℓ -sum

for particle number current $\left\langle \hat{J}^r \right\rangle^{U^- - B}$

Properties:

- Empty at radial infinity (as opposed to bosons)

- Divergence in ergosphere but regular outside it



Hartle-Hawking for Fermions

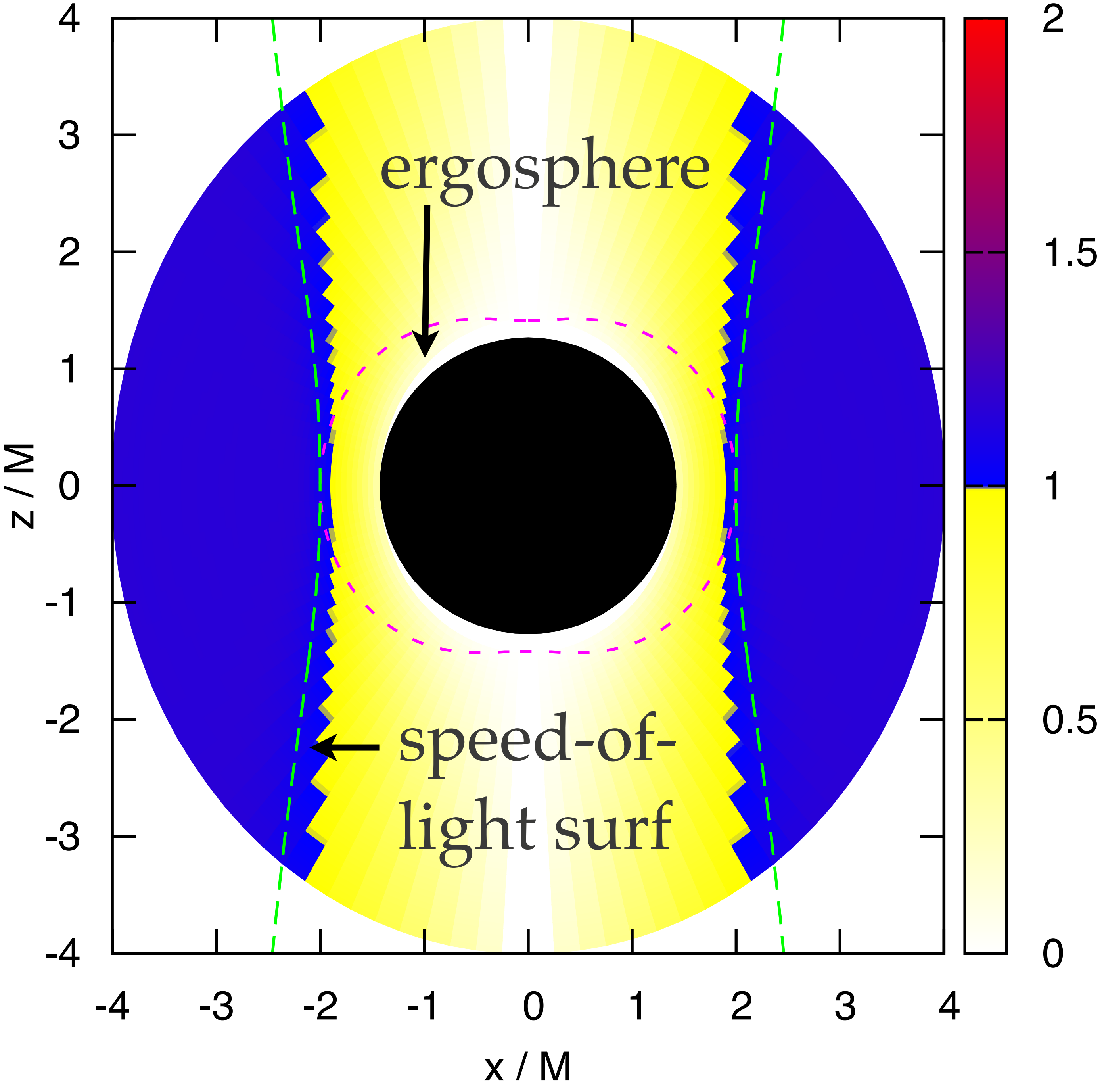
$a \approx 0.91M$

Ratio test in ℓ-sum for $\langle \hat{T}_{\theta\theta} \rangle_{ren}^{H-U^-}$

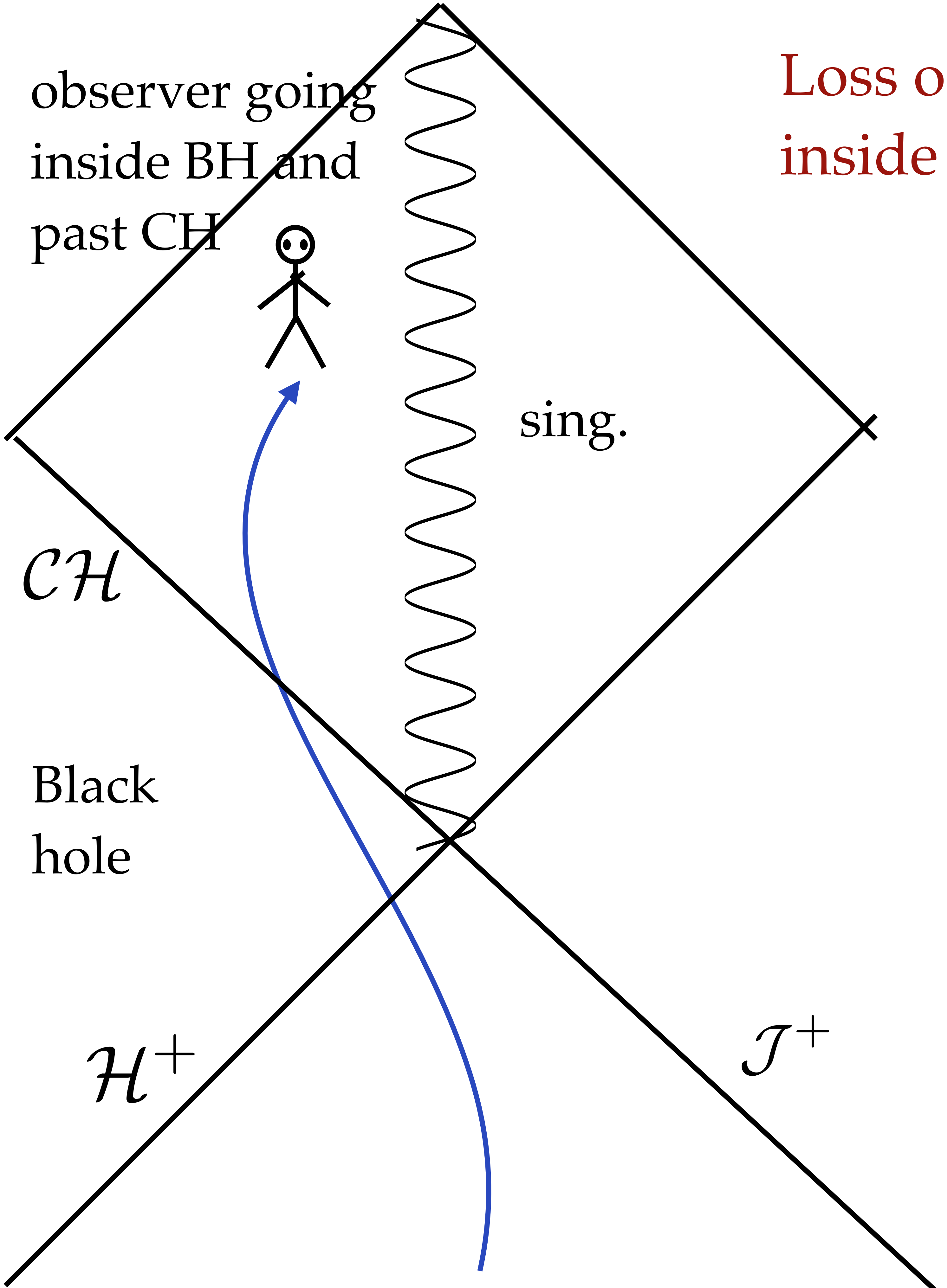
Properties:

- It's well-defined and regular inside SOL

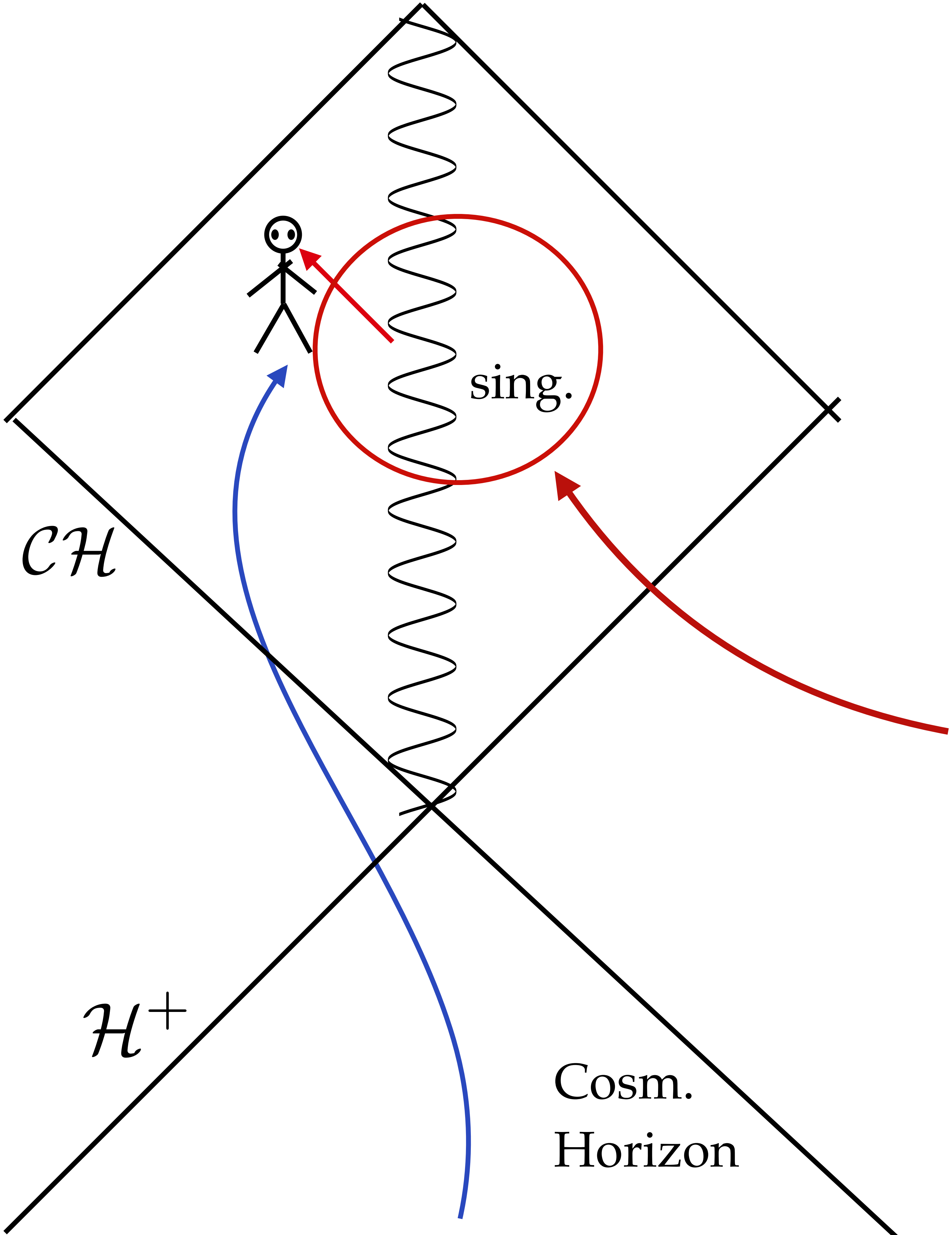
- Divergence on SOL due to large-modes: ℓ -> thermal bath rotating with horizon?



Loss of predicability inside the BH



observer going inside BH and past CH

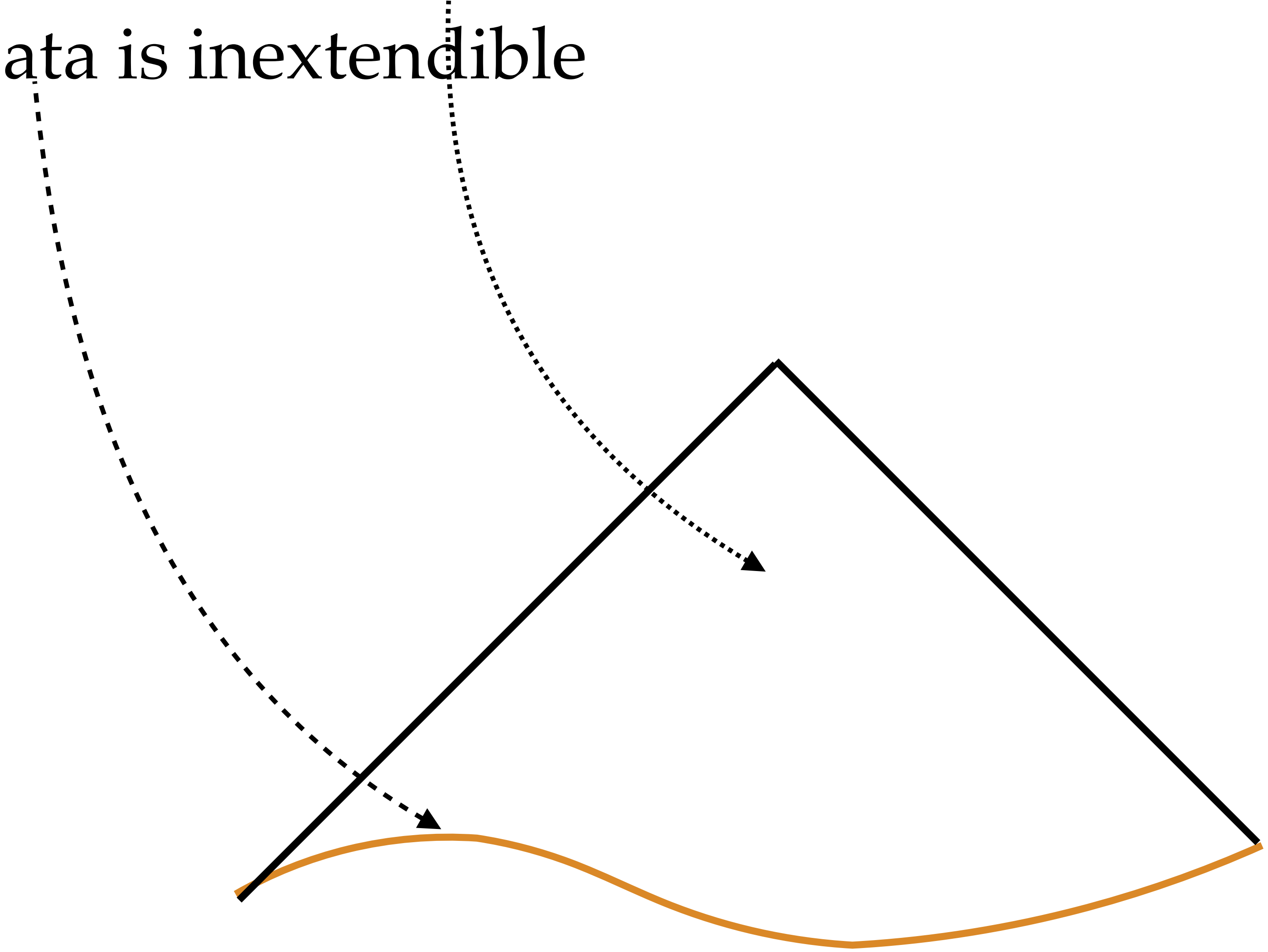


this is a *timelike* singularity, and so it's *visible* to an observer going into the BH

Unpredictability: the Cauchy ("initial") Value Problem is not well posed

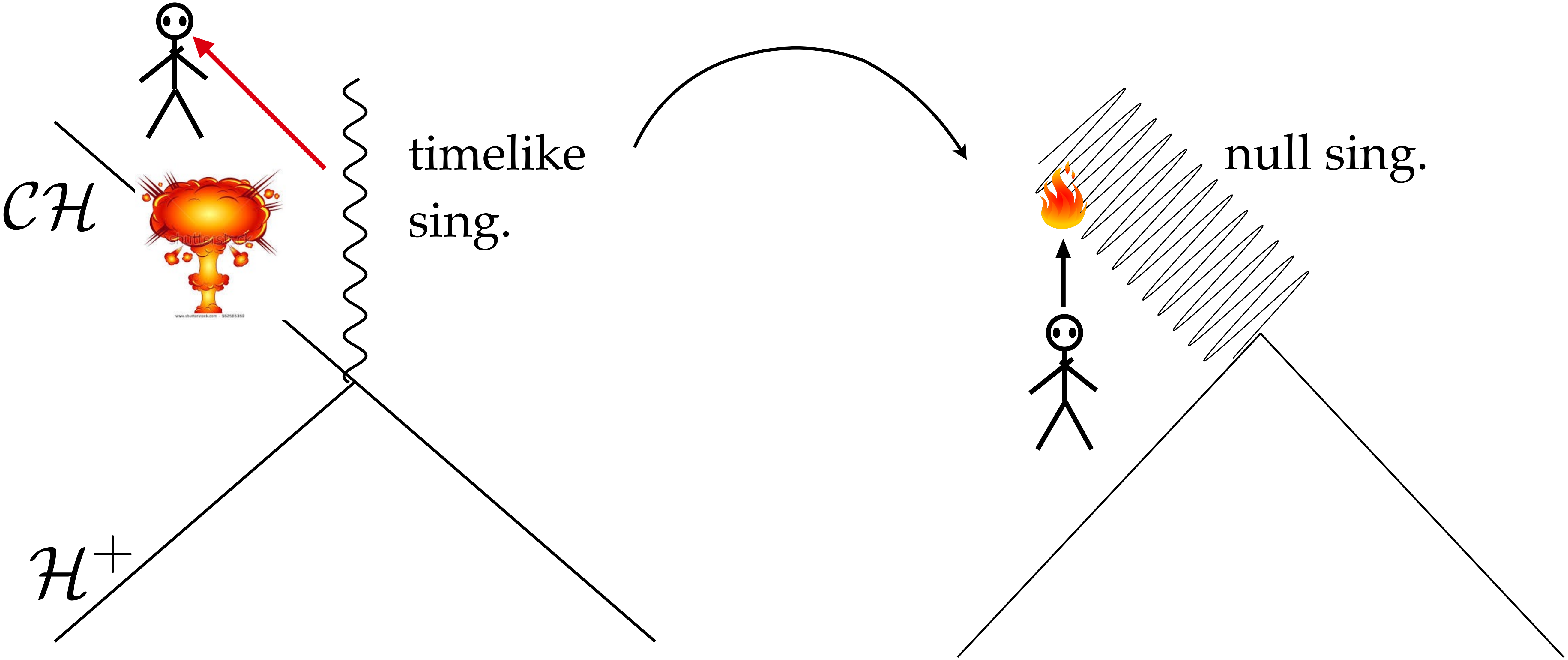
Strong Cosmic Censorship hypothesis

Strong Cosmic Censorship (SCC) Hypothesis by Penrose’72, essentially: the maximal Cauchy development via Einstein's equations of generic initial data is inextendible



So, if BHs that exist in Nature possess singularities in their inside, then they’re not visible even to observers inside (i.e., they’re not timelike)

SCC could be upheld if the Cauchy horizon is “destroyed” by field perturbations or semiclassical effects



observer can see singularity

observer cannot see singularity and crashes into it in the future

But it's a hypothesis - it needs to be verified!

Gajic's talk: Classically, the CH of Kerr is extendible in $\mathcal{C}^0$ and conjecture that it's not in $\mathcal{C}^2$

What about semiclassical effects on the CH?

Quantum backreaction on CH

evaluation
on CH

If $\langle T^{-}_{vv} \rangle_{\rm ren} \neq 0$, then $\langle \hat{T}^{-}_{VV} \rangle_{ren}$ diverges

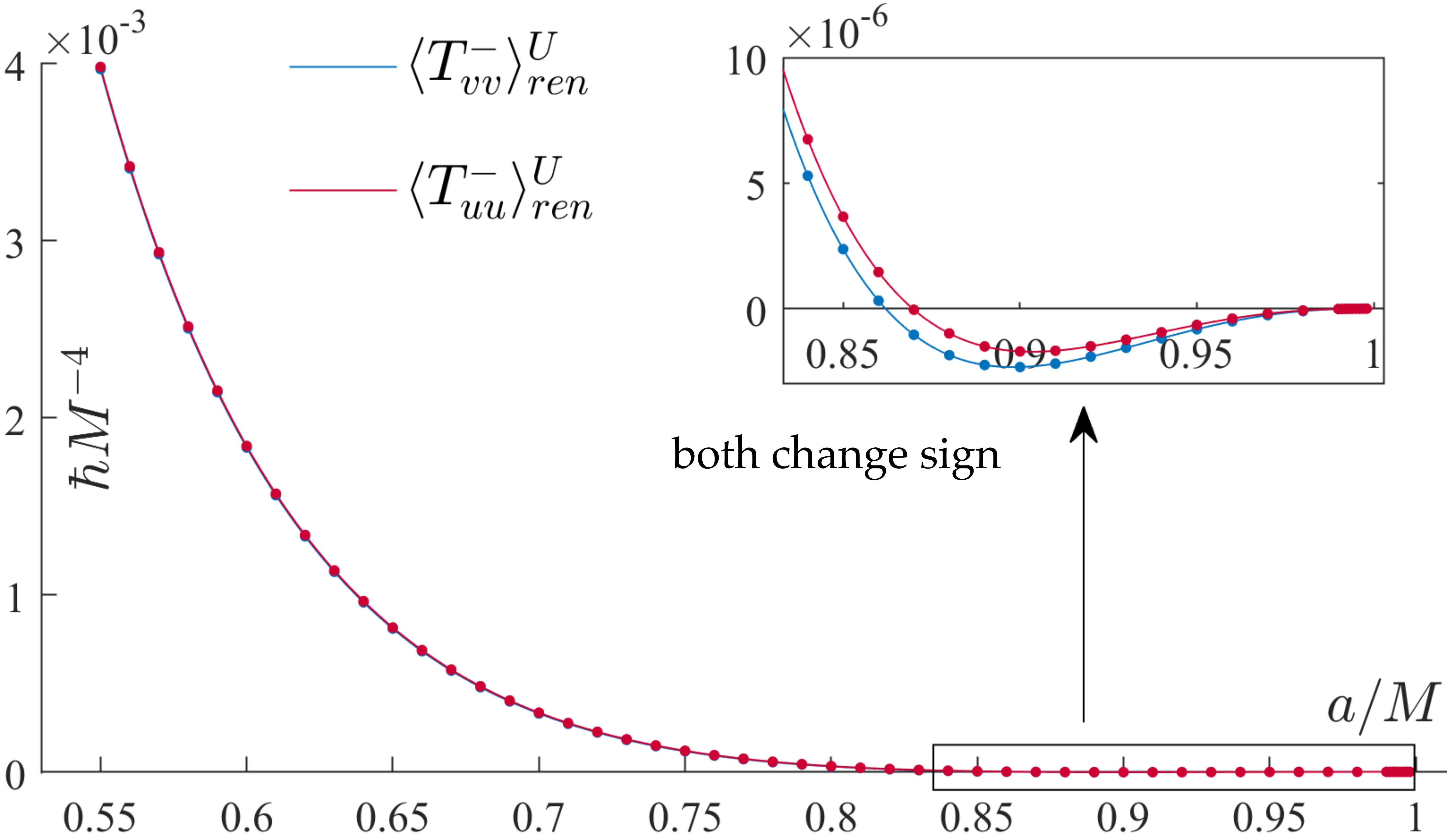
-> curvature singularity & tidal deformation of observer crossing the CH:

$\langle T^{-}_{vv} \rangle_{\rm ren} > 0$ -> contraction of observer
$<$ expansion

At least for a spherical & charged BH,

$\langle T^{-}_{uu} \rangle_{\rm ren} > 0$ -> contraction of CH
$<$ expansion

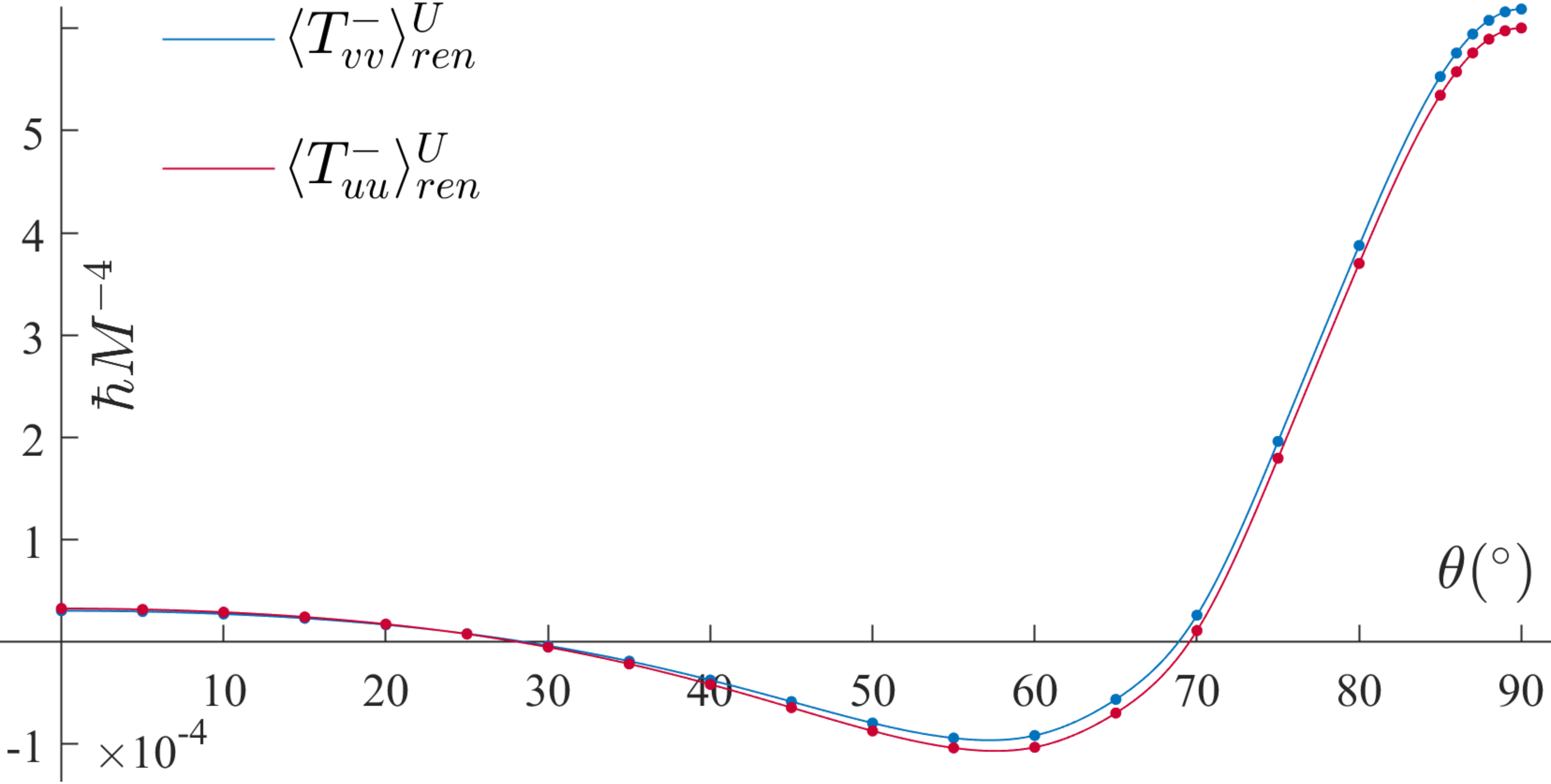
Fluxes at $\theta = 0$ of the CH [Zilberman, Casals, Ori & Ottewill'22]



both change sign

$\langle T_{vv}^{-}\rangle_{\text{ren}} \neq 0$ -> CH becomes irregular

Fluxes on the CH for $a = 0.8M$ [Zilberman, Casals, Ori & Ottewill'22]



Summary

- Concept of ‘vacuum’ is observer-dependent -> Unruh effect

- Quantum BHs emit Hawking radiation -> evaporation -> information paradox

- Rotating BHs:

For bosons, there’s no empty state at infinity (‘Boulware') nor a state in thermal equilibrium (‘Hartle-Hawking’)

For fermions, there’s empty state at infinity but diverges in ergosphere and there’s a thermal state but diverges on speed-of-light surf.

Cauchy Horizon becomes irregular -> no loss of predictability

Vielen Dank!