Gravitational Favorites:
Formulations, Trumpets and Critical Phenomena

Thomas Baumgarte

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Once upon a proper time. . .

Critical Phenomena in gravitational collapse

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Universality and Scaling in Gravitational Collapse of a Massless Scalar Field

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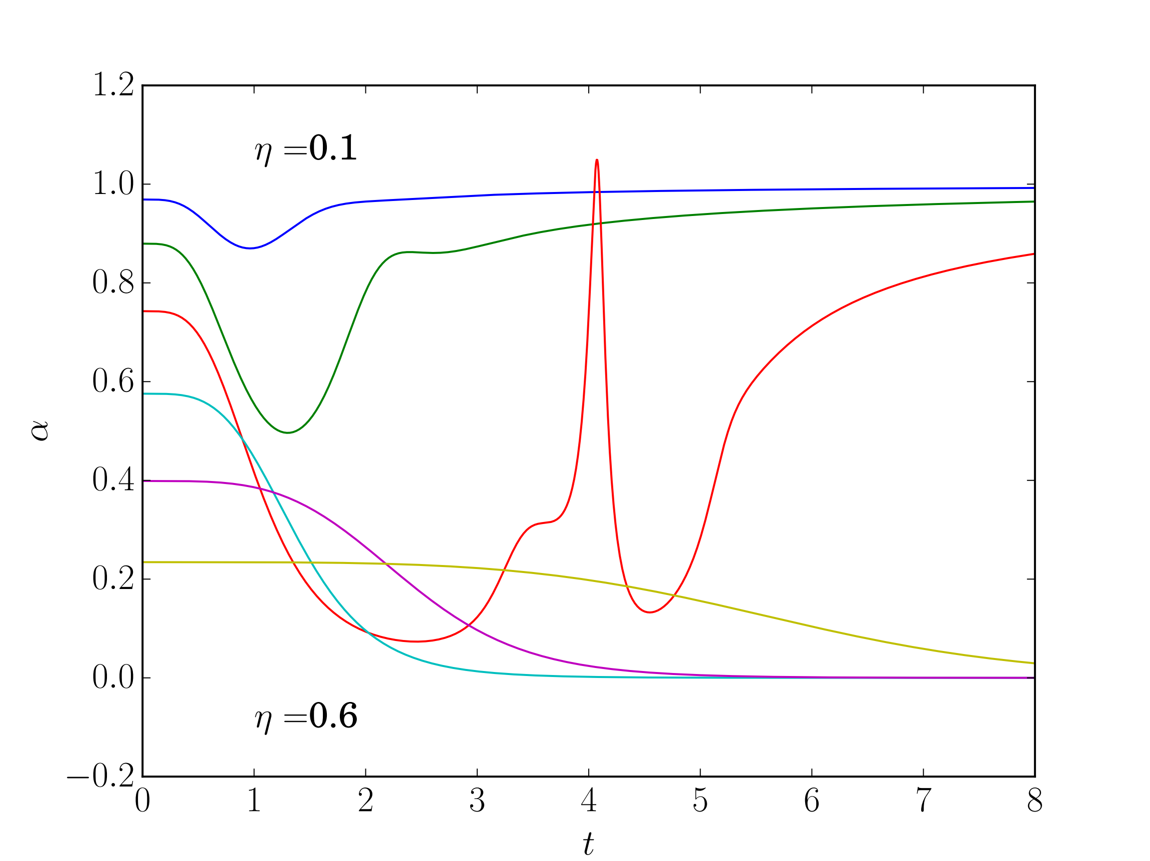
(Received 22 September 1992)

I summarize results from a numerical study of spherically symmetric collapse of a massless scalar field. I consider families of solutions, $\mathcal{S}[p]$, with the property that a critical parameter value, $p^\star$, separates solutions containing black holes from those which do not. I present evidence in support of conjectures that (1) the strong-field evolution in the $p \to p^\star$ limit is universal and generates structure on arbitrarily small spatiotemporal scales and (2) the masses of black holes which form satisfy a power law $M_{\rm BH} \propto |p - p^\star|^\gamma$, where $\gamma \approx 0.37$ is a universal exponent.

A numerical experiment...



- Consider scalar field

$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...

Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3 < \eta_* < 0.4$$

A numerical experiment...

- Let's say scalar field

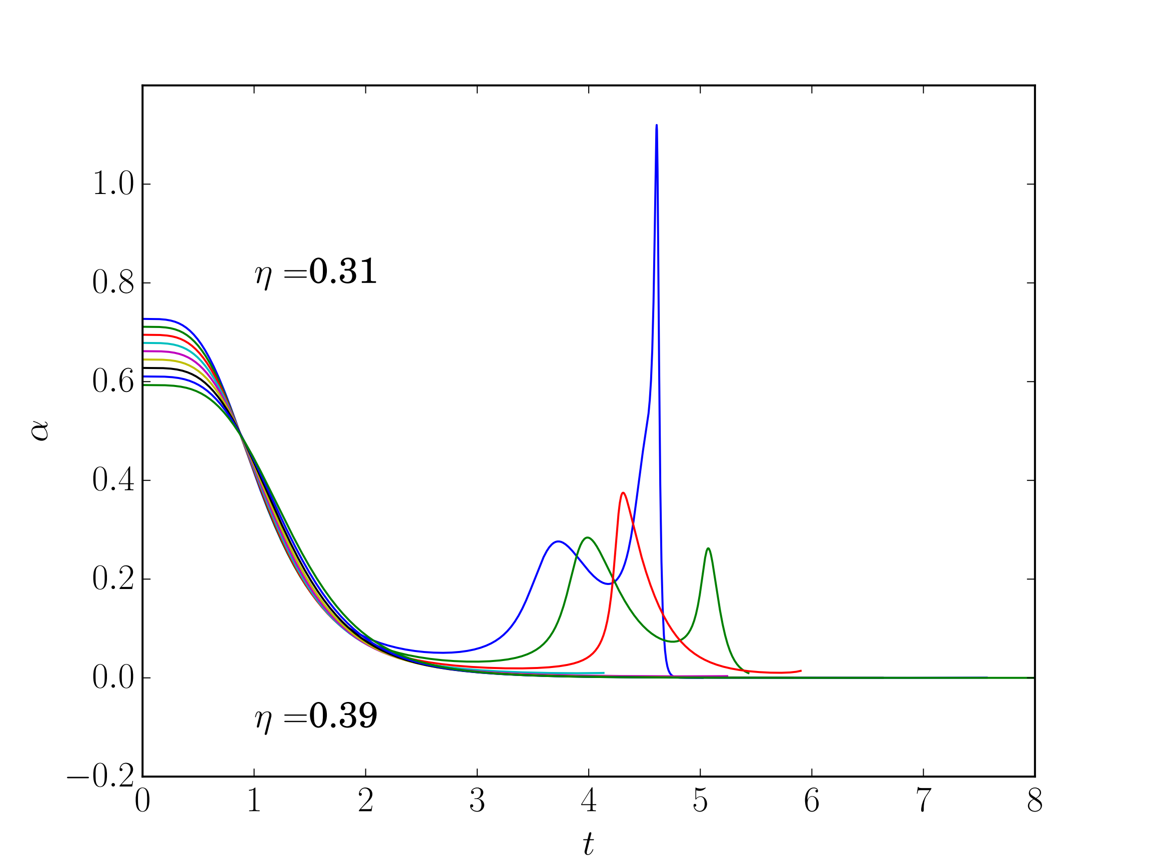
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3 < \eta_* < 0.31$$

A numerical experiment...

- Let's say scalar field

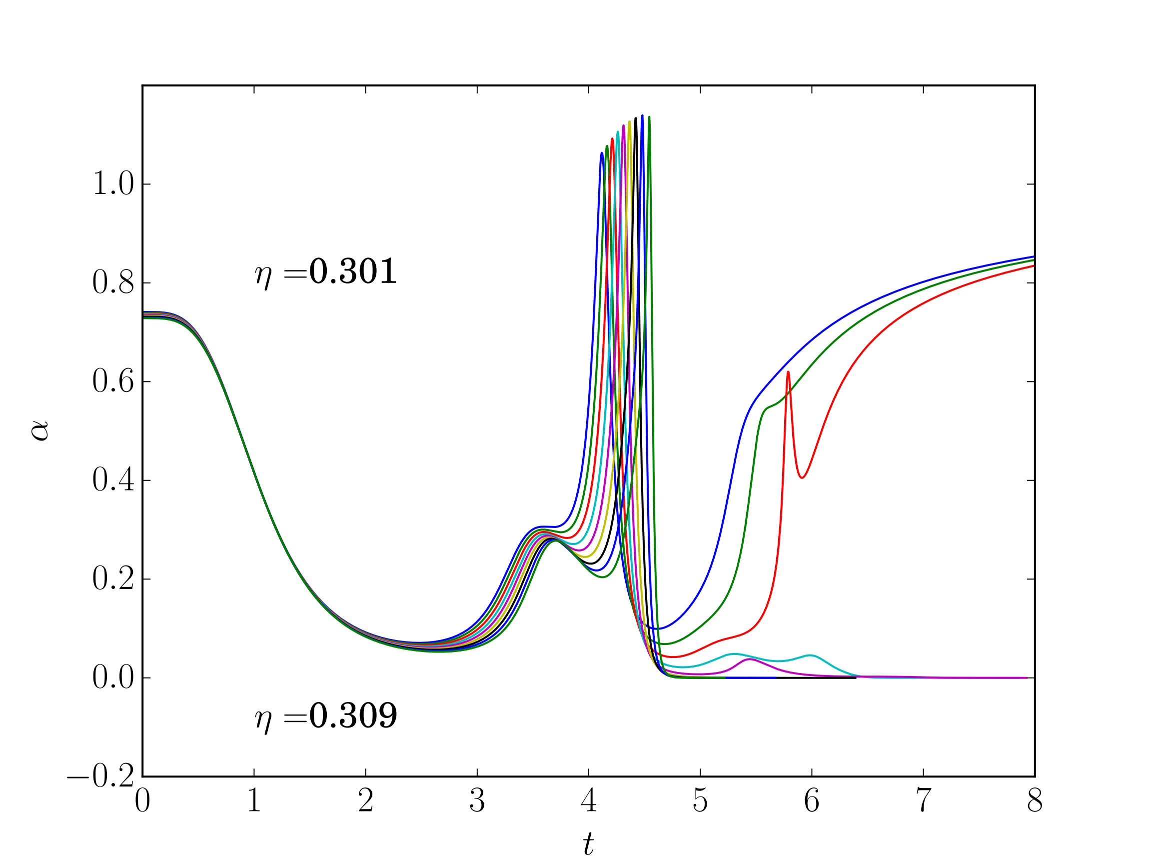
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303 < \eta_* < 0.304$$

A numerical experiment...

- Let's say scalar field

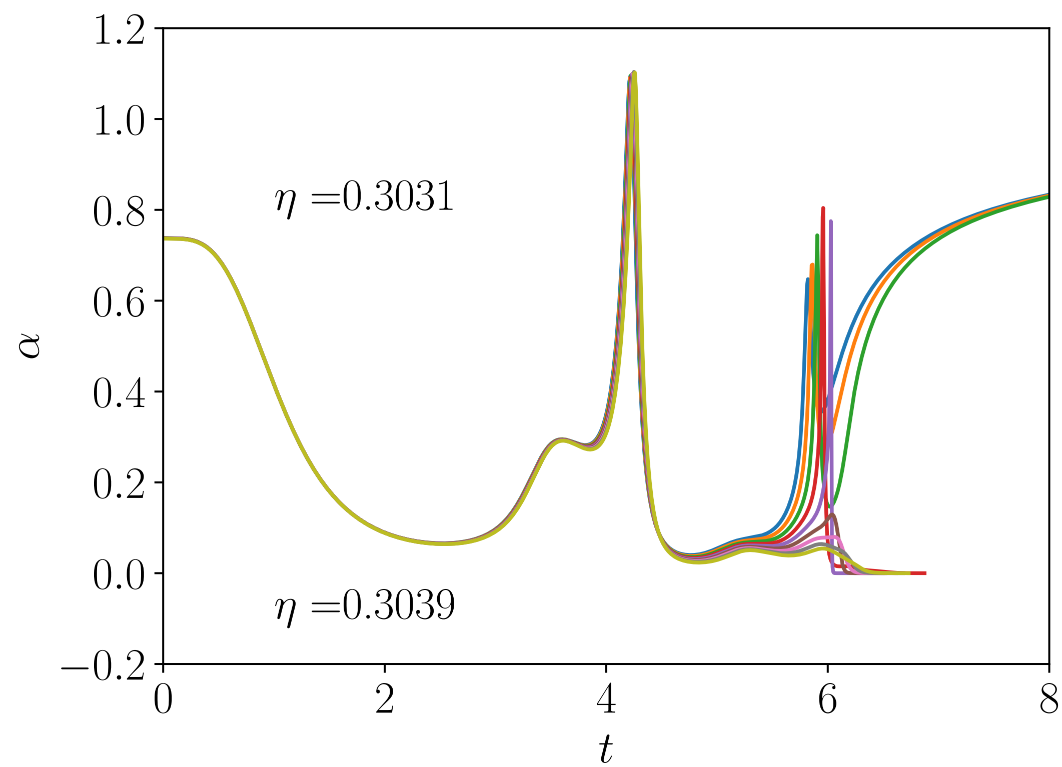
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3033 < \eta_* < 0.3034$$

A numerical experiment...

- Let's say scalar field

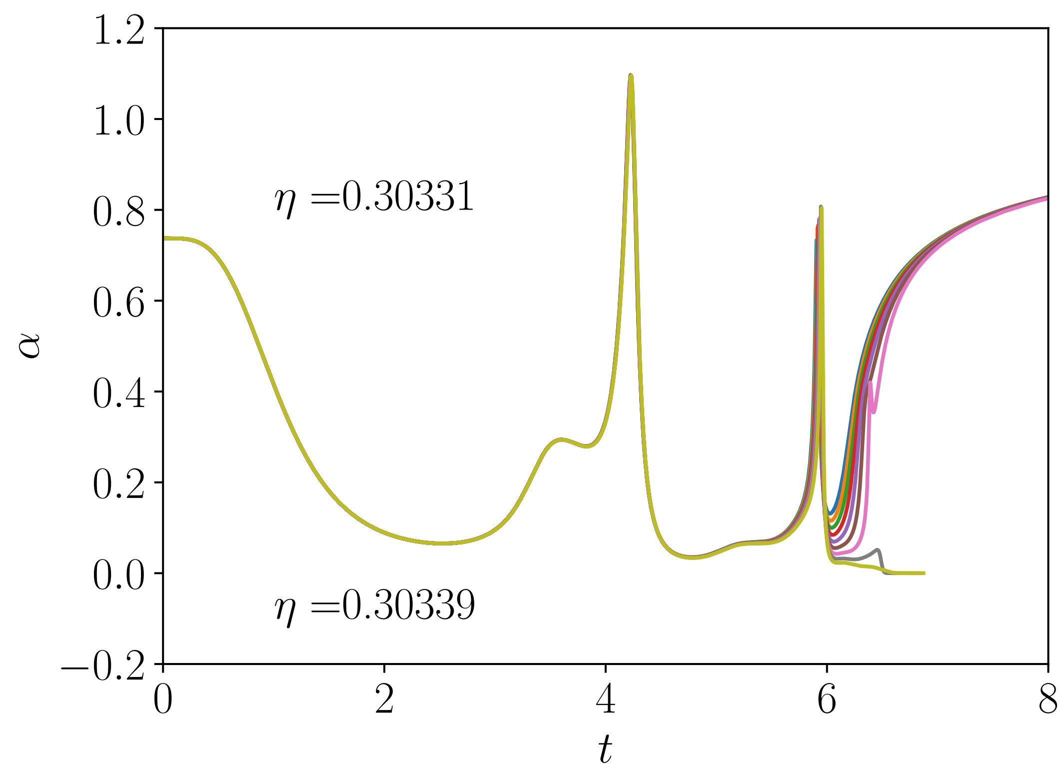
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.30337 < \eta_* < 0.30338$$

A numerical experiment...

- Let's say scalar field

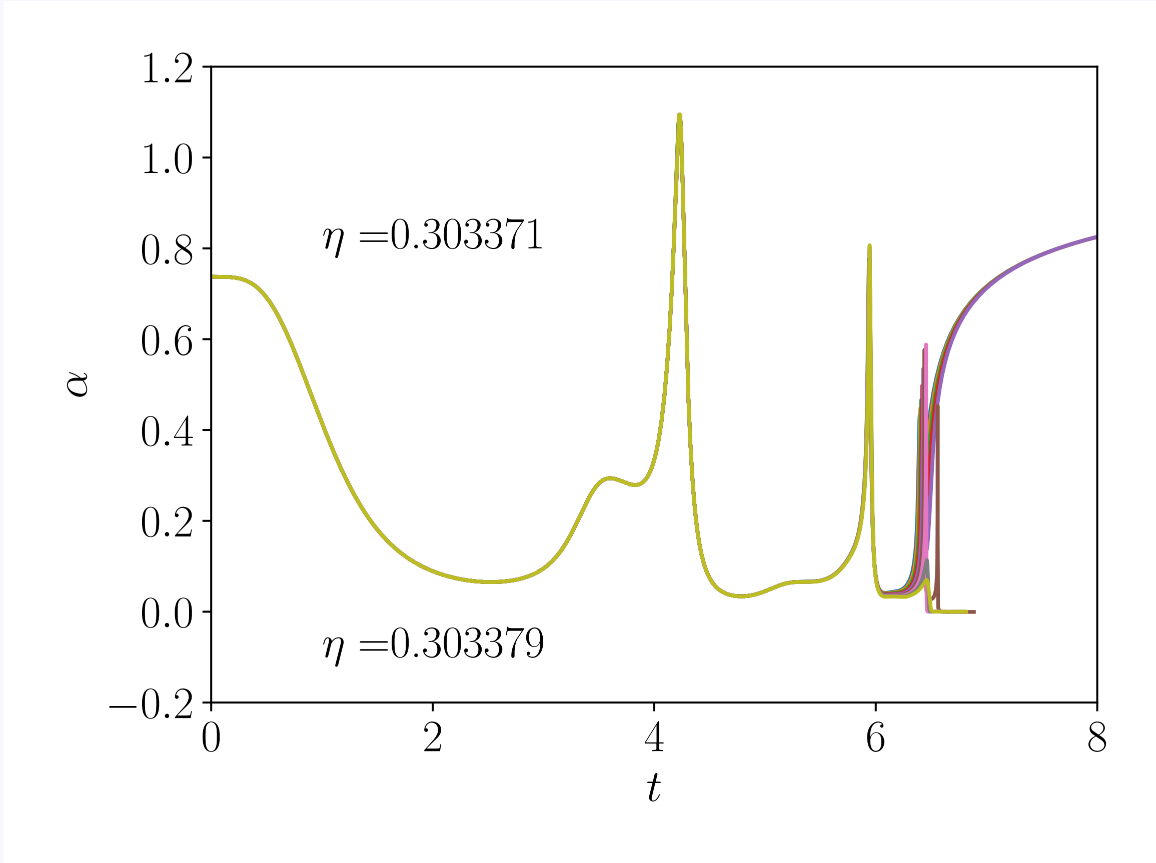
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta \exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

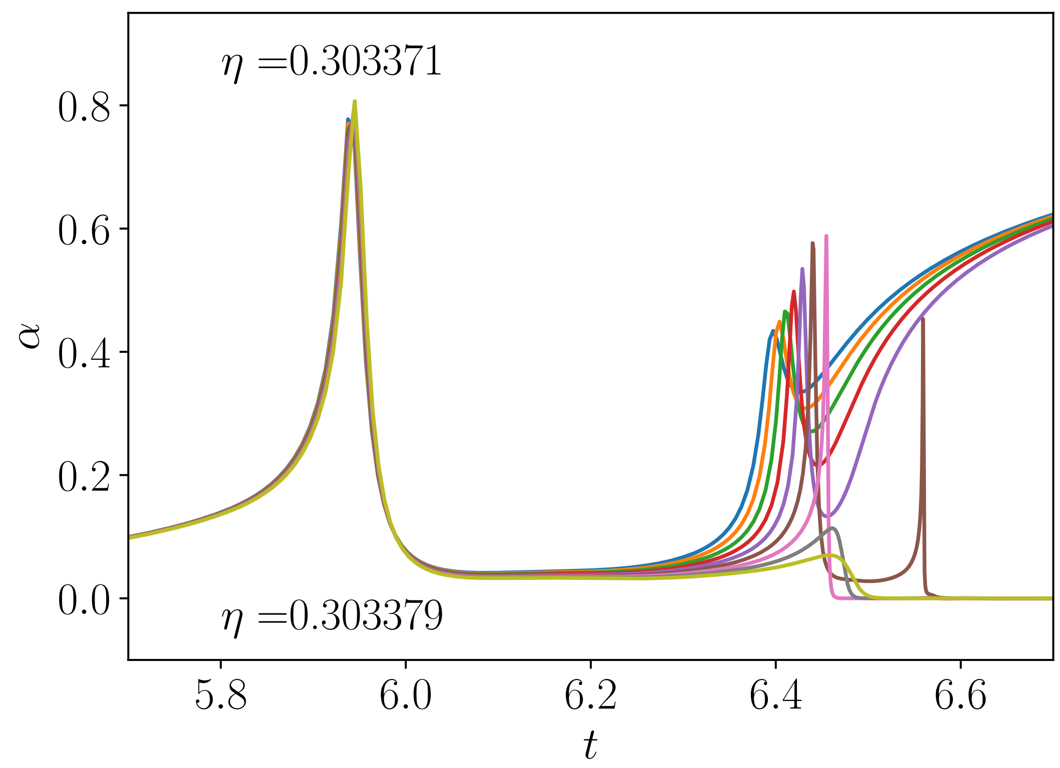
$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303375 < \eta_* < 0.303376$$

A numerical experiment...



- Let's say scalar field

$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta \exp(-R^2/R_0^2)$$

- try out different η...

Have critical value η_* so that

$\eta < \eta_*$ $\alpha \to 1$ end up with flat space

$\eta > \eta_*$ $\alpha \to 0$ end up with black hole

Black-hole threshold

$$0.303375 < \eta_* < 0.303376$$

A numerical experiment...

- Let's say scalar field

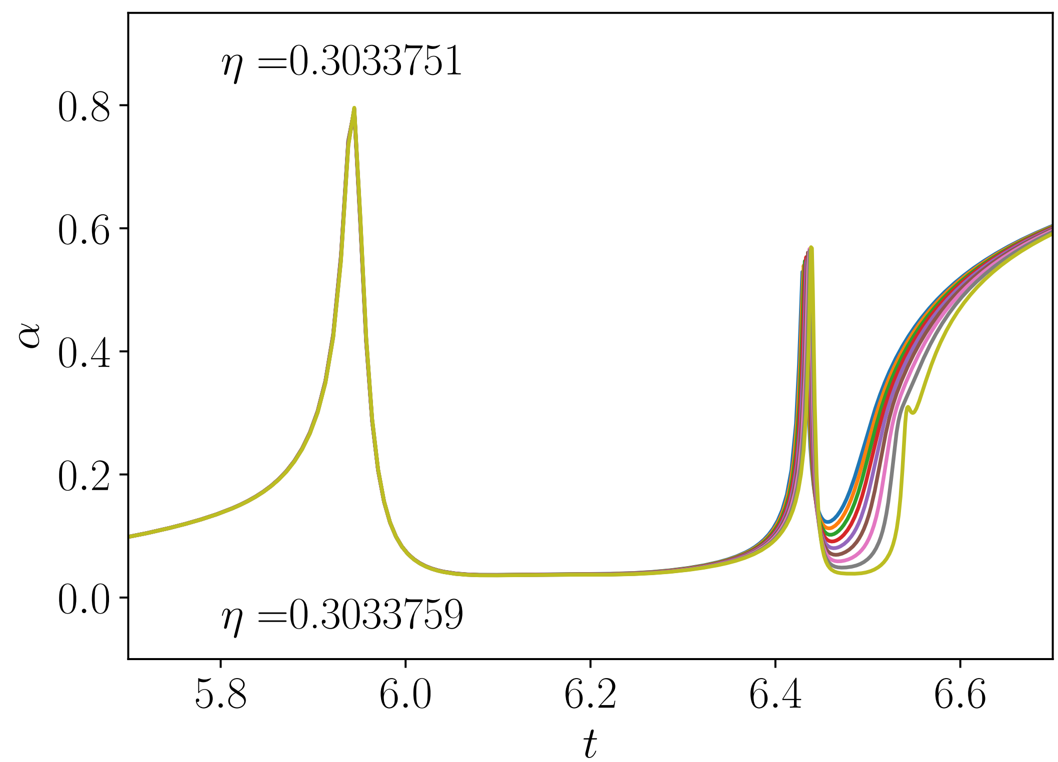
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta \exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3033759 < \eta_* < 0.3033760$$

A numerical experiment...

- Let's say scalar field

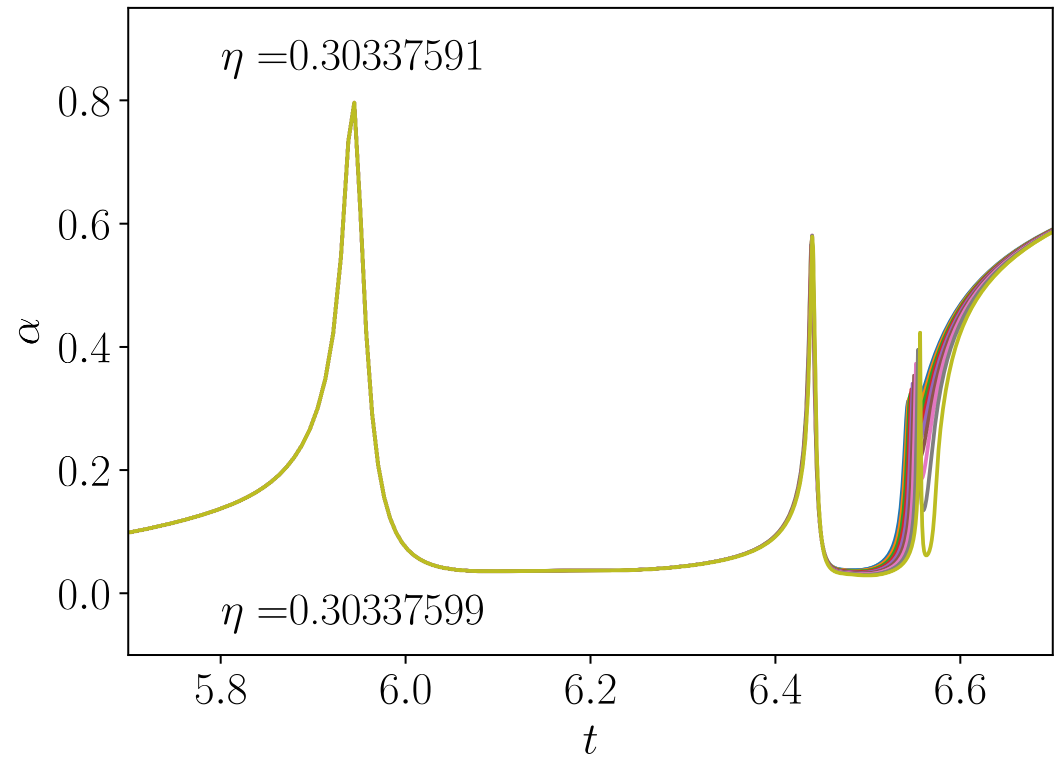
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.30337599 < \eta_* < 0.30337600$$

A numerical experiment...

- Let's say scalar field

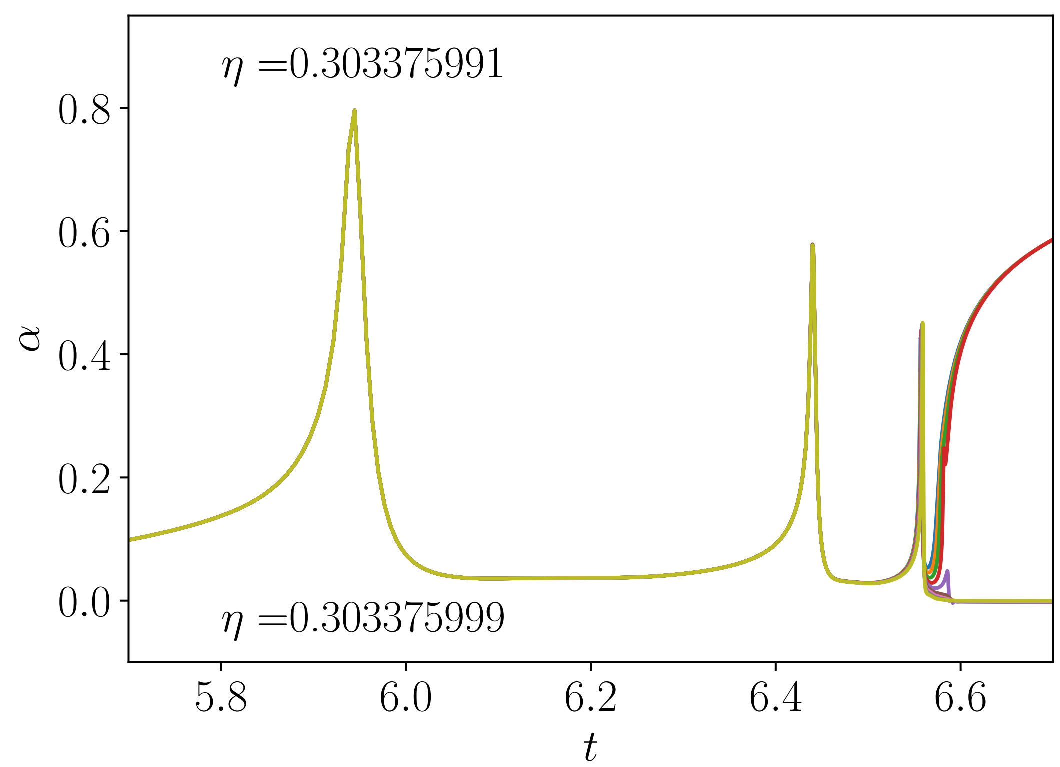
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta \exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303375994 < \eta_* < 0.303375995$$

A numerical experiment...

- Let's say scalar field

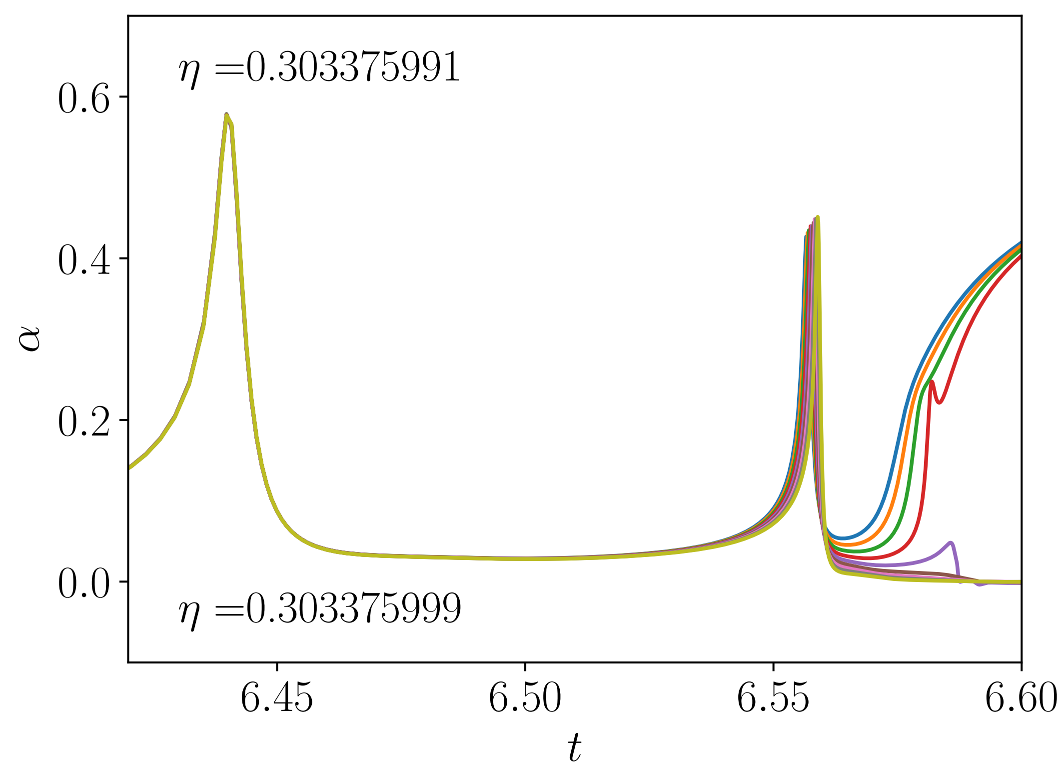
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303375994 < \eta_* < 0.303375995$$

A numerical experiment...

- Let's say scalar field

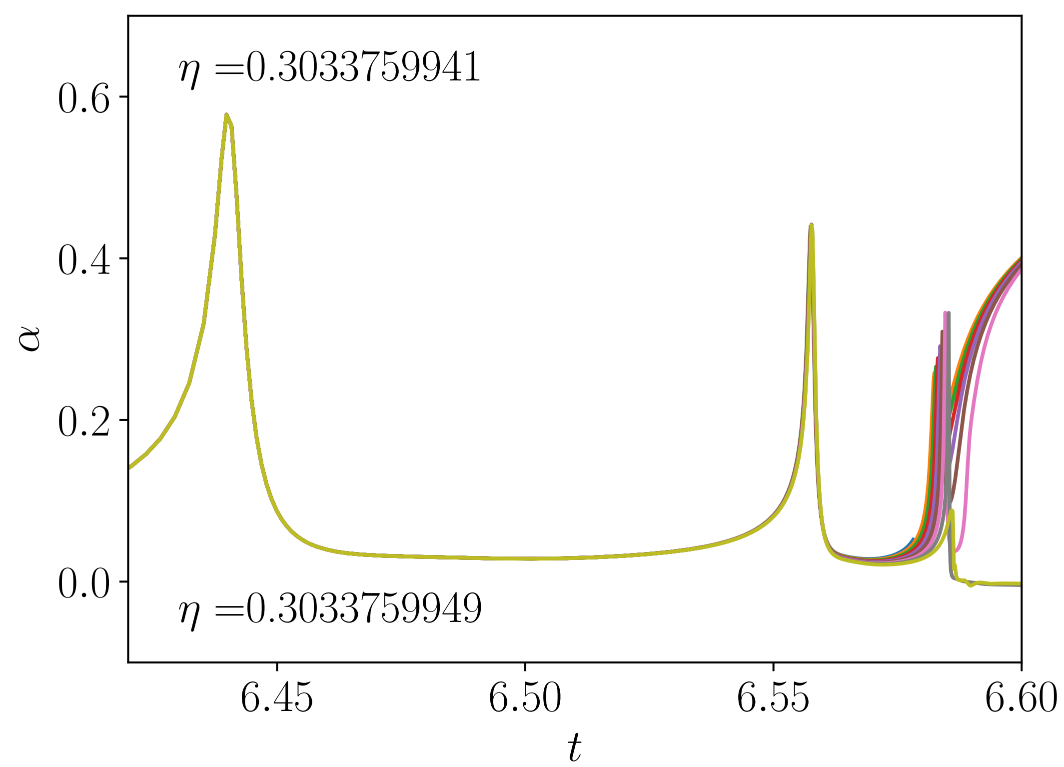
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3033759947 < \eta_* < 0.3033759948$$

A numerical experiment...

- Let's say scalar field

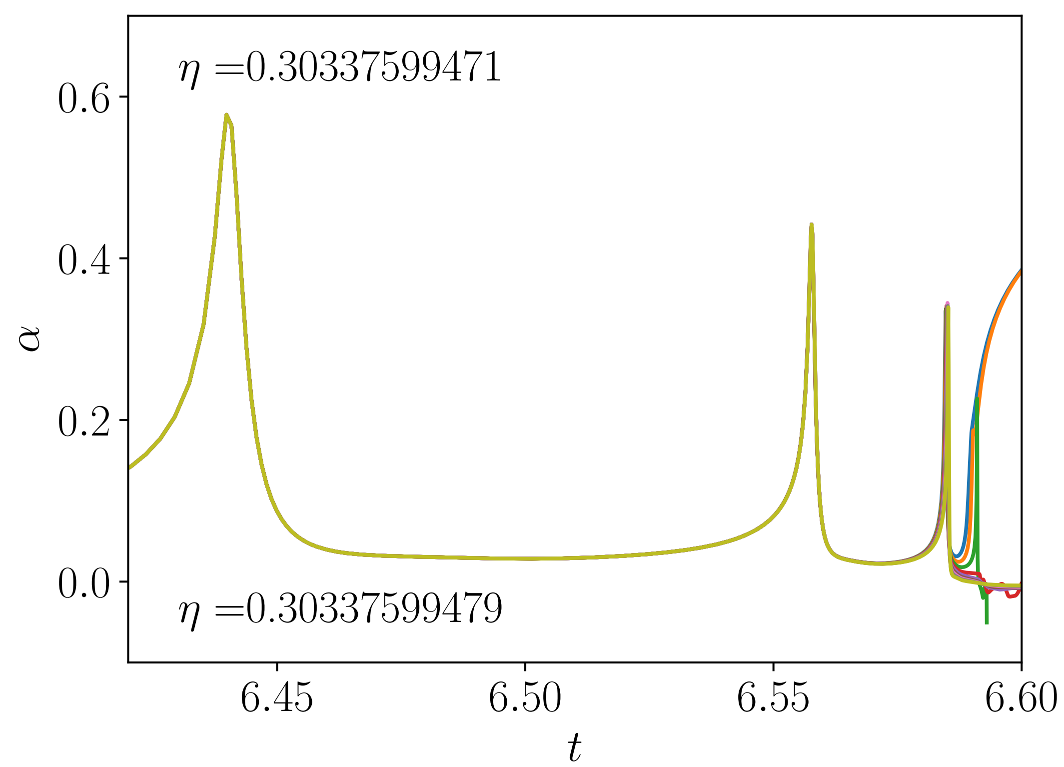
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.30337599472 < \eta_* < 0.30337599473$$

A numerical experiment...

- Let's say scalar field

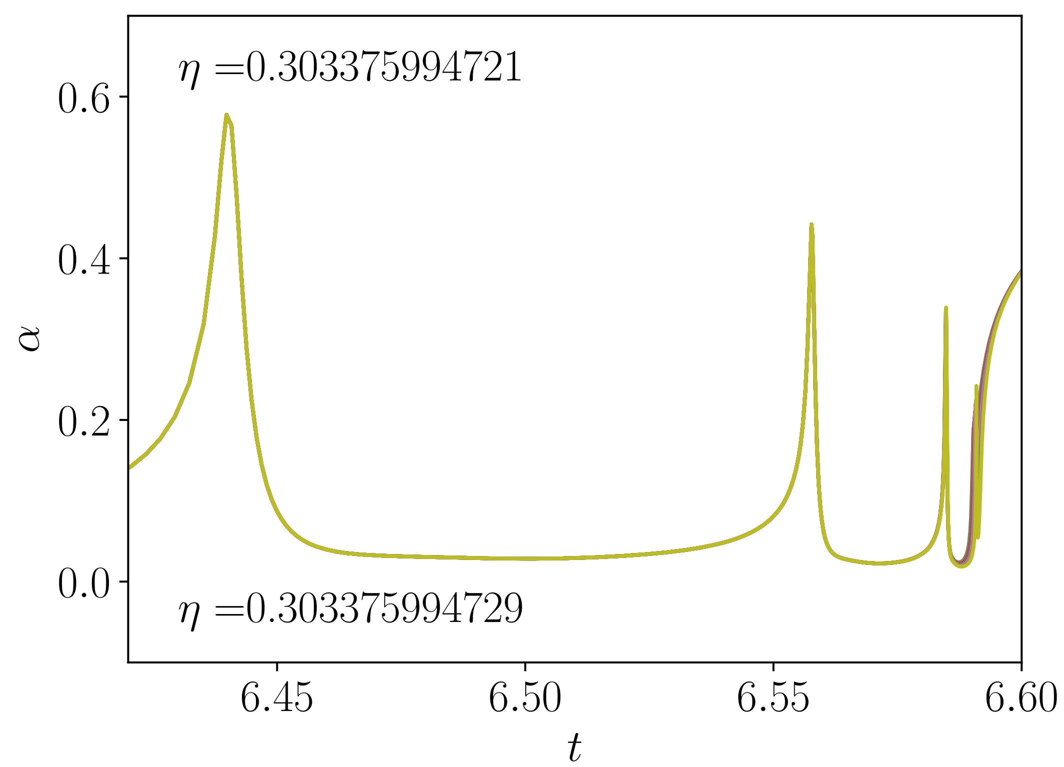
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303375994729 < \eta_* < 0.303375994730$$

A numerical experiment...

- Let's say scalar field

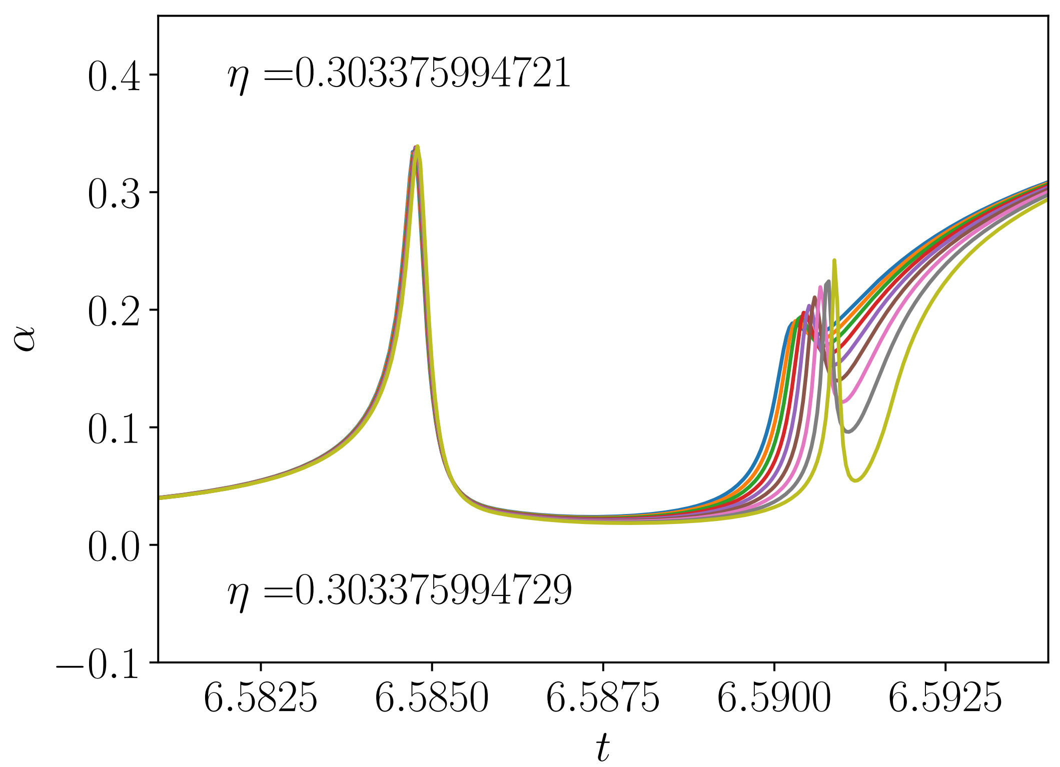
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.303375994729 < \eta_* < 0.303375994730$$

A numerical experiment...

- Let's say scalar field

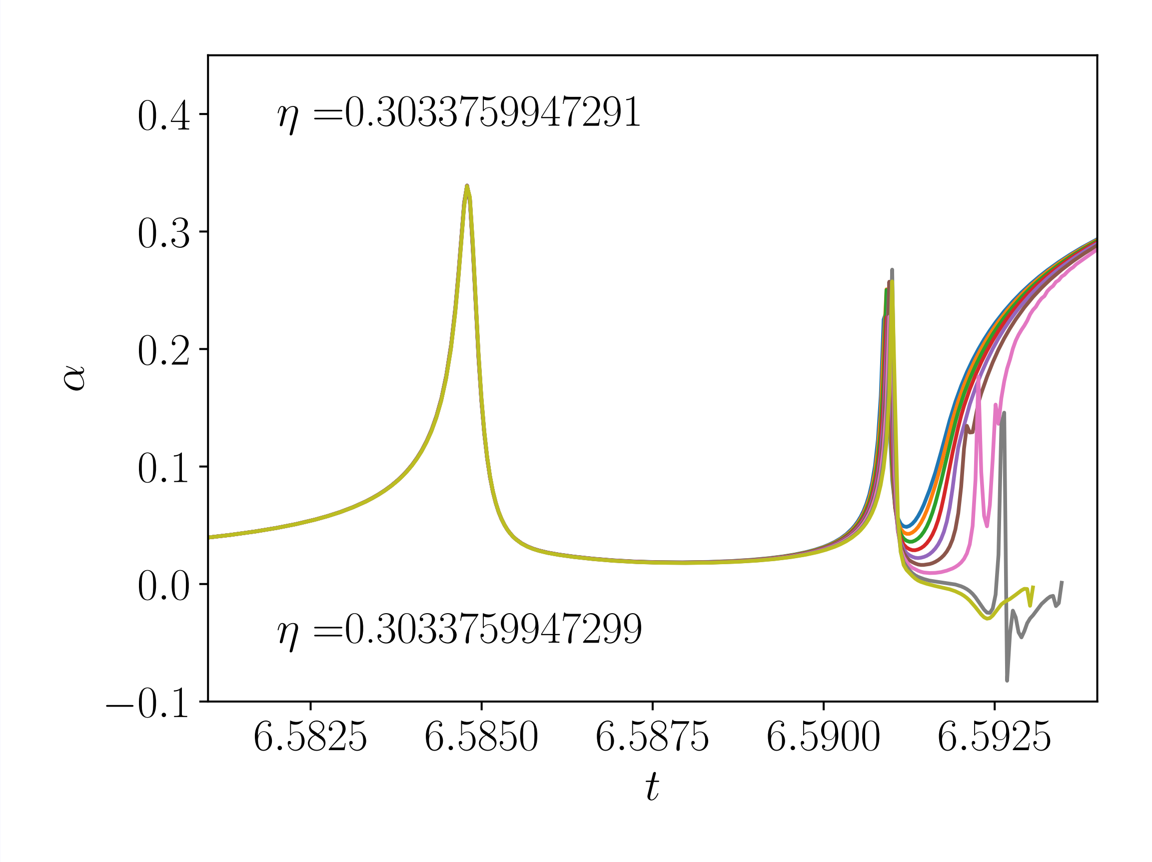
$$\Box\varphi \equiv g^{ab}\nabla_a\nabla_b\,\varphi = 0$$

coupled to Einstein's equations

- Initial data

$$\varphi = \eta\exp(-R^2/R_0^2)$$

- try out different η...



Have critical value η_* so that

$$\eta < \eta_* \qquad \alpha \to 1 \qquad \text{end up with flat space}$$

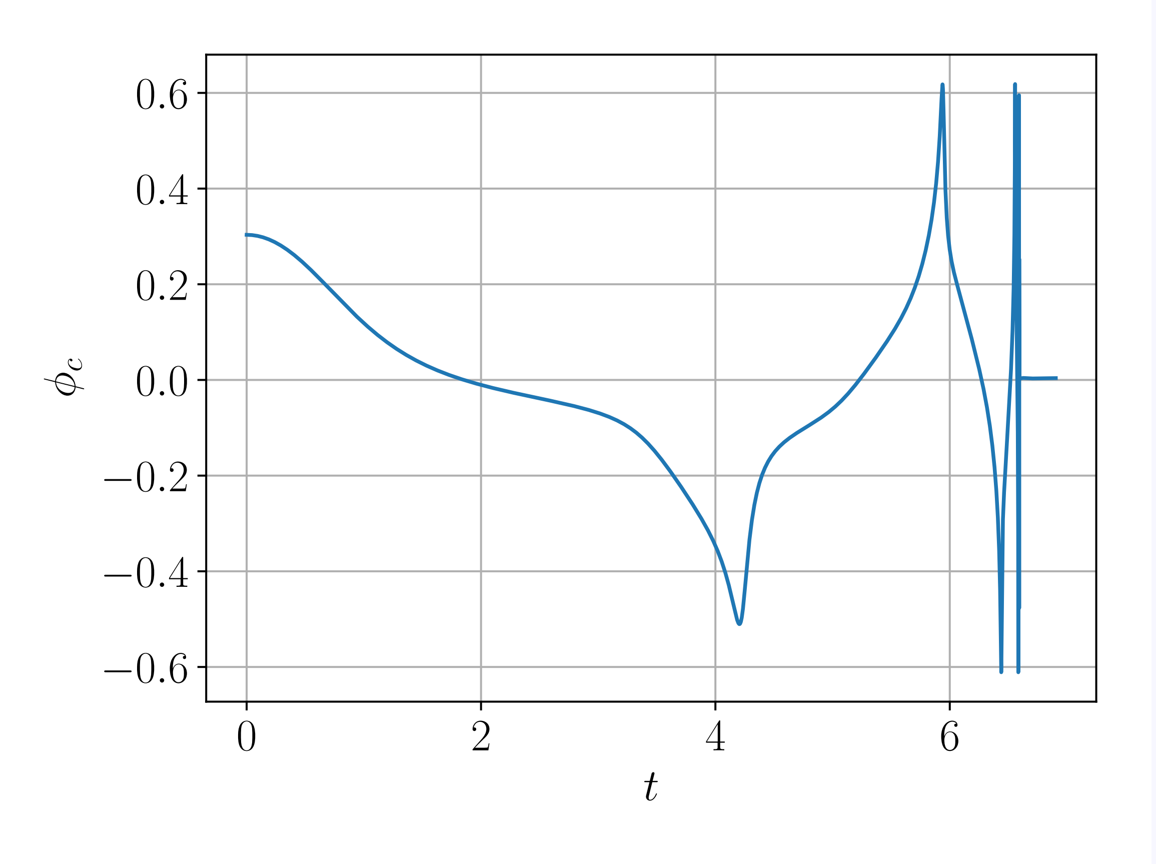
$$\eta > \eta_* \qquad \alpha \to 0 \qquad \text{end up with black hole}$$

Black-hole threshold

$$0.3033759947297 < \eta_* < 0.3033759947298$$

Critical Solution

- Let's look at φ for $\eta \approx \eta_*$ at $r = 0$



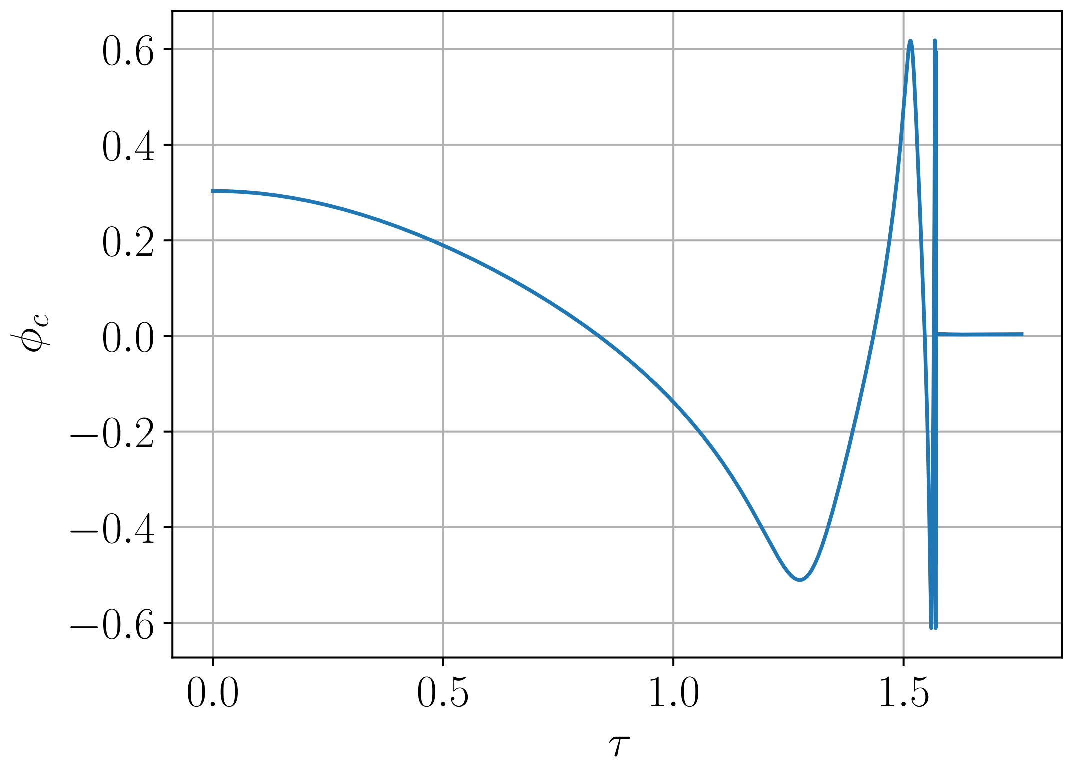
Critical Solution

- Let's look at φ for $\eta \approx \eta_*$ at $r = 0$

- plot as function of proper time τ

$\Longrightarrow$ oscillations "accumulate" at "accumulation" time

$$\tau_* \approx 1.5698$$



Critical Solution

- Let's look at φ for $\eta \approx \eta_*$ at $r = 0$
- plot as function of proper time τ

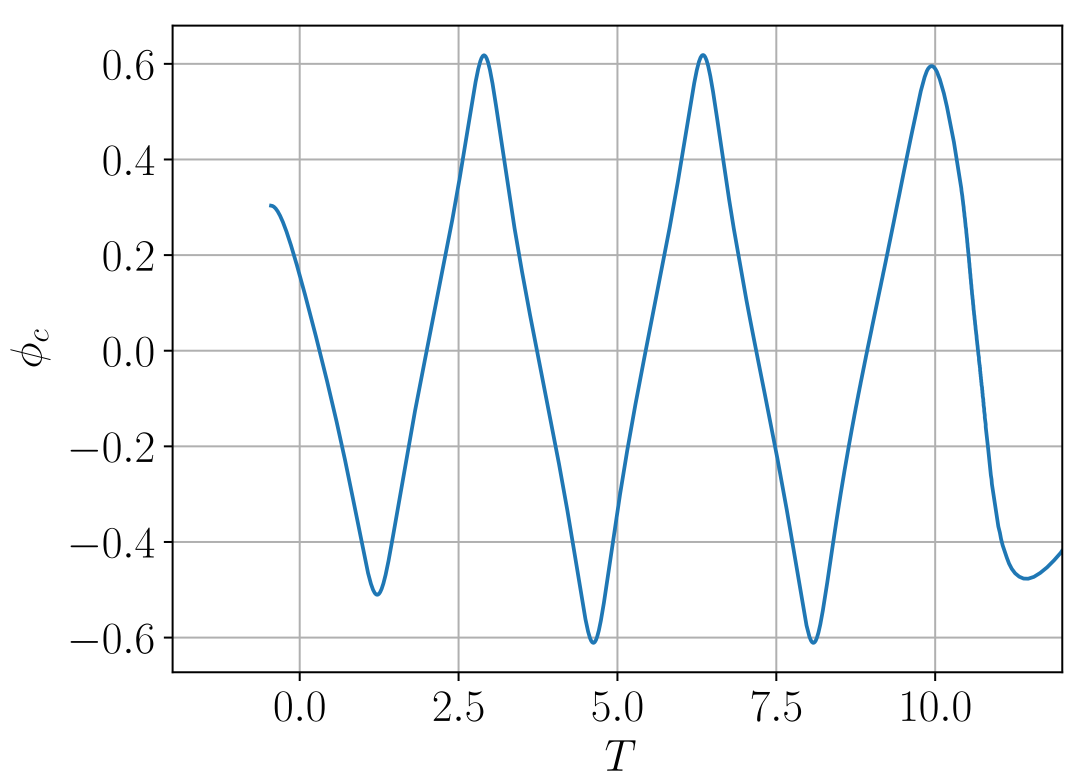
$\Longrightarrow$ oscillations "accumulate" at "accumulation" time

$$\tau_* \approx 1.5698$$

- plot as function of

$$T \equiv -\log(\tau_* - \tau)$$

- "Choptuik spacetime"



A few details

- Solve Einstein's equations

$$G_{ab} = 8\pi T_{ab}$$

- Choose matter model...
  - ...scalar field

    equation of motion:

$$g^{ab}\nabla_a\nabla_b\varphi = 0$$

    stress-energy tensor:

$$T_{ab} = \nabla_a\varphi\nabla_b\varphi - \frac{1}{2}g_{ab}\nabla^c\varphi\nabla_c\varphi$$

  - ...ultra-relativistic fluid (special case: radiation fluid: $\kappa = 1/3$)

$$P = \kappa\rho$$

    equation of motion: relativistic equations of hydrodynamics

    stress-energy tensor:

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab}$$

Radiation fluids

- radiation fluid

$$P = \rho/3$$

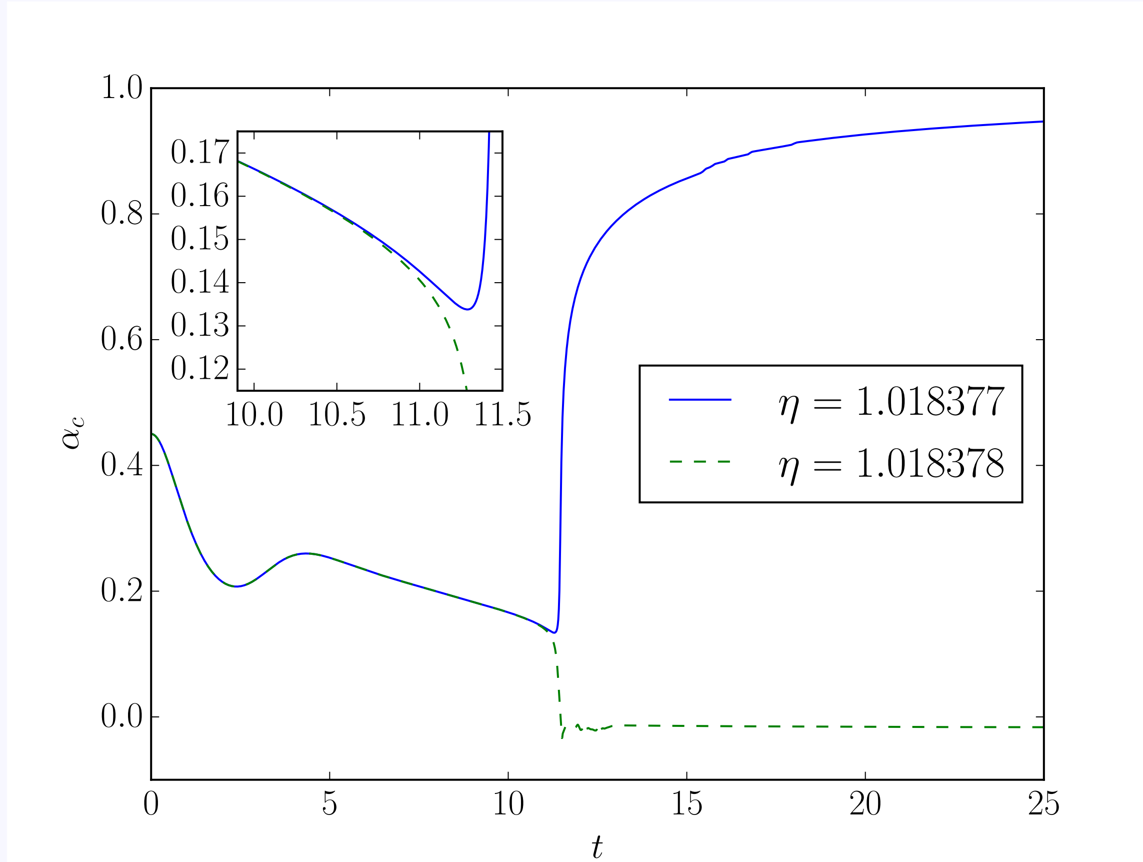
- initial data

$$\rho(R) \propto \eta \exp(-R^2/R_0^2)$$

- find similar behavior, with

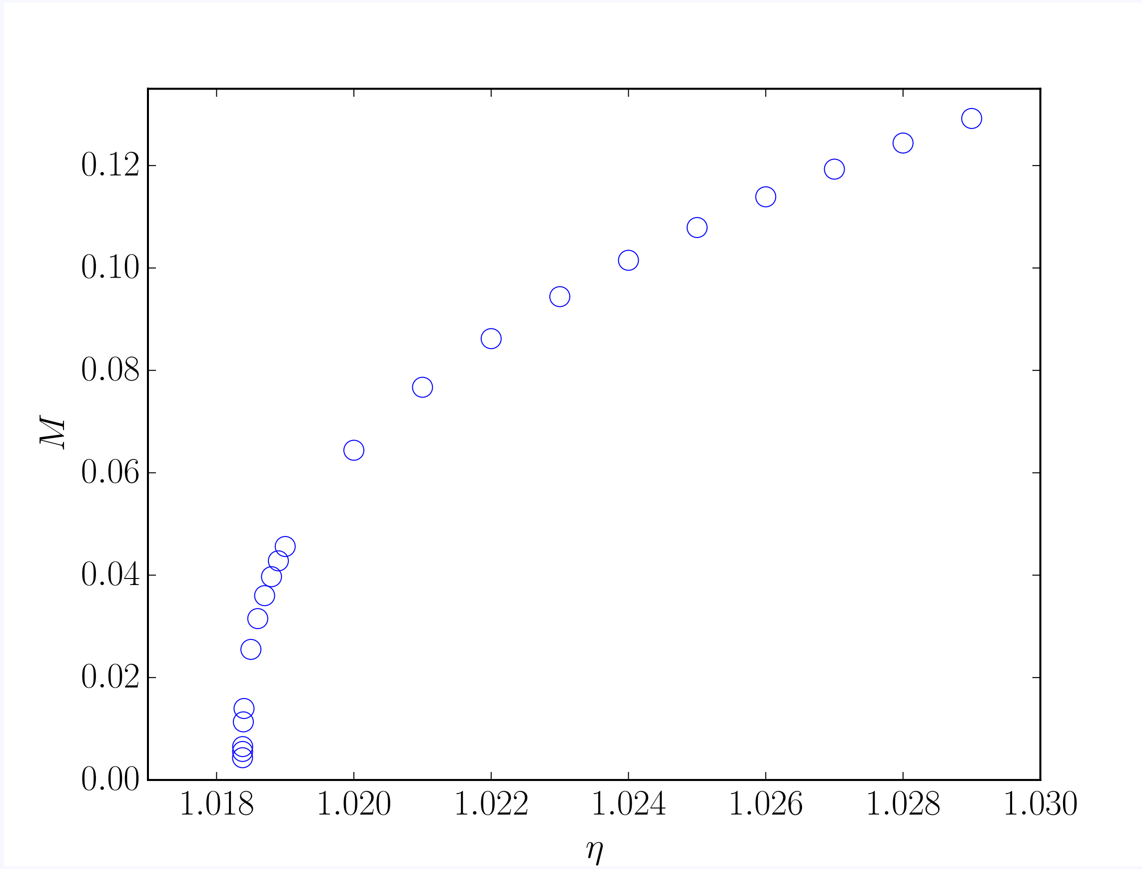
$$\eta_* \approx 1.01838$$

[Evans & Coleman, 1994]



Can we form arbitrarily small black holes??

Consider black-hole mass M as $\eta \to \eta_*$



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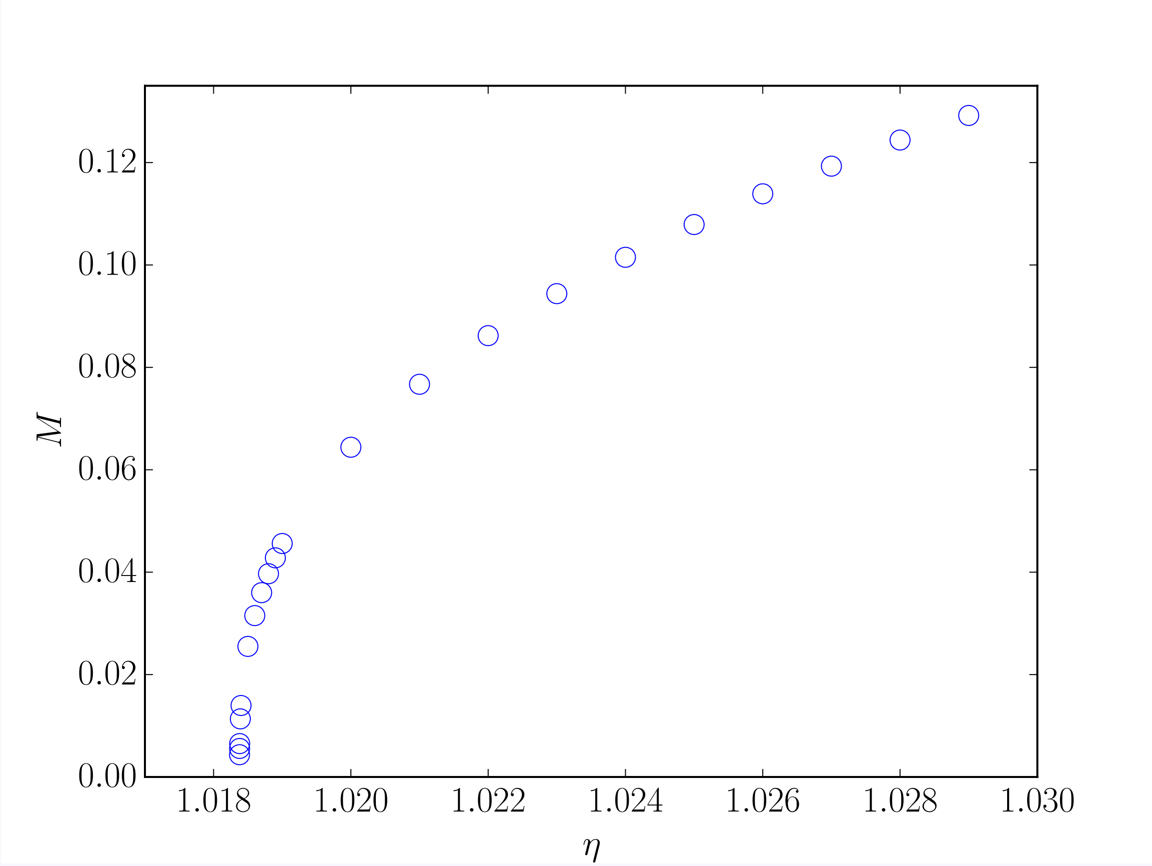
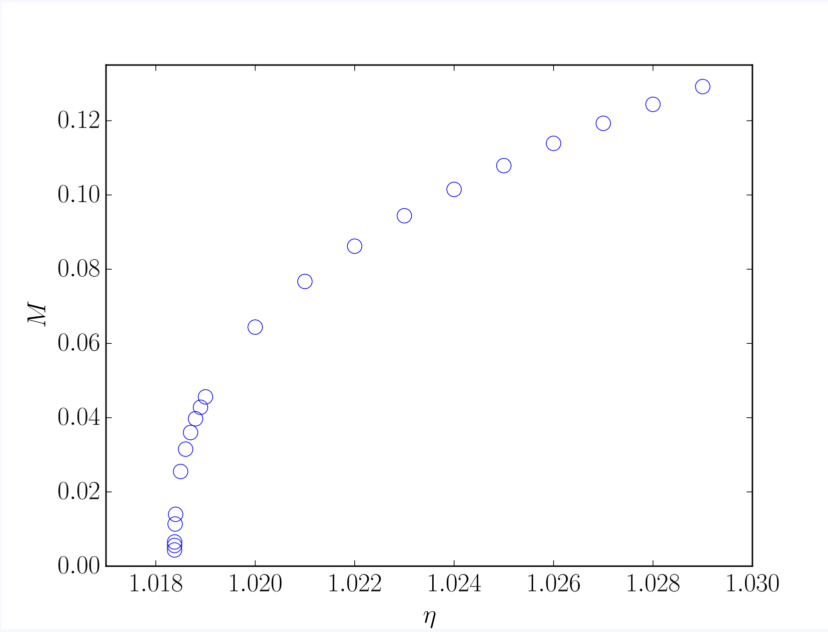
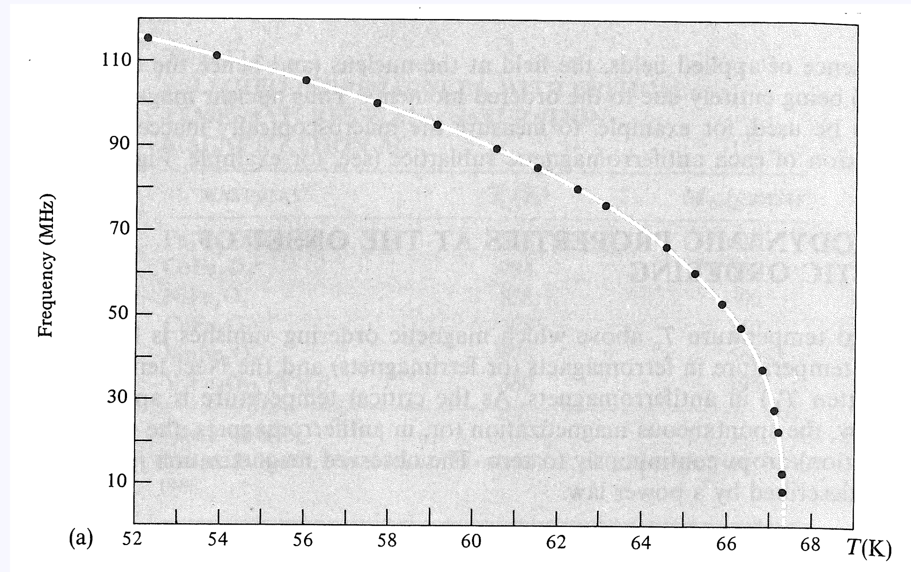


Looks familiar???

Can we form arbitrarily small black holes??

Consider black-hole mass M as $\eta \to \eta_*$

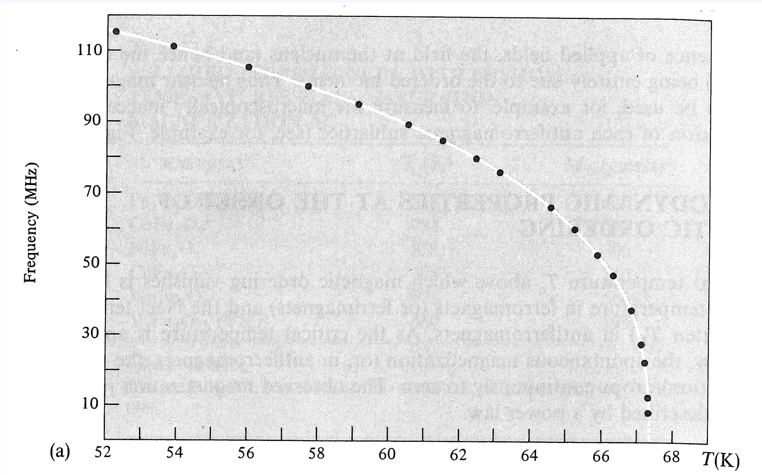




Looks familiar???

Critical Phenomena...

Magnetic field close to critical temperature:



[Ashcroft & Mermin, *Solid State Physics*, 1976]

Thermodynamic Properties at the Onset of Magnetic Ordering 699

in the absence of applied fields, the field at the nucleus (and hence the resonance frequency) being entirely due to the ordered moments. Thus nuclear magnetic resonance can be used, for example, to measure the macroscopically inaccessible net magnetization of each antiferromagnetic sublattice (see, for example, Figure 33.4).

THERMODYNAMIC PROPERTIES AT THE ONSET OF MAGNETIC ORDERING

The critical temperature T_c above which magnetic ordering vanishes is known as the Curie temperature in ferromagnets (or ferrimagnets) and the Néel temperature (often written T_N) in antiferromagnets. As the critical temperature is approached from below, the spontaneous magnetization (or, in antiferromagnets, the sublattice magnetization) drops continuously to zero. The observed magnetization just below T_c is well described by a power law.

$$M(T) \sim (T_c - T)^{\beta}, \qquad \textbf{(33.1)}$$

where β is typically between 0.33 and 0.37 (see Figure 33.4).

The onset of ordering is also signaled as the temperature drops to T_c from above,

Critical Phenomena:

- appear close to phase transitions
- result in scaling laws

Critical Phenomena in Gravitational Collapse

Consider initial matter distribution parametrized by η (say density) and evolve...

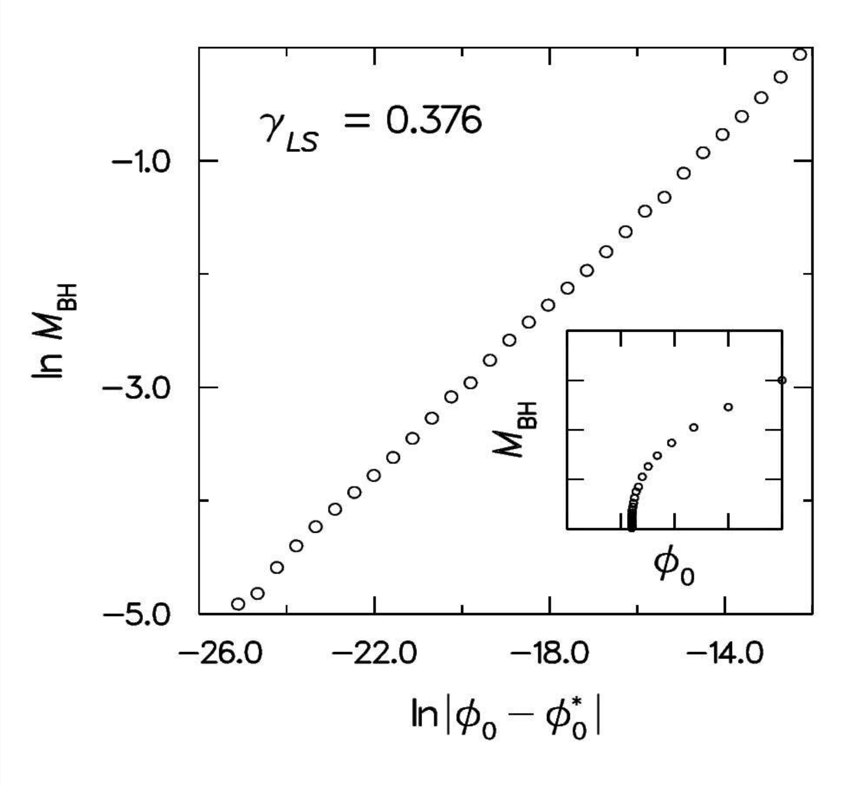
Then critical parameter η_* separates
- ◦ supercritical data: form black hole
- ◦ subcritical data: don't

Close to η_* observe critical phenomena:

- black hole formed from supercritical data has mass

$$M \simeq |\eta - \eta_*|^{\gamma_M}$$

where γ_M is universal

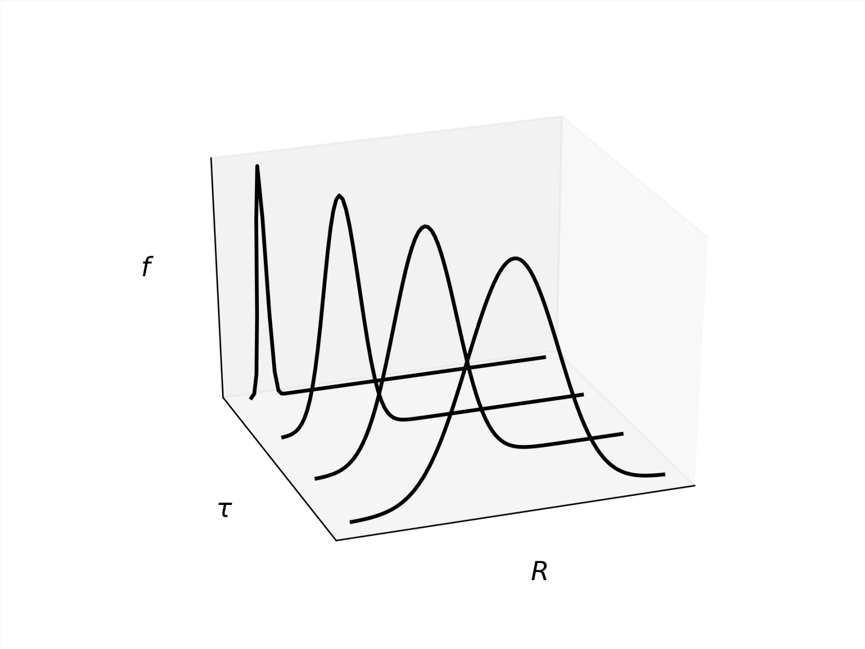


[Choptuik, 1998]

- spacetime approaches self-similar critical solution

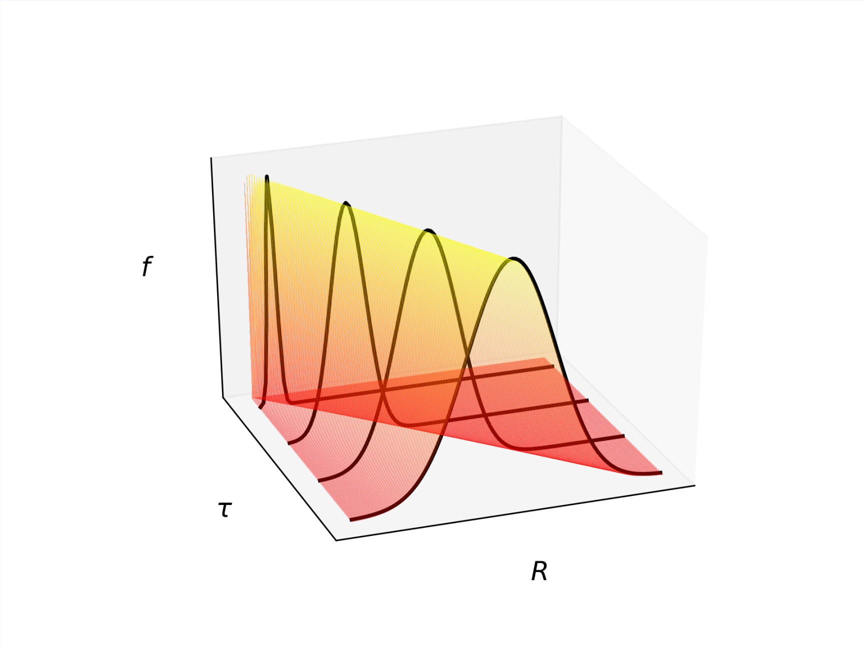
[Choptuik, 1993]

Self-similarity



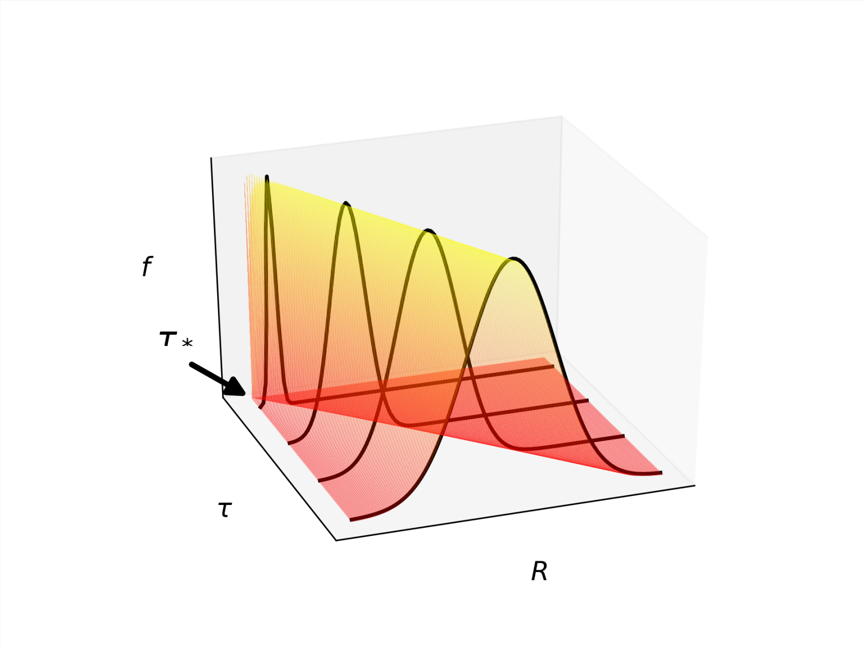
- Solution contracts without changing shape. . .

Self-similarity



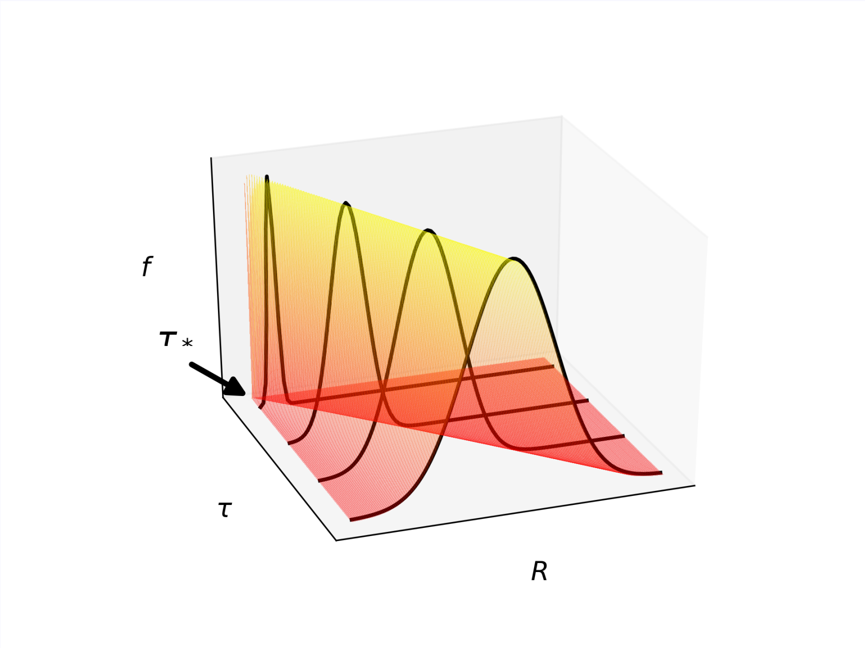
- Solution contracts without changing shape. . .

Self-similarity



- Solution contracts without changing shape. . .
- . . . towards accumulation event at $\tau = \tau_*$

Self-similarity



- Solution contracts without changing shape. . .
- . . . towards accumulation event at $\tau = \tau_*$
- radius R proportional to $\tau_* - \tau$,

$$R \simeq (\tau_* - \tau)$$

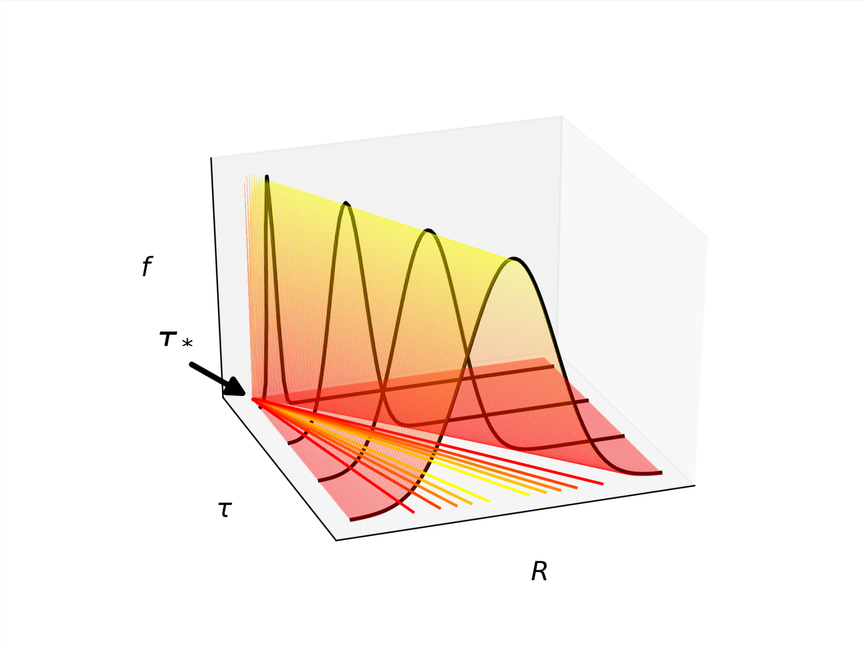
$\Longrightarrow$ dimensionless quantities are functions of

$$\xi \equiv \frac{R}{\tau_* - \tau}$$

only, i.e.

$$Z = Z_*(\xi)$$

Self-similarity



- Solution contracts without changing shape. . .
- . . . towards accumulation event at $\tau = \tau_*$
- radius R proportional to $\tau_* - \tau$,

$$R \simeq (\tau_* - \tau)$$

$\Longrightarrow$ dimensionless quantities are functions of

$$\xi \equiv \frac{R}{\tau_* - \tau}$$

only, i.e.

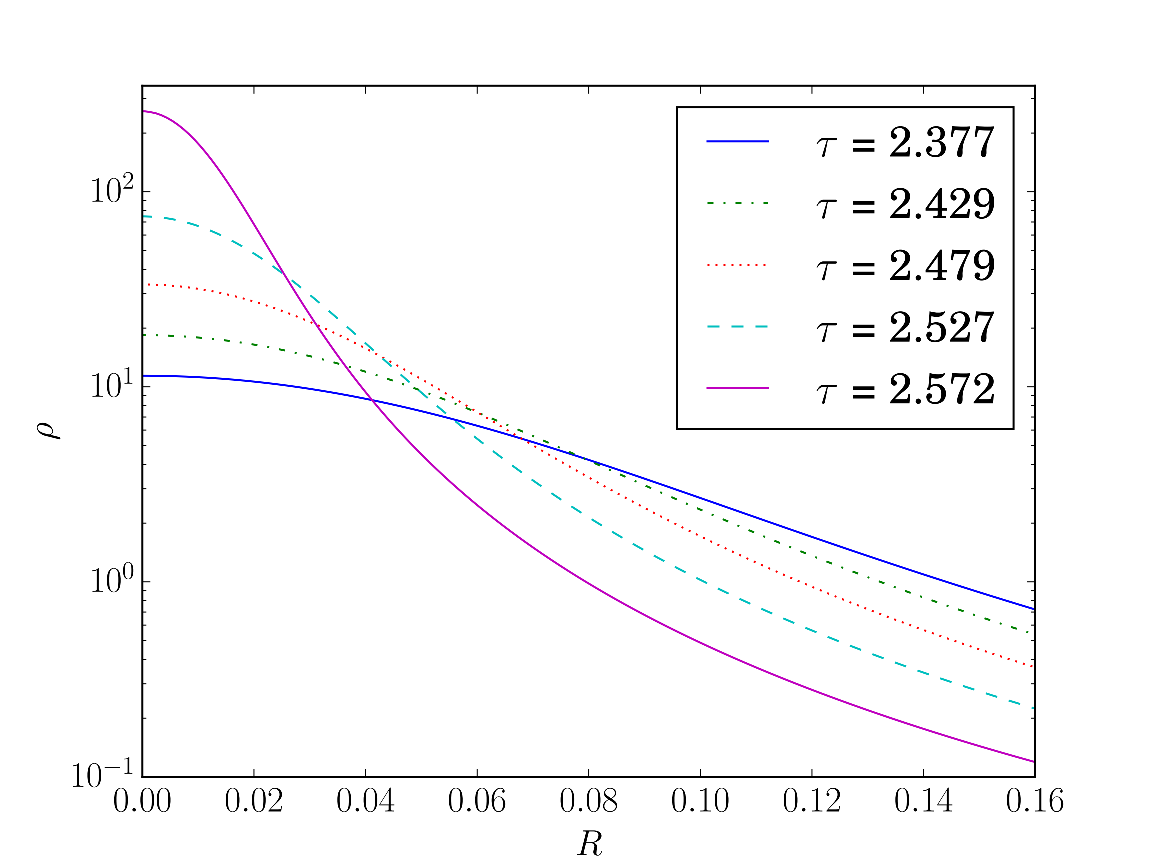
$$Z = Z_*(\xi)$$

Numerical example

Now choose

$$\eta \simeq \eta_*$$

and look for self-similarity



Numerical example

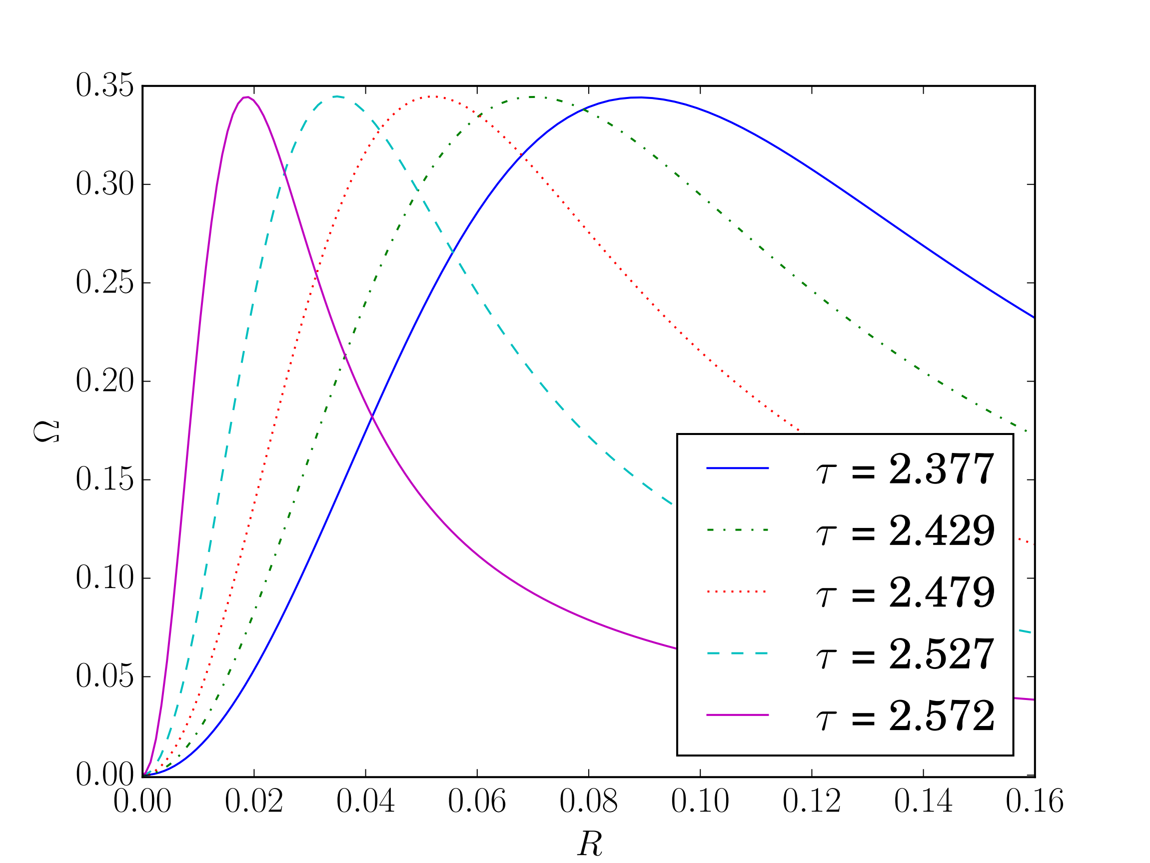
Now choose

$$\eta \simeq \eta_*$$

and look for self-similarity

- instead of ρ, consider

$$\Omega \equiv 4\pi R^2 \rho$$



Numerical example

Now choose

$$\eta \simeq \eta_*$$

and look for self-similarity

- instead of ρ, consider

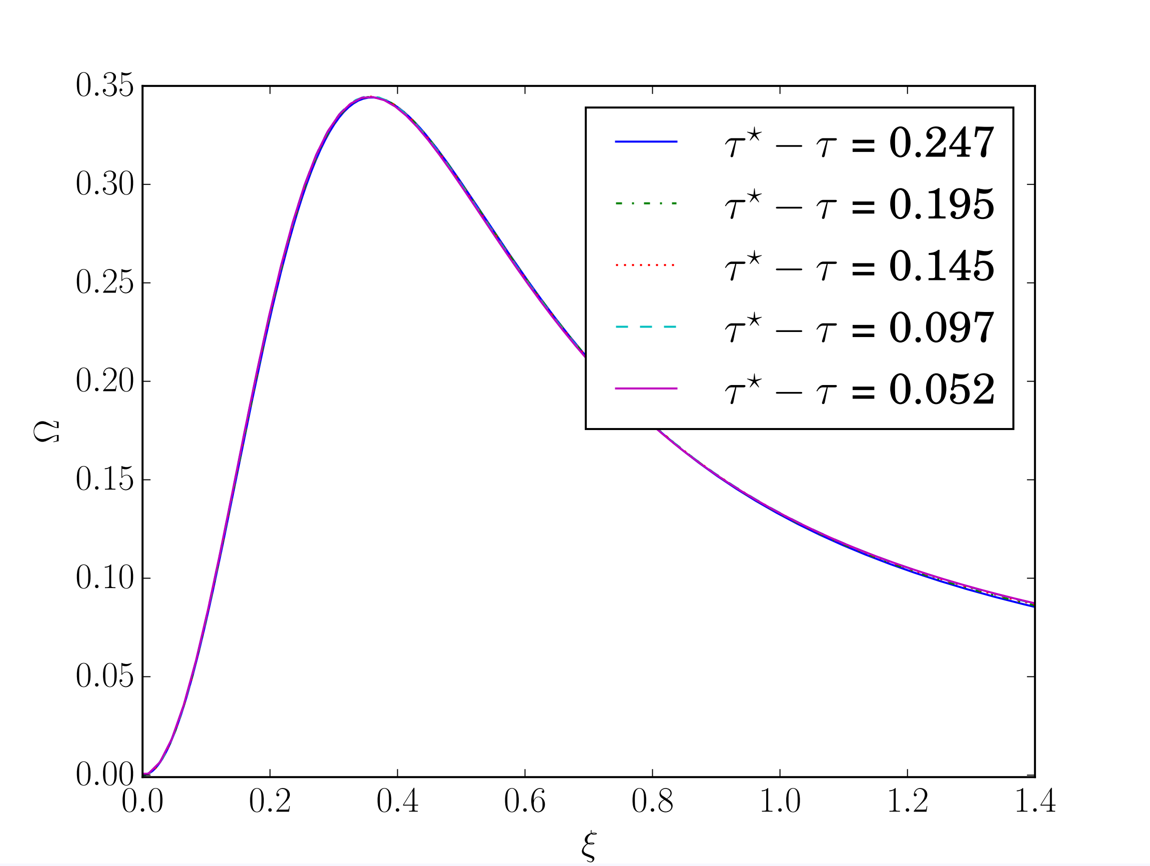
$$\Omega \equiv 4\pi R^2 \rho$$

- plot as function of

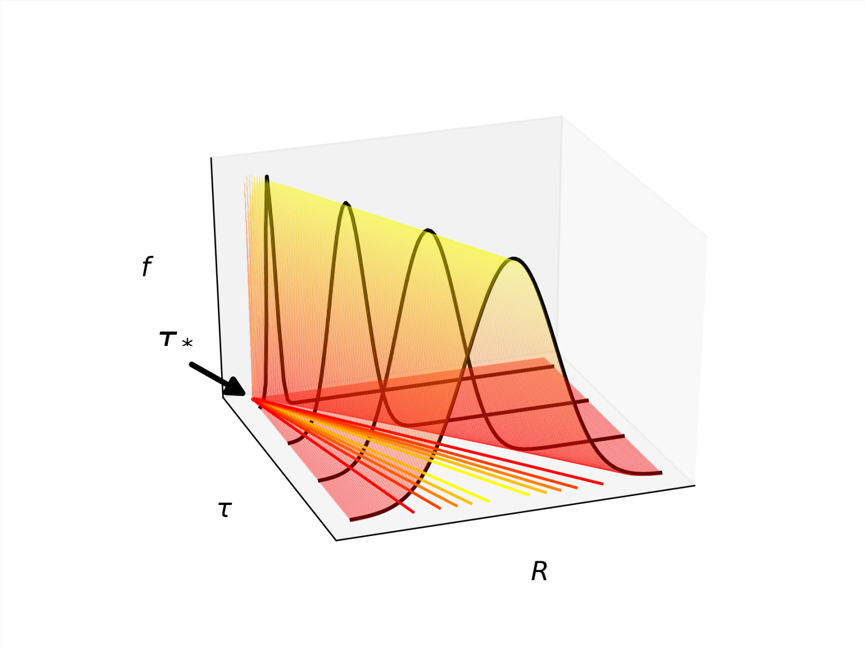
$$\xi \equiv \frac{R}{\tau_* - \tau}$$

with $\tau_* = 2.624$

$\Longrightarrow$ self-similarity evident



Self-similarity



- Solution contracts without changing shape. . .
- . . . towards accumulation event at $\tau = \tau_*$
- radius R proportional to $\tau_* - \tau$,

$$R \simeq (\tau_* - \tau)$$

$\Longrightarrow$ dimensionless quantities are functions of

$$\xi \equiv \frac{R}{\tau_* - \tau}$$

only, i.e.

$$Z = Z_*(\xi)$$

$\Longrightarrow$ no preferred global length scale

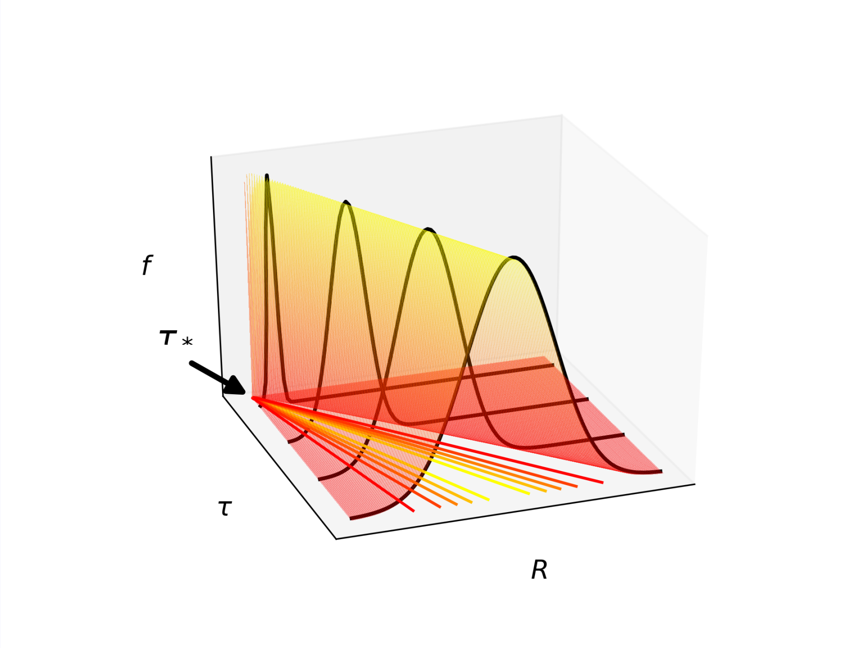
What sets scale of forming black holes?

Three phases of evolution

- Phase I:
  from initial data to something close to critical solution
  (how close? depends on degree of fine-tuning)
- Phase II:
  critical solution plus perturbation
  (until perturbation becomes nonlinear)
- Phase III:
  collapse to black hole or disperse

$\Longrightarrow$ length scale set by size of self-similar solution at transition from Phase II to III

Phase II: Perturbations of Critical Solutions



- Consider perturbations ζ of critical solution

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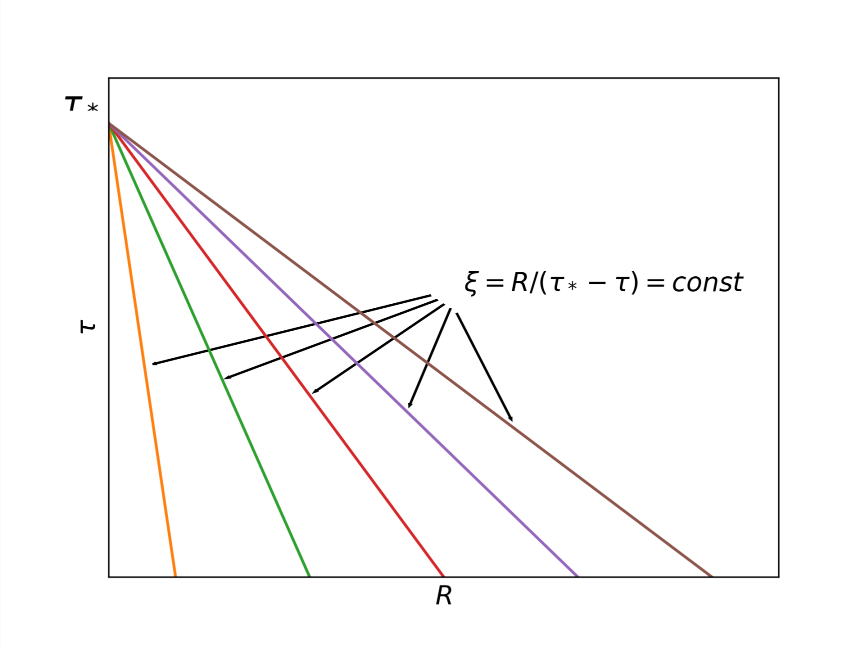

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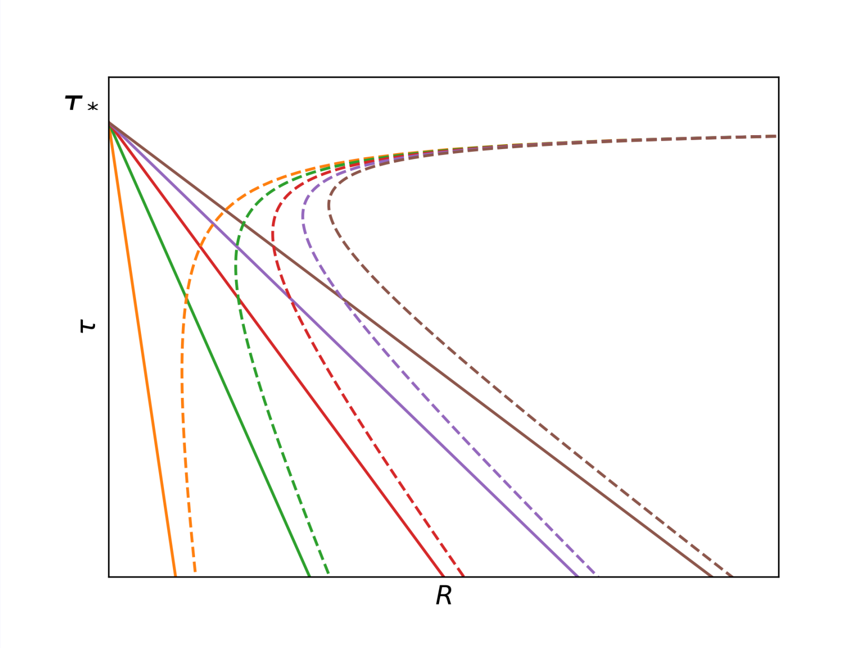

- Consider perturbations ζ of critical solution

- assume that only one mode is unstable

$\Longrightarrow$ grows at rate λ in $T = -\log(\tau_* - \tau)$

$$\zeta \propto \exp(\lambda T) = (\tau_* - \tau)^{-\lambda}$$

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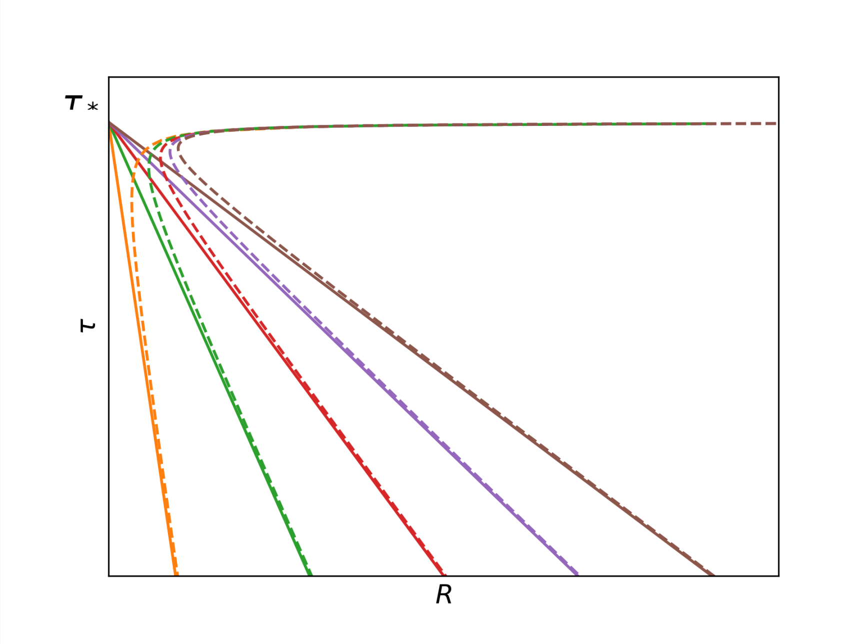

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$$\zeta \propto (\eta - \eta_*)(\tau_* - \tau)^{-\lambda}$$

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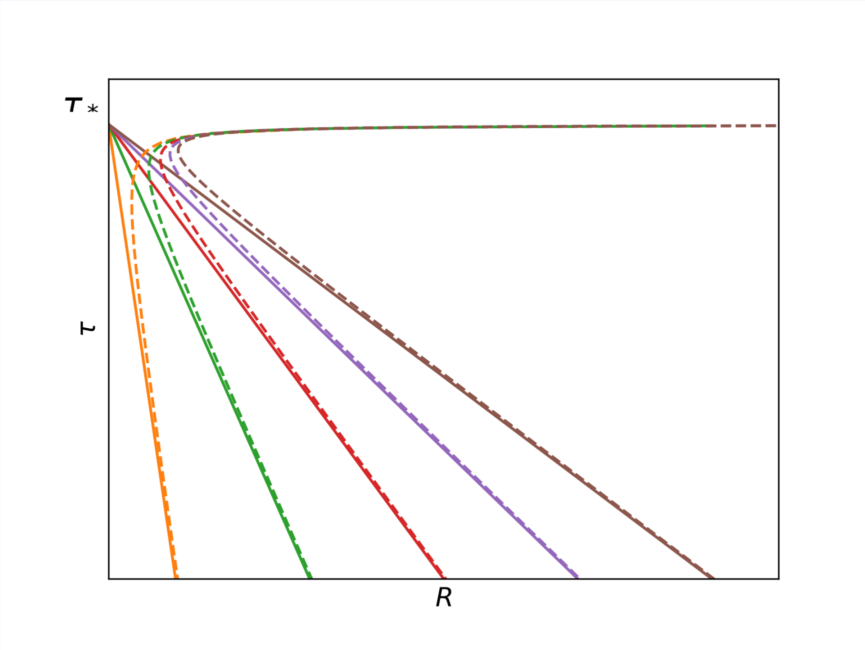

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Mode becomes nonlinear when $\zeta = const$
$\Longrightarrow$ determines length scale

$$R \propto (\tau_* - \tau) \propto (\eta - \eta_*)^{1/\lambda}$$

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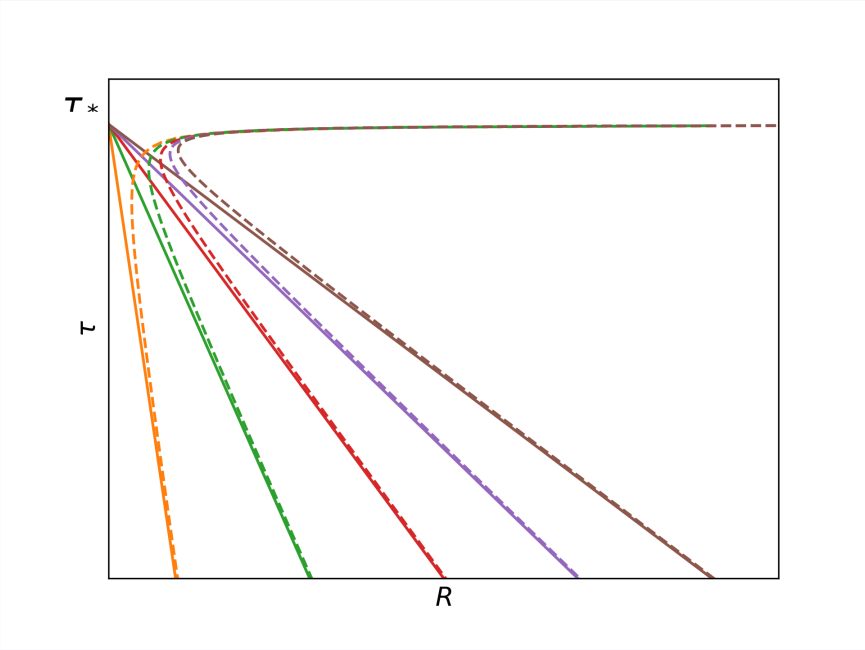

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Mode becomes nonlinear when $\zeta = const$

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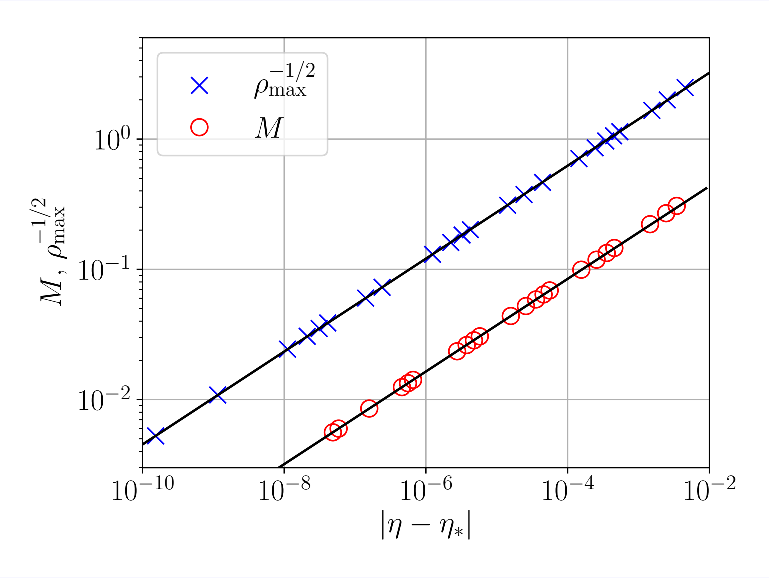
$$R \propto (\tau_* - \tau) \propto (\eta - \eta_*)^{1/\lambda}$$

$\Longrightarrow$ scaling laws, e.g.

$$M \propto (\eta - \eta_*)^{\gamma}$$

with $\gamma = 1/\lambda$

[Koike *et.al.*, 1995; Maison 1995]



$1/\lambda = 0.3558$ $\gamma = 0.356$

Brief History

- Original discovery:
  - ○ scalar fields
  - ○ numerical simulations in spherical symmetry

  [Choptuik 1993]
- Other matter models:
  - ○ vacuum (gravitational waves) [Abrahams & Evans, 1993]
  - ○ radiation fluids [Evans & Coleman, 1994]
  - ○ etc...
- Perturbative calculations

  [Koike $et.al.$, 1995; Maison, 1995; Martín-García & Gundlach, 1999; ...]

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But, until recently, very few numerical studies in absence of spherical symmetry, despite progress in 3D numerical relativity

Effects of asphericity in critical collapse

- What is effect of aspherical perturbations? Are they stable or unstable?
  [TWB & Montero, 2015; Celestino & TWB, 2018]
- What is role of angular momentum?
  [TWB & Gundlach, 2016; Gundlach & TWB, 2016, 2018]

Numerical code

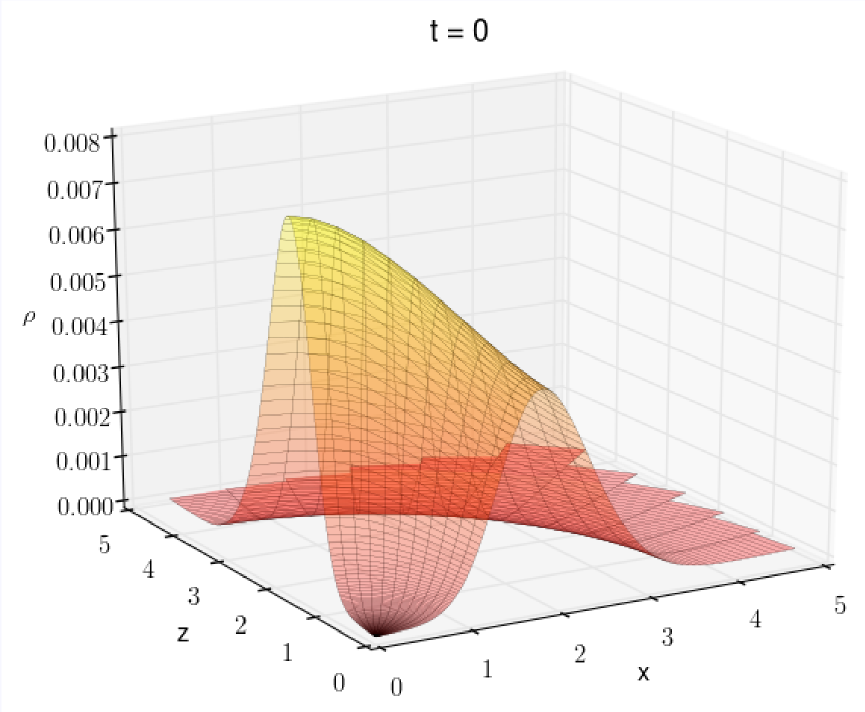
- adopts BSSN formulation in reference-metric form
  [Nakamura *et.al.*, 1987; Shibata & Nakamura, 1995; TWB & Shapiro, 1998; Brown, 2007]
- adopts spherical polar coordinates without symmetry assumptions
  [TWB, Montero, Cordero-Carrión & Müller, 2013; Montero, TWB & Müller, 2014]
- "moving puncture gauge": 1+log slicing and Gamma driver
  [Bona *et.al.*, 1995; Alcubierre *et.al.*, 1999]
- uses logarithmic radial coordinate and regridding

Aspherical deformations of ultrarelativistic fluids

Recall ultrarelativistic fluid $P = \kappa\rho$

Consider two-parameter family of initial data

- Gaussian density distribution centered on R_c
- parameterized by
  - η: strength of data
  - ϵ: eccentricity (proportional to $\ell = 2$)



Evolution

- for given value of κ choose ϵ, then fine-tune η to black-hole threshold

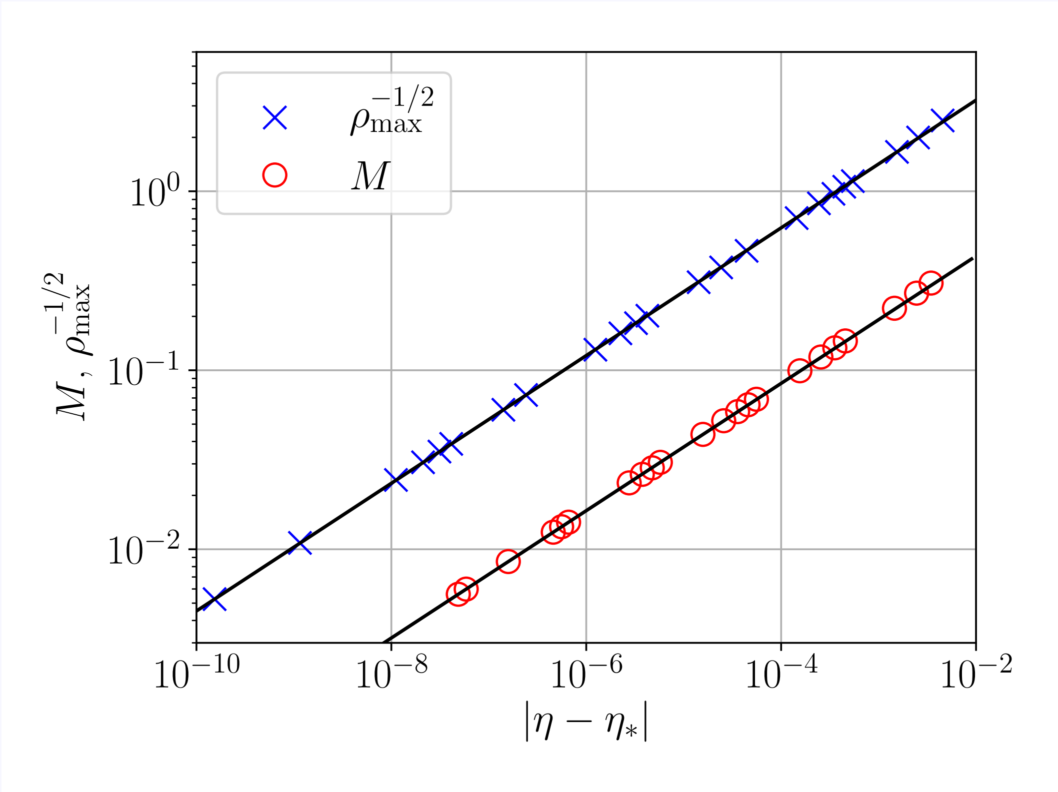
$\Longrightarrow$ confirm scaling laws

$$M \simeq (\eta - \eta_*)^\gamma$$

$$\rho_c^{-1/2} \simeq (\eta_* - \eta)^\gamma$$

$\Longrightarrow$ measure critical exponents

- excellent agreement with perturbative values
- at most little dependence on ϵ



$\kappa = 1/3$, $\epsilon = 1.0$

Deviations from sphericity

- Measure dimensionless "density variable"

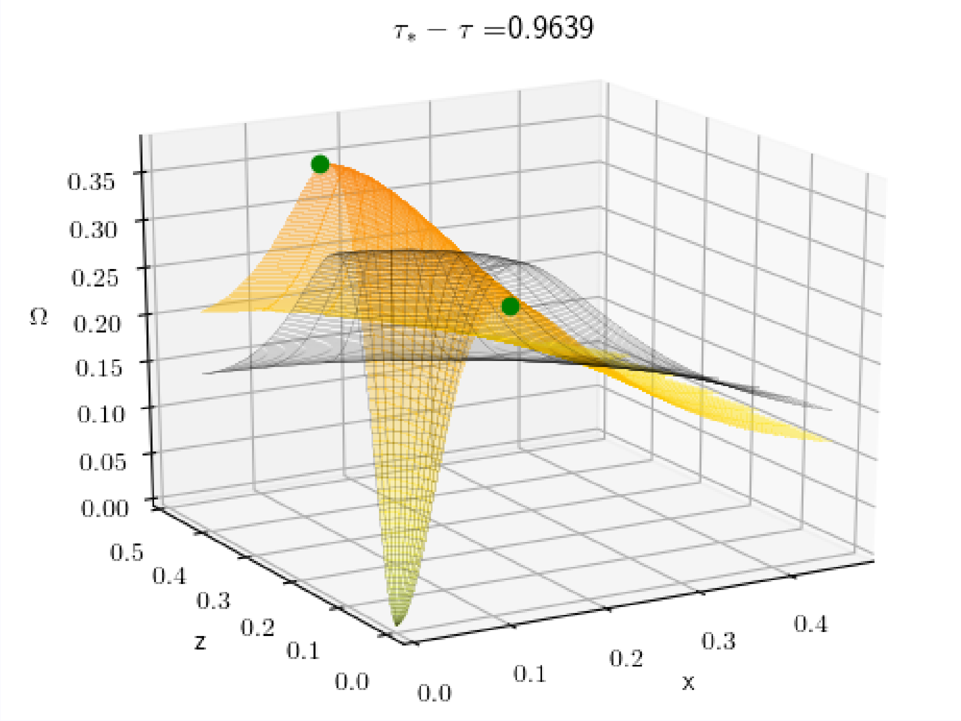
$$\Omega \equiv 4\pi R^2 \rho$$

  - in spherical symmetry, during self-similar phase $\Omega = \Omega(\xi)$
  - Track

$$\Delta\Omega \equiv \Omega_{\rm max,ax} - \Omega_{\rm max,eq}$$

    as measure of asphericity

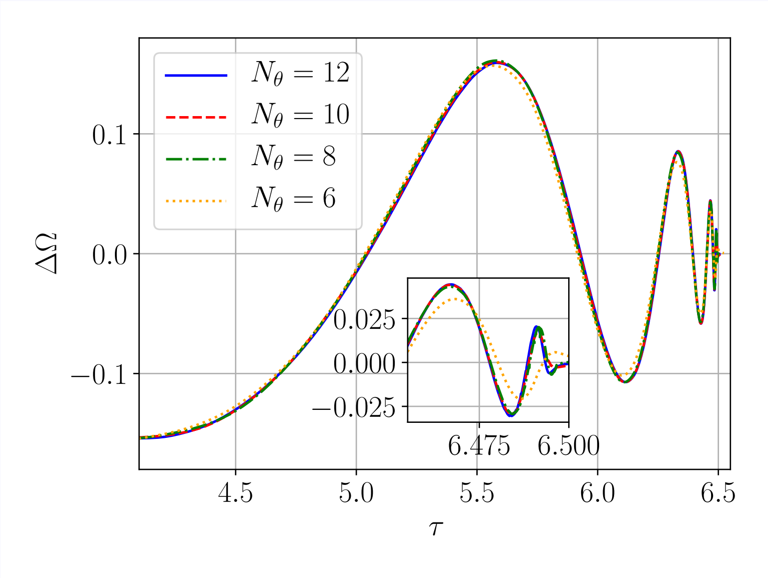
- Example:
  - Radiation fluid, $\kappa = 1/3$
  - $\epsilon = 1.0$



Pass the popcorn...

Fits

- Plot $\Delta\Omega$ as function of τ



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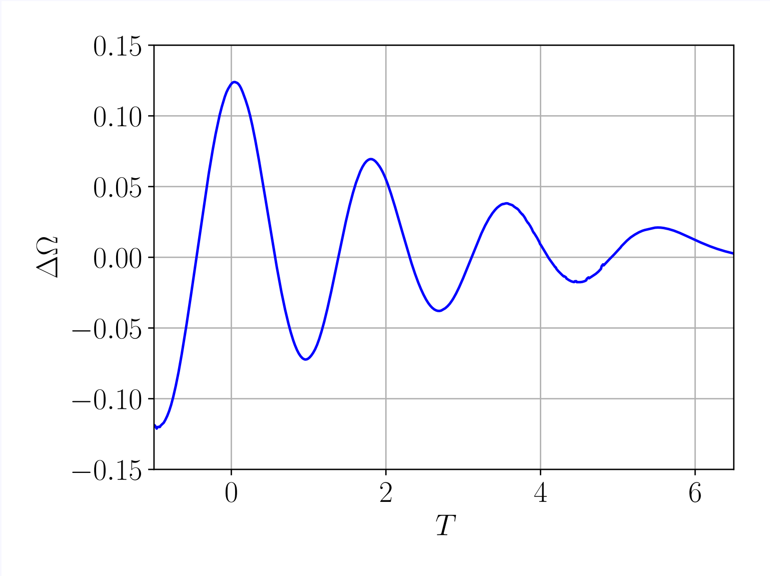


Fits

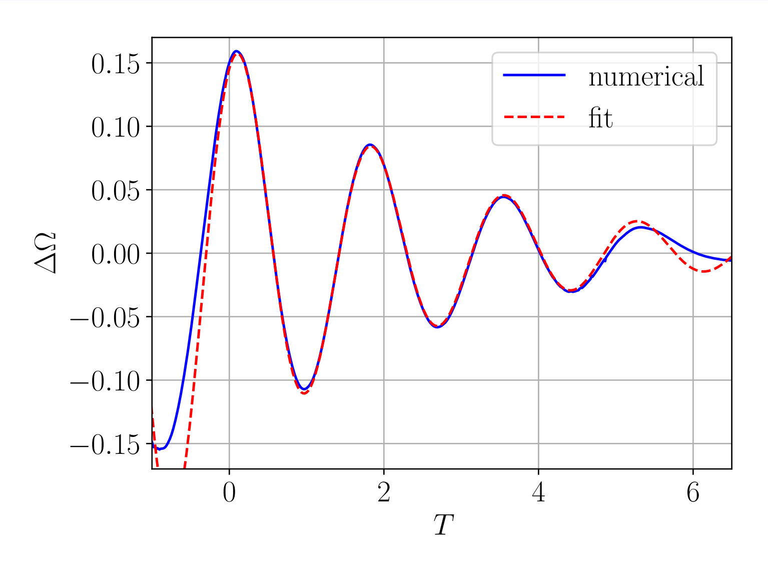
- Plot $\Delta\Omega$ as function of τ
- instead, plot as function of

$$T \equiv -\log(\tau_* - \tau)$$

$\Longrightarrow$ damped oscillations of form

$$\Delta\Omega \simeq e^{\lambda T}\cos(\omega T + \phi)$$

[Gundlach, 2002]



$\kappa = 1/3$						
ϵ	η_*	τ_*	γ_M	γ_ρ	λ	ω
perturbative			0.3558		-0.3846	3.6158
0	0.124087	6.449	0.357	0.357	–	–
0.01	0.124087	6.449	0.355	0.356	-0.36	3.64
0.1	0.124098	6.450	0.357	0.356	-0.36	3.64
0.5	0.124444	6.460	0.356	0.357	-0.36	3.64
1.0	0.125544	6.496	0.356	0.357	-0.37	3.65

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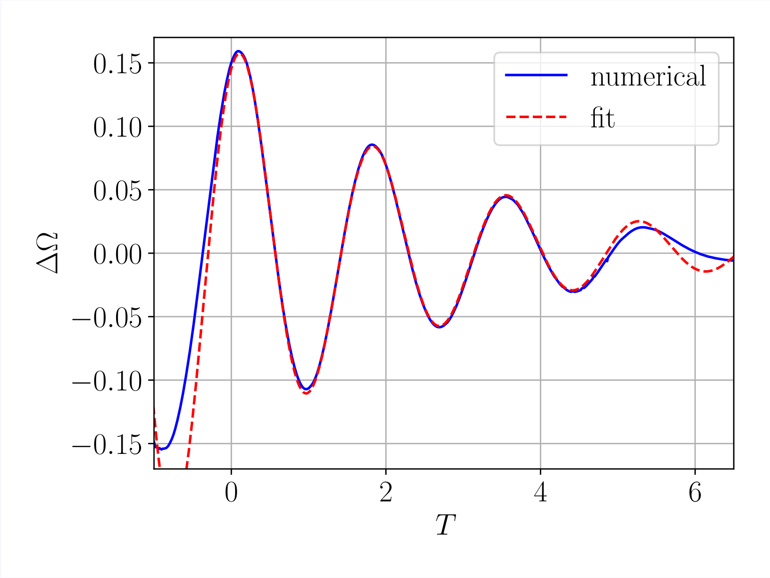

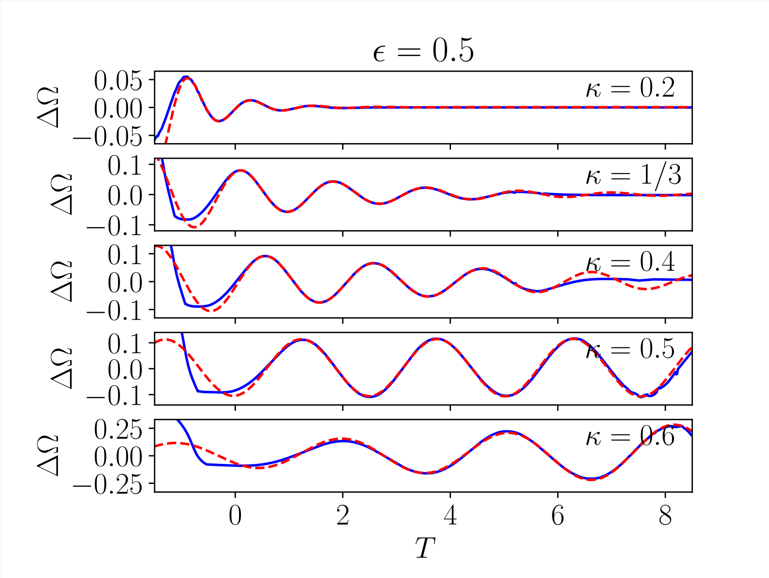

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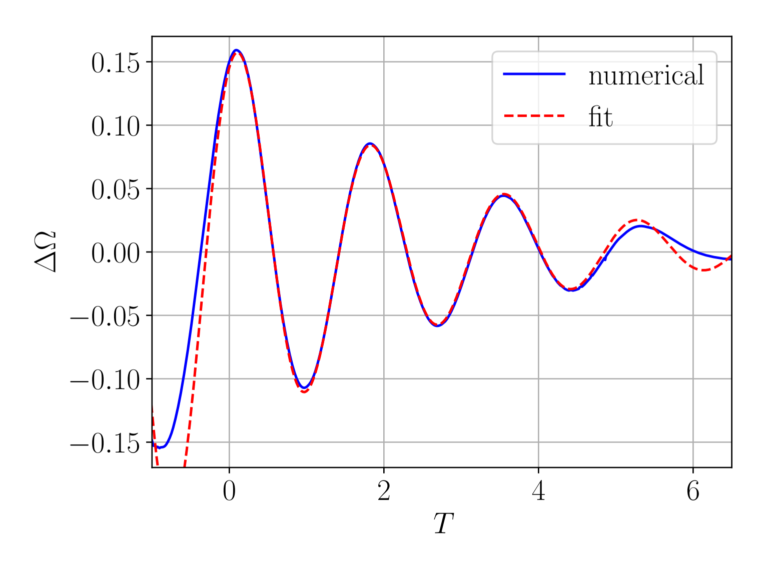


$\Longrightarrow$ modes become unstable for $\kappa \gtrsim 0.49$

$\Longrightarrow$ expect break-down of scaling at small scales

Critical Collapse with Rotation

Critical collapse with rotation leads to rotating black hole:

- Kerr black hole [Kerr, 1963]
- characterized by mass M and angular momentum J with

$$\frac{J}{M^2} < 1$$

$\Longrightarrow$ how does J behave as $M \to 0$ in critical collapse??

$\Longrightarrow$ what is role of angular momentum in critical collapse??

[Choptuik $et.al.$, 2004]

Critical Collapse with Rotation: Perturbative Results

Angular momentum scales with

$$J \simeq |\eta - \eta_*|^{\gamma_J}$$

Combine with $M \simeq |\eta - \eta_*|^{\gamma_M}$ to find

$$\frac{J}{M^2} \simeq M^{(\gamma_J - 2\gamma_M)/\gamma_M}$$

For perfect fluid

$$P = \kappa\rho \qquad \text{radiation fluid: } \kappa = 1/3$$

with $1/9 < \kappa \lesssim 0.49$ expect

$$\gamma_J = \frac{5\,(1+3\kappa)}{3\,(1+\kappa)}\,\gamma_M \qquad \text{radiation fluid: } \gamma_J = 2.5\,\gamma_M$$

[Gundlach, 1998; 2002]

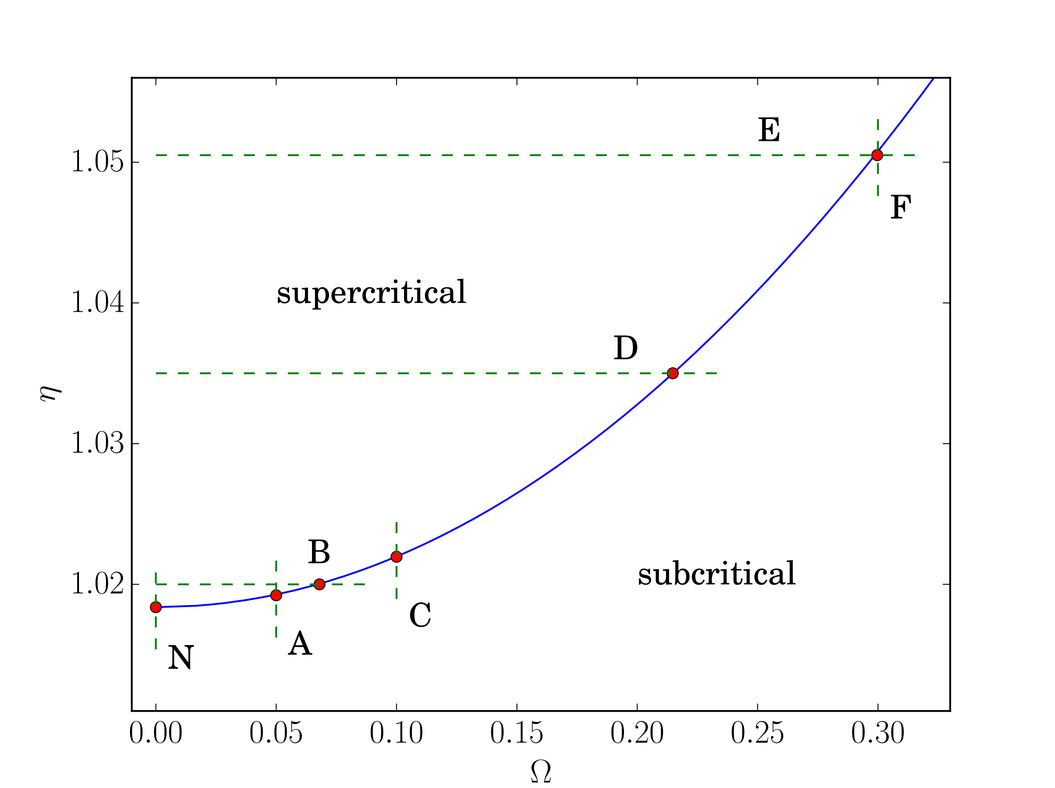
$\Longrightarrow$ For $\kappa > 1/9$ have $\gamma_J > 2\,\gamma_M$ and hence expect

$$\frac{J}{M^2} \rightarrow 0 \quad \text{as} \quad M \rightarrow 0$$

$\Longrightarrow$ explore numerically...

Initial Data

- Gaussian density distribution ρ, parameterized by:
  - amplitude η
  - spin velocity Ω



$\Longrightarrow$ explore sequences through two-dimensional parameter space:
- locate critical curve
- study scaling close to criticality
- generalize power-law behavior "globally"

Rotating Data

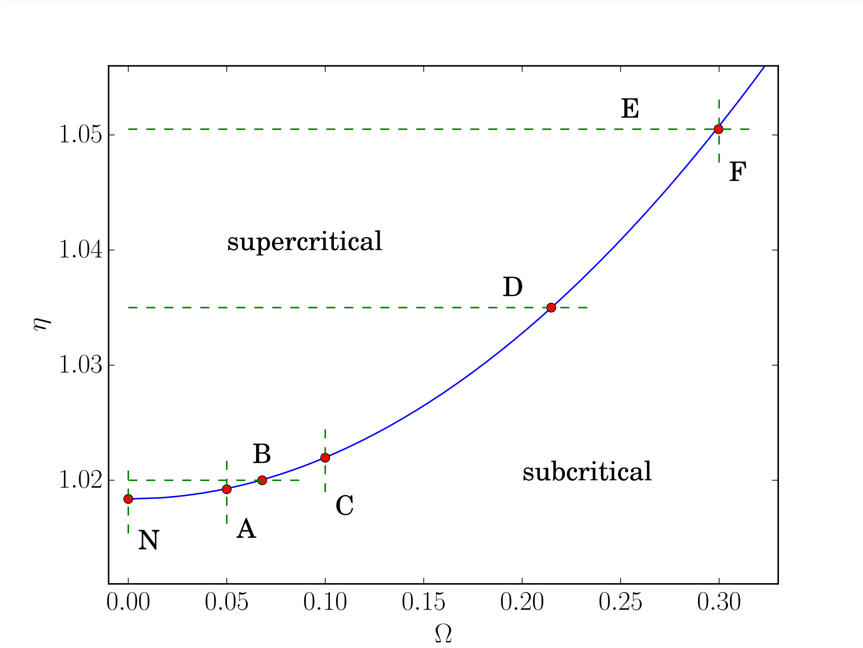
Sequences for

- constant η
- constant Ω

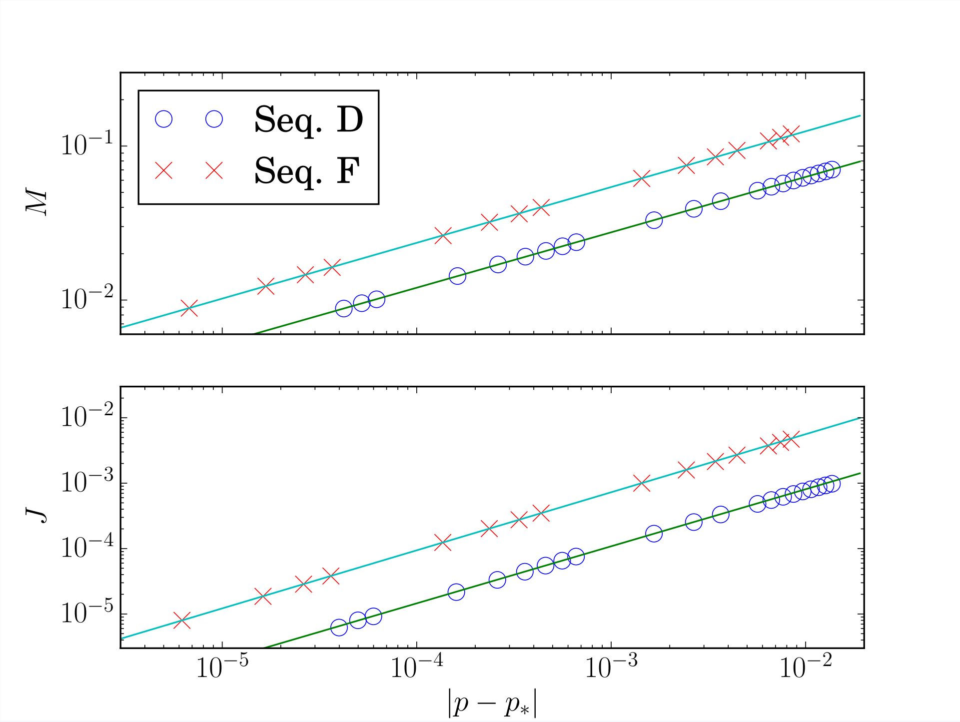
Rotation provides centrifugal support

$\Longrightarrow$ critical curve:

$$\eta_* \simeq \eta_{*0} + 0.36\,\Omega_*^2$$



[TWB & Gundlach, 2016]



For all sequences

$$\gamma_M \simeq \gamma_\rho \simeq 0.36$$

$$\gamma_J \simeq 0.87 \simeq 2.43\,\gamma_M$$

- independent of Ω and "direction" across critical curve
- critical black hole is non-spinning

Scaling laws

Include rotational $\ell = 1$ term in perturbative treatment

$\Longrightarrow$ derive extended scaling laws

$$\boxed{M \simeq \left(\eta - \eta_{*0} - K\Omega^2\right)^{\gamma_M}}$$

and

$$\boxed{J \simeq \Omega \left(\eta - \eta_{*0} - K\Omega^2\right)^{\gamma_J}}$$

Extended power-law relations

- Extended scaling laws for M and J

$$M \simeq \left(\eta - \eta_{*0} - K\Omega^2\right)^{\gamma_M}$$
$$J \simeq \Omega \left(\eta - \eta_{*0} - K\Omega^2\right)^{\gamma_J}$$

- for example, consider "horizontal" $\eta = const$ sequences

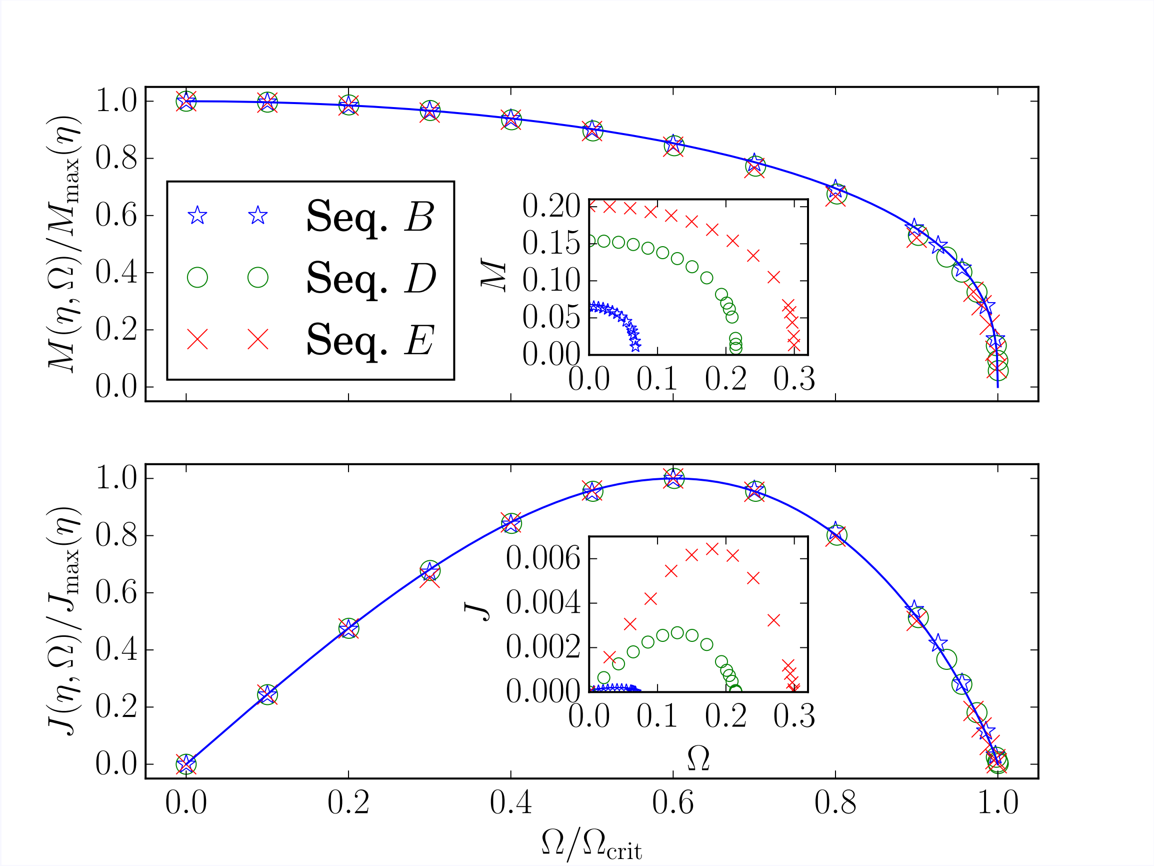
- normalize by maximum values

$$\frac{M(\eta,\Omega)}{M_{\max}(\eta)} \simeq (1-x^2)^{\gamma_M}$$
$$\frac{J(\eta,\Omega)}{J_{\max}(\eta)} \simeq \frac{x}{C}(1-x^2)^{\gamma_J}$$

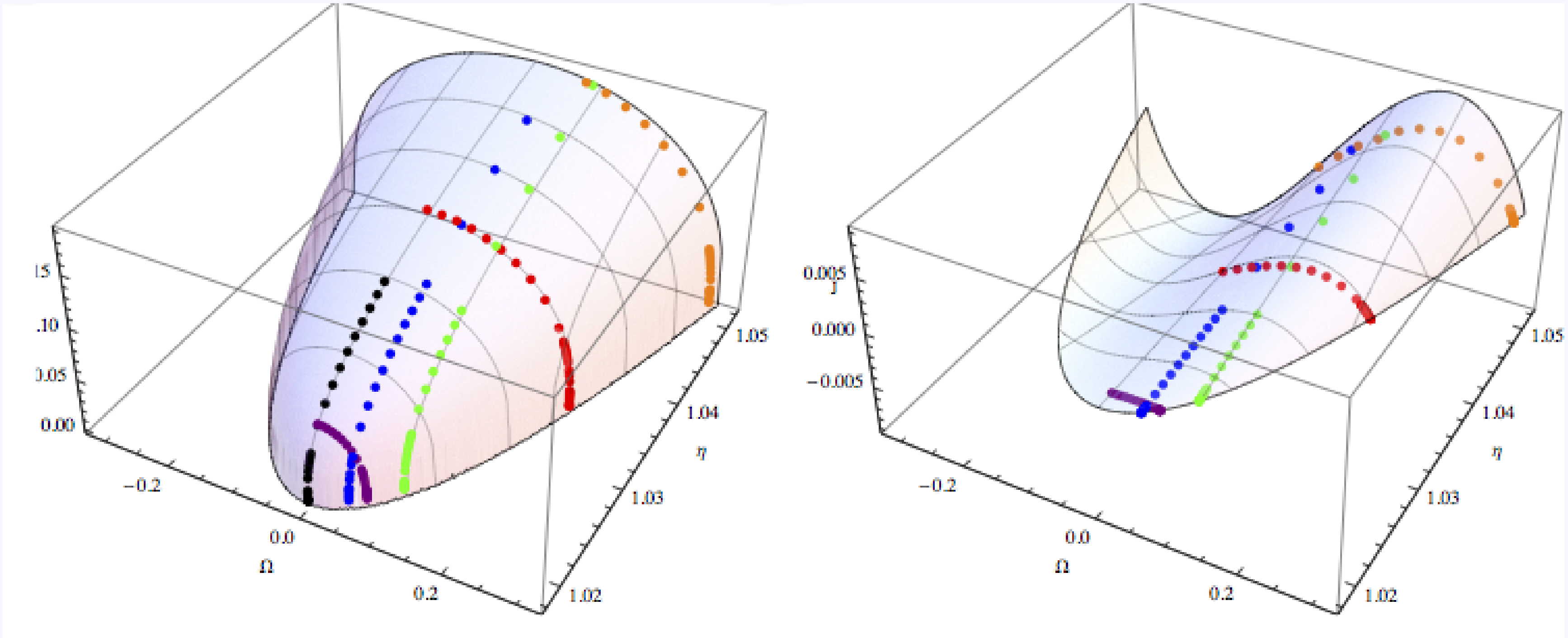
where $x = \Omega/\Omega_*(\eta)$

$\Longrightarrow$ excellent agreement

[TWB & Gundlach, 2016]



Predictions for mass and angular momentum



M J

[Gundlach & TWB, 2016]

... the unreasonable effectiveness of perturbation theory ...

Summary

Numerical simulations of critical collapse in the absence of spherical symmetry

- use code that adopts spherical polar coordinates
- study effects of
  - aspherical deformations
  - rotation
- find excellent agreement with perturbation theory
- aspherical modes are stable in regime $1/9 < \kappa \lesssim 0.49$ only
  $\Longrightarrow$ scaling-laws to smallest scales in this regime only
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