

# Fluidlike behavior in small atomic systems

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5<sup>th</sup> October, 2022

**Based on:** [Floerchinger, Giacalone, Heyen, Tharwat, **PRC 105, 044908 (2022)**]  
[Brandstetter, Heintze, Lunt, Subramanian, Holten, Jochim, Heyen, Giacalone, Floerchinger, **in preparation**]



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1.

Collectivity under extreme conditions

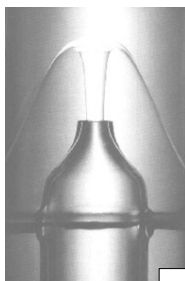
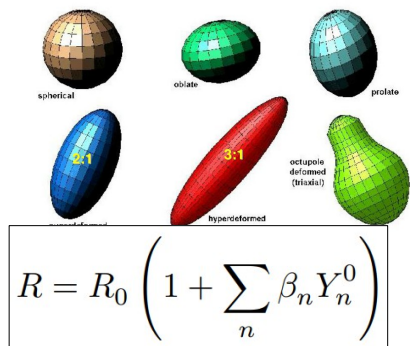
Emergent phenomena are among the most interesting in Nature.

“More is different”, [P. Anderson, 1972]

<https://en.wikipedia.org/wiki/Emergence>

## Examples from nuclear / cold atom physics:

### Nuclear deformations



$$\eta = 0$$

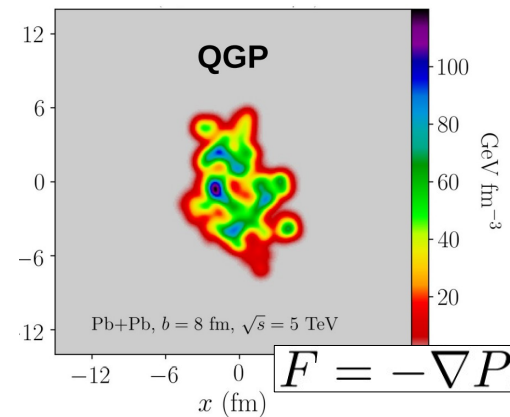
Superfluidity

### Superconductivity



$$R = U/I = 0$$

### quark-gluon plasma



**Focus of this talk:** Hydrodynamics, a prime example of emergent (macroscopic) behavior.

$$\begin{array}{ccc} \text{global equilibrium} & \longrightarrow & \text{local equilibrium} \\ \rho, T = \text{const} & \longrightarrow & \rho(t, x), T(t, x) \end{array}$$

+ conservation laws for mass / momentum / energy

$$\begin{aligned} \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{u}) &= 0 \\ \rho (\partial_t + \vec{u} \cdot \vec{\nabla}) \vec{u} + \vec{\nabla} P &= 0 \\ (\partial_t + \vec{u} \cdot \vec{\nabla}) e + (e + P) (\vec{\nabla} \cdot \vec{u}) + \vec{\nabla} \cdot \vec{q} &= 0 \end{aligned}$$

—► few assumptions for a very powerful framework

**Focus of this talk:** Hydrodynamics, a prime example of emergent (macroscopic) behavior.

$$F = -\nabla P.$$

Emergence in a particle system via collisions (kinetic theory).

The *pressure tensor* is defined as the fluctuation of the velocities of the ensemble from the mean velocity, i.e. as the 2-nd order moment:

$$P = m \int (\mathbf{v} - \mathbf{v}_b)(\mathbf{v} - \mathbf{v}_b) f(\mathbf{v}) d^3v$$

Emergence of superfluid motion in BEC (no collisions, but due to interactions in a Fermi gas).

$$\frac{\partial}{\partial t} n + \nabla \cdot (\mathbf{v}_S n) = 0$$

$$m \frac{\partial}{\partial t} \mathbf{v}_S + \nabla \cdot \left( \frac{1}{2} m \mathbf{v}_S^2 + \mu(n) + V_{ext} \right) = 0$$

pressure ↗

Hydrodynamic equations of superfluids (T=0)  
Closed equations for  $n$  and  $\mathbf{v}_S$

[from S. Stringari,  
Lectures at Collège de France (2004/2005)]

Both situations require a macroscopic scenario, i.e., very large particle numbers.

**Frontier:** behavior of mesoscopic systems? What if the particle number is small?

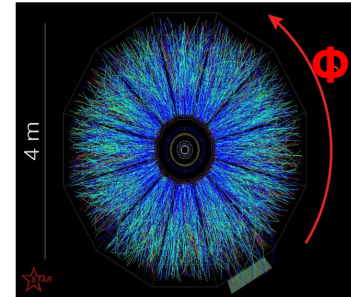
**Tool to probe hydrodynamic behavior:** Elliptic flow. Shape inversion of the gas due to asymmetry in pressure-gradient force.

[Ollitrault, PRD 46 (1992) 229-245]

Does not really matter whether system is superfluid or collisional.

**In heavy-ion collisions:**  
2<sup>nd</sup> Fourier harmonic of the azimuthal particle distribution.

$$V_2 = \frac{1}{N} \int_{\mathbf{p}_t} \frac{dN}{d^2\mathbf{p}_t} e^{-i2\phi_p}$$



**Realistic application:** ideal Fermi gas in 2D at zero temperature.

2.

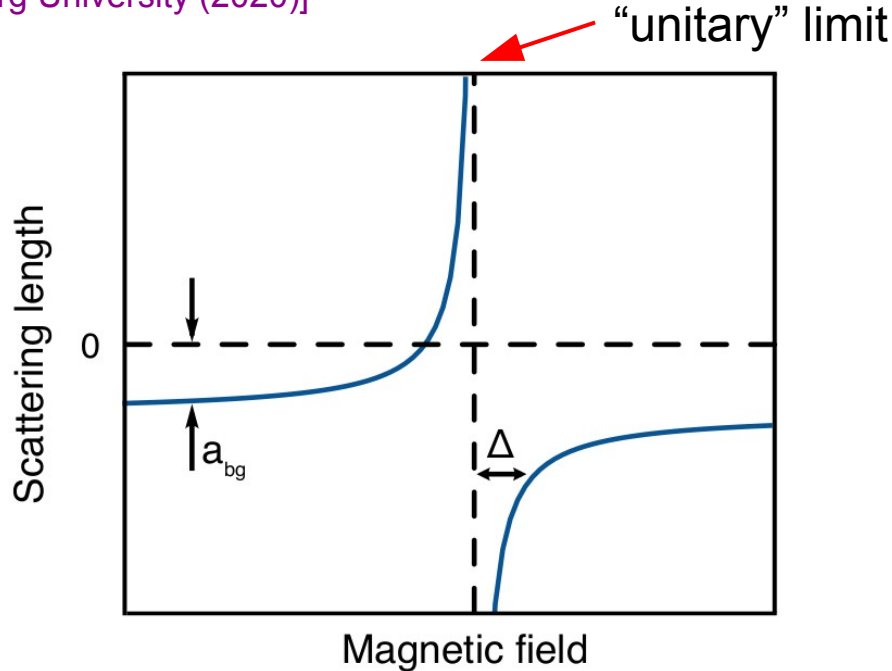
Cold atom gases as a probe of hydrodynamics

## Why ultracold atom gases? **First reason:** interactions are tunable.

Interactions at low momenta described by an s-wave scattering length parameter.

Tunable via a Feshbach resonance in presence of an external magnetic field.

[from Luca Bayha, PhD thesis,  
Heidelberg University (2020)]



$$a_{3D} = a_{bg} \left( 1 + \frac{\Delta}{B - B_0} \right)$$

**Values for lowest states of  ${}^6\text{Li}$ :**

$$a_{bg} = -2100 a_{\text{Bohr}}$$

$$B_0 = 690 \text{ G}$$

$$\Delta = 200 \text{ G}$$

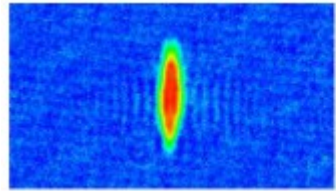
**We can move from non-interacting to strongly-interacting systems.**



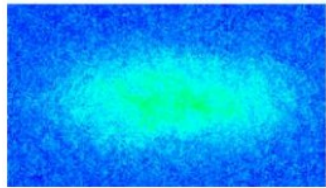
**Second reason:** we can play with the geometry of the system.

**Elliptic flow used to reveal superfluid behavior of an ultracold Fermi gas.**

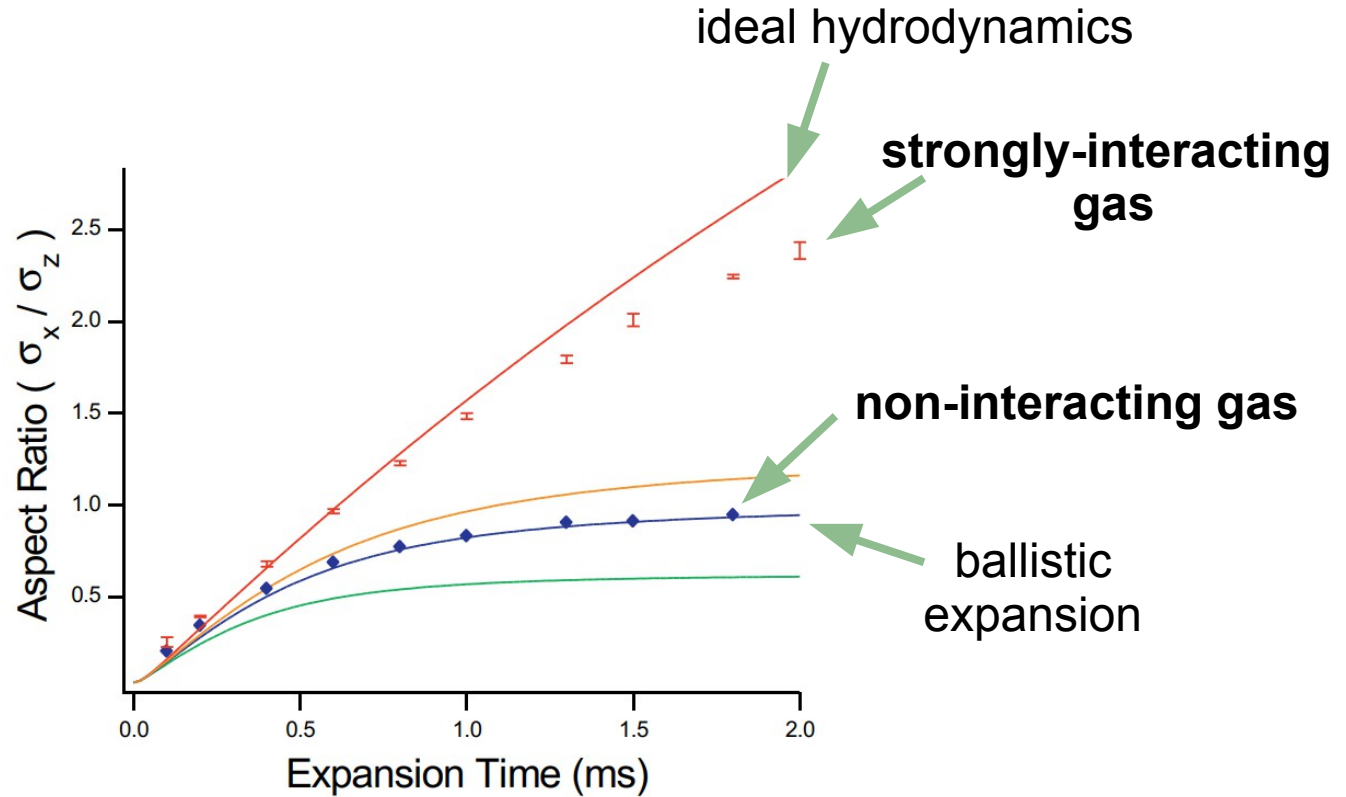
**NB:** at low temperature, strongly-interacting Fermi gas is a gas of pairs (effective bosons).



100  $\mu\text{s}$



2000  $\mu\text{s}$

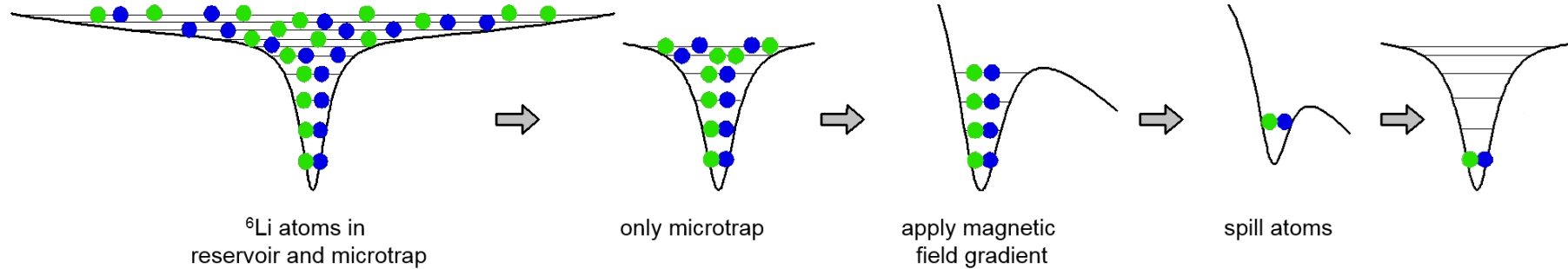


[Menotti, Pedri, Stringari, *PRL* **89**, 250402 (2002)]  
[O'Hara et al., *Science* **298** (2002) 2179-2182]

## Third reason [focus of this talk]: much less explored, the particle number is tunable!

Going “cold” brings dramatic advantages. Effective control over the number of particles.

[Serwane et al., *Science* 332 (2011) 6027]



[from <http://ultracold.physi.uni-heidelberg.de/02research/>]

## Transition from few-body to many-body physics.

### Our proposal:

Study elliptic flow to assess emergent hydrodynamic behavior as a function of particle number (in two dimensions).

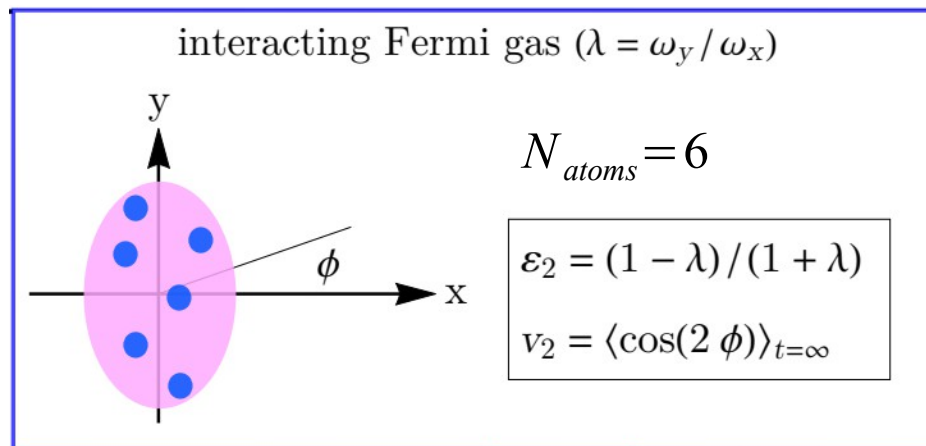
3.

## Elliptic flow as a function of particle number

[Floerchinger, Giacalone, Heyen, Tharwat, **PRC 105, 044908 (2022)**]

## Measuring elliptic flow in mesoscopic samples.

- 1 – Resort to a statistical description, i.e., repeat the experiment many times like in heavy-ion collisions.
- 2 – Unlike in heavy-ion collisions, the orientation of the ellipse and the initial ellipticity,  $\epsilon_2$ , can be chosen.
- 3 – Let the system expand and measure the anisotropy of the system (e.g.  $\langle \cos 2\Phi_p \rangle$ ) with respect to the fixed axis.
- 4 – Repeat the experiments for different number of atoms in the cloud.



Imposing an elliptical potential has a strong impact on the initial momentum distribution.

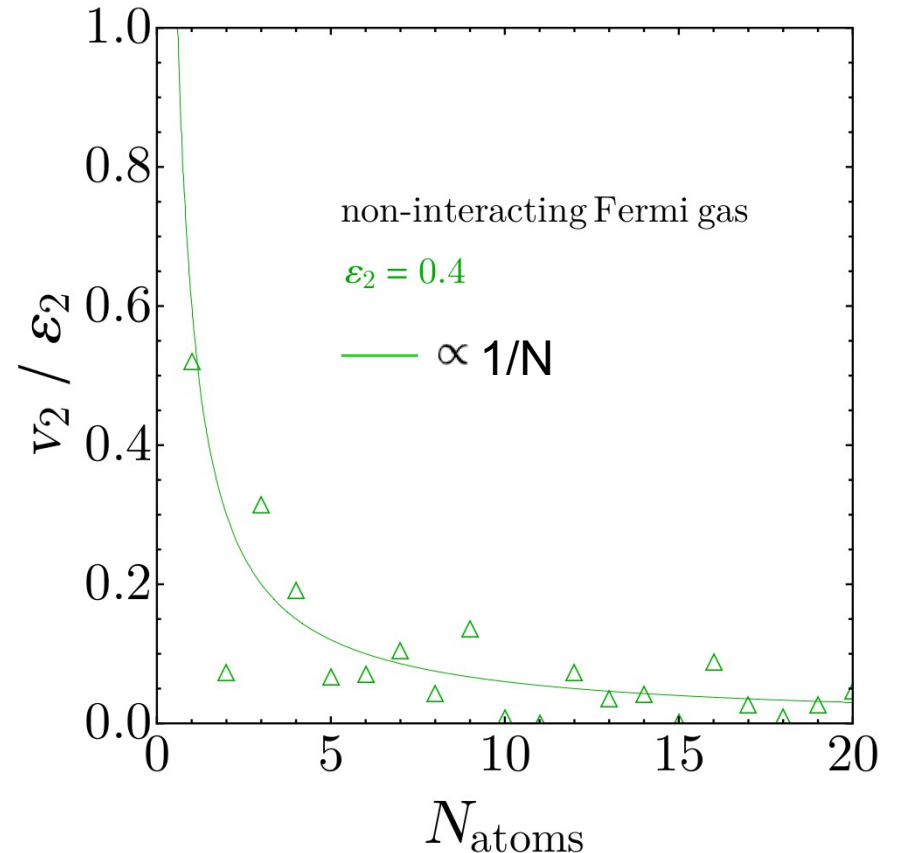
$$\Delta p_x \Delta x \geq \frac{h}{4\pi}$$

Calculate  $v_2$  from the quantum harmonic oscillator (initial momentum anisotropy).

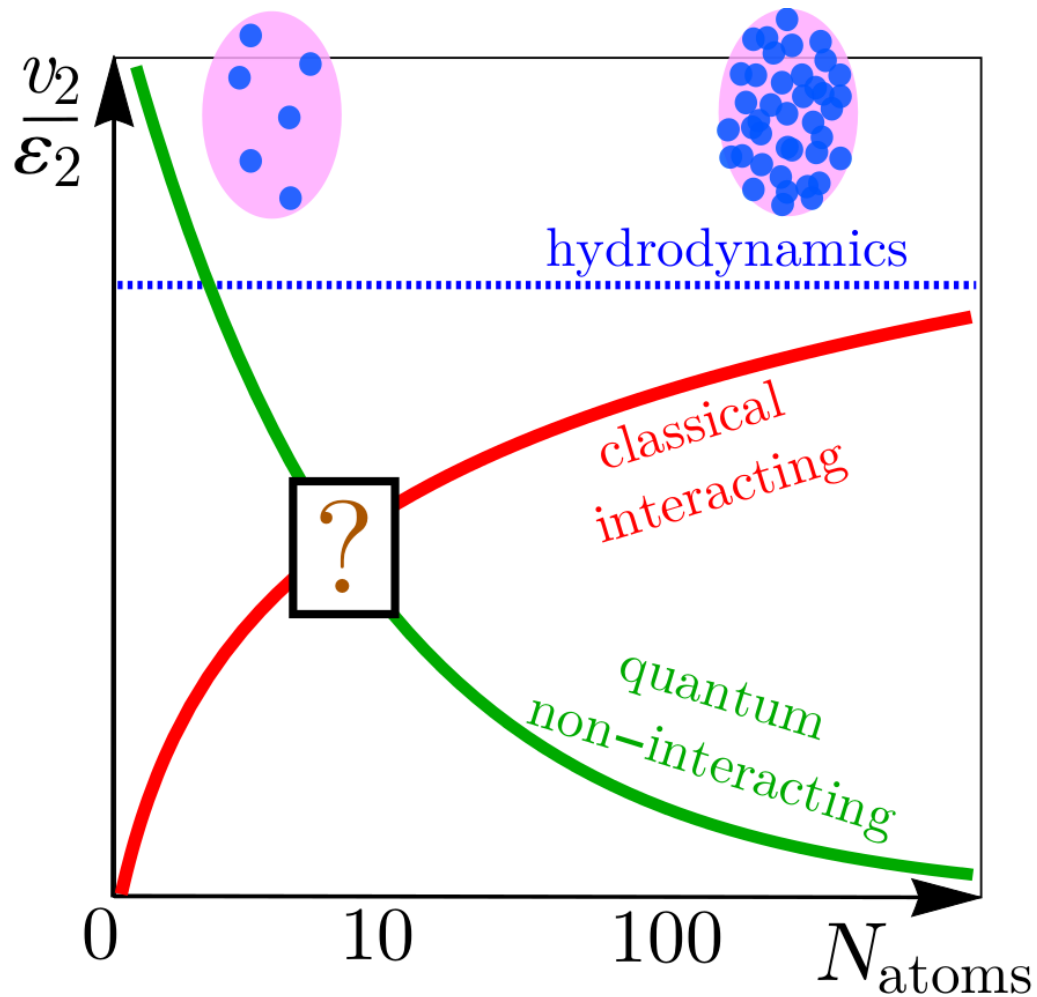
$$v_2 = \langle \cos(2\phi_p) \rangle_{\Psi}$$

trapped  
non-interacting  
fermions

Very important if we only have 2-3 particles.  
**However, it disappears quickly, like  $1/N$ .**



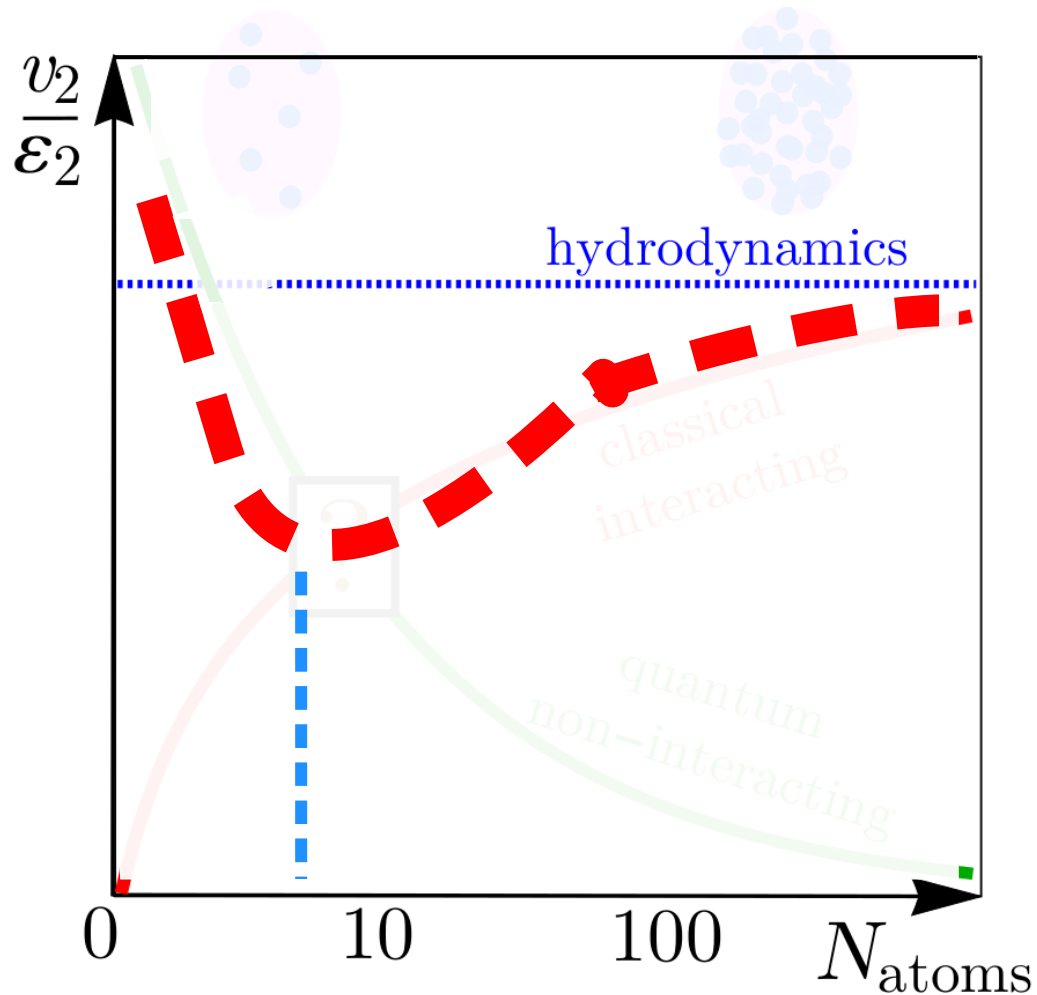
Qualitative expectations.



Combining the curves...

Could there be a minimum?

Transition from quantum effects  
to interaction effects?



# 4. Experiment





Few-body experiments run at the  
Physics Institute of Heidelberg University.

<http://ultracold.physi.uni-heidelberg.de/>

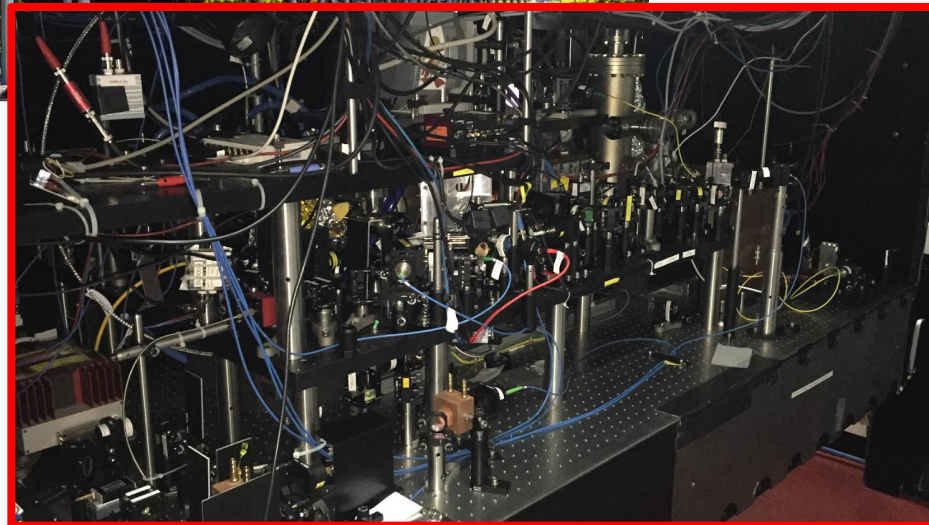
**Main collaborators:**

Selim Jochim (PI)

Sandra Brandstetter (PhD student)

Carl Heintze (PhD student)

Philipp Lunt (PhD student)



Atoms trapped on  
this optical bench.

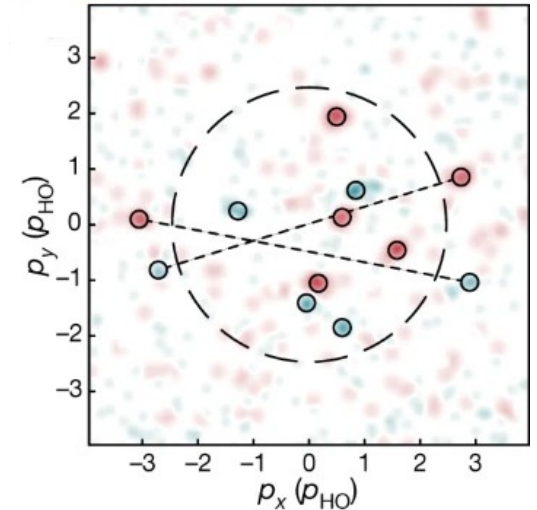
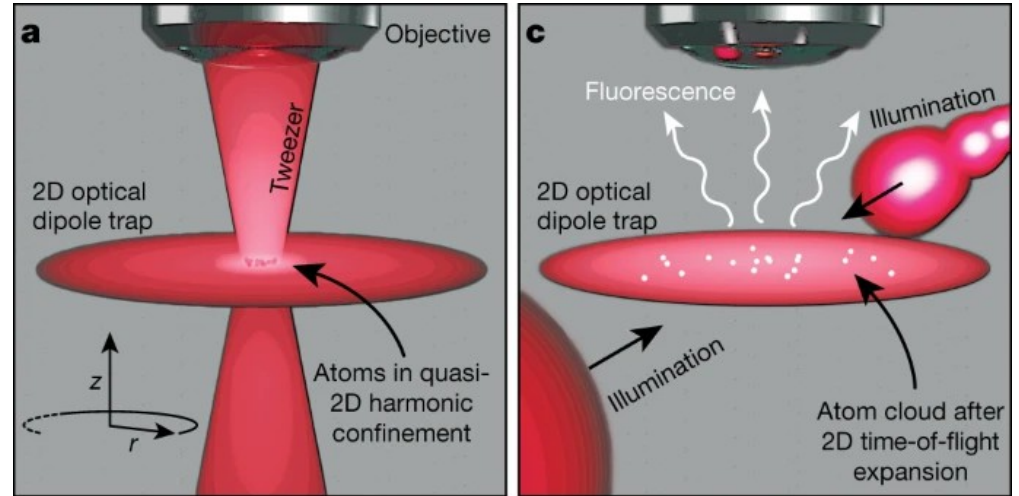
[from Holten et al., *Nature* 606, 287-291 (2022)]

Unique method to determine atom positions and momenta in an expanding cloud.

[Bergschneider et al., *PRA* 97, 063613 (2018)]

For each atom one detects about 20 photons per  $20\mu\text{s}$  of exposure.

**Localization fidelity:**  $99.4 \pm 0.3\%$

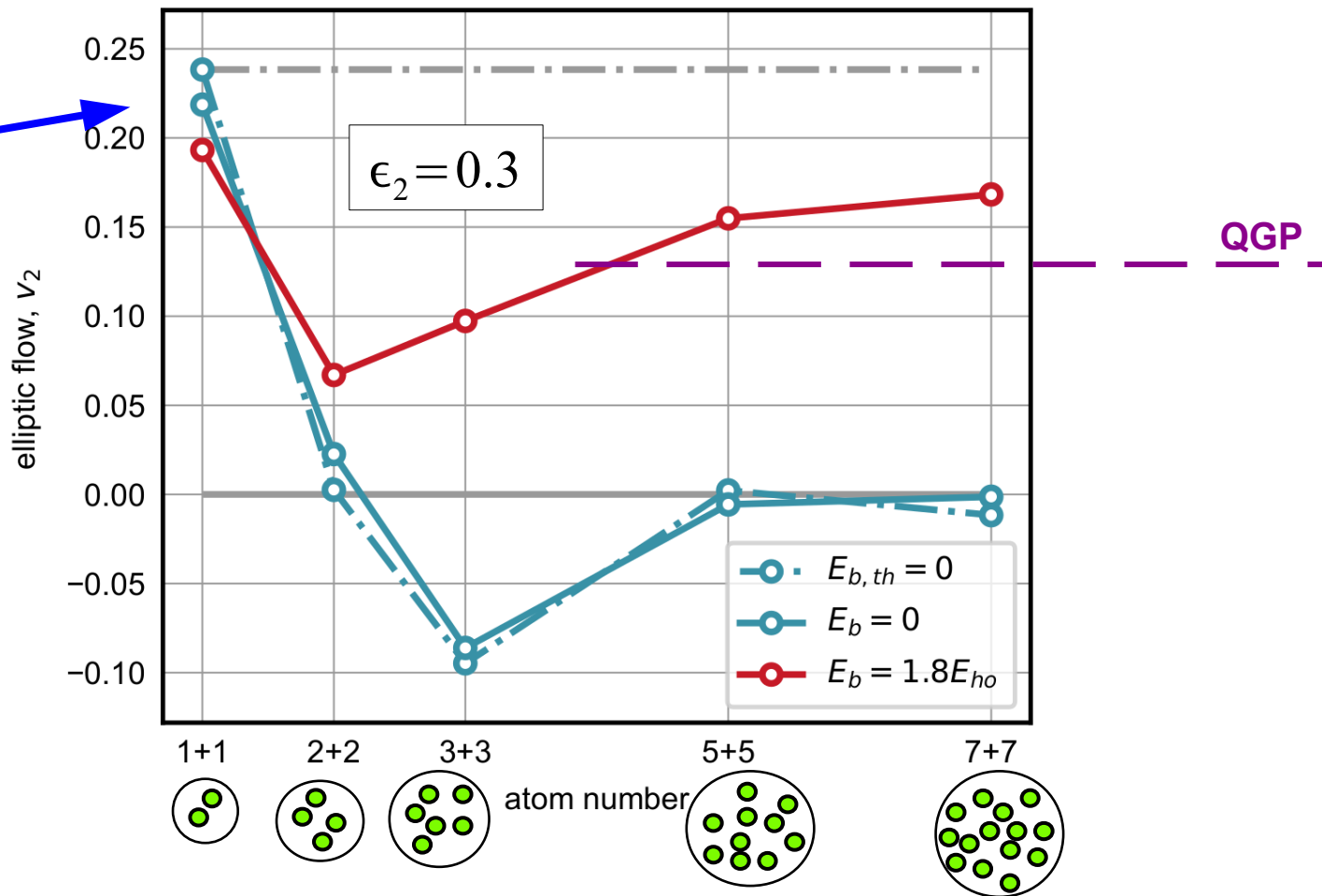


5.  
Preliminary results

## $v_2$ as a function of particle number.

The conjectured minimum is observed!

QUANTUM  
EFFECTS

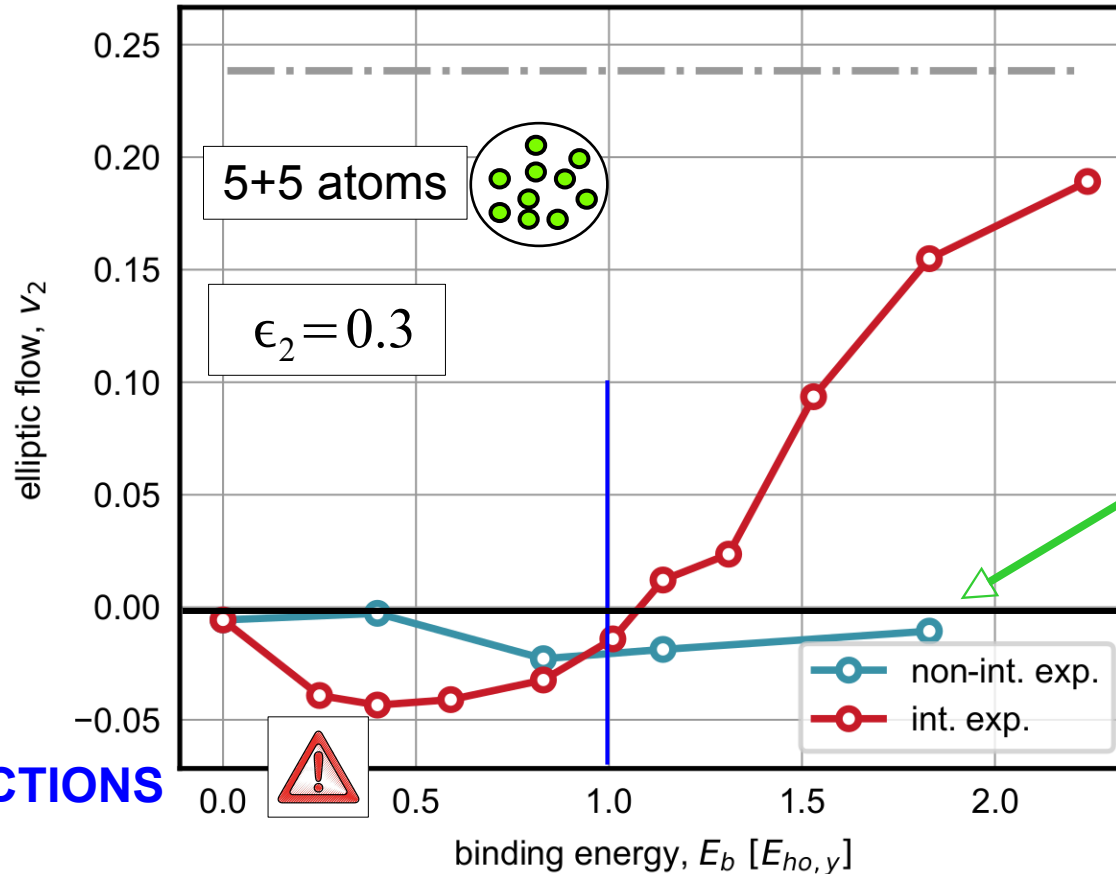


# Emergence of $v_2$ from interactions.

$E_b$  = binding energy of one pair  
 $E_{ho}$  = harmonic oscillator energy

When  $E_b = E_{ho}$ , bosonic pairs appear. Onset of superfluidity?

Energy dependence of effective viscosity? (superfluid = zero viscosity)



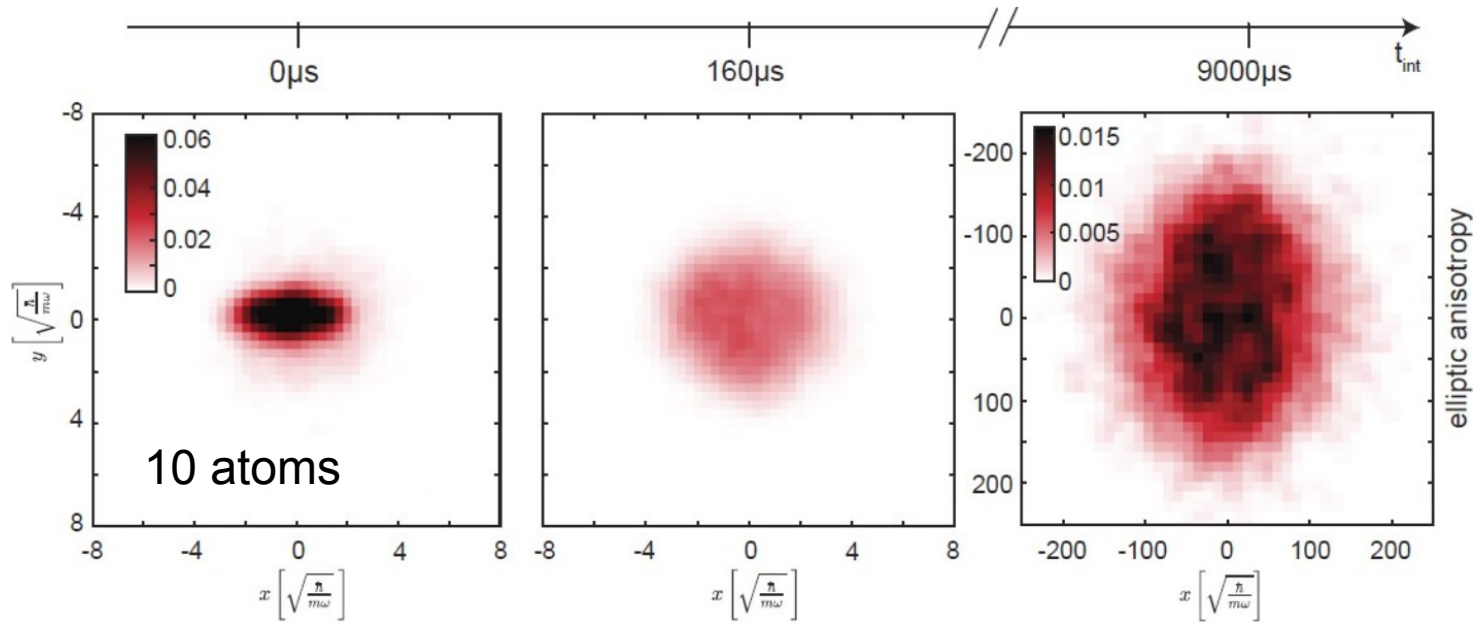
Interactions switched off right after trap is released.

Remaining  $v_2$  is from the initial state, i.e., the initial momentum anisotropy.

**NO INTERACTIONS**

**STRONG INTERACTIONS**

# Temporal evolution of the shape of the average cloud. (coordinate space)



Is this a fluid expansion?

Hydrodynamic prediction?

$$F = -\nabla P.$$

We only need the pressure.

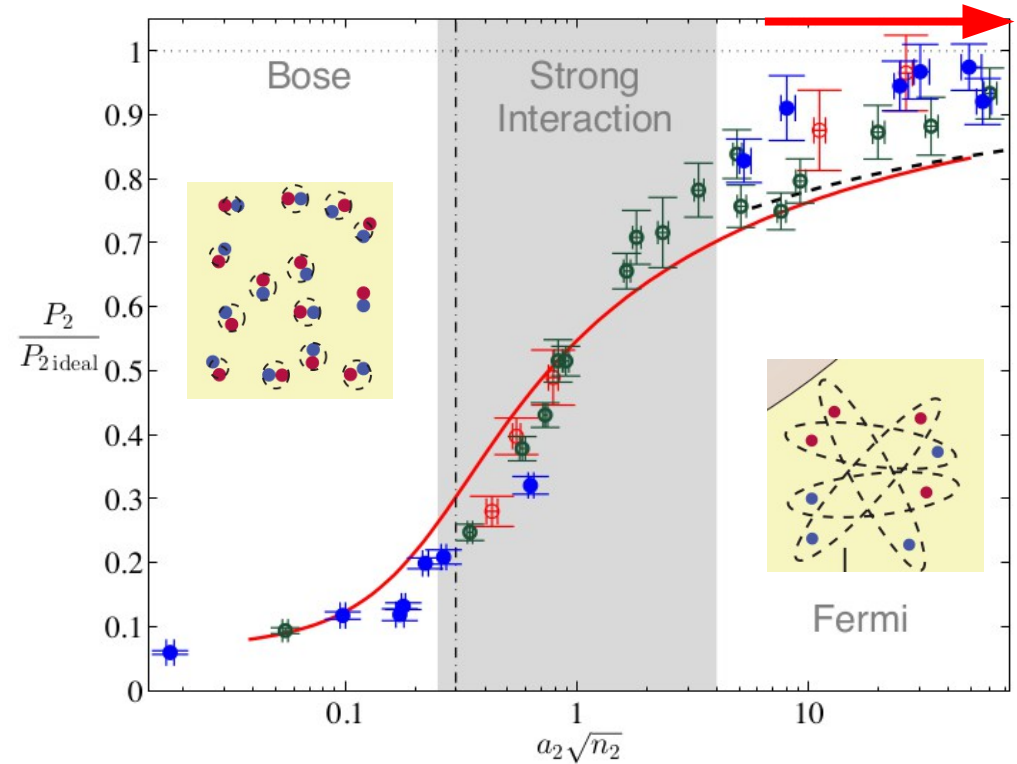
For a 2D Fermi gas at zero temperature, this has been extensively studied.

[Levinsen, Parish, *Annual Review of Cold Atoms and Molecules*, arXiv:1408.2737]

EoS for ideal Fermi gas (Pauli pressure):

$$P_{\text{ideal}} = \frac{\pi \hbar^2}{2M} n^2$$

→ Mass of  ${}^6\text{Li}$



**In the experiment:**

$$a_{2D} \approx 1 \mu\text{m}$$
$$n \approx N/\pi \approx 3 \mu\text{m}^{-2}$$

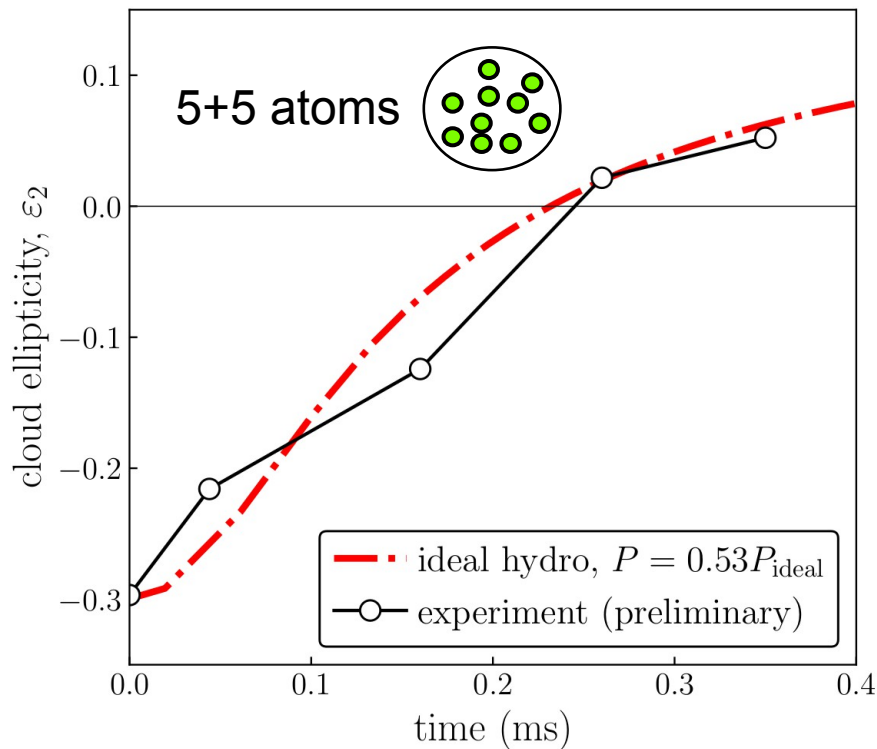
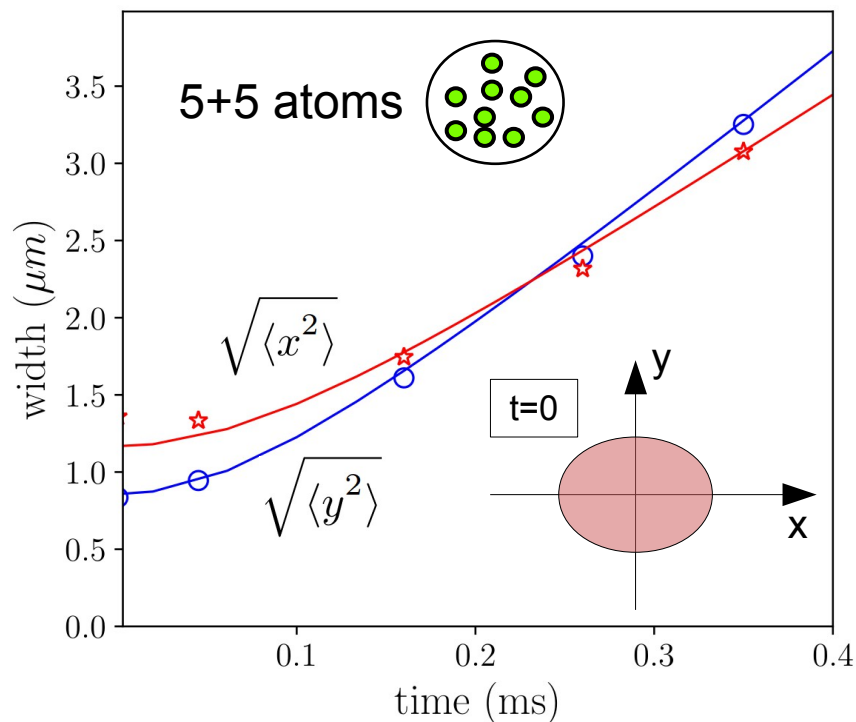
→

$$P \approx 0.53 P_{\text{ideal}}$$

## HYDRODYNAMIC RESULTS

Average shape directly comparable to experimental data.

$$\varepsilon_2 = \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



**A mesoscopic fluid.**

Compressible hydro solver developed at Stony Brook:  
<https://pyro2.readthedocs.io/en/latest/index.html#>



# CONCLUSION

- Hydrodynamics is an emergent behavior observed in systems across vastly different energy scales (superfluids at  $T=0$ , QGP at  $T\sim 10^{12}$  K)
- Cold atom experiments permit us to study emergent hydrodynamic behavior as a function of particle number and tunable interactions.
- Quantum effects leading to elliptic flow vanish quickly with the particle number and as soon as interactions are turned on.
- We observe a large elliptic flow driven by interactions in a cloud of  $N\sim 10$  strongly interacting fermions. **Ideal hydrodynamic results naturally explain the data.**

# PROSPECTS

- Study more observables (e.g. triangular flow, mean momentum).
- Further signals of superfluidity (rotational properties).
- Alternative theoretical descriptions? Microscopic dynamics?

**THANK YOU!**  
**(and stay tuned)**

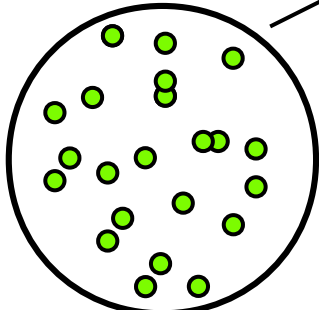
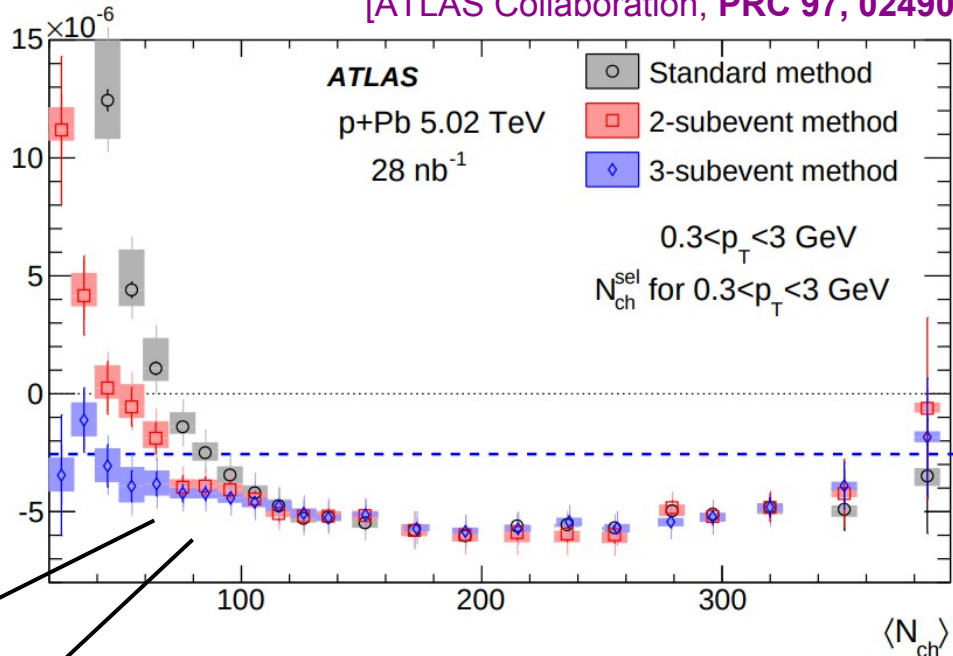
# Backup Slides

**Why mesoscopic systems?** Interesting motivation from high-energy collisions. Signals of collective particle emission at low multiplicities.

$2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle = c_2\{4\}$

genuine 4-body (non-Gaussian) correlation.

[ATLAS Collaboration, PRC 97, 024904 (2018)]

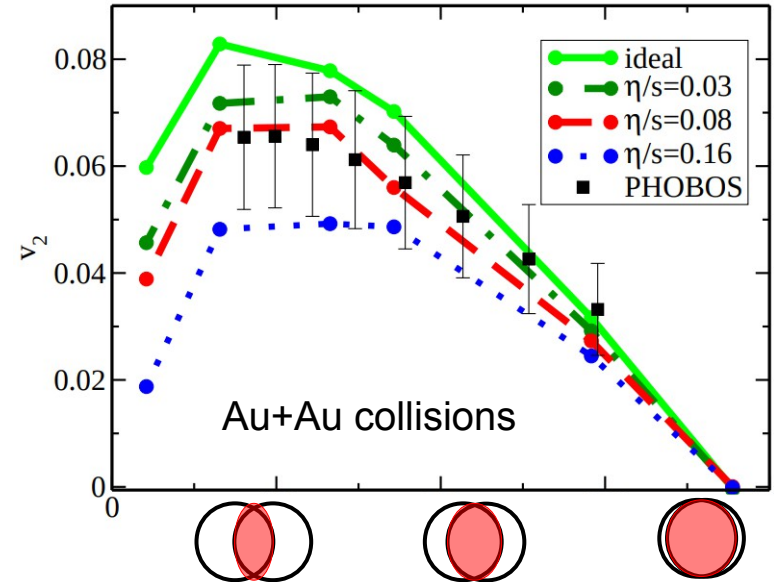


**About 20 strongly-interacting particles per unit rapidity. Can that make a fluid?**

Elliptic flow is sensitive to viscous corrections.

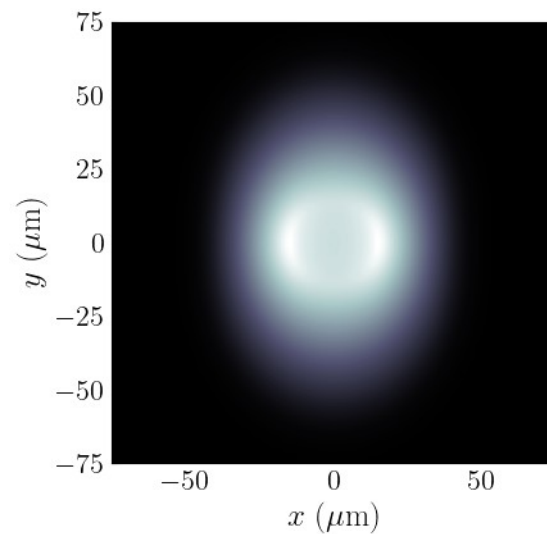
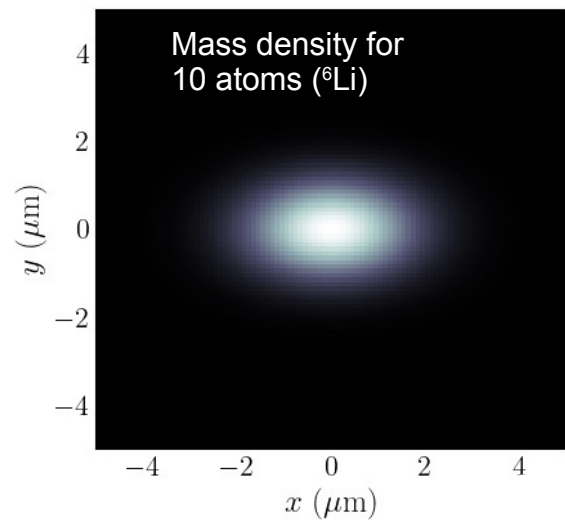
$$\rho \frac{d\mathbf{v}}{dt} = \underbrace{-\vec{\nabla} P}_{1/R} + \underbrace{\eta \vec{\nabla}^2 \mathbf{v} + \vec{\nabla} \left[ \vec{\nabla} \cdot \mathbf{v} \left( \zeta + \frac{2}{3} \eta \right) \right]}_{1/R^2}$$

[Romatschke & Romatschke, PRL 99, 172301 (2007)]



Provides convincing evidence that the QGP behaves like a nearly ideal fluid.

## Examples of hydrodynamic simulation output



## HYDRODYNAMIC RESULTS

At what time scale is  $v_2$  created?

Accurate estimate for relativistic QGP:

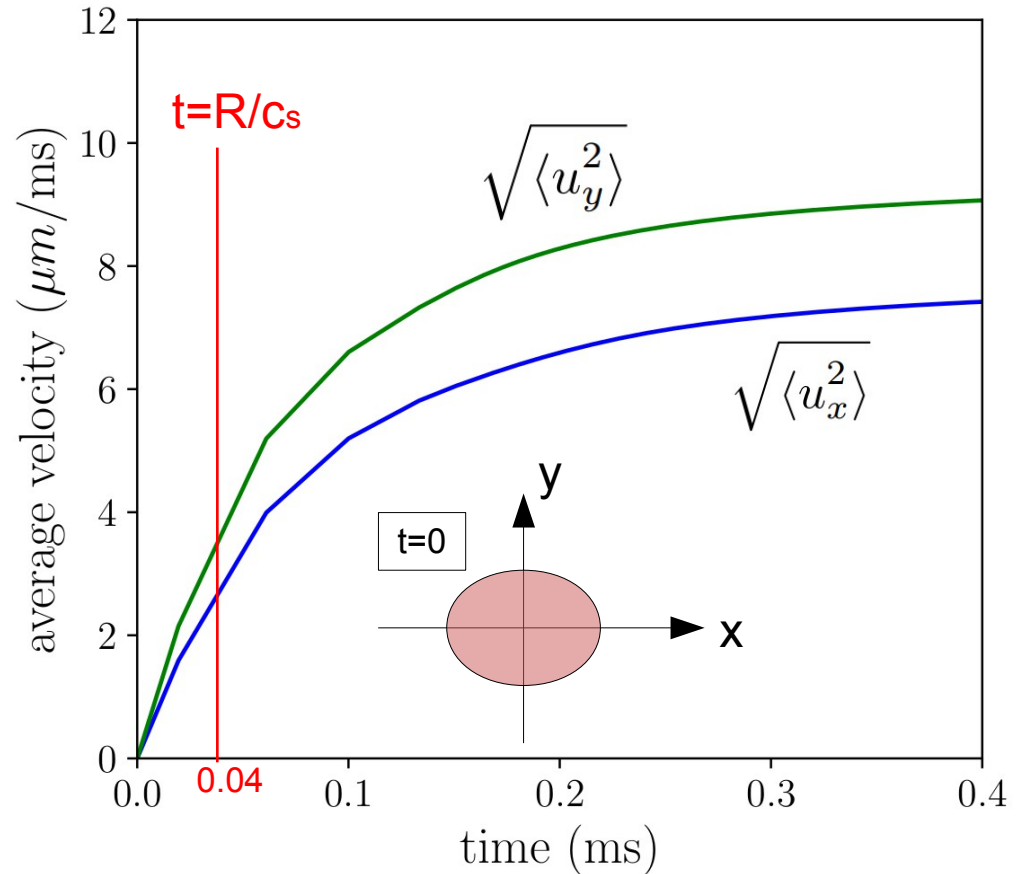
$$R/c_s$$

[Teaney, Yan, **PRC 83 (2011) 064904**]  
[Bhalerao et al., **PLB 627 (2005) 49-54**]

Wrong estimate for a system where  
typical velocity is lower than speed of sound.  
(Mach number  $\sim 0.5$ )

$R \sim 1 \mu\text{m}$      $c_s \sim 25 \mu\text{m/ms}$  ➔

Compressible hydro solver developed at Stony Brook:  
<https://pyro2.readthedocs.io/en/latest/index.html#>



**Momentum in the fluid is built in a time scale  $R/u$  ( $u$  is typical velocity).**



## APPLICATION OF SINGLE ATOM IMAGING:

Observing the Pauli exclusion principle “by eye” (**Pauli crystals**).

[Holten et al., *PRL* 126, 020401 (2021)]

Identify (non-interacting) atoms after some time of flight (magnification).

Rotate images and shift center-of-masses to common reference.

Exclusion principle drives the average geometry away from isotropy.

