Inconsistency of binary systems as sources of g-waves in a first order approach to the weak gravity limit

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The typical example of application of the Pauli-Fierz equations which is found in the books of general relativity is the study of the gravitational radiation emitted by binary star systems. In this work, however, I show that this system, as it is studied in the books, is not consistent with a first order approach to the weak-gravity limit. To convince the reader that a problem exists, I will show that, if one computes the reduced perturbation to the metric, $\bar{h}$, in two equivalent ways, he will obtain two different results, whose discrepancy is not a higher order of the theory, but is comparable with $\bar{h}$ itself. In the last part of the paper I will explain the origin of the contradiction and why binary systems do not properly emit gravitational radiation at the first order.

Brief review of first order g-waves theory

First of all, let us briefly summarise some key steps of the study of the gravitational radiation as a perturbative approach to general relativity in an almost flat spacetime.

One starts imposing the metric to be the sum of the flat metric[1] and a small perturbation h:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| << 1.$$

Keeping only the first order in h, which is considered an infinitesimal perturbation, it is immediate to show that

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu},$$

where the indices of h should be raised and lowered with g, but, since we keep only the first order, this is equivalent to using η. It is useful to define the reduced perturbation $\bar{h}$, which satisfies the two equivalent conditions

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\bar{h}^{\sigma}{}_{\sigma}\eta_{\mu\nu} \qquad\qquad \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h^{\sigma}{}_{\sigma}\eta_{\mu\nu}.$$

Notice that $h^{\sigma}{}_{\sigma} = -\bar{h}^{\sigma}{}_{\sigma}$ is the reason why the two equations above are equivalent and that this is true only in four dimensions.

Considering the choice of the chart as a sort of gauge freedom for h, it is immediate to show that, given b^{μ} an arbitrary infinitesimal vector-field, there always exists a change of coordinates such that

$$g'_{\mu\nu} = \eta_{\mu\nu} + h'_{\mu\nu} \qquad \textit{with} \qquad h'_{\mu\nu} = h_{\mu\nu} - \partial_{\mu}b_{\nu} - \partial_{\nu}b_{\mu}.$$

Under this transformation $\bar{h}$ becomes

$$\bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - \partial_{\mu}b_{\nu} - \partial_{\nu}b_{\mu} + \partial_{\sigma}b^{\sigma}\eta_{\mu\nu}.$$

Manipulating the Einstein field equations, it has been shown that you can find a chart in which the system

$$\begin{cases} \partial_{\mu}\bar{h}^{\mu\nu} = 0 & (\textit{Lorentz Gauge}) \\ \\ \Box\bar{h}_{\mu\nu} = -16\pi T_{\mu\nu} & (\textit{Pauli} - \textit{Fierz Equations}) \end{cases}$$

constitutes the first-order dynamics of h (we always employ geometrical units: $c = 1$, $G = 1$).

[1]the Minkowsky metric $\eta_{\mu\nu} = diag(-1, +1, +1, +1)$

Remark: if we take the four-divergence of the second equation of the system and use the first we find that

$$-16\pi\partial_\mu T^{\mu\nu} = \partial_\mu \Box \bar{h}^{\mu\nu} = \Box \partial_\mu \bar{h}^{\mu\nu} = 0 \qquad \Longrightarrow \qquad \partial_\mu T^{\mu\nu} = 0.$$

This result, on the other hand, holds for any gauge. Consider, in fact, the exact Einstein equations

$$(8\pi)^{-1} G_{\mu\nu} = T_{\mu\nu}.$$

Applying the covariant four-divergence we get

$$0 = \nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \Gamma^\mu{}_{\mu\lambda} T^{\lambda\nu} + \Gamma^\nu{}_{\mu\lambda} T^{\mu\lambda}.$$

The terms with the Christoffel sysmbols are at least second orders in h, so they can be neglected, leaving

$$\partial_\mu T^{\mu\nu} = 0,$$

which is what we wanted to prove. $\Box$

Going back to our system, the solution, using the the d'Alembert operator's Green function, is

$$\bar{h}_{\mu\nu}(\mathbf{x}, t) = \int_{\mathbb{R}^3} \frac{4T_{\mu\nu}(\mathbf{y}, t_R)}{|\mathbf{x} - \mathbf{y}|} d_3 y \qquad \textit{with} \qquad t_R = t - |\mathbf{x} - \mathbf{y}|.$$

t_R is called retarded time and its presence encodes a delay in the transmission of information. It informs us that gravity travels at the speed of light and, thus, that an amount of time is required by signals to travel from a place to an other.

In the case of interest for our work, it is assumed that the source of the waves is a small distribution of matter located around the origin O and that the evaluation point has a distance r from O which is enormous compared with the scale of the source, allowing us to write

$$\frac{1}{|\mathbf{x} - \mathbf{y}|} \approx \frac{1}{r}.$$

This is the far-field limit. The second approximation consists of assuming that the system is sufficiently small and its motion is sufficiently slow that, in the time needed by the waves to cross it, the distribution of matter does not change considerably, leading us to the simplification

$$t_R \approx t - r$$

in the integral, which is the slow-source limit. This means that, in the calculations, the variable t_R is kept constant and the delay is an overall shift of times. Plugging our approximation in the expression of $\bar{h}$, we get

$$\bar{h}_{\mu\nu}(\mathbf{x}, t) = \frac{4}{r} \int_{\mathbb{R}^3} T_{\mu\nu}(\mathbf{y}, t_R) d_3 y.$$

If one considers the Lorentz gauge equation, he will notice that it must be true that

$$\bar{h}^{0\nu}(\mathbf{x}, t) = \bar{h}^{0\nu}(\mathbf{x}, 0) - \int_0^t \partial_j \bar{h}^{j\nu}(\mathbf{x}, t') dt',$$

therefore, all the information about the dynamics is contained in

$$\bar{h}^{jk} = \frac{4}{r} \int_{\mathbb{R}^3} T^{jk}(\mathbf{y}, t_R) d_3 y, \tag{1}$$

which is the first version of the equation of the gravitational radiation.

There is a useful way to rewrite it. You only have to take the two following steps

Step1: Consider the chain of identities

$$\partial_l(y^jT^{lk}) = \partial_l y^j T^{lk} + y^j\partial_l T^{lk} = T^{jk} + y^j\partial_\mu T^{\mu k} - y^j\partial_0 T^{0k}.$$

Isolating T^{jk} we get

$$T^{jk} = \partial_0(y^jT^{0k}) + \partial_l(y^jT^{lk}) - y^j\partial_\mu T^{\mu k},$$

which, integrated, becomes

$$\int_{\mathbb{R}^3} T^{jk} d_3y = \frac{d}{dt}\int_{\mathbb{R}^3} y^jT^{0k}d_3y - \int_{\mathbb{R}^3} y^j\partial_\mu T^{\mu k}d_3y.$$

Considering that $\partial_\mu T^{\mu\nu} = 0$, it is clear that the second term in the right-hand side is zero. So we have

$$\bar{h}^{jk} = \frac{4}{r}\frac{d}{dt}\int_{\mathbb{R}^3} y^jT^{0k}d_3y \qquad \bar{h}^{kj} = \frac{4}{r}\frac{d}{dt}\int_{\mathbb{R}^3} y^kT^{0j}d_3y.$$

But $\bar{h}^{jk} = \bar{h}^{kj}$, thus

$$\bar{h}^{jk} = \frac{\bar{h}^{jk} + \bar{h}^{kj}}{2} = \frac{2}{r}\frac{d}{dt}\int_{\mathbb{R}^3}(y^jT^{0k} + y^kT^{0j})d_3y.$$

Step2: Consider, now, this identity

$$\partial_l(y^jy^kT^{0l}) = y^jT^{0k} + y^kT^{0j} + y^jy^k\partial_l T^{0l}.$$

Remembering that $\partial_\mu T^{0\mu} = 0$, we find that

$$y^jT^{0k} + y^kT^{0j} = \partial_l(y^jy^kT^{0l}) + y^jy^k\partial_0T^{00},$$

which leads us to the final result

$$\bar{h}^{jk} = \frac{2}{r}\frac{d^2}{dt^2}\int_{\mathbb{R}^3} y^jy^kT^{00}d_3y.$$

Now we define the quadrupole moment of the distribution of matter as

$$Q^{jk}(t) := 3\int_{\mathbb{R}^3} y^jy^kT^{00}(\mathbf{y},t)d_3y,$$

giving

$$\bar{h}^{jk} = \frac{2}{3r}\frac{d^2}{dt^2}Q^{jk}(t_R). \tag{2}$$

This is the second formula for the gravitational radiation we are interested in. Remember that the equations 1 and 2 are equivalent.

Binary systems

Now we are ready to study the gravitational radiation emitted by binary systems. We consider the simple situation presented in figure 1: two bodies of equal mass M (this simplifies calculations) which follow a circular motion of radius R around their collective center of mass, which is located in the origin O. We build the axes in a way that their orbit lies in the $x^1 - x^2$ plane. The two objects are studied on a scale which allows us to consider them as point-particles.
Solving the Newtonian problem one finds that the angular velocity of the bodies is

$$\Omega = \sqrt{\frac{M}{4R^3}}$$

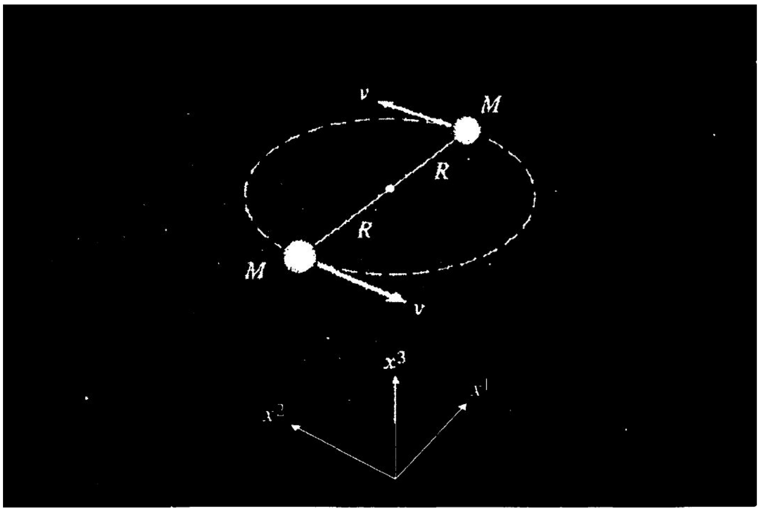

Figure 1: Visual representation of our binary star system.

and the tangential velocity is obviously

$$v = \Omega R = \sqrt{\frac{M}{4R}}.$$

Thus, once M and R are known, all the information, in the Newtonian limit, is given. In an almost flat spacetime the energy-momentum tensor of a system of point-particles is

$$T^{\mu\nu}(\mathbf{x}, t) = \sum_{\alpha} m_{(\alpha)} \gamma_{(\alpha)}(t) v^{\mu}_{(\alpha)}(t) v^{\nu}_{(\alpha)}(t) \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}(t)),$$

where α is a particle counter, $m_{(\alpha)}$ is the mass of the α-th particle, $\mathbf{r}_{(\alpha)}(t)$ its position and the three-vector $\mathbf{v}_{(\alpha)}(t) := \dot{\mathbf{r}}_{(\alpha)}(t)$ its velocity, while

$$\gamma_{(\alpha)}(t) := \frac{1}{\sqrt{1 - |\mathbf{v}_{(\alpha)}(t)|^2}} \qquad \textit{and} \qquad v^{\mu}_{(\alpha)}(t) = (1, \mathbf{v}_{(\alpha)}(t))^T$$

are respectively the Lorentz factor and the non-normalized four-velocity. In our case of interest we assume that the speed of the objects is small compared with the speed of light, so we can neglect the $\gamma_{(\alpha)}$. It is not a necessary requirement for our calculations, but it simplifies the notation and the interpretations of the results. We set the origin of time in a way that, when $t = 0$, one of the two masses, which will be considered the mass 1, lies on the positive x^1 semi-axis. Our test, now, will consist of calculating the emitted gravitational radiation using the two different equations we derived and verifying that the results do not coincide.

1) First of all we consider that an integral over $\mathbb{R}^3$ cancels a δ_3, thus

$$\int_{\mathbb{R}^3} T^{jk} d_3 y = \sum_{\alpha} m_{(\alpha)} v^j_{(\alpha)} v^k_{(\alpha)}.$$

The three-velocities of the two bodies are always opposite because the center of mass is at rest, so

$$\int_{\mathbb{R}^3} T^{jk} d_3 y = 2M v^j_{(1)} v^k_{(1)}$$

and we can focus our attention on the first star. Obviously it is true that

$$v^j_{(1)} = \Omega R \begin{pmatrix} -\sin(\Omega t) \\ \cos(\Omega t) \\ 0 \end{pmatrix}, \qquad \textit{thus} \qquad v^j_{(1)} v^k_{(1)} = \Omega^2 R^2 \begin{bmatrix} \sin^2(\Omega t) & -\sin(\Omega t)\cos(\Omega t) & 0 \\ -\sin(\Omega t)\cos(\Omega t) & \cos^2(\Omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We finally arrive at

$$\bar{h}^{jk}_{(T)} = \frac{4}{r}\int_{\mathbb{R}^3} T^{jk}(\mathbf{y}, t_R) d_3 y = \frac{8M\Omega^2 R^2}{r}\begin{bmatrix} \sin^2(\Omega t_R) & -\sin(\Omega t_R)\cos(\Omega t_R) & 0 \\ -\sin(\Omega t_R)\cos(\Omega t_R) & \cos^2(\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The subscript (T) reminds us that we derived this expression using the equation 1, which involves the explicit form of the energy-momentum tensor. We can rewrite the equation in a more interesting way using the Newtonian prescription for Ω and the duplication formulas of trigonometry:

$$\bar{h}^{jk}_{(T)} = \frac{M^2}{Rr}\begin{bmatrix} 1-\cos(2\Omega t_R) & -\sin(2\Omega t_R) & 0 \\ -\sin(2\Omega t_R) & 1+\cos(2\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

2) The second way requires the computation of the quadrupole moment:

$$Q^{jk} = 3\int_{\mathbb{R}^3} x^j x^k T^{00} d_3 x.$$

We notice that all the distribution of matter lies in the $x^1 - x^2$ plane, thus $Q^{j3} = 0$. The computation of the integral, using the general expression of the energy-momentum tensor for a system of point particles, leads us to

$$\int_{\mathbb{R}^3} x^j x^k T^{00} d_3 x = \sum_\alpha m_{(\alpha)} r^j_{(\alpha)} r^k_{(\alpha)}.$$

The positions of the two bodies are always opposite because the center of mass is in the origin, so

$$Q^{jk} = 6M r^j_{(1)} r^k_{(1)}$$

and we can focus our attention on the first star. Obviously it is true that

$$r^j_{(1)} = R\begin{pmatrix} \cos(\Omega t) \\ \sin(\Omega t) \\ 0 \end{pmatrix}, \qquad \textit{thus} \qquad r^j_{(1)} r^k_{(1)} = R^2\begin{bmatrix} \cos^2(\Omega t) & \sin(\Omega t)\cos(\Omega t) & 0 \\ \sin(\Omega t)\cos(\Omega t) & \sin^2(\Omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The quadrupole moment is, therefore,

$$Q^{jk} = 3MR^2\begin{bmatrix} 1+\cos(2\Omega t) & \sin(2\Omega t) & 0 \\ \sin(2\Omega t) & 1-\cos(2\Omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and its second derivative is

$$\ddot{Q}^{jk} = 3MR^2(2\Omega)^2\begin{bmatrix} -\cos(2\Omega t) & -\sin(2\Omega t) & 0 \\ -\sin(2\Omega t) & \cos(2\Omega t) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We, thus, arrive at the reduced perturbation of the metric

$$\bar{h}^{jk}_{(Q)} = \frac{2}{3r}\frac{d^2}{dt^2}Q^{jk}(t_R) = \frac{8M\Omega^2 R^2}{r}\begin{bmatrix} -\cos(2\Omega t_R) & -\sin(2\Omega t_R) & 0 \\ -\sin(2\Omega t_R) & \cos(2\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where the subscript (Q) reminds us that we derived this expression using the expression 2, which involves the quadrupole moment. Expliciting Ω^2 we get

$$\bar{h}^{jk}_{(Q)} = \frac{M^2}{Rr}\begin{bmatrix} -2\cos(2\Omega t_R) & -2\sin(2\Omega t_R) & 0 \\ -2\sin(2\Omega t_R) & 2\cos(2\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Comparison of the results

Now that we have calculated $\bar{h}^{jk}$ using the two equations we can subtract the second to the first, arriving at

$$\bar{h}^{jk}_{(T)} - \bar{h}^{jk}_{(Q)} = \frac{M^2}{Rr}\begin{bmatrix} 1+\cos(2\Omega t_R) & \sin(2\Omega t_R) & 0 \\ \sin(2\Omega t_R) & 1-\cos(2\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

which is, clearly, different from zero. The discrepancy has the same order of $\bar{h}^{jk}$, so the error cannot be neglected. We proved, however, that the two ways of calculating the reduced perturbation must be equivalent, so... what went wrong? The only ingredient which was used to pass from the first to the second, apart from simple algebra, was the condition $\partial_\mu T^{\mu\nu} = 0$. Let us study its validity.[2]

$$\partial_\mu T^{\mu\nu} = \sum_\alpha m_{(\alpha)} \partial_\mu \left[v^\mu_{(\alpha)} v^\nu_{(\alpha)} \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}) \right].$$

Separating the temporal index from the spatial ones, we get

$$\partial_\mu T^{\mu\nu} = \sum_\alpha m_{(\alpha)} \left[v^\nu_{(\alpha)} v^j_{(\alpha)} \partial_j \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}) + \partial_t v^\nu_{(\alpha)} \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}) - v^\nu_{(\alpha)} \dot{r}^j_{(\alpha)} \partial_j \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}) \right].$$

Clearly the first and the third term inside the square brackets cancel out, leaving

$$\partial_\mu T^{\mu\nu} = \sum_\alpha m_{(\alpha)} \partial_t v^\nu_{(\alpha)} \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}).$$

Notice that, by definition, $v^0_{(\alpha)} = 1$, thus it is always true that $\partial_\mu T^{\mu 0} = 0$. Since this is the equation which was used in the step 2, we can conclude that the problem is not there. On the other hand, if the particles do not occupy the same point, it is clear that

$$\partial_\mu T^{\mu j} = 0 \qquad \Longleftrightarrow \qquad \partial_t v^j_{(\alpha)} = 0 \quad \forall \alpha.$$

The interpretation of this result is simple: $\partial_\mu T^{\mu j} = 0$ is the conservation of momentum in absence of gravity and for a point particle this means that it follows a uniform rectilinear motion. This condition was invoked in the step 1, but it is not satisfied by the binary system. In fact, the two bodies follow a circular motion, governed by a centripetal acceleration which is a pure product of gravity, so we are not allowed to neglect the connection coefficients in $\nabla_\mu T^{\mu j} = 0$.

So, if we go back to the step 1, and assume that the divergence of the energy-momentum tensor does not vanish, we should obtain a prediction for the discrepancy between $\bar{h}^{jk}_{(T)}$ and $\bar{h}^{jk}_{(Q)}$, given by the formula

$$\bar{h}^{jk}_{(T)} = \bar{h}^{jk}_{(Q)} - \frac{4}{r} \int_{\mathbb{R}^3} x^j \partial_\mu T^{\mu k} d_3 x.$$

Let us calculate the second term in the right-hand side and verify that we get exactly the error we got computing $\bar{h}^{jk}_{(T)}$ and $\bar{h}^{jk}_{(Q)}$ separately. The acceleration is centripetal, thus

$$\partial_t v^k_{(\alpha)} = -\Omega^2 x^k$$

and

$$x^j \partial_\mu T^{\mu k} = -M\Omega^2 \sum_\alpha x^j x^k \delta_3(\mathbf{x} - \mathbf{r}_{(\alpha)}).$$

Performing the integration, using the duplication trigonometric formulas and expliciting Ω^2, we get

$$\bar{h}^{jk}_{(T)} - \bar{h}^{jk}_{(Q)} = \frac{M^2}{Rr}\begin{bmatrix} 1+\cos(2\Omega t_R) & \sin(2\Omega t_R) & 0 \\ \sin(2\Omega t_R) & 1-\cos(2\Omega t_R) & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

[2] We remain in the approximation $\gamma_{(\alpha)} \approx 1$, which simplifies the calculations.

which is exactly the discrepancy we found. So the problem is that, being the system self-gravitating, we cannot neglect in the study of $\bar{h}^{jk}$ the role played by gravity in the motion of the bodies.

Explanation of the inconsistency

The Pauli-Fierz equatios hold as the first order of Einstein equations in the Lorentz gauge. On the other hand, the terms with the Christoffel symbols in the equation

$$-\partial_\mu T^{\mu j} = \Gamma^\mu{}_{\mu\lambda} T^{\lambda j} + \Gamma^j{}_{\mu\lambda} T^{\mu\lambda}$$

are a second order, thus in the limit of weak gravity, we expect that their role vanishes. However, we saw above that their existence is the origin of the discrepancy between $\bar{h}^{jk}_{(T)}$ and $\bar{h}^{jk}_{(Q)}$ and that their contribution is always comparable with $\bar{h}^{jk}$ itself. In fact it is true that

$$\frac{\bar{h}^{12}_{(T)} - \bar{h}^{12}_{(Q)}}{\bar{h}^{12}_{(T)}} = -1,$$

which does not vanish in the weak gravity limit. Therefore there seems to be a mathematical contradiction because we have that the ratio between a second order and a first order, both different from zero, does not go to zero in the weak gravity limit, but remains fixed at -1. The final (and most important) step of this brief work consists of solving this contradiction and explaining why the Pauli-Fierz equations cannot be applied to binary systems in the way we, and the books, did.

In general, a perturbative analysis is performed considering a fundamental scale parameter, studying how the system behaves when we vary it and taking the limit in which it is sufficiently small to consider every power of the parameter negligible with respect to the previous one. Since we are interested in the weak gravity limit, our natural scale parameter is the mass M of the bodies, because, if it goes to zero, the energy-momentum tensor, and therefore the perturbation to the metric, vanishes. Now, let us write the energy-momentum tensor. For simplicity we consider only the contribution given to it by the first body because the other is analogous:

$$T^{\mu\nu}_{(1)} = M\delta_3(\mathbf{x} - \mathbf{r}_{(1)}) \begin{bmatrix} 1 & -R\Omega s & R\Omega c & 0 \\ -R\Omega s & R^2\Omega^2 s^2 & -R^2\Omega^2 sc & 0 \\ R\Omega c & -R^2\Omega^2 sc & R^2\Omega^2 c^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where s is an abbreviation for $\sin(\Omega t)$ and c is an abbreviation for $\cos(\Omega t)$. The unwary reader, looking at the expression above, noticing that the right-hand side of the equation is proportional to M, may conclude that $T^{\mu\nu}_{(1)}$ can be considered a first order in the mass. There is, however, the problem that Ω is not independent of M, but it must be true that

$$\Omega = \sqrt{\frac{M}{4R^3}}.$$

Therefore we have that

$$T^{00}_{(1)} \propto M \qquad\qquad T^{0j}_{(1)} \propto M^{3/2} \qquad\qquad T^{jk}_{(1)} \propto M^2.$$

If we keep only the first order in M we must write

$$T^{\mu\nu}_{(1)} = M\delta_3(\mathbf{x} - \mathbf{r}_{(1)}) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The substance is that if we want to consider the mass an infinitesimum, but at the same time we want that the two bodies follow a circular motion, then we must impose that also the velocity goes to zero, otherwise the stars would escape the attraction. Thus the first order of $\bar{h}$ is the Newtonian long range attraction of the binary system seen as a single object of mass $2M$. The effects due to the rotation are a higher correction and cannot be studied as solutions of the Pauli-Fierz equations. In fact, if you look at the final expressions of $\bar{h}^{jk}_{(T)}$ and $\bar{h}^{jk}_{(Q)}$, they are both proportional to M^2 and not to M. The apparent paradox

$$\frac{\bar{h}^{12}_{(T)} - \bar{h}^{12}_{(Q)}}{\bar{h}^{12}_{(T)}} = -1,$$

now is solved, because we showed that both the numerator and the denominator are a second order in M, eliminating the contradiction.

The conclusion is that a priori, if Ω was free, T^{jk} would have the same order of T^{00}, however we make the speeds depend on the intensity of gravity itself and, in particular, we choose to work with some velocities which are so small to make the perturbation of the metric generated by the rotational motion exactly comparable with the second order of the theory. Therefore we may say that binary systems are forced, in order to avoid to decompose, to be so slow that their emission is negligible at the first order.