UNIVERSITÄT
LEIPZIG

Master thesis

Fermi-sea excitations in chiral one-dimensional quantum channels

Felix Puster

15.06.2022

Outline

- Introduction of the system
- Results for the injected electron
- Distribution for the Fermi-sea excitations

Motivation: Relaxation and integrable systems

Many body systems typically thermalize

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Special case: Integrable systems

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- Or not show relaxation (non-interacting systems)

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- Known to reach quasi-thermal states
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In the following: Integrable system with partial relaxation

=> Emergence of far-from-equilibrium state

1D

In 1D electrons at low energy cannot pass through each other (Pauli principle)

=> Only collective low energy excitations

Chiral one-dimensional quantum channels

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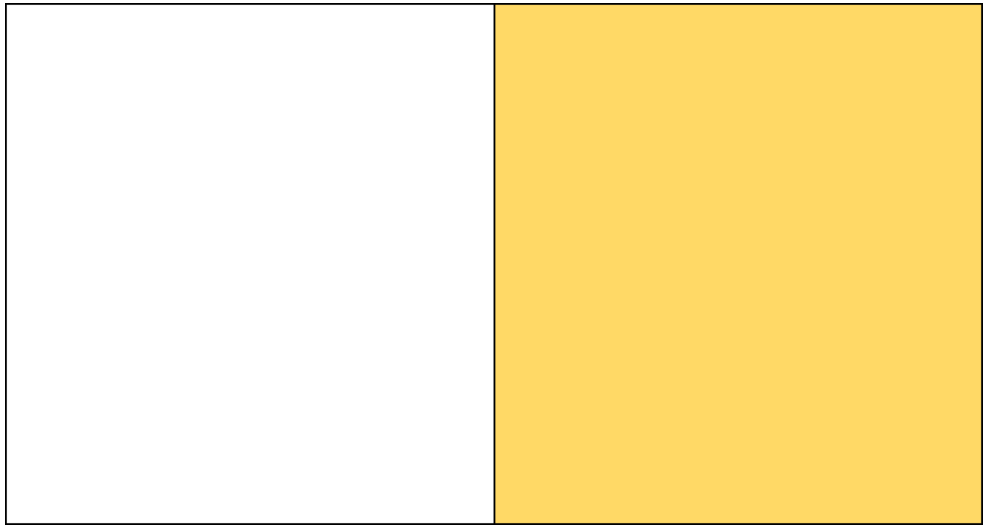
One-dimensional systems can exist at the boundary of two-dimensional systems



Chiral one-dimensional quantum channels

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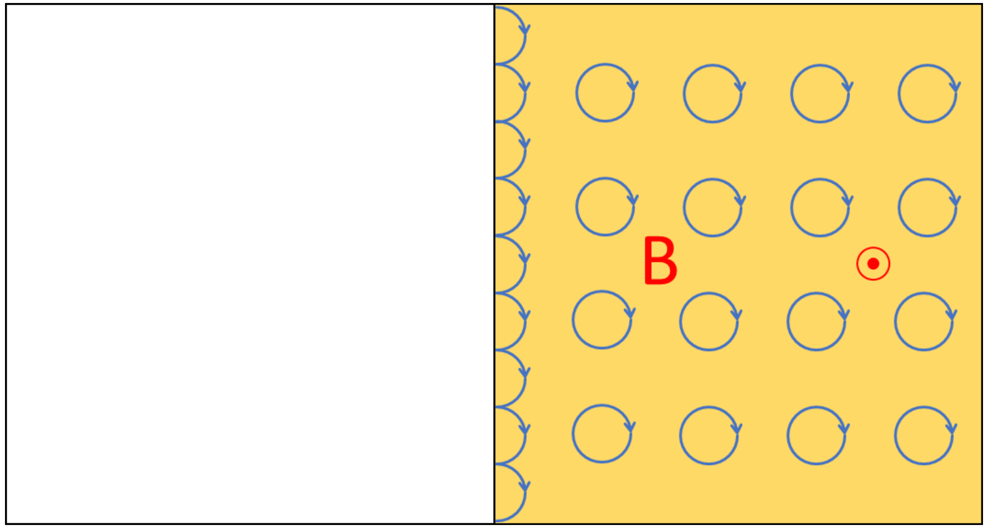
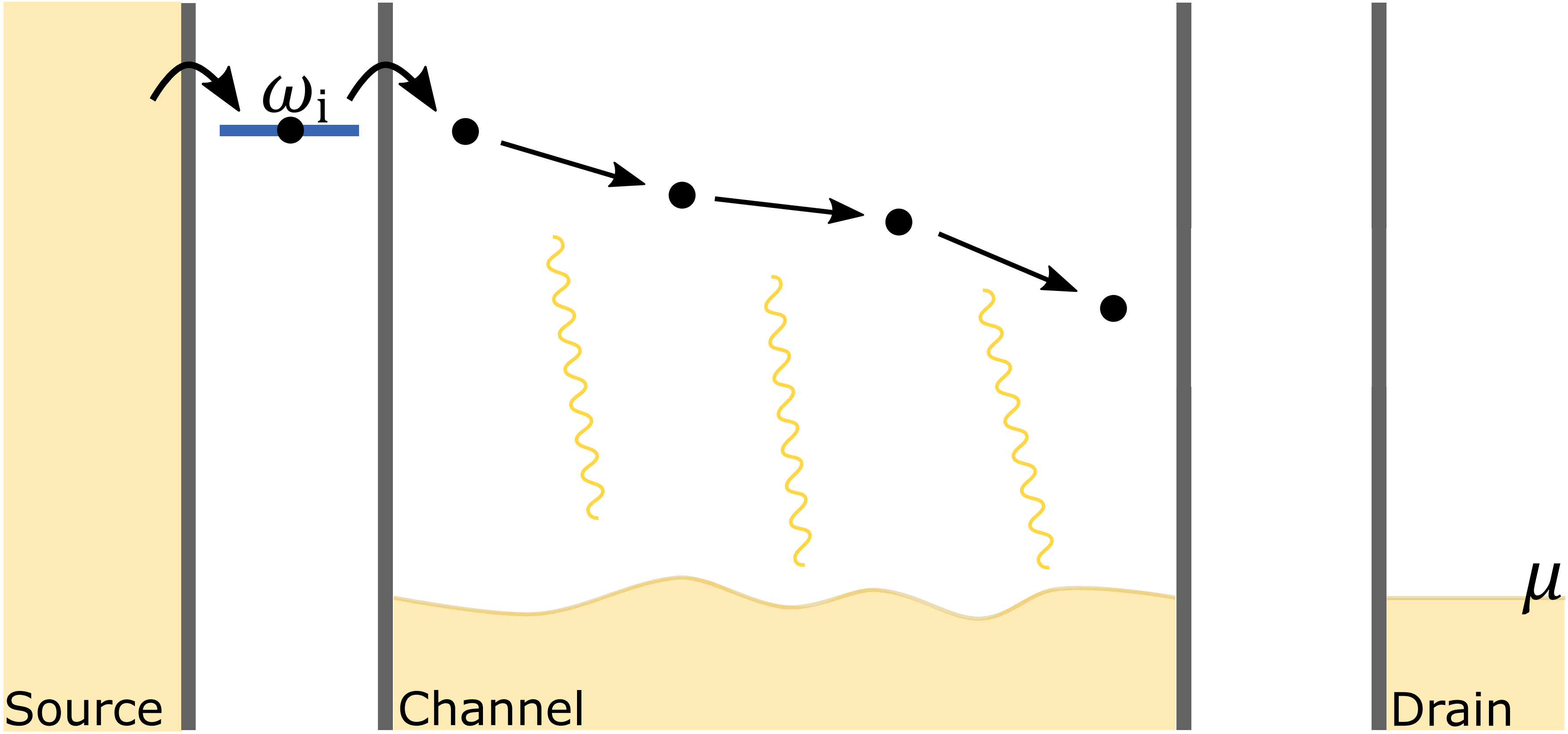
Integer quantum Hall effect:
Chiral electron states at the boundary



Setup



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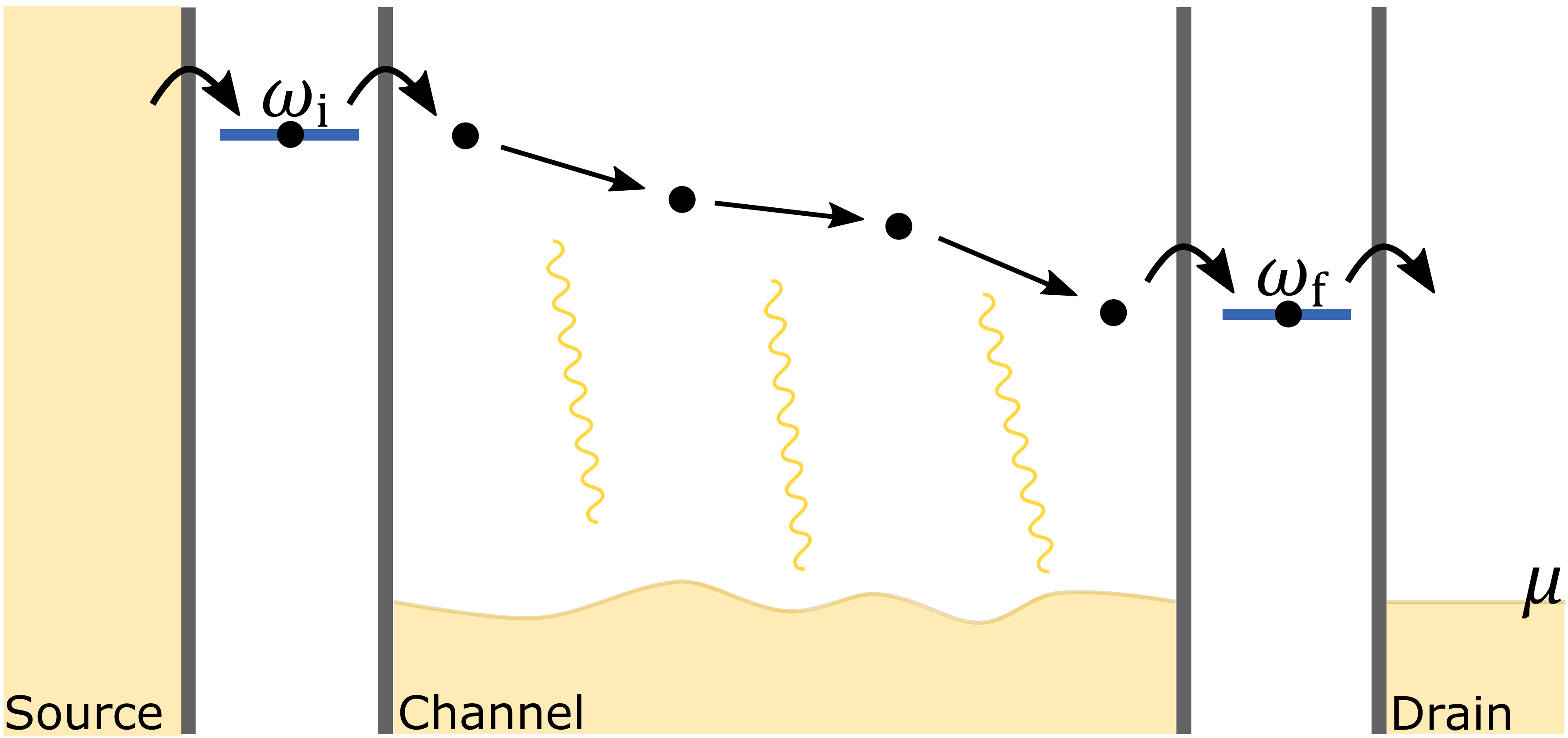

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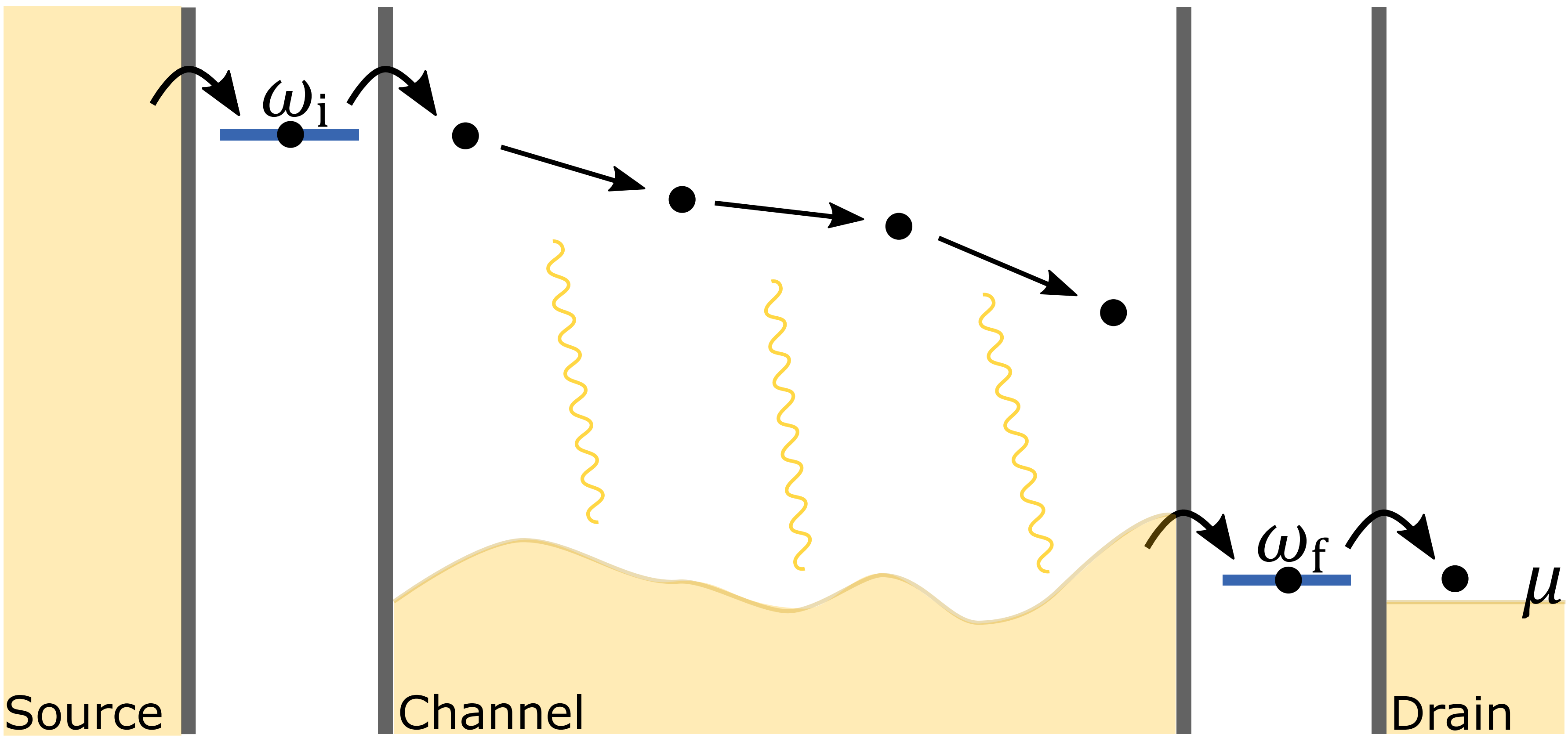

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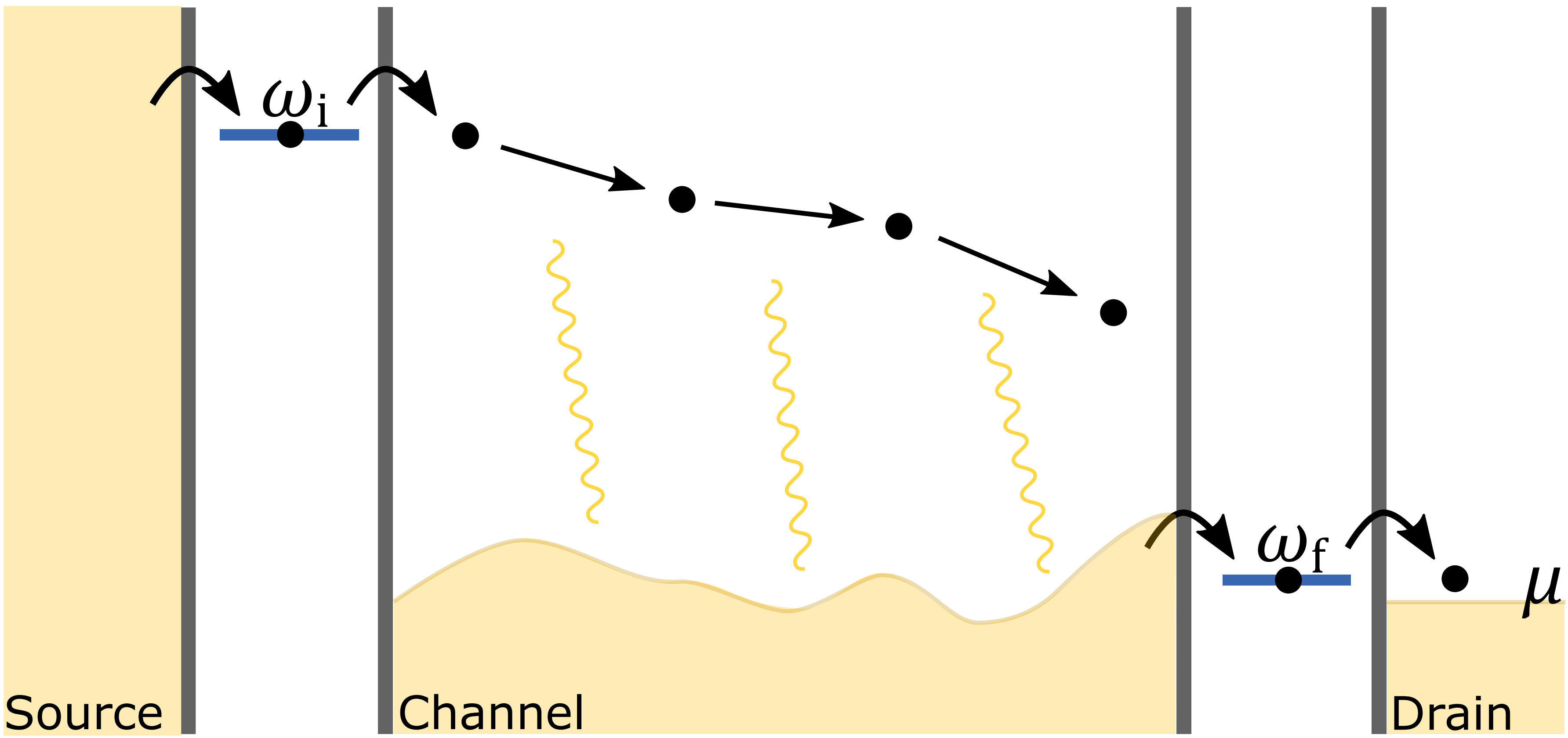
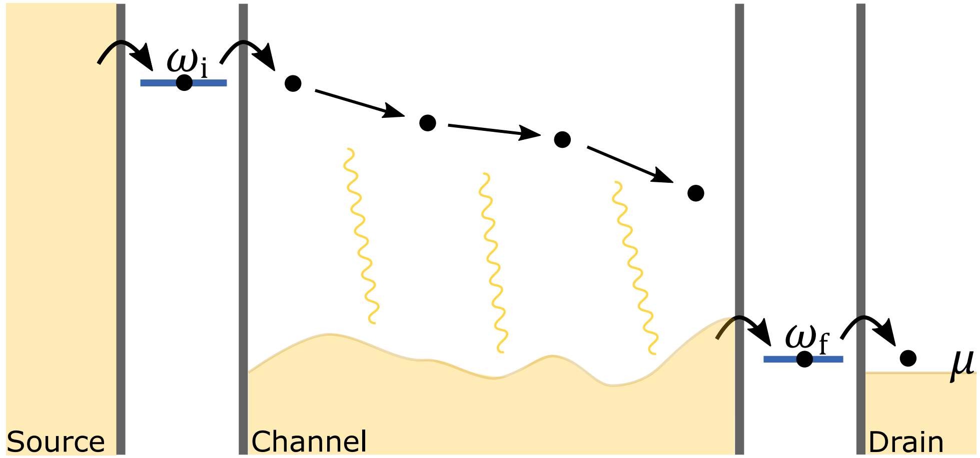


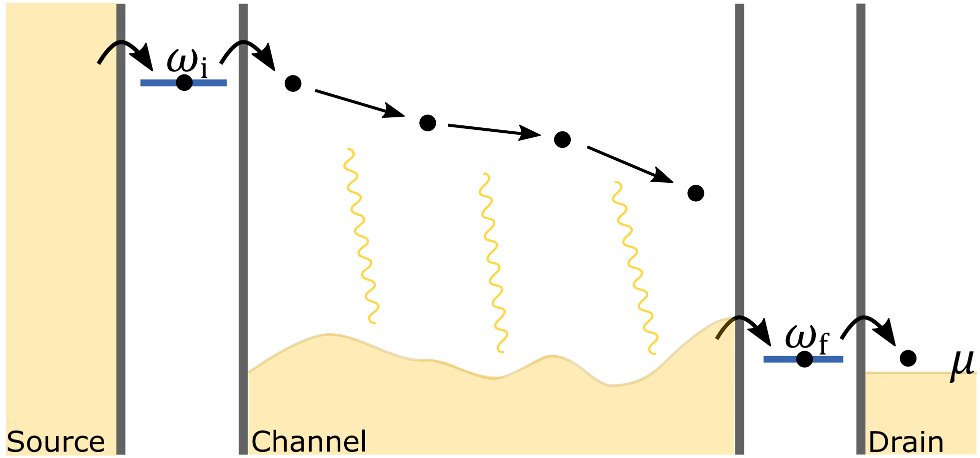
$$H_c = \sum_k vk\, c_k^\dagger c_k + \frac{1}{2L} \sum_{k,k',q\neq 0} V_q c_{k+q}^\dagger c_{k'-q}^\dagger c_{k'} c_k$$

Bosonization

Fermi sea excitations are plasmons



Bosonization



Fermi sea excitations are plasmons

Plasmons are generated by boson operators:

$$b_q^\dagger = \sqrt{\frac{2\pi}{Lq}} \sum_k c_{k+q}^\dagger c_k, \; b_q = \sqrt{\frac{2\pi}{Lq}} \sum_k c_{k-q}^\dagger c_k$$

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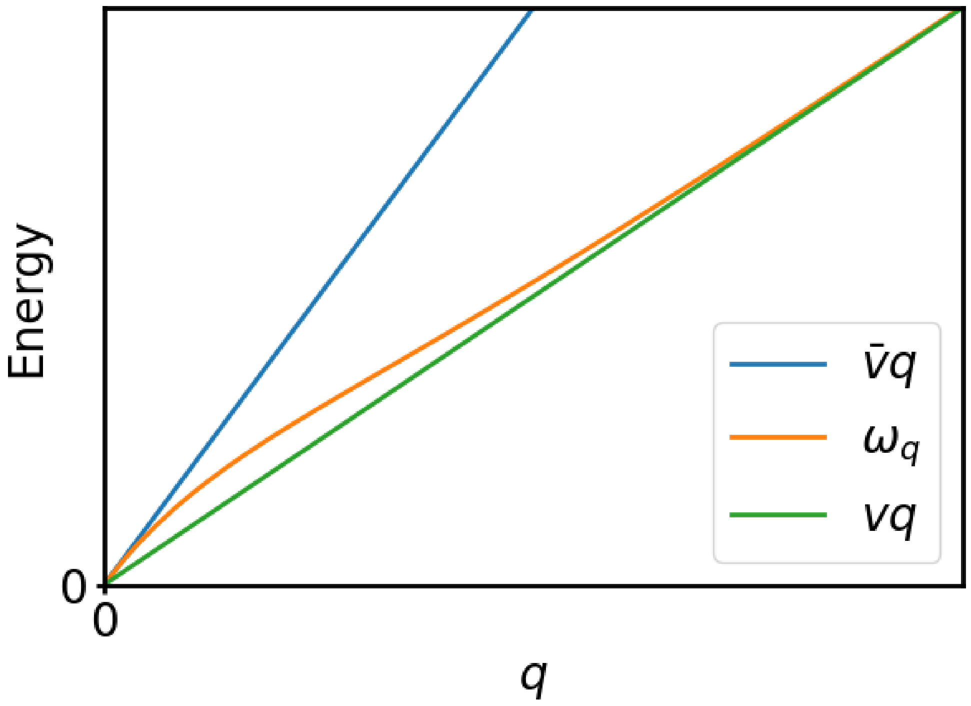

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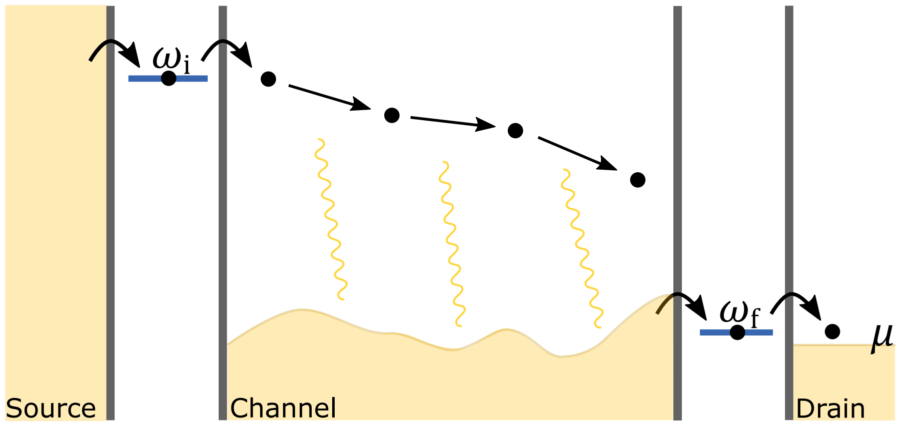
System can be expressed in terms of boson operators:

$$H_c = \sum_{q>0} \omega_q b_q^\dagger b_q, \text{ where } \omega_q = q\left[v + \frac{V_q}{2\pi}\right]$$

Current through the detector dot

Goal: Distribution of electrons and holes



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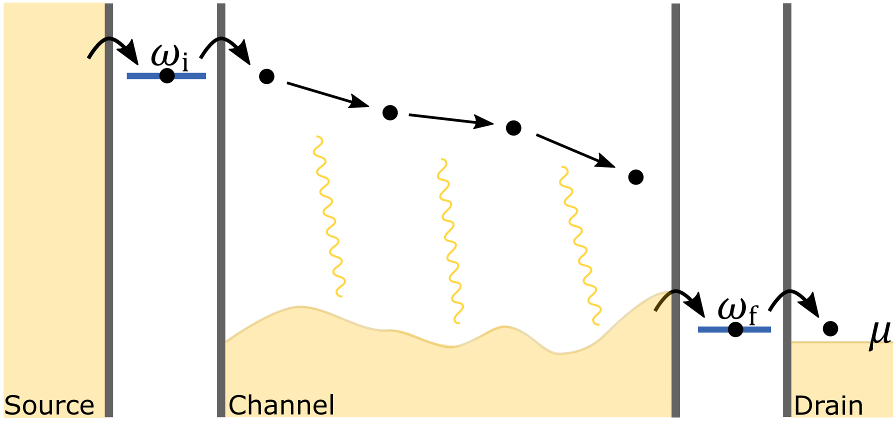

Start from the current through the detector dot:

$$
\begin{aligned}
I(t) &= -e\left\langle \frac{d}{dt}\left(d_{\mathrm{f}}^{\dagger}(t)\, d_{\mathrm{f}}(t)\right)\right\rangle \\
&= -e\frac{i}{\hbar}\sum_{k}\left(\langle d_{\mathrm{f}}^{\dagger}(t) t_{\mathrm{fc}}(k) c_{k}(t)\rangle - \langle c_{k}^{\dagger}(t) t_{\mathrm{cf}}(k) d_{\mathrm{f}}(t)\rangle\right)
\end{aligned}
$$

General form of the distribution

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1. Include tunneling up to lowest non-vanishing order

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Assuming strict chirality:

$$p(\omega_\mathrm{i},\omega_\mathrm{f}) = \frac{v^2}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t_0 \left\{ \left[g_\mathrm{f}^<(-t_0)G_\mathrm{c}^<(0,-t_0) - g_\mathrm{f}^>(-t_0)G_\mathrm{c}^>(0,-t_0)\right] \right.$$
$$\left. \times \int_{-\infty}^{\infty}\mathrm{d}t_1\int_{-\infty}^{\infty}\mathrm{d}t_2\, g_\mathrm{i}^<(t_1-t_2)G_\mathrm{c}^<(0,t_1-t_2)\frac{G_\mathrm{c}^<(x,-t_2)G_\mathrm{c}^>(x,t_0-t_1)}{G_\mathrm{c}^<(x,t_0-t_2)G_\mathrm{c}^>(x,-t_1)} \right\}$$

Lesser/greater Green functions: $G^<(x,t) = i\langle\psi^\dagger(0,0)\psi(x,t)\rangle,\ G^>(x,t) = -i\langle\psi(x,t)\psi^\dagger(0,0)\rangle$

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Two velocity model

Consider a model where all plasmons move with velocity $\bar{v}$

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Then:

$$G_{\mathrm{c}}^{\gtrless}(x,t) = \frac{1}{2\pi}\frac{1}{x - vt \pm i\delta}\frac{x - vt \pm i\lambda_c}{x - \bar{v}t \pm i\lambda_c}$$

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$$p_{\mathrm{inelastic}}(\omega_{\mathrm{f}} \gg \bar{v}/\lambda_c) = \frac{x_s^2}{x_s^2 + \lambda_c^2}\frac{\lambda_c^2}{\bar{v}^2}\omega_{\mathrm{if}}\exp\left[-\omega_{\mathrm{if}}\frac{\lambda_c}{\bar{v}}\right]$$

$$\bar{v} = v + \frac{\nu}{2\pi},\ x_s = x\frac{\nu}{2\pi v},\ \omega_{\mathrm{if}} = \omega_{\mathrm{i}} - \omega_{\mathrm{f}}$$

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Consider a scaling limit: $\nu \to 0,\ x \to \infty,\ x_s = \mathrm{const}$

Exponential interaction

Exponential interaction

General solution via bosonization:

$$G^{\gtrless}(x,t) = \frac{1}{2\pi}\frac{1}{x - vt \pm i\delta}\exp\left[\int_0^\infty \frac{\mathrm{d}q}{q}\left(e^{\pm iq[x-(v+\frac{V_q}{2\pi})t]} - e^{\pm iq(x-vt)}\right)\right]$$

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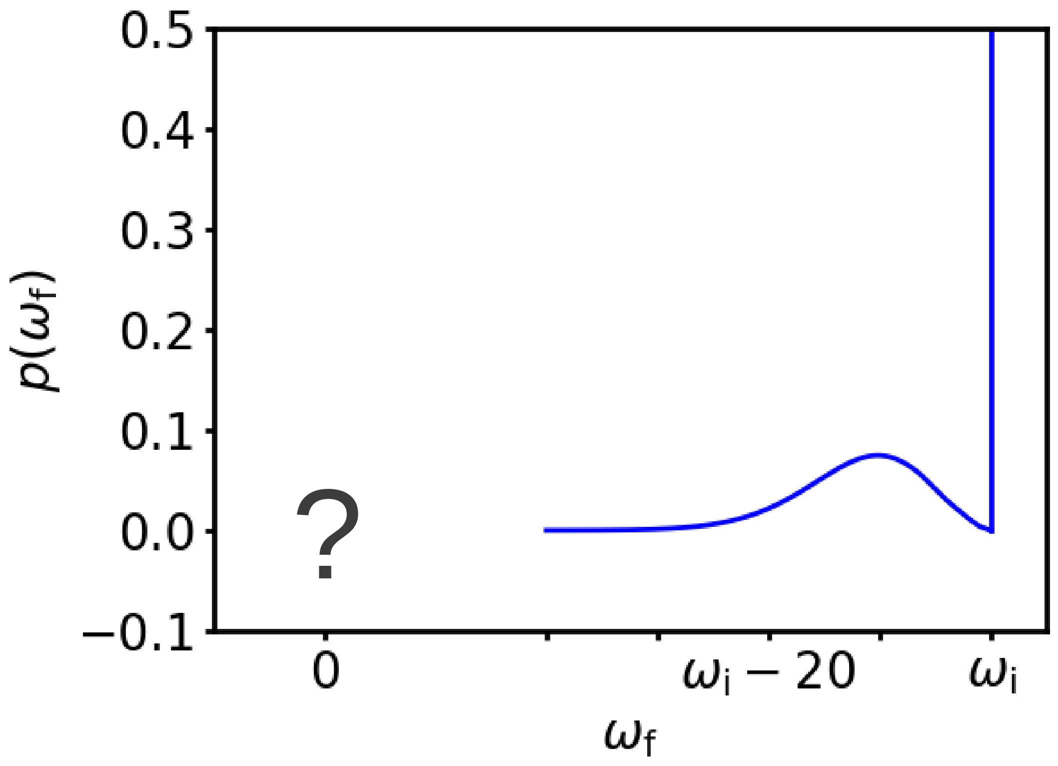
Assume an exponential interaction $V_q = \nu e^{-\lambda|q|}$

Scaling limit $\nu \to 0,\, x \to \infty,\, x_s = x\frac{\nu}{2\pi v} = \text{const.}$

$$S^{\gtrless}(x, t + \frac{x}{v}) \xrightarrow[\text{limit}]{\text{scaling}} \sum_{n=1}^{\infty} \frac{1}{n}\frac{(-x_s)^n}{(vt \mp i\lambda n)^n}$$

Result for the injected electron

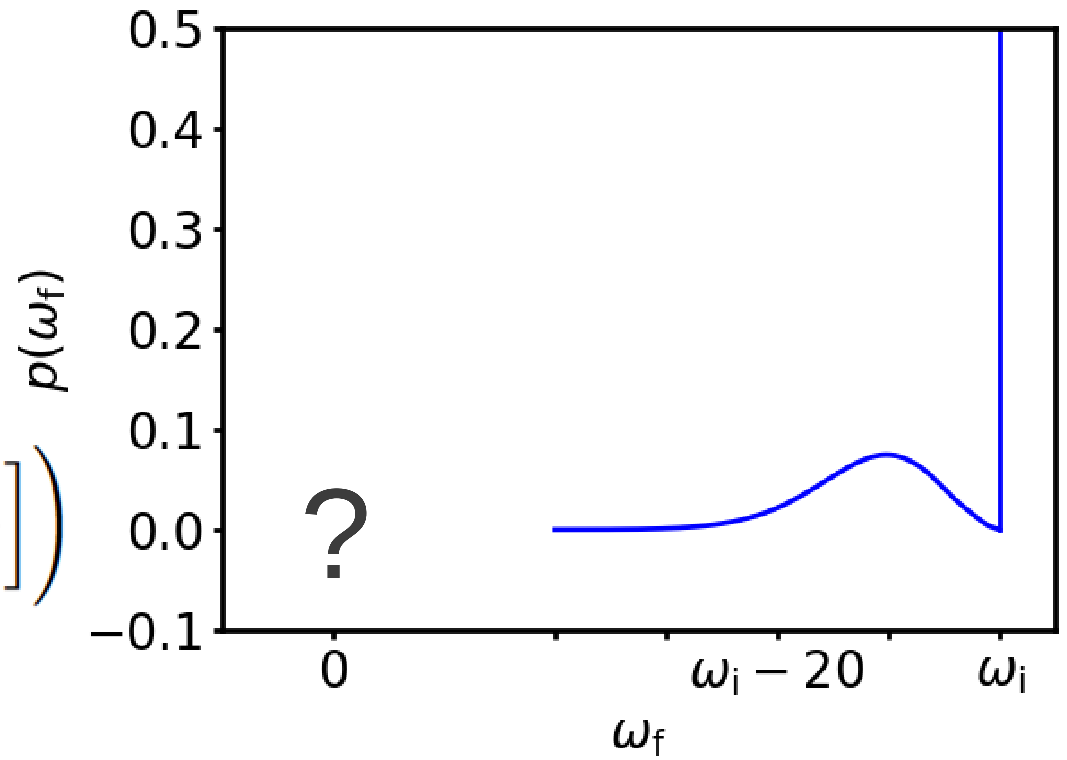
Use the limit: $\omega_\mathrm{i}, \omega_\mathrm{f} \gg v/\lambda$



Result for the injected electron

Use the limit: $\omega_\mathrm{i}, \omega_\mathrm{f} \gg v/\lambda$

$$p_\mathrm{HE}(\omega_\mathrm{if}, x_s) = \delta(\omega_\mathrm{if})\frac{\lambda}{v}\exp\left(\sum_{n=1}^{\infty}\frac{x_s^{2n}}{n}\frac{1}{(2n\lambda i)^{2n}}\right)$$
$$+\frac{\lambda}{v}\int_{\square}\frac{\mathrm{d}t}{2\pi}e^{i\omega_\mathrm{if}t}\exp\left(\sum_{n=1}^{\infty}\frac{x_s^{2n}}{n}\left[\frac{1}{(2n\lambda i)^{2n}}-\frac{1}{(t-2n\lambda i)^{2n}}\right]\right)$$



S. G. Fischer, Y. Meir, Y. Gefen, and B. Rosenow, arXiv:2108.00685

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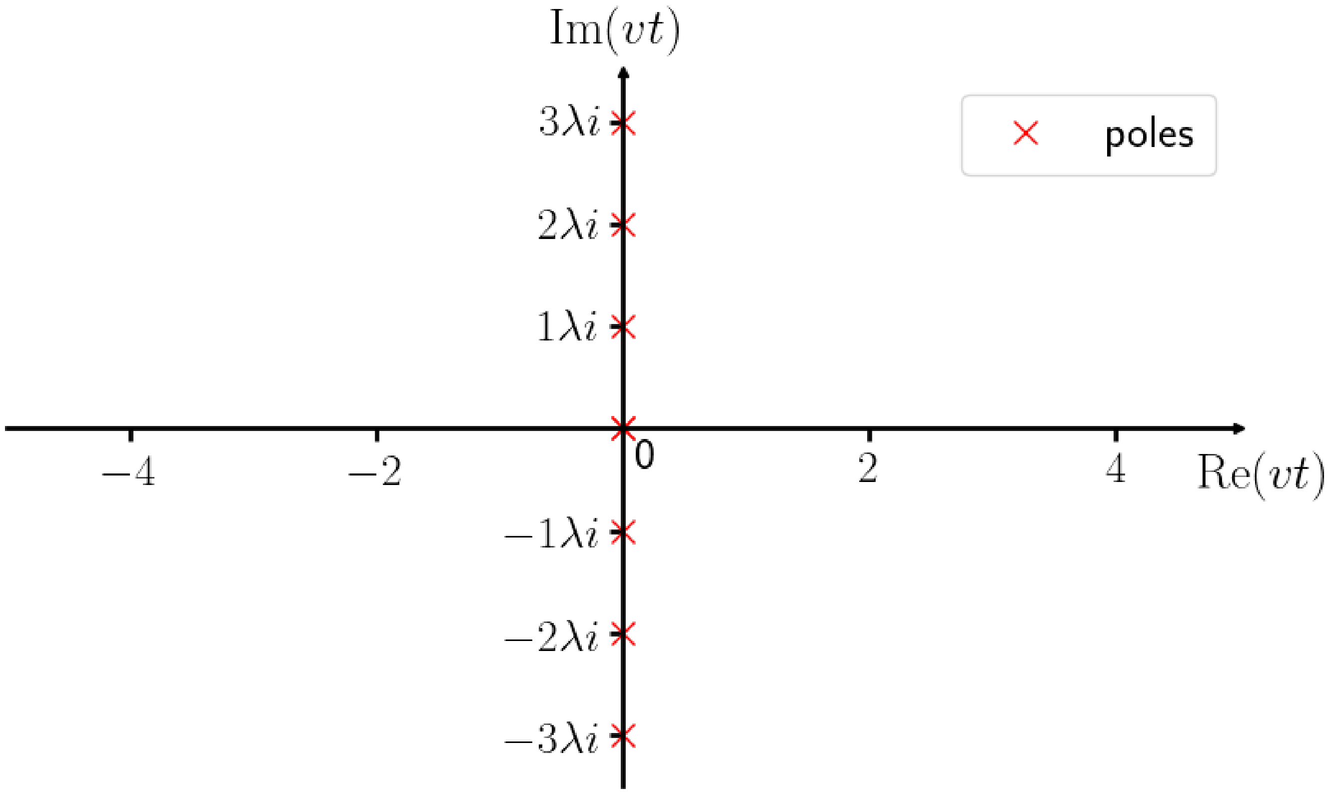
Reminder

$$p(\omega_\mathrm{i},\omega_\mathrm{f},x_s)=\frac{v^2}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t_0\,\Bigg\{\left[g_\mathrm{f}^<(-t_0)G_\mathrm{c}^<(0,-t_0)-g_\mathrm{f}^>(-t_0)G_\mathrm{c}^>(0,-t_0)\right]$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}t_1\int_{-\infty}^{\infty}\mathrm{d}t_2\,g_\mathrm{i}^<(t_1-t_2)G_\mathrm{c}^<(0,t_1-t_2)\frac{G_\mathrm{c}^<(x,-t_2+\frac{x}{v})G_\mathrm{c}^>(x,t_0-t_1+\frac{x}{v})}{G_\mathrm{c}^<(x,t_0-t_2+\frac{x}{v})G_\mathrm{c}^>(x,-t_1+\frac{x}{v})}\Bigg\}$$

With: $g_\mathrm{i}^<(t_1-t_2)=ie^{-i\omega_\mathrm{i}(t_1-t_2)}$ and $g_\mathrm{f}^{\gtrless}(t)=\mp i\Theta(\pm\omega_\mathrm{f})e^{-i\omega_\mathrm{f}t}$

Limit of high injection energies



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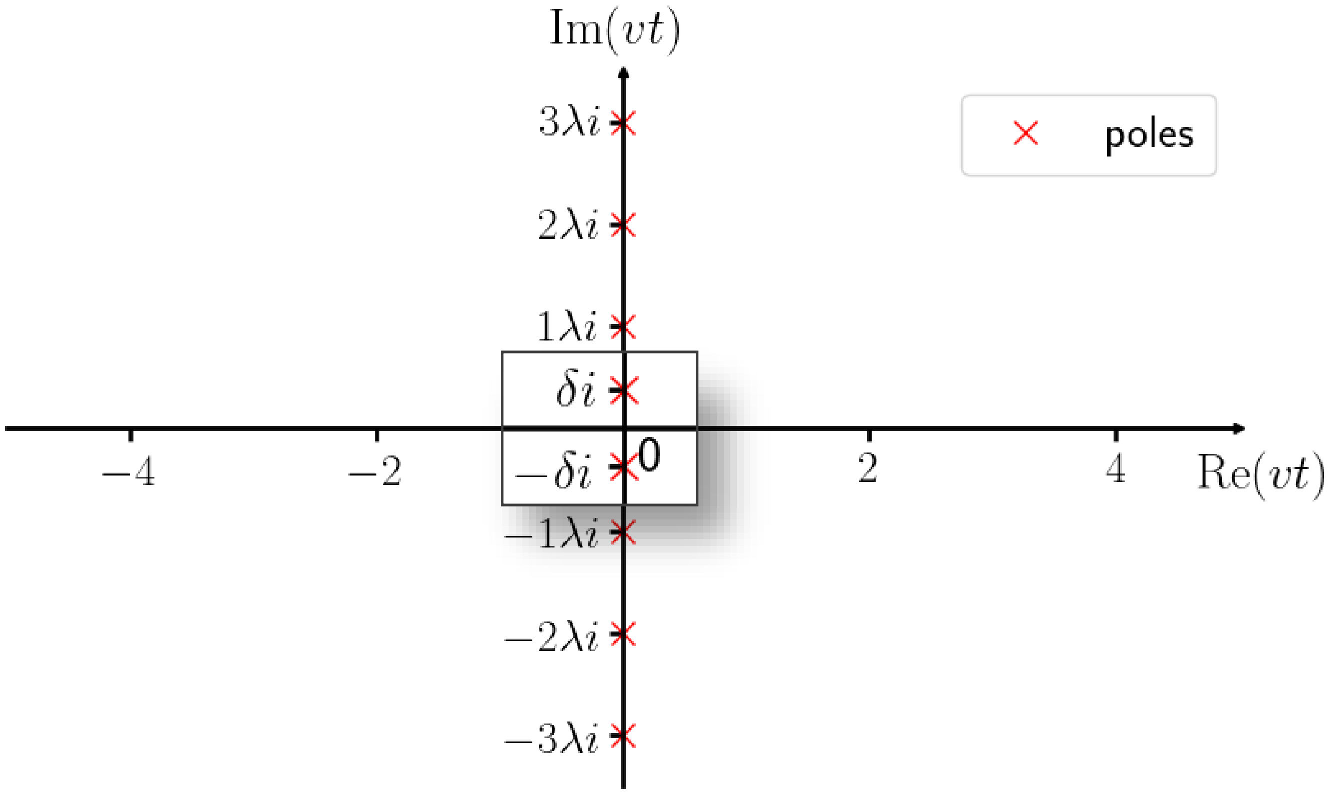

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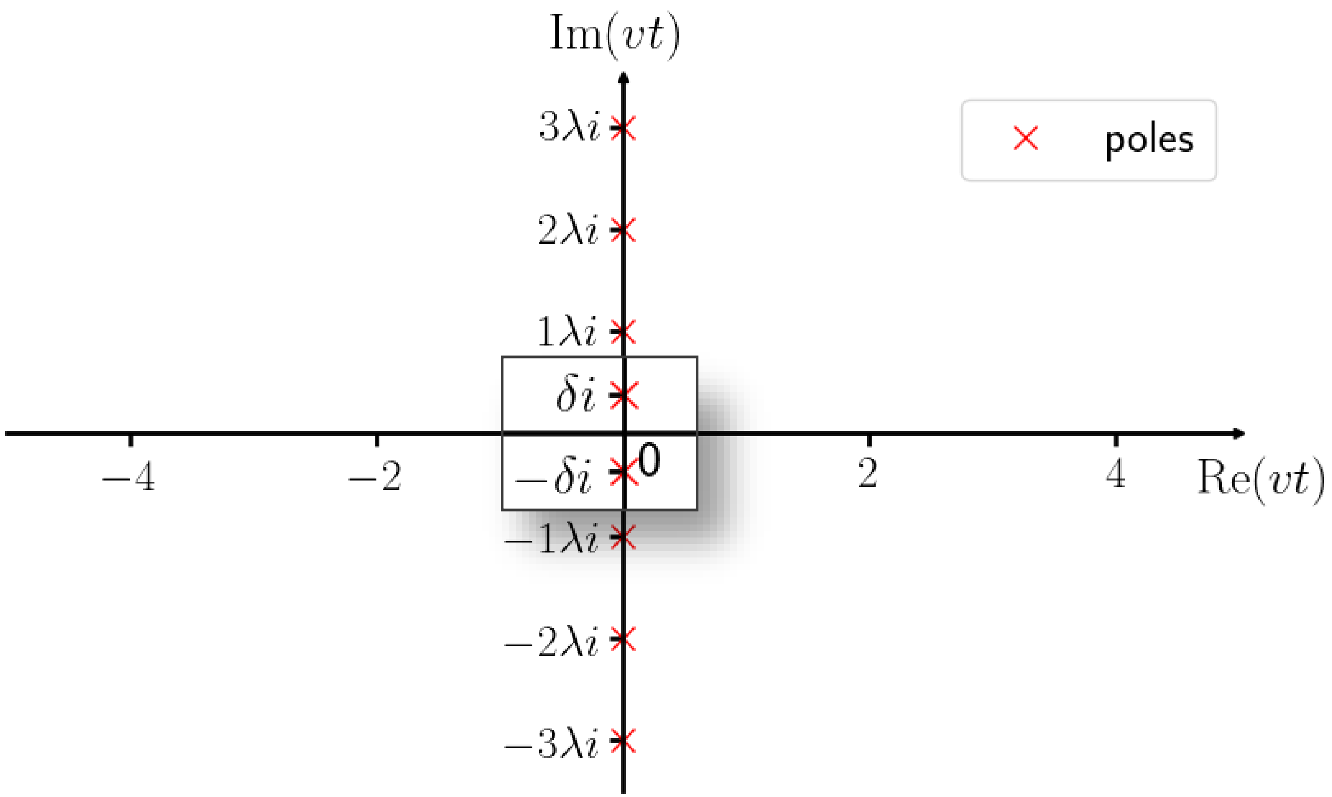

$$\mathrm{Res}_a(f) \propto \exp\left[-\omega_\mathrm{i}\lambda/v \cdot \mathrm{Im}(a)\right]$$

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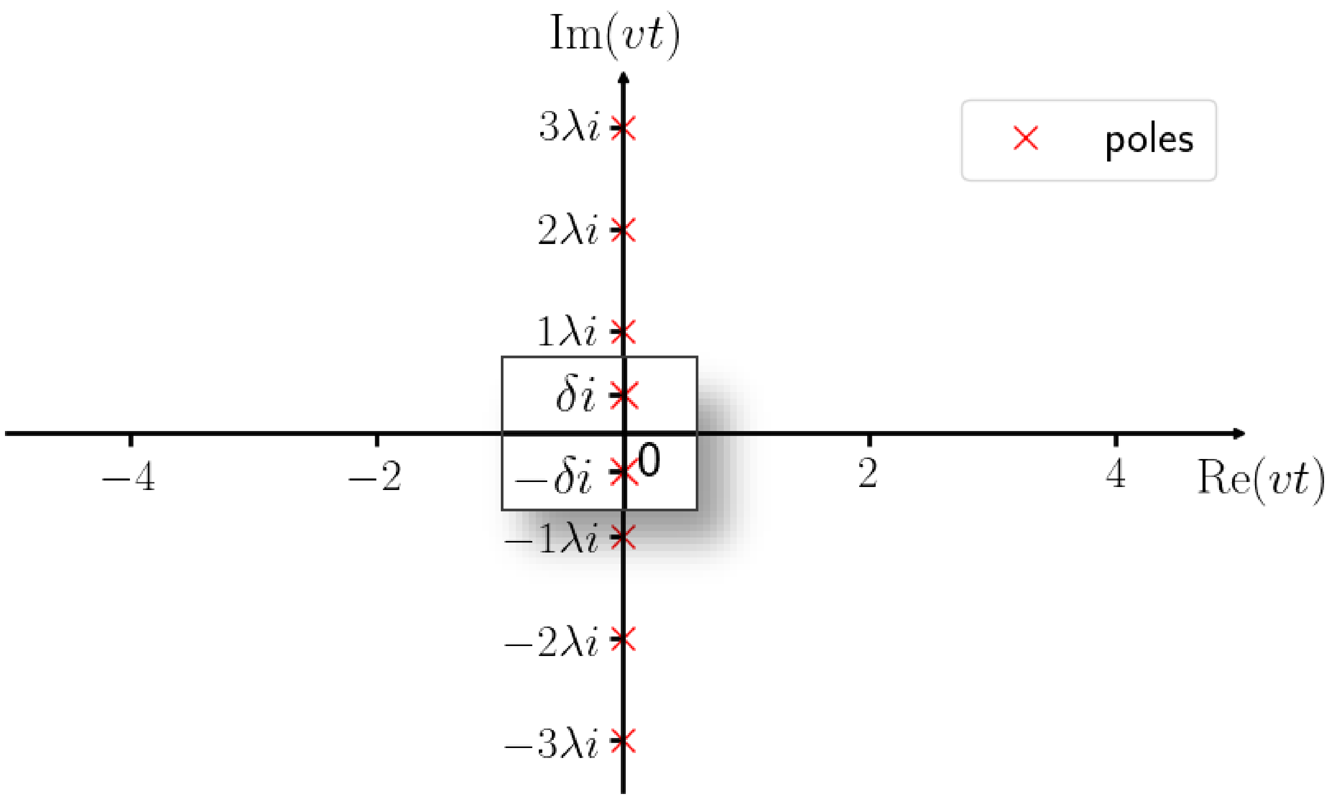

$$\mathrm{Res}_a(f) \propto \exp\left[-\omega_\mathrm{i}\lambda/v \cdot \mathrm{Im}(a)\right]$$

=> In the limit $\omega_\mathrm{i} \gg v/\lambda$

Neglect all residues with $\mathrm{Im}(a) \propto \lambda$

Limit of high injection energies

$$p_{\mathrm{FS}}(\omega_{\mathrm{f}}, x_s) = i\frac{v}{4\pi^2}\int_{-\infty}^{\infty}\mathrm{d}\tilde{t}_0\int_{-\infty}^{\infty}\mathrm{d}\tilde{t}_1\left[\frac{\Theta(\omega_{\mathrm{f}})e^{i\omega_{\mathrm{f}}(\tilde{t}_0-\tilde{t}_1)}}{(v(\tilde{t}_0-\tilde{t}_1)+i\delta)} + \frac{\Theta(-\omega_{\mathrm{f}})e^{i\omega_{\mathrm{f}}(\tilde{t}_0-\tilde{t}_1)}}{(v(\tilde{t}_0-\tilde{t}_1)-i\delta)}\right]$$
$$\times\exp\left(\sum_{n=1}^{\infty}\frac{(-x_s)^n}{n}\left[\frac{1}{(v\tilde{t}_1+i\lambda n)^n}-\frac{1}{(v\tilde{t}_1-i\lambda n)^n}\right]\right)$$
$$\times\exp\left(\sum_{n=1}^{\infty}\frac{(-x_s)^n}{n}\left[\frac{1}{(v\tilde{t}_0-i\lambda n)^n}-\frac{1}{(v\tilde{t}_0+i\lambda n)^n}\right]\right)$$

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$$\times \exp\left(\sum_{n=1}^{\infty}\frac{(-x_s)^n}{n}\left[\frac{1}{(v\tilde{t}_0-i\lambda n)^n} - \frac{1}{(v\tilde{t}_0+i\lambda n)^n}\right]\right)$$

Rewrite: $$\frac{1}{(\tilde{t}_0-\tilde{t}_1)\pm i\frac{\delta}{v}} = \mp i\int_0^{\infty}\mathrm{d}\omega\, e^{\pm i\omega((\tilde{t}_0-\tilde{t}_1)\pm i\frac{\delta}{v})}$$

Decoupling of the time integrals

Truncate sums

$$p_{\mathrm{FS}}(\omega_{\mathrm{f}}, x_s) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \mathrm{d}\tilde{t}_0 \int_{-\infty}^{\infty} \mathrm{d}\tilde{t}_1 \int_0^{\infty} \mathrm{d}\omega \left[\Theta(\omega_{\mathrm{f}}) e^{i(\omega_{\mathrm{f}}+\omega)(\tilde{t}_0-\tilde{t}_1)+i\frac{\delta}{v}} - \Theta(-\omega_{\mathrm{f}}) e^{-i(|\omega_{\mathrm{f}}|+\omega)(\tilde{t}_0-\tilde{t}_1)-i\frac{\delta}{v}} \right]$$
$$\times \exp\left(\sum_{n=1}^{N} \frac{(-x_s)^n}{n} \left[\frac{1}{(v\tilde{t}_1 + i\lambda n)^n} - \frac{1}{(v\tilde{t}_1 - i\lambda n)^n} \right] \right)$$
$$\times \exp\left(\sum_{n=1}^{N} \frac{(-x_s)^n}{n} \left[\frac{1}{(v\tilde{t}_0 - i\lambda n)^n} - \frac{1}{(v\tilde{t}_0 + i\lambda n)^n} \right] \right)$$

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Truncate sums

1. Rewrite time integrals as contour integrals

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$$\times \exp\left(\sum_{n=1}^{N} \frac{(-x_s)^n}{n} \left[\frac{1}{(v\tilde{t}_1 + i\lambda n)^n} - \frac{1}{(v\tilde{t}_1 - i\lambda n)^n}\right]\right)$$
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Decoupling of the time integrals

Truncate sums

1. Rewrite time integrals as contour integrals
2. Exchange the ω integral with the time integrals

$$p_{\mathrm{FS}}(\omega_{\mathrm{f}}, x_s) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \mathrm{d}\tilde{t}_0 \int_{-\infty}^{\infty} \mathrm{d}\tilde{t}_1 \int_0^{\infty} \mathrm{d}\omega \left[\Theta(\omega_{\mathrm{f}}) e^{i(\omega_{\mathrm{f}}+\omega)(\tilde{t}_0-\tilde{t}_1)+i\frac{\delta}{v}} - \Theta(-\omega_{\mathrm{f}}) e^{-i(|\omega_{\mathrm{f}}|+\omega)(\tilde{t}_0-\tilde{t}_1)-i\frac{\delta}{v}} \right]$$
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Decoupling of the time integrals

1. Truncate sums to finite number of poles
2. Rewrite time integrals as contour integrals
3. Exchange the ω integral with the time integrals
4. Identify the time integrals as complex conjugate of one another

$$p_{\mathrm{FS}}(\omega_{\mathrm{f}}, x_s) = \frac{\Theta(\omega_{\mathrm{f}})}{4\pi^2} \int_0^\infty \mathrm{d}\omega \left| \int_{\square} \mathrm{d}\tilde{t}\, e^{i(\omega_{\mathrm{f}}+\omega)\tilde{t}} \exp\left(\sum_{n=1}^{N} \frac{(-x_s)^n}{n} \left[\frac{1}{(v\tilde{t} - i\lambda n)^n} - \frac{1}{(v\tilde{t} + i\lambda n)^n} \right] \right) \right|^2$$

$$- \frac{\Theta(-\omega_{\mathrm{f}})}{4\pi^2} \int_0^\infty \mathrm{d}\omega \left| \int_{\square} \mathrm{d}\tilde{t}\, e^{i(|\omega_{\mathrm{f}}|+\omega)\tilde{t}} \exp\left(\sum_{n=1}^{N} \frac{(-x_s)^n}{n} \left[\frac{1}{(v\tilde{t} + i\lambda n)^n} - \frac{1}{(v\tilde{t} - i\lambda n)^n} \right] \right) \right|^2 .$$

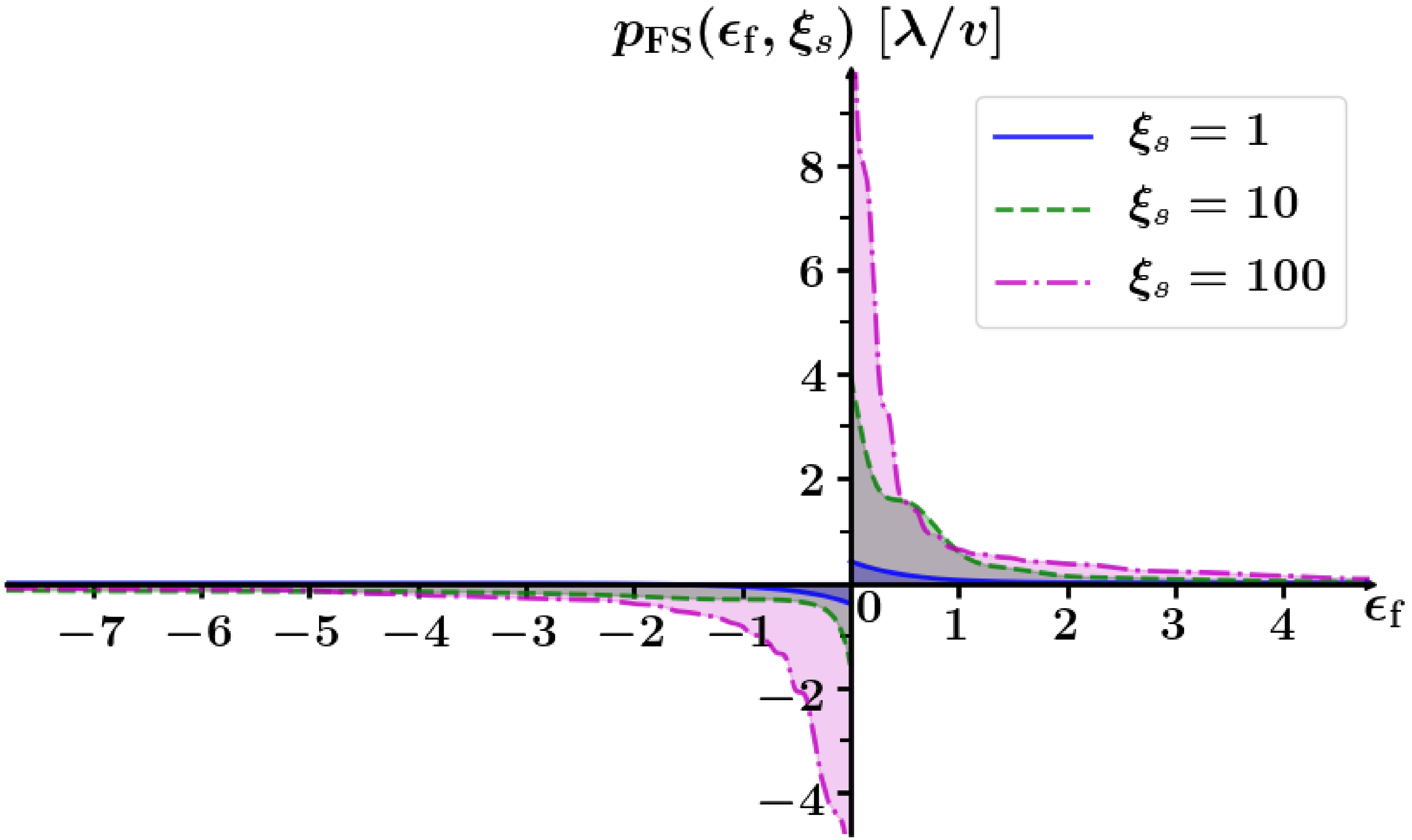
Decoupling of the time integrals

1. Truncate sums to finite number of poles
2. Rewrite time integrals as contour integrals
3. Exchange the ω integral with the time integrals
4. Identify the time integrals as complex conjugate of one another
5. Use dimensionless variables $w = \omega\lambda v^{-1}$ $Y = v\tilde{t}\lambda^{-1}$ $\varepsilon_{\mathrm{f}} = \frac{\omega_{\mathrm{f}}\lambda}{v}$ $\xi_s = \frac{x_s}{\lambda}$

$$p_{\mathrm{FS}}(\varepsilon_{\mathrm{f}}, \xi_s) = \frac{\Theta(\varepsilon_{\mathrm{f}})}{4\pi^2}\frac{\lambda}{v}\int_0^\infty \mathrm{d}w \left| \int_\square \mathrm{d}Y\, e^{i(\varepsilon_{\mathrm{f}}+w)Y} \exp\left(\sum_{n=1}^{N} \frac{(-\xi_s)^n}{n}\left[\frac{1}{(Y-in)^n} - \frac{1}{(Y+in)^n}\right]\right)\right|^2$$

$$- \frac{\Theta(-\varepsilon_{\mathrm{f}})}{4\pi^2}\frac{\lambda}{v}\int_0^\infty \mathrm{d}w \left| \int_\square \mathrm{d}Y\, e^{i(|\varepsilon_{\mathrm{f}}|+w)Y} \exp\left(\sum_{n=1}^{N} \frac{(-\xi_s)^n}{n}\left[\frac{1}{(Y+in)^n} - \frac{1}{(Y-in)^n}\right]\right)\right|^2$$

Fermi-sea distribution



Mixing of one- and two-plasmon states

Consider a state $|q, q', q''\rangle = [b_q^\dagger b_{q'}^\dagger + A \cdot b_{q''}^\dagger]|0\rangle_0$

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The electron hole distribution is then

$$p(k; q, q', q'') = \langle q, q', q''| {}^*_* c_k^\dagger c_k {}^*_* |q, q', q''\rangle$$, with ${}^*_* c_k^\dagger c_k {}^*_* = \Theta(k) c_k^\dagger c_k - \Theta(-k) c_k c_k^\dagger$

Mixing of one- and two-plasmon states

Consider a state $|q, q', q''\rangle = [b_q^\dagger b_{q'}^\dagger + A \cdot b_{q''}^\dagger]|0\rangle_0$

The electron hole distribution is then

$$p(k; q, q', q'') = \langle q, q', q''|{}^*_* c_k^\dagger c_k {}^*_*|q, q', q''\rangle$$, with $${}^*_* c_k^\dagger c_k {}^*_* = \Theta(k) c_k^\dagger c_k - \Theta(-k) c_k c_k^\dagger$$

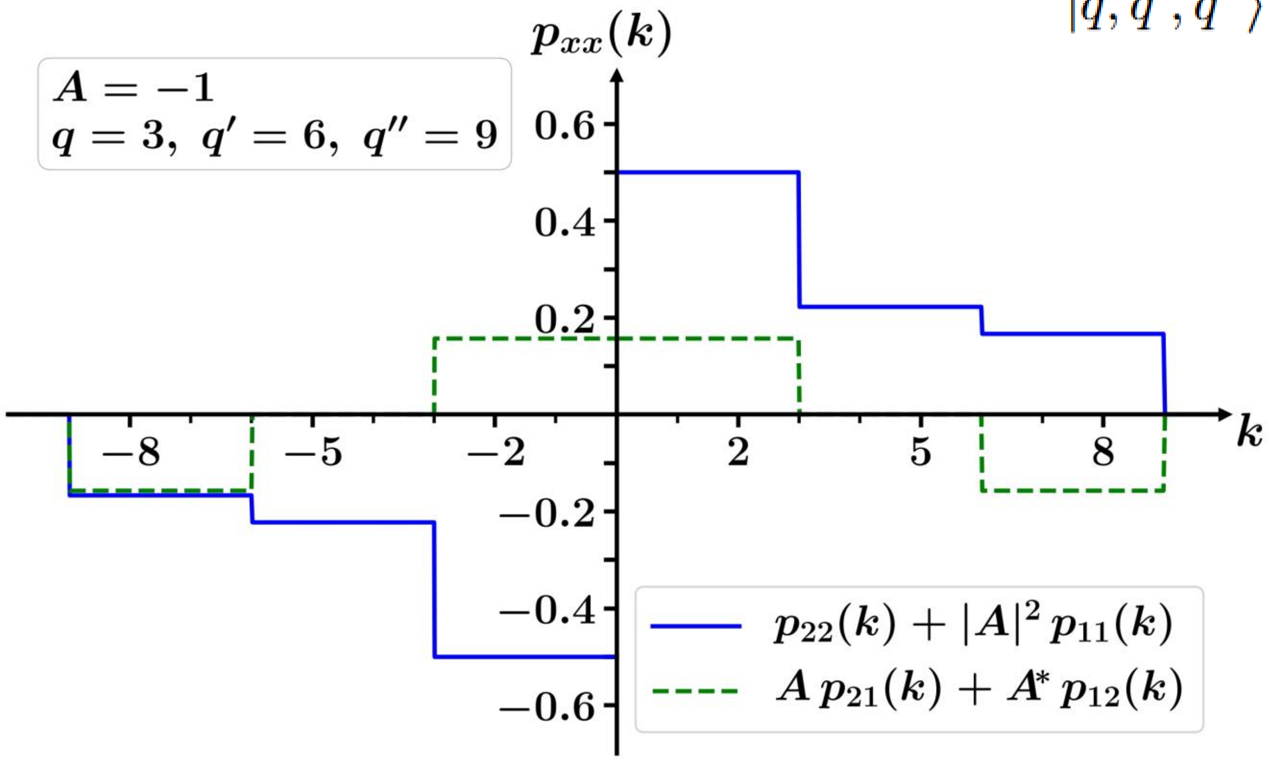
There are four terms:

$$p_{22}(k) = {}_0\langle 0|b_{q'} b_q {}^*_* c_k^\dagger c_k {}^*_* b_q^\dagger b_{q'}^\dagger |0\rangle_0 \qquad p_{11}(k) = {}_0\langle 0|b_{q''} {}^*_* c_k^\dagger c_k {}^*_* b_{q''}^\dagger |0\rangle_0$$

$$p_{12}(k) = {}_0\langle 0|b_{q''} {}^*_* c_k^\dagger c_k {}^*_* b_q^\dagger b_{q'}^\dagger |0\rangle_0 \qquad p_{21}(k) = {}_0\langle 0|b_{q'} b_q {}^*_* c_k^\dagger c_k {}^*_* b_{q''}^\dagger |0\rangle_0$$

Mixing of one- and two-plasmon states

$$|q, q', q''\rangle = [b_q^\dagger b_{q'}^\dagger + A \cdot b_{q''}^\dagger]|0\rangle_0$$



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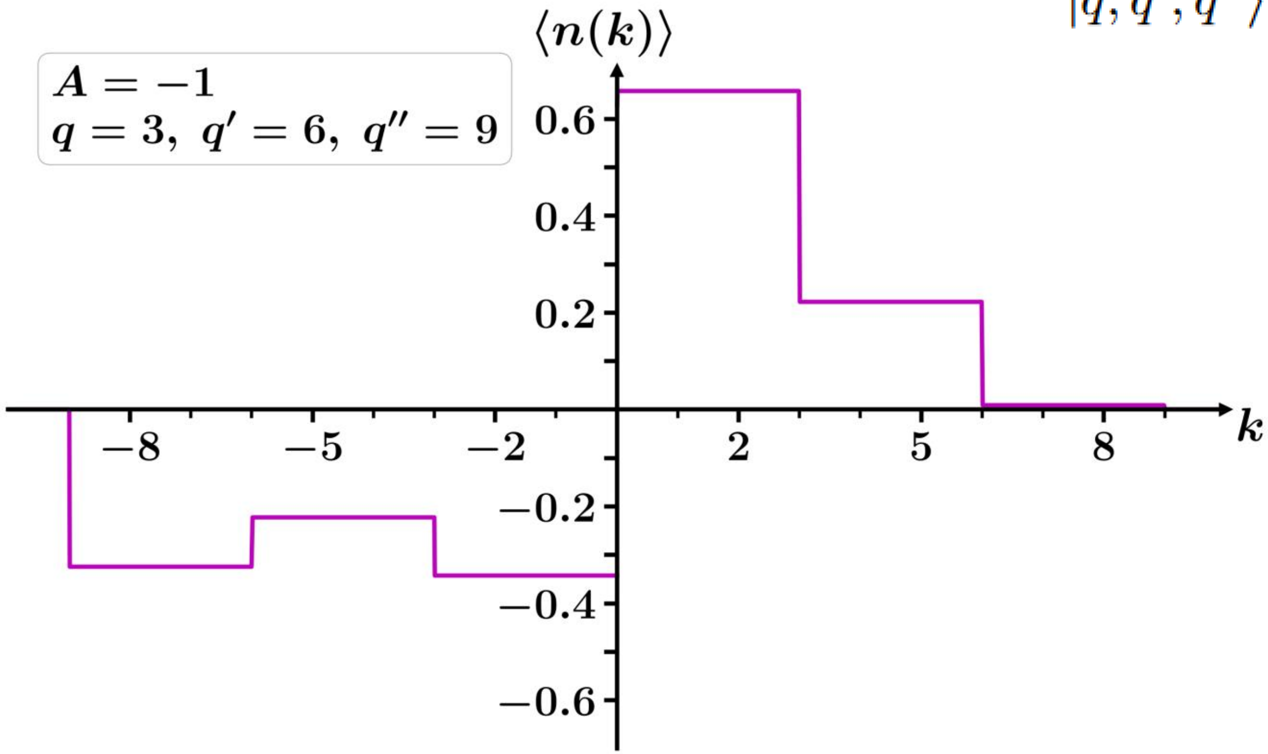

Summary

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- Numerically computed the distribution of excited electrons and holes
- Found a surprising asymmetry between the distribution of electrons and holes, whose physical interpretation remains open for further investigation
- Showed that an asymmetry can already arise when mixing one- and two-plasmon states

Thank you for your attention

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- Distribution of the injected electron
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Particle number conservation

$$\int_{-\infty}^{\infty} \mathrm{d}\varepsilon_{\mathrm{f}}\, \pi_{\mathrm{FS}}(\varepsilon_{\mathrm{f}}, \xi_s) = 0,\ \pi_{\mathrm{FS}} = \frac{v}{\lambda} p_{\mathrm{FS}}$$

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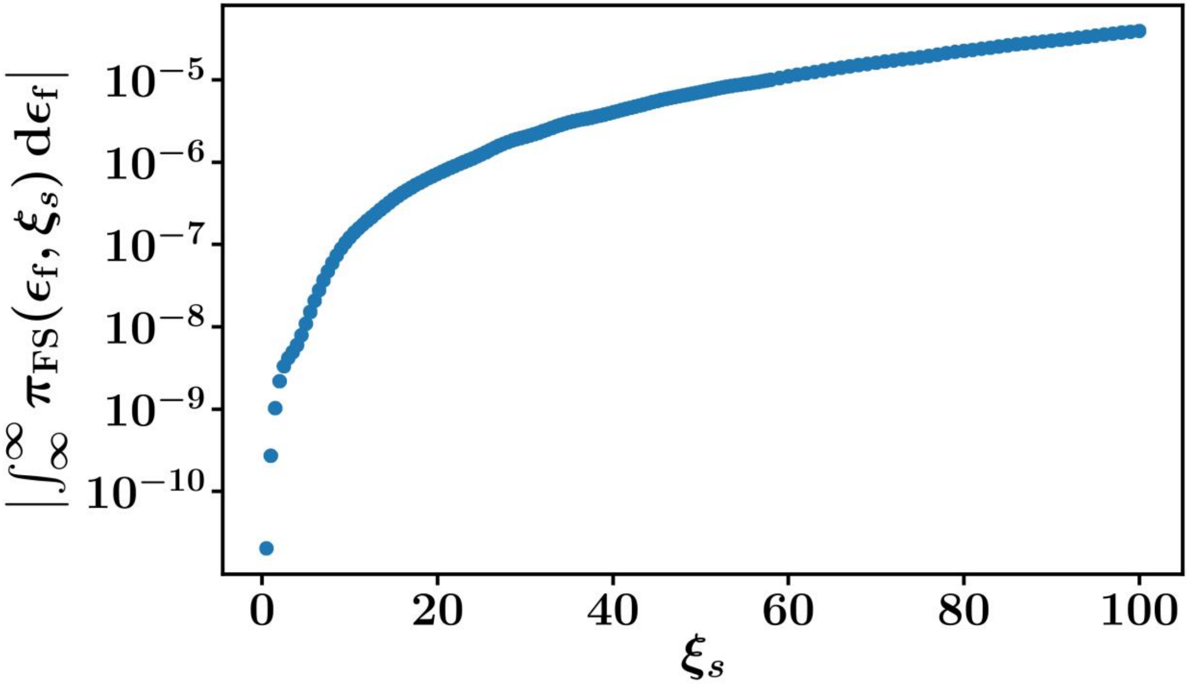

Energy conservation

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The energy stored in the Fermi-sea excitations should conincide with energy loss of the injected electron:

$$\int_{-\infty}^{\infty} \mathrm{d}E_{\mathrm{f}}\, E_{\mathrm{f}} \cdot p_{\mathrm{FS}}(E_{\mathrm{f}}, x_s) = \int_{-\infty}^{\infty} \mathrm{d}E_{\mathrm{if}}\, E_{\mathrm{if}} \cdot p_{\mathrm{HE}}(E_{\mathrm{if}}, x_s)$$

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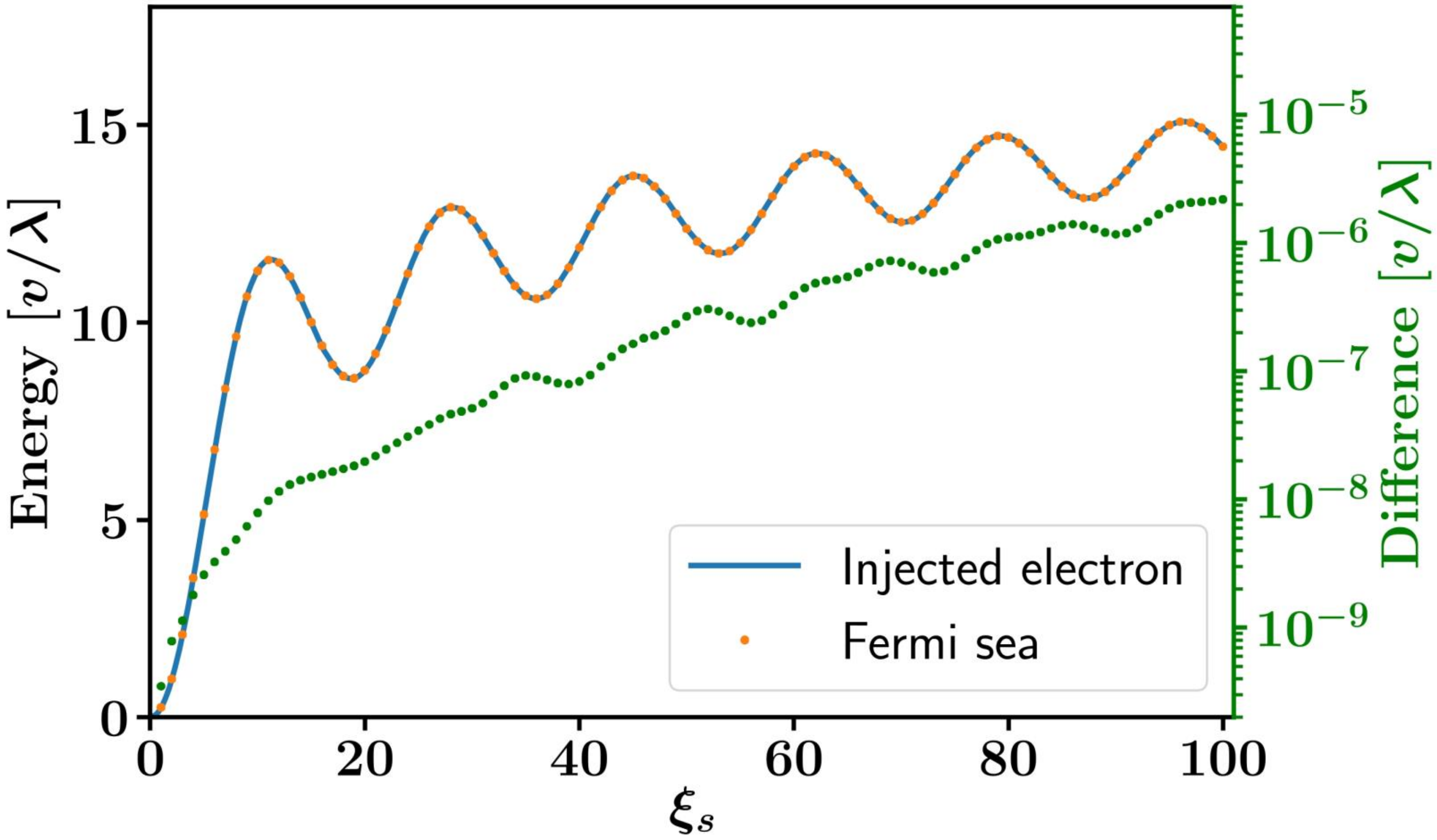
From S. Fischer et al.:

$$p_{\mathrm{HE}}(\varepsilon_{\mathrm{if}}, \xi_s) = \delta(\varepsilon_{\mathrm{if}}) \frac{\lambda}{v} \exp\left(\sum_{n=1}^{\infty} \frac{\xi_s^{2n}}{n} \frac{1}{(2ni)^{2n}} \right) + \frac{\lambda}{v} \int_{\square} \frac{\mathrm{d}Y}{2\pi} e^{i\varepsilon_{\mathrm{if}} Y} \exp\left(\sum_{n=1}^{\infty} \frac{\xi_s^{2n}}{n} \left[\frac{1}{(2ni)^{2n}} - \frac{1}{(Y-2ni)^{2n}} \right] \right)$$

Energy conservation II



Non-equilibrium Green functions

Contour-ordered Green function at zero temperature:

$$G(x,x',t,t') = -i\langle T_{\mathcal{C}}[\psi(x,t)\psi^\dagger(x',t')]\rangle$$

$T_{\mathcal{C}}$ sorts operators that come earlier on the contour to the right

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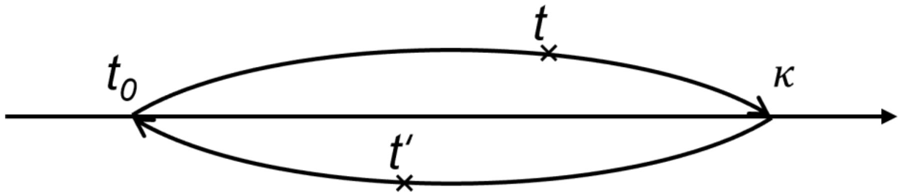

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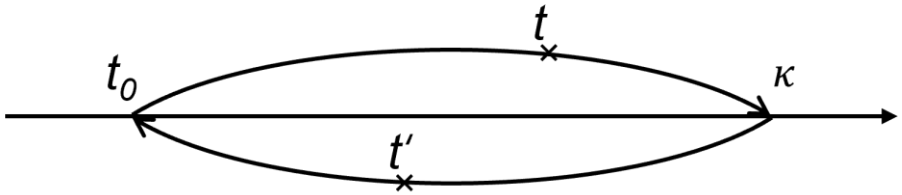

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$$G^a(x,x',t,t') = G^t_c(x,x',t,t') - G^>_c(x,x',t,t') = G^{\bar{t}}_c(x,x',t,t') - G^<_c(x,x',t,t')$$

Appendix: Two velocity model

v

$\bar{v}$

λ_c

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High-energy electrons move with the Fermi velocity v, low-energy bosonic excitations move with renormalized velocity $\bar{v}$

λ_c

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Consider weak interactions with effective range λ_c and assume the channel Green functions are given by

$$\tilde{G}_{\mathrm{c}}^{\gtrless}(x,t) = \frac{1}{2\pi} \frac{1}{x - vt \pm i\delta} \frac{x - vt \pm i\lambda_c}{x - \bar{v}t \pm i\lambda_c}$$

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In the limit of high injection energies, the current is then

$$I(t) = \frac{e}{\hbar}\eta_1^2\eta_2^2\left[4\pi\Theta(E_{\mathrm{f}})\frac{e^{-E_{\mathrm{f}}\frac{2\lambda_c}{\bar{v}}}}{3v\bar{v}^3}\frac{\lambda_c(x^2+\lambda_c^2)(\bar{v}-v)^2(2v+\bar{v})}{[x(\bar{v}-v)]^2+[\lambda_c(\bar{v}+v)]^2}\right.$$
$$\left.-12\pi\Theta(-E_{\mathrm{f}})\frac{\lambda_c(x^2+\lambda_c^2)(\bar{v}-v)^2 e^{E_{\mathrm{f}}\frac{2\lambda_c}{v}}}{v^2\,(2\bar{v}+v)\,[x^2(\bar{v}-v)^2+\lambda_c^2(\bar{v}+v)^2]}\right]$$

Appendix: Two velocity model

Define $x_s = x(\bar{v} - v)/v$

Then the scaling limit is $\bar{v} \to v,\, x \to \infty,\, x_s = \text{const}$

And the distribution of excited electrons and holes becomes

$$p_{\text{FS}}(E_\text{f}, x_s) = \left[\Theta(E_\text{f}) \frac{2\lambda_c}{v} \frac{x_s^2}{x_s^2 + 4\lambda_c^2} e^{-E_\text{f}\frac{2\lambda_c}{v}} - \Theta(-E_\text{f}) \frac{2\lambda_c}{v} \frac{x_s^2}{x_s^2 + 4\lambda_c^2} e^{E_\text{f}\frac{2\lambda_c}{v}}\right]$$

Appendix: Two velocity model

For small ξ_s the two velocity solution is in good agreement with the one of the exponential interaction model