Equation of state and neutron star properties constrained by nuclear physics and observations

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Astro-PF Polish-French collaboration in astrophysics

National Science Centre Poland

PHAROS The multi-messenger physics and astrophysics of neutron stars

Neutron stars: general aspects

Discovery of neutron stars (NSs)

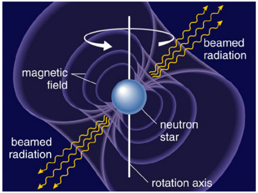
Yakovlev et al., arXiv:1210.0682 (2012); Haensel et al.'s book (2007)

From theoretical predictions ...

- Feb. 1931: anticipation of the idea of NSs by Lev Landau.
- Jan. 1932: experiments by Chadwick and discovery of the neutron.
- Dec. 1933: Baade & Zwicky: "*supernovæ represent the transitions from ordinary stars to neutron stars, which in their final stages consist of extremely closely packed neutrons*".

... to observations

- 1967: observation by chance by Bell (Hewish's graduate student) of very stable radio pulses with $P = 1.3373012$ s. The source is called "pulsar" meaning "Pulsating Source of Radio".
- 1974: Nobel Prize to Hewish (only) for the discovery of pulsars.
- May 1968 : Gold, Nature : pulsar = rotating NS.



Lighthouse model

Period of the pulses = spin period P of the pulsar.
All PSRs are NSs but not all NSs are seen as PSRs.

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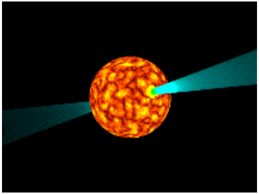

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Origin

Remnant from the gravitational collapse of a ~ 10 $M_\odot$ star during a Type II, Ib, Ic supernova event.

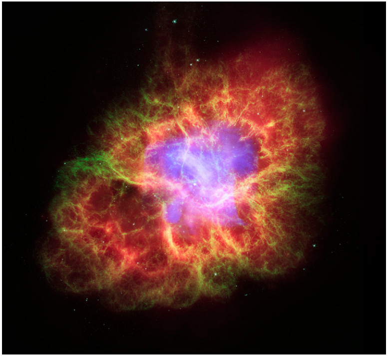
Orders of magnitude

- mass $M \sim 1.4\, M_\odot$ ($M_\odot \simeq 10^{30}$ kg $= 10^{33}$ g),
- radius $R \sim 10$ km $= 10^6$ cm,
- magnetic field $B \sim 10^4 - 10^{14}$ T.
- compactness $\frac{GM}{Rc^2} \sim 0.2$, GR effects needed to model macrophysical properties,
- total number of nucleons $A = M_\odot/m_{\rm N} \sim 10^{57}$!
- temperatures $T \sim 10^6 - 10^9$ K inferred from X-ray observations.
- mean mass density $\bar{\rho} \sim 5 \times 10^{14}$ g cm^{-3}.

NS vs. atomic nuclei

- A nucleons
- radius: $r_{\rm nucleus} = Ar_0$ with $r_0 \simeq 1.25$ fm$= 1.25 \times 10^{-13}$ cm,
- $m_{\rm nucleus} = Am_{\rm N}$ with the nucleon mass $m_{\rm N} = 1.67 \times 10^{-24}$ g
- (mass)-density of nucleons in a nucleus: $\rho_0 \simeq m_{\rm nucleus}/(4/3\pi r^3_{nucleus}) = 2.8 \times 10^{14}$ g cm^{-3}, $n_0 = 0.16$ fm^{-3}

Crab Nebula hosting a pulsar



Credits : NASA/ESA.

Multi-messenger observations

~ 3000 NSs from radio to γ-rays, a majority as radio pulsars, $\sim 5\%$ of them in a binary with a companion star.
Gravitational waves emitted by a binary NS merger observed in August 2017.

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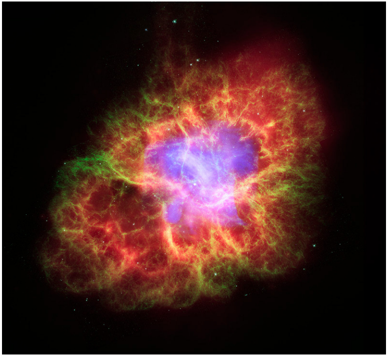
One of the many NS puzzles:

What are NSs made of?

Crab Nebula hosting a pulsar



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From the EoS to the $M - R$ relation

(Too-)simple EoS

Degenerate, ideal Fermi gas of neutrons

- non-interacting particles
- at $T = 0$: Fermi temperature $T = \frac{\hbar^2}{2m_N k_\mathrm{B}} \left(3\pi^2 n_0\right)^{2/3} \sim 10^{11}$ K, with $n_0 = 0.16$ fm^{-3} the NS mean density, much larger the $T \sim 10^6 - 10^9$ K inside a NS;
- relation between the pressure P and the density n, a so-called **equation of state** (EoS), or equivalently between P and the mass-energy density ε using the first law of thermodynamics:

$$\mathrm{d}\left(\frac{\varepsilon}{n}\right) = -P\mathrm{d}\left(\frac{1}{n}\right)$$

- Let us consider non-relativistic neutrons hence a polytropic EoS $P = Kn^\Gamma$ with $\Gamma = 5/3$.

How to obtain the properties of the NS, in particular the relation between the mass M and the radius R?

Tolman-Oppenheimer-Volkoff equations

Hydrostatic equilibrium in GR.
Einstein equation:

$$\underbrace{\boldsymbol{R} - \frac{1}{2}R\boldsymbol{g}}_{\text{spacetime}} = \underbrace{\frac{8\pi G}{c^4}\boldsymbol{T}}_{\text{matter}}.$$

- spherically symmetric star (effects of rotation neglected) $\rightarrow$ Schwarzschild metric
- perfect fluid: no viscosity, no shear stresses, no heat conduction $\rightarrow$ stress-energy tensor

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$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\varepsilon}{r^2}\left(1 + \frac{P}{\varepsilon c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

with $P(r)$, $m(r)$ and $\varepsilon(r) = \varepsilon(P(r))$.

GR corrections to hydrostatic equilibrium.

- boundary conditions: $m(r = 0) = 0$ $P(r = 0) = P_\mathrm{c}$ a chosen value of the central pressure.
- radius R of the star where $P(r = R) = 0$
- *gravitational* mass M of the star $M = m(r = R)$.

$\rightarrow$ profiles $P(r)$ and $m(r)$.

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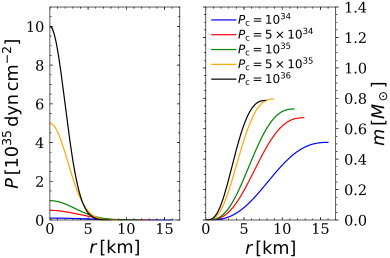
For a given central P_c (or n, ε), solve the TOV eq. using the EoS.



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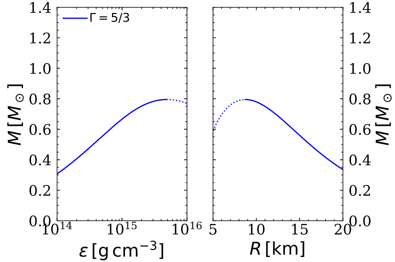

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GR corrections to hydrostatic equilibrium.

Maximum mass

- purely relativistic effect, not existing in Newtonian physics,
- marks the onset of an instability w.r.t small perturbations,
- $\mathrm{d}M/\mathrm{d}\varepsilon < 0 \rightarrow$ unstable;
- $\mathrm{d}M/\mathrm{d}\varepsilon > 0 \rightarrow$ stable in general (see discussion in HPY);
- for higher densities collapse to a black hole

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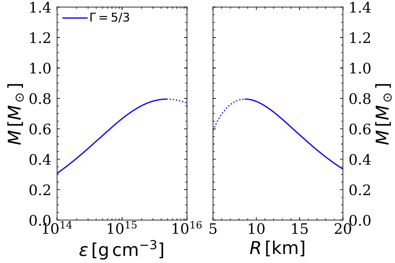

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GR corrections to hydrostatic equilibrium.

Maximum mass... problem

- $M_{\rm max} = 0.79\, M_\odot$ ...inconsistent with observations of $1 - 2\, M_\odot$ NSs!
- EoS is ruled out by observations...
- Fermi gas of relativistic neutrons at high density: $P \propto n \rightarrow$ $M_{\rm max} = 0.71\, M_\odot$
- in other words a NS is not composed of Fermi gas of non-interacting neutrons.
- **Which ingredient is missing?**

The neutron star equation of state

NS matter and nuclear interactions

NS equilbrium

balance between the attractive gravitational force & the repulsive nuclear force, **not** the Fermi pressure of degenerate neutrons!

NS matter

Non-accreting NS: matter in complete thermodynamic equil., in its ground state with the lowest possible energy.

- Cold ($T = 0$) β-equilibrated matter (stable against neutron β-decay)
- neutron-rich: $n_\mathrm{n} \sim (5 - 10)n_\mathrm{p}$ with n_i the n, p number densities
- charge neutral at the global scale
- without neutrinos: few min after the supernova mean free path becomes larger than the NS R as T decreases
- many-body system of strongly-interacting particles.

Two approaches to the EoS

In principle one would want to describe NS matter using QCD...but there are no ab-initio QCD calculations available describing NS matter.

- phenomenological models with effective interactions with parameters adjusted to nuclear and astrophysical measurements or calculations; eg. (non-relativistic hence not necessarily causal) Skyrme, relativistic mean-field (RMF), quark-meson coupling,...models
- ab-initio approaches: 'solving' the many body problem starting with few (=2, 3)-body interactions; eg. (Dirac)-Brueckner-Hartree-Fock approach, ...

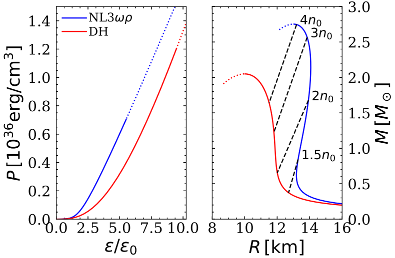
Equation of state

EoS

- Describes the composition and properties of NS matter;
- typically written as a relation between P and n_{B} or ε.

Mass-radius diagram

An EoS + TOV equations = a specific $M - R$ relation.



- each $M - R$ point corresponds to a given central density.
- each EoS gives a unique $M - R$ relation.
- each $M - R$ has a maximum mass $M_{\max}$
- "Soft" EoS = compressible $\rightarrow$ small $M_{\max}$ and R
- "Stiff" EoS = less compressible $\rightarrow$ large $M_{\max}$ and R

Which of the two EoS is the stiffest one?

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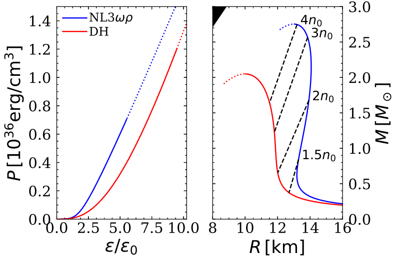

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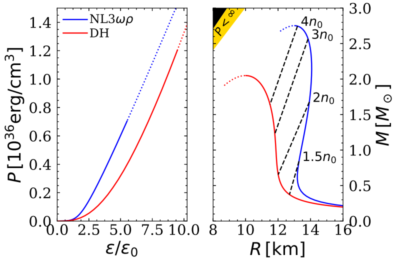

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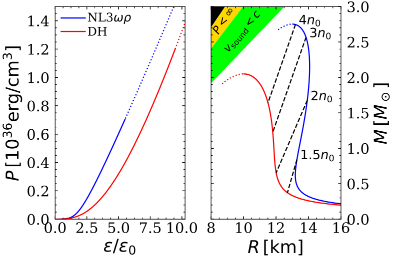

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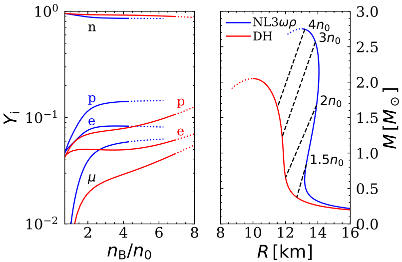

Composition

Fraction of a given species (i=n, p, e,...): $Y_{\mathrm{i}} = n_{\mathrm{i}}/n_{\mathrm{B}}$ with the number density of a given species n_{i}, the baryon number density $n_{\mathrm{B}} = n_{\mathrm{p}} + n_{\mathrm{n}} + \ldots$.

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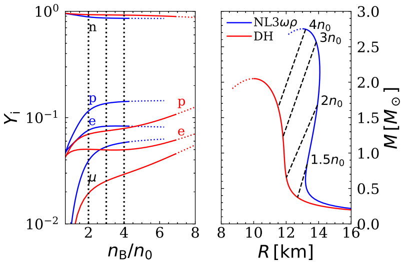

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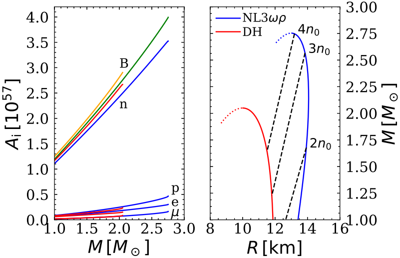

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$$A_\mathrm{i} = \int_0^R n_\mathrm{i}/\left(1 - 2Gm/(c^2 r)\right)^{1/2} 4\pi r^2 \mathrm{d}r.$$

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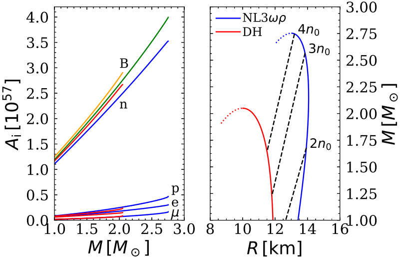


Composition

A NS is mostly composed of . . . neutrons!

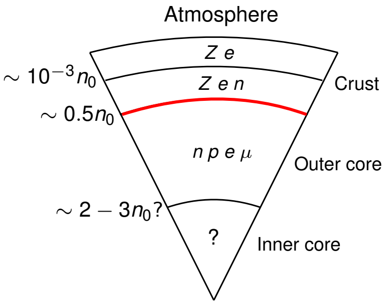
Structure

Neutronization

- take n, p, e^- completely degenerate at $T = 0$;
- β-equilibrium: $e^- + p \leftrightarrows n$
- $Q = (m_{\rm n} - m_{\rm p})c^2 \sim 1.3$ MeV
- e^- capture on p if $m_{\rm e}c^2(= 0.5\ \text{MeV})+$ kinetic energy$> Q$
- $\rho_\beta > 1.2 \times 10^7$ g cm^{-3} (as e^- are ultra-relativistic and matter is neutral)
- n cannot decay back because of Pauli blocking
- $\rightarrow$ neutronized state of matter is stable.

Atmosphere

- Plasma whose composition determines the spectrum of the NS emission.



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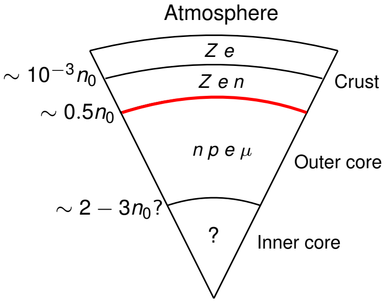
Outer-crust

- Gas of electrons,
- lattice of nuclei: ^{56}Fe and then more and more n-rich with increasing ρ

Neutron-drip density: $\rho \simeq 4 \times 10^{11}$ g cm^{-3}



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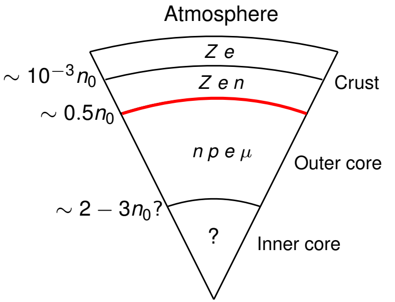
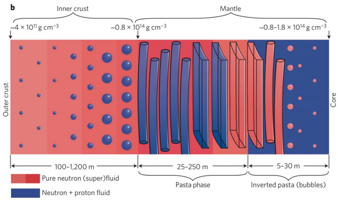


Nuclear saturation density: $n_0 = 0.16$ fm^{-3}

Inner-crust

- Gas of electrons,
- lattice of n-rich nuclei
- more and more unbound (superfluid) neutrons with increasing ρ
- at the bottom, pasta phase? ()

Core-crust transition: $\rho \simeq 1.4 \times 10^{14}$ g cm^{-3}



W. Newton, Nature (2013)

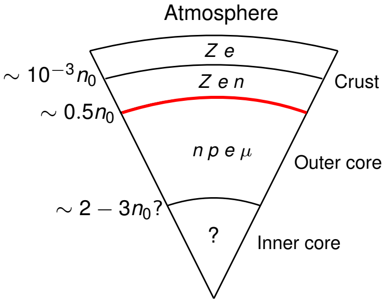
Structure

Neutronization

- take n, p, e^- completely degenerate at $T = 0$;
- β-equilibrium: $e^- + p \leftrightarrows n$
- $Q = (m_\mathrm{n} - m_\mathrm{p})c^2 \sim 1.3$ MeV
- e^- capture on p if $m_\mathrm{e} c^2 (= 0.5 \text{ MeV}) +$ kinetic energy$> Q$
- $\rho_\beta > 1.2 \times 10^7$ g cm^{-3} (as e^- are ultra-relativistic and matter is neutral)
- n cannot decay back because of Pauli blocking
- $\rightarrow$ neutronized state of matter is stable.

Outer core

- Free neutrons and protons (superfluid?),
- electrons,
- muons.



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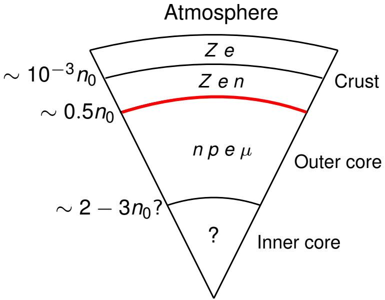
Inner core

- nucleons,
- hyperons (baryons with a least one s quark),
- quark matter (deconfined d, u and s),
- pion or kaon condensation, ...



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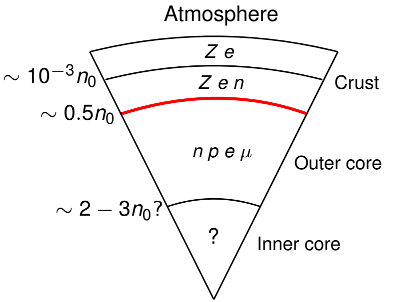
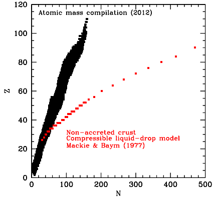


Nuclear saturation density: $n_0 = 0.16$ fm^{-3}

Problem

most of NS matter not accessible in terrestrial laboratories ...

Nuclei in lab. vs. NS crust



Key point

How to constrain the EoS and thus understand what is inside NSs?

Astrophysical constraints

Constraints from mass measurements

See eg. Özel & Freire, ARAA (2016)

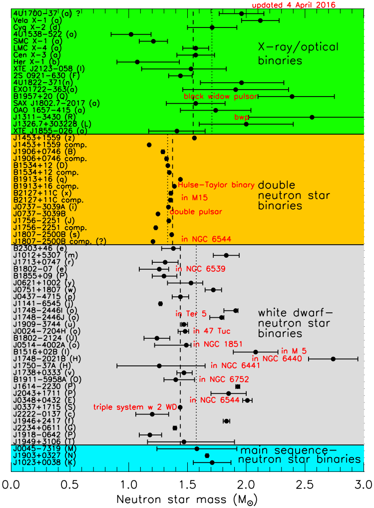
Keplerian orbital elements

- orbital period,
- time of periastron passage,
- eccentricity,
- projected semi-major axis,
- angle of periastron;

$\Rightarrow$ mass function $f_1(M, m_c, i)$.

+ 2 additional quantities

- Post Keplerian parameters:
  - precession of periastron,
  - orbital decay,
  - Einstein delay,
  - Shapiro delay;
- Spectroscopy:
  - orbital velocity,
  - H lines in the white dwarf atmosphere;
- Eclipse modeling.



https://stellarcollapse.org/nsmasses

Mass measurements

Theory

- each EoS has a maximum mass $M_{\max}$;
- $M_{\max} \geq M_{\max}^{\rm obs}$.

PSR J1614-2230

Fonseca et al., ApJ (2016)
Shapiro delay parameters:

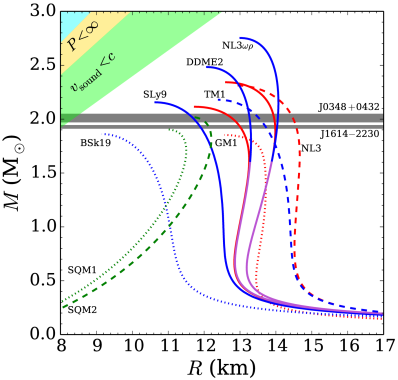
$$M_{\max}^{\rm obs} = 1.928 \pm 0.017\ \mathrm{M}_{\odot}.$$

PSR J0348+0432

Antoniadis et al., Science (2013) WD spectroscopy:

$$M_{\max}^{\rm obs} = 2.01 \pm 0.04\ \mathrm{M}_{\odot}.$$

Mass-radius diagram



EoSs for nucleonic matter (blue), exotic matter (pink) and strange quark matter (green).

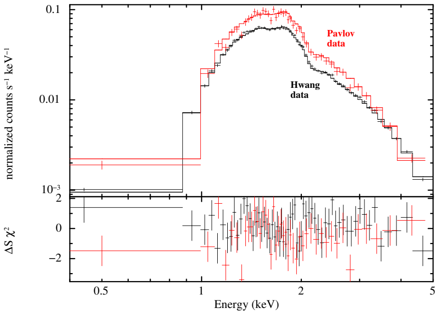
Radius measurements: isolated NSs

Thermal emission

Modeling of the X-ray spectra using atmosphere models.
Determination of the radius observed at infinity :

$$R_\infty = \frac{R}{\sqrt{1-2GM/(Rc^2)}}$$

Cas A NS (Ho & Heinke, Nature 2009)



Composition	H	He	C
$R_{\rm emission}$ (km)	4	5	12

No pulsation $\rightarrow$ emitting region = whole NS.
$\rightarrow$ NS with a C atmosphere.

Radius measurements: isolated NSs

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Modeling of the X-ray spectra using atmosphere models.
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Limitations:

- unknown chemical composition of the envelope,
- distance to the source,
- magnetic field B,
- ...

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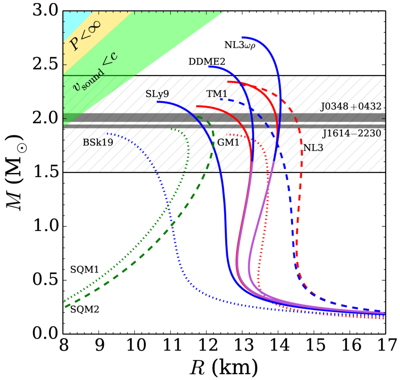


Radius measurements: accreting NSs

Properties

- Low B
- accreted atmosphere $\rightarrow$ H, He
- if NS in a globular cluster, distance accurately known.

Quiescence phase= no accretion

see eg. Heinke+ MNRAS (2014)
Limitations:

- H or He atmosphere? R up to 50% larger
- Lack for precise distance measurements. Athena and Gaia may help.
- . . .

X-bursts

eg. Steiner et al., EPJA (2016)
Suleimanov et al., EPJA (2016)
Özel et Freire, ARAA (2016)

Photospheric radius expansion bursts: strong enough to lift up the outer layers of the NS.
Limitations:

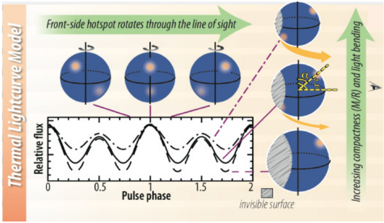
- uncertainties in the modelling of the burst, the burst selection, and the composition of the atmosphere.

Radius measurements: X-ray pulse profile of ...

X-ray emission from radio millisecond pulsars

PSR J0437−4715 (Bogdanov, ApJ 2013)

- ► pulsations due to magnetic polar caps
- \+ mass known from radio observations: $M = 1.76 \pm 0.2$ $M_\odot$.
- → $R > 12.29$ km (2σ)
- ► new mass measurement from Reardon et al., MNRAS (2016): $M = 1.44 \pm 0.07$ $M_\odot$



accreting millisecond X-ray pulsars

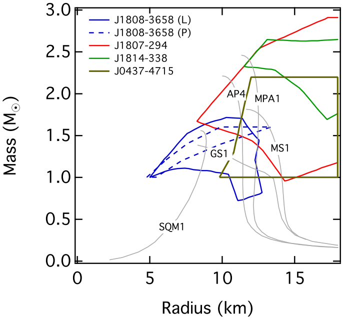
e.g. SAX J1808.4-3658 (Morsink & Leahy, ApJ 2011)

- ► pulsations due to accretion onto the NS magnetic poles

Limitations

Özel et Freire, ARAA (2016)

- ► hot spot modeling (shape)
- ► geometry of the system

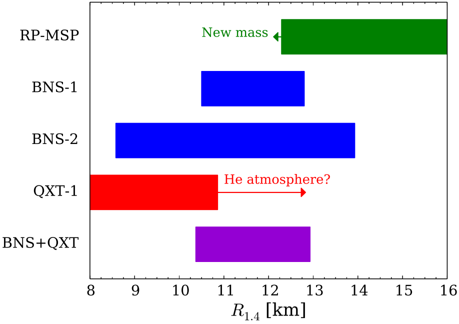


Özel et Freire, ARAA (2016)

Radius measurements

Fitting the spectrum of

- X-ray emission from radio millisecond pulsars (RP-MSP);
- the quiescent thermal emission of accreting NSs (QXT);
- X-bursts from accreting NSs (BNS).



Summary

Adapted from Fortin et al. A&A (2015)

- RP-MSP: Bodganov, ApJ (2013)
- BNS-1: Nättilä et al. AA (2016)
- BNS-2: Güver & Özel, ApJ (2013)
- QXT-1: Guillot & Rutledge, ApJ (2014)
- BNS+QXT: Steiner et al., ApJ (2013)

Conclusion

- many remaining uncertainties in the modelling,
- inclusion of rotation: effect $\simeq$ 10%.

Current consensus

$$R = 9 - 14 \text{ km}.$$

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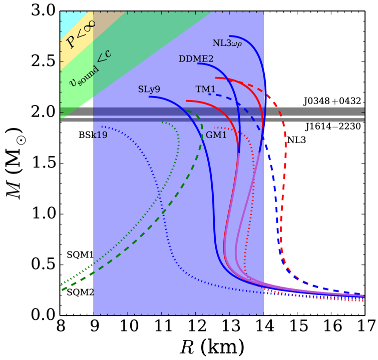

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- inclusion of rotation: effect $\simeq$ 10%.

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$$R = 9 - 14 \text{ km}.$$

Radius measurements

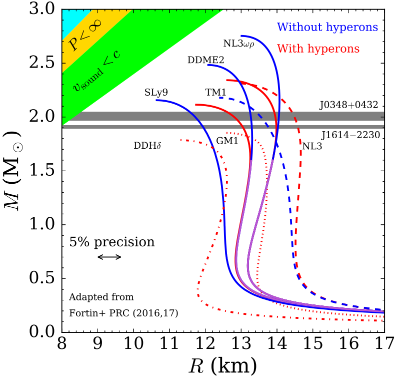
NICER

- Neutron star Interior Composition ExploreR Mission
- NASA project
- On the ISS, operating since July 2017
- Rotating hot spots from non-accreting MSPs
- $M - R$ constraints with a precision of $\sim 5\%$ for few NS.

Athena

- Advanced Telescope for High ENergy Astrophysics
- ESA project
- L2 point
- in 2028
- X-ray emission from MSPs;
- quiescent thermal emission of accreting NSs;
- PRE bursts from accreting NSs.

M − *R* measurements



- rule out EoS
- reconstruct the EoS.

Nuclear constraints

Nuclear parameters

- nuclear matter: idealised infinite uniform system of nucleons with $E_{\rm Coulomb} = 0$;
- liquid-drop model of nuclei: energy per nucleon $E/A(n_{\rm p}, n_{\rm n})$
- asymmetry $\delta = (n_{\rm n} - n_{\rm p})/n_{\rm B}$ (in NSs: $\delta \simeq 1$) & nucleon (or baryon) number density $n_{\rm B} = n_{\rm p} + n_{\rm n} \to E/A(n_{\rm B}, \delta)$
- symmetric nuclear matter: $n_{\rm p} = n_{\rm n}$, $\delta = 0$; simplest approx. for heavy nuclei
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$$\begin{aligned}
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E_0(n_{\rm B}) &= -B_{\rm sat} + K/2u^2 + \dots \\
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\end{aligned}$$

with $u = (n_{\rm B} - n_{\rm sat})/3n_{\rm sat}$.

Nuclear parameters

- $n_{\rm sat}$ the saturation density
- $B_{\rm sat}$ the binding energy,
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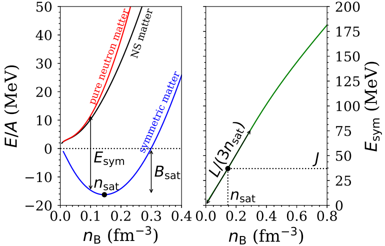

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Experimental constraints

Experimentally measured nuclear masses

- $n_{\rm sat} = 0.16 \pm 0.01$ fm^{-3}
- $B_{\rm sat} = -16.0 \pm 1.0$ MeV

Isoscalar giant monopole resonance in heavy nuclei:

- $K = 240 \pm 10$ MeV

Active debates: generally accepted

- $J = 30 - 34$ MeV
- $L = 35 - 70$ MeV
- $K_{\rm sym}$=?

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Experimental constraints

Symmetry energy J and its slope L at n_{sat}:

- neutron skin thickness of ^{208}Pb
- heavy ion collisions (HIC)
- electric dipole polarizability α_{D}
- giant dipole resonance of ^{208}Pb
- measured nuclear masses
- isobaric analog states (IAS)

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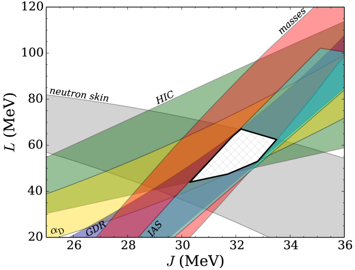

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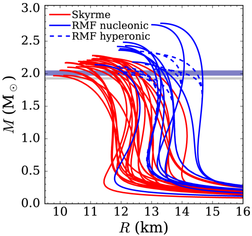
eg. Fortin+ PRC 94 (2016): 33 EoS with $M_{\rm max} \geq 2M_{\odot}$



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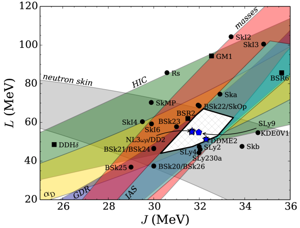

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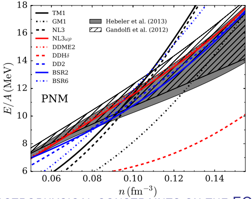
Theoretical constraints

Ab-initio calculations somewhat easier for pure neutron matter up to n_0, e.g. QMC or chiral effective field theory calculations...



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- liquid-drop model of nuclei: energy per nucleon $E/A(n_{\rm p}, n_{\rm n})$
- asymmetry $\delta = (n_{\rm n} - n_{\rm p})/n_{\rm B}$ (in NSs: $\delta \simeq 1$) & nucleon (or baryon) number density $n_{\rm B} = n_{\rm p} + n_{\rm n} \to E/A(n_{\rm B}, \delta)$
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- pure neutron matter: $n_{\rm p} = 0$, $n_{\rm n} = n_{\rm B}$, $\delta = 1$

$$E/A(n_{\rm B}, \delta) = \underbrace{E/A(n_{\rm B}, \delta = 0)}_{\text{symmetric matter}} + \underbrace{E_{\rm sym}(n_{\rm B})}_{\text{symmetry energy}} \delta^2 + \mathcal{O}(\delta^4)$$

$$E_0(n_{\rm B}) = -B_{\rm sat} + K/2u^2 + \dots$$

$$E_{\rm sym}(n_{\rm B}) = J + Lu + K_{\rm sym}/2u^2 + \dots$$

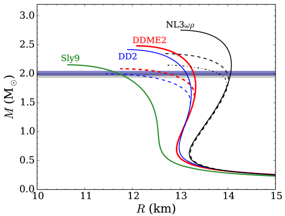
with $u = (n_{\rm B} - n_{\rm sat})/3n_{\rm sat}$.

Nuclear parameters

- $n_{\rm sat}$ the saturation density
- $B_{\rm sat}$ the binding energy,
- K the incompressibility;

- J the symmetry energy at $sn_{\rm sat}$
- L its slope at $n_{\rm sat}$
- $K_{\rm sym}$ its curvature at $n_{\rm sat}$
- ...

eg. Fortin+ PRC 94 (2016): 33 EoS with $M_{\rm max} \geq 2M_\odot$



$R_{1.4} = 12.45 - 13.75$ km

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Neutron skin in neutron-rich nuclei

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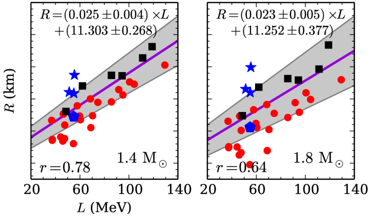

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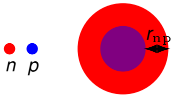


Neutron-rich nuclei

- To be measured for ^{208}Pb with PREX-II and ^{48}Ca with CREX.
- the larger L the thicker $r_{\rm np}$,

$\Rightarrow$ correlation between $r_{\rm np}$ and R.

Return of the astrophysical constraints

Thermal evolution of accreting NSs

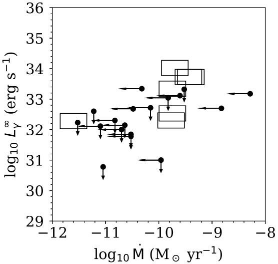
Soft X-ray Transients

NSs in close binaries with a low-mass companion undergoing:

- repeated short periods of accretion;
- long quiescent phases.

Heating

- Deep crustal heating: nuclear reactions in the crust as the accreted matter sinks into deeper into it.
- $\propto$ accretion rate $\dot{M}$.



Luminosity in quiescent state

Emission of photons at the surface
$=$ Heat generated in the interior by nuclear reactions
$-$ Emission of neutrinos from the whole interior.

Neutrino emission

Direct Urca process:

$$n \to p + e + \bar{\nu}_e, \qquad p + e \to n + \bar{\nu}_e.$$

- Pauli blocking $\to$ allowed for neutrons close (within $\sim kT$) to their Fermi surface.
- Momentum conservation: $p_\mathrm{p}^\mathrm{F} + p_\mathrm{e}^\mathrm{F} \geq p_\mathrm{n}^\mathrm{F}$ with $p_\mathrm{i}^\mathrm{F} \propto n_\mathrm{i}^{1/3}$.
- Charge neutrality: $n_\mathrm{e} = n_\mathrm{p}$

$\to$ DUrca is on if $n_\mathrm{n} \leq 8 n_\mathrm{p}$ or $Y_\mathrm{p} \geq 11\%$

$+$ similar process with muons.

- most efficient neutrino process in nucleonic NSs.

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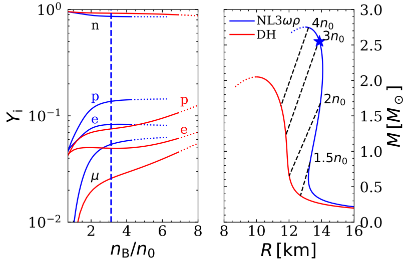

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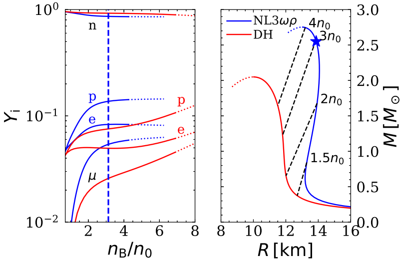

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Other processes:

- modified Urca $n + N \rightarrow p + N + l + \nu_l$ with N a spectator nucleon to ensure momentum conservation.
- NN-bremsstrahlung $N + N \rightarrow N + N + \nu_l + \bar{\nu}_l$
- Much less efficient.

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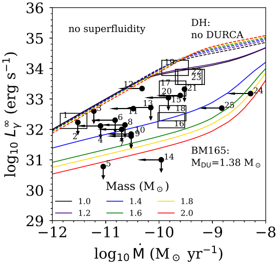

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- Luminous objects: low-mass NSs;
- Less luminous ones: high-mass NSs.

NSs with a very-low luminosity $\to$ DUrca operates?

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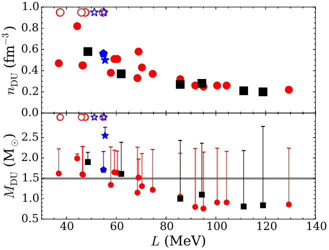
Fortin et al. PRC, 2016



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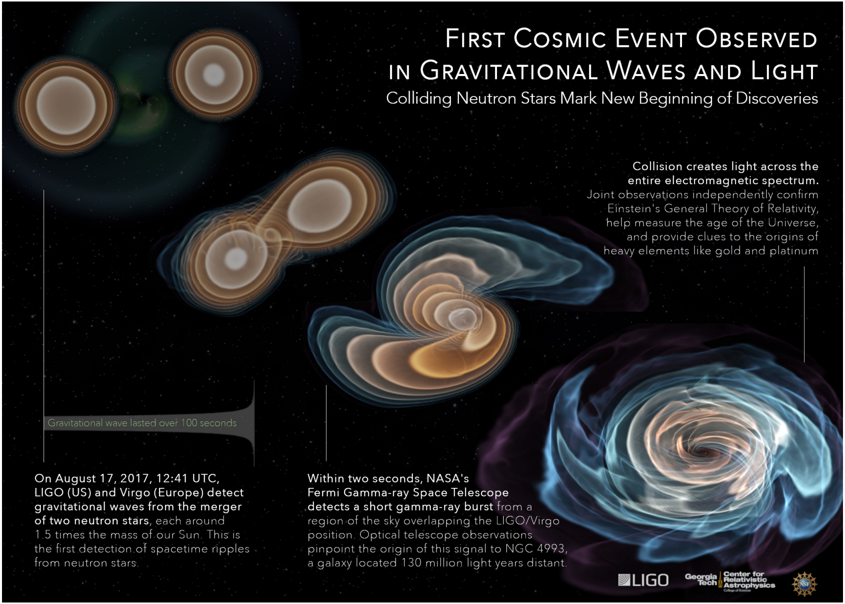
Gravitational wave detection



Gravitational wave detection and constraint on the EOS

Tidal deformability Λ

- during the last stage of the inspiral, each NS develops a mass quadrupole due to the extremely strong tidal gravitational field induced by the other NS
- Λ measures the degree of deformation of a NS due to the tidal field of the companion NS
- LIGO-Virgo paper: $\Lambda(M = 1.4\, M_\odot) < 800$

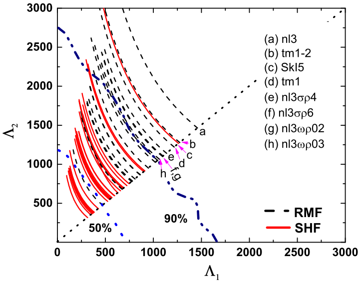
Constraint on R

e.g. Annala+; Fattoyev+ PRL (2018):

$\rightarrow$ $R(M = 1.4\, M_\odot) < 13.7\,\mathrm{km}$.

Constraint on the EoS

e.g. Malik, Alam, Fortin+ PRC (2018)
42 EoS all consistent with 2 $M_\odot$



GW170817 limit on Λ

In fact, EoS that are excluded have a very large L and are excluded because of nuclear constraints!

Perspectives

- BNS mergers expected from the LIGO-Virgo observational with more stringent constraints

Thermodynamic consistency

Thermodynamic consistency and NS radius

Thermodynamic consistency

- first law of thermodynamics
  $\mathrm{d}(\varepsilon/n) = -P\mathrm{d}(1/n)$
- chemical potential
  $\mu = (P + \varepsilon)/n = \mu(P)$
- hence $n = \mathrm{d}P/\mathrm{d}\mu$
- n increasing $\rightarrow P(\mu)$ (continuous) increasing and convex in the absence of phase transition

NS crust

Core is homogeneous but the crust is a lattice of nuclei $\rightarrow$ non-uniform.

- no ab-initio many-body calculations for inhomogeneous matter.
- single nucleus approx.: one nucleus, energetically favored, at a given density
- Wigner-Seitz cell: matter divided in charged-neutral cells
- techniques: Liquid-drop, Thomas-Fermi models, . . .

$\Rightarrow$ many more core EoS than crust EoS.

Example: $M - R$ relation for the NL3 EoS

core EoS: RMF code
crust: ??.

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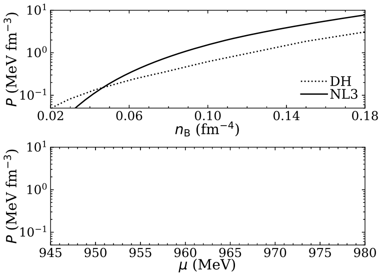

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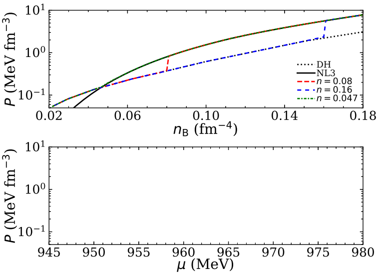

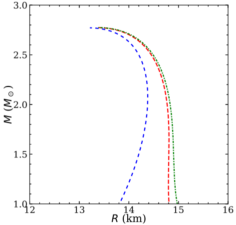

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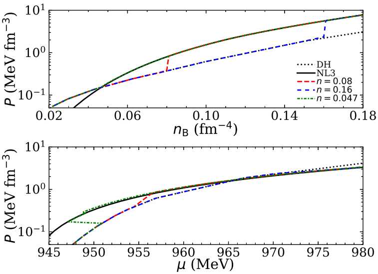

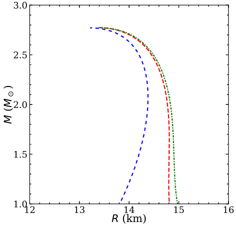

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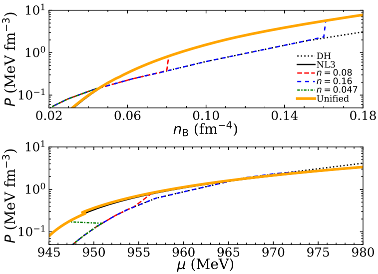

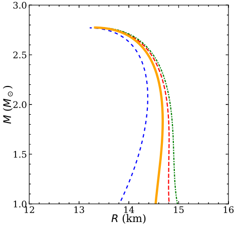

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Core-crust matching

- can introduce an ‘uncertainty’ of up $\sim$ 4% (up to $\sim$ 30% on the crust thickness),
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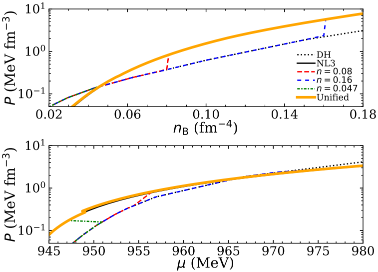

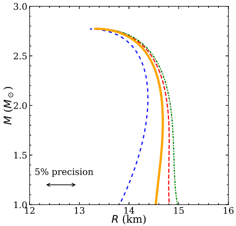

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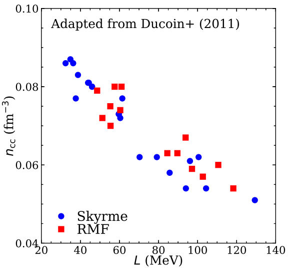
Core-crust transition

- when uniform matter is unstable wrt variations in the particle densities.
- various techniques: (thermo)dynamical spinodals, RPA, . . .
- Transition density: $n_{\rm cc} \sim 0.05 - 0.09$ fm^{-3} $n_{\rm cc} \sim (0.3 - 0.6)n_0$;
- $L \nearrow$, $n_{\rm cc} \searrow$



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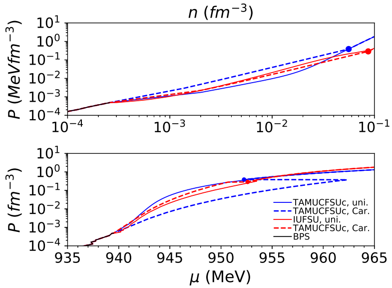
Polytropes

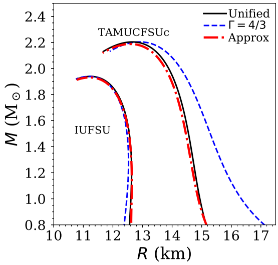
e.g. Carriere+ ApJ (2003)

- BPS for the outer crust
- core for $n < n_{\mathrm{cc}}$
- in between: $P(\varepsilon) = K\varepsilon^{4/3} + \varepsilon_{\mathrm{oc}}$
- $\rightarrow$ TOV eq. with $P(\varepsilon) \rightarrow M - R$ relations ...
- BUT with $\mathrm{d}n/n = \mathrm{d}\varepsilon/(P+\varepsilon) \rightarrow n$
- but μ is not continuous!
- NOT thermodynamically consistent
- hence 'uncertainty' on R!

One needs to be careful and always rederive quantities from basic principles.

Polytropes





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- how to, if not solve, at least handle this problem?

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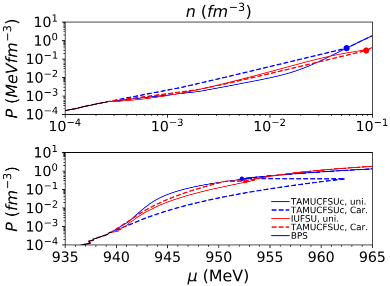

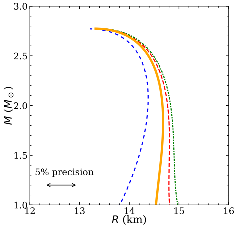

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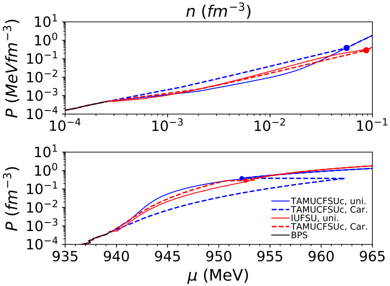
- first law of thermodynamics
  $\mathrm{d}\left(\varepsilon/n\right) = -P\mathrm{d}\left(1/n\right)$
- chemical potential
  $\mu = (P+\varepsilon)/n = \mu(P)$
- hence $n = \mathrm{d}P/\mathrm{d}\mu$
- n increasing $\rightarrow$ $P(\mu)$ (continuous) increasing and convex in the absence of phase transition

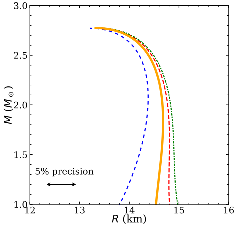
Core-crust matching

- can introduce an ‘uncertainty’ of up $\sim$ 4% (up to $\sim$ 30% on the crust thickness),
- with NICER, Athena: expected precision $\sim$ 5% . . . .
- how to, if not solve, at least handle this problem?

1. unified EoSs: Skyrme: Douchin & Haensel (2001), BSk models; RMF: Fortin+ (2016), Providência+ al. (2019); Sharma+ (2015), . . .
2. approximate approach to the crust. . .

Polytropes

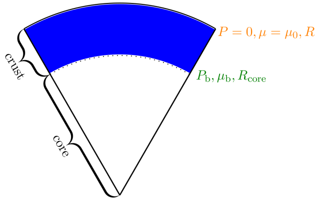




Approximate formula for the radius and crust thickness

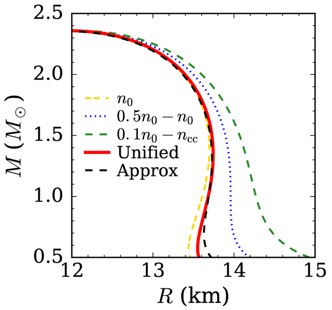
Zdunik, Fortin, and Haensel, A&A (2017)

- All you need is ...: the core EOS down to a chosen density $n_{\rm b}$ with $\mu(n_{\rm b}) = \mu_{\rm b}$.
- Obtain the $M(R_{\rm core})$ relation solving the TOV equations.
- Obtain $M(R)$ with
$$R = R_{\rm core} \Big/ \left(1 - \left(\frac{\mu_{\rm b}{}^2}{\mu_0{}^2} - 1\right)\left(\frac{R_{\rm core}c^2}{2GM} - 1\right)\right).$$



2 unknowns

- $\mu_0 = 930.4$ MeV - minimum energy per nucleon of a bcc lattice of ^{56}Fe.
- $\mu_{\rm b}$ at the core-crust transition?
- $\mu_{\rm b} = (P+\rho)/n$ at $n_0/2 = 0.08$ fm^{-3}



Results

- $\Delta R \lesssim 0.2\%$ for $M > 1\ M_\odot$
- $\Delta l^{\rm cr} \lesssim 1\%$ for $M > 1\ M_\odot$

+ Formulas for NSs with an accreted crust.

Exotic phases?

Exotic phases in NSs

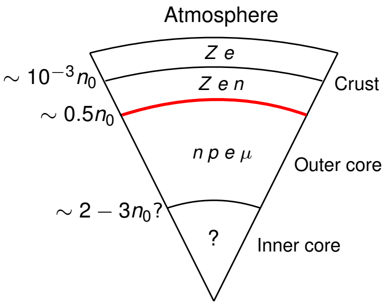
Inner core

- nucleons,
- hyperons (baryons with a least one s quark),
- quark matter (deconfined d, u and s),
- pion or kaon condensation, . . .

Consequences

- Additional species without an (repulsive) interaction included
- replacement of neutrons with a large Fermi energy by new species with a lower Fermi energy
- lower pressure hence a softer EoS
- lower maximum mass

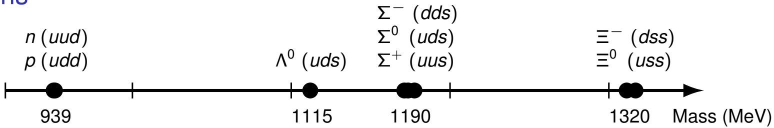
Let us focus on hyperons as an example.



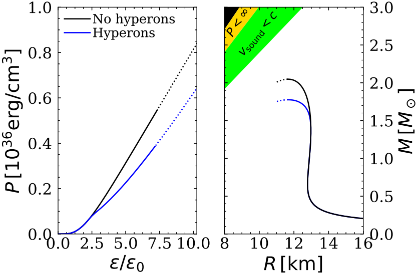
Nuclear saturation density: $n_0 = 0.16\ \mathrm{fm}^{-3}$

Hyperonic equations of state

Hyperons



Hyperon softening



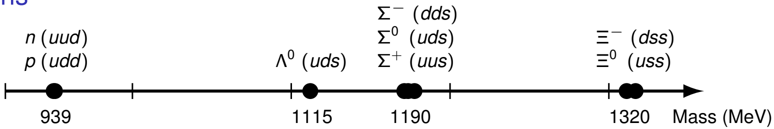
- on Earth not stable, decay into nucleons via weak interaction
- in NSs, Pauli blocking preventing them from decaying

Hyperon puzzle

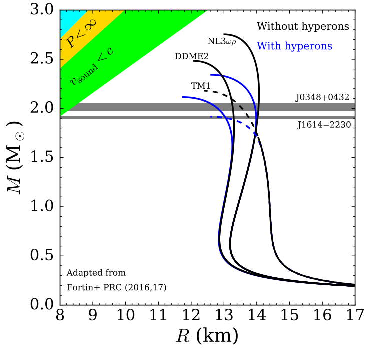
- M_{max} reduced when hyperons are included;
- Can hyperons be present in NSs and yet $M_{\mathrm{max}} \geq M^{\mathrm{obs}}_{\mathrm{max}}$ with $M^{\mathrm{obs}}_{\mathrm{max}} \simeq 2\ M_\odot$?

Hyperonic equations of state

Hyperons



Hyperon softening



Hyperon puzzle

- $M_{\rm max}$ reduced when hyperons are included;
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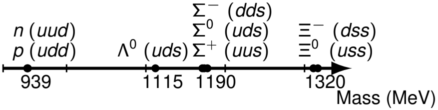
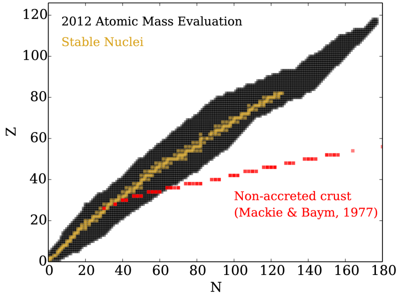
Hyperons in the lab

Hyperons

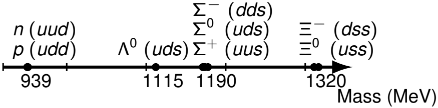


From nuclei to hypernuclei

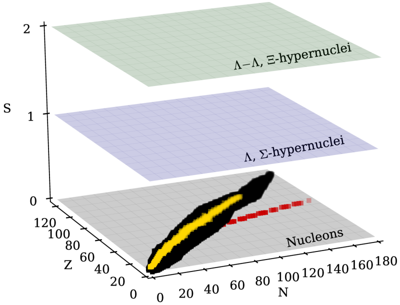


Hyperons in the lab

Hyperons



From nuclei to hypernuclei



Hyperons in the lab

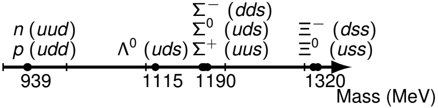
Experimental hypernuclei data

Gal et al., RMP (2016)

- ~ 40 Λ-hypernuclei
  + measurement of binding energy B_Λ
- few Ξ-hypernuclei
  but no measurement of binding energy
- no Σ-hypernuclei
  repulsive Σ-nucleon interaction?
- only one unambiguous $\Lambda\Lambda$-hypernuclei:
  measurement of the bond energy:

$$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 0.67 \pm 0.17 \text{ MeV}.$$

Hyperons



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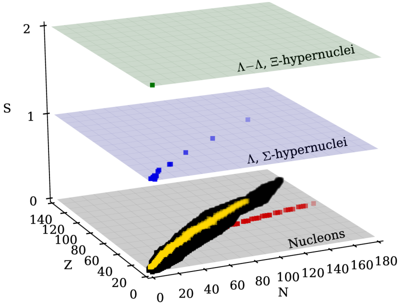

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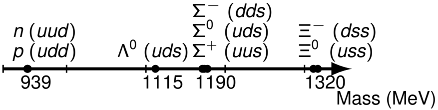
Experimental calibrated RMF EoSs

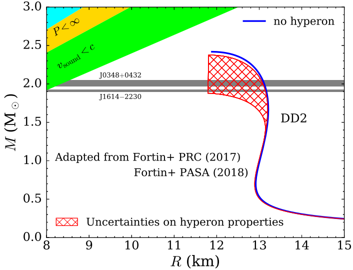
Fortin+ (2017,2018), Providência+ (2019)

- parameters of the models adjusted to experimental data on hyperons;
- all EoSs are consistent with $2M_\odot$,
- because too little experimental data on hyperons.

Hyperons



From hypernuclei to NSs



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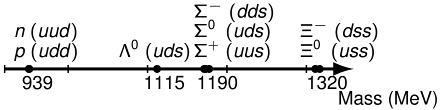
  measurement of the bond energy:

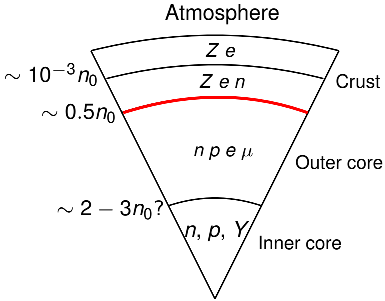
$$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) = 0.67 \pm 0.17 \text{ MeV}.$$

Ab-initio calculations

- (D)BHF calculations: 3-body force not strong to obtain 2 $M_\odot$ NSs for EoSs with nuclear properties in agreement with experimental constraints
- Quantum Monte Carlo calculations: possible to get an EoS stiff enough to reach 2 $M_\odot$

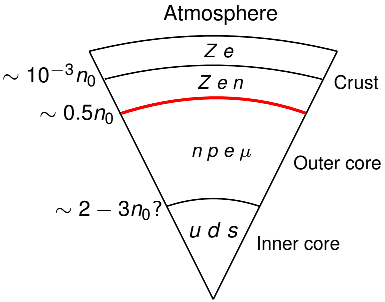
Hyperons





Many open questions about the presence of hyperons and other additional non-nucleonic species in NSs.

Quark core?

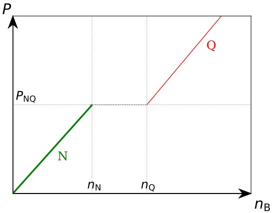


Deconfined u, d and s quarks.

Phase transition

Generally assumed to be a first order phase transition.

Global & local charge neutrality:

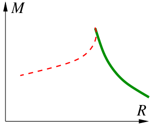


N: normal phase; Q: quark phase

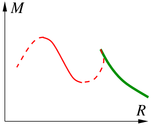
Stability

$\varepsilon_{\mathrm{Q}}/\varepsilon_{\mathrm{N}} > \lambda_{\mathrm{crit}} = 3/2(1 + P_{\mathrm{NQ}}/\varepsilon_{\mathrm{N}})$
$\rightarrow$ star destabilized by the phase transition.

$M - R$ figures adapted from Alford+ (2013)

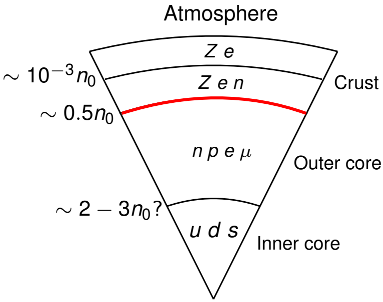


Too soft Q EoS $\rightarrow$ no hybrid star



Q EoS stiffens at higher density $\rightarrow$ twin branch

Quark core?



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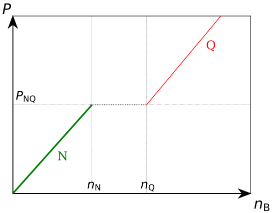
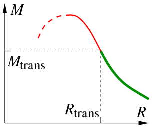


N: normal phase; Q: quark phase

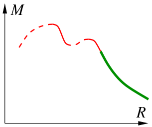
Stability

$\varepsilon_Q/\varepsilon_N < \lambda_{crit} = 3/2(1 + P_{NQ}/\varepsilon_N)$
$\rightarrow$ star not destabilized by the phase transition

$M - R$ figures adapted from Alford+ (2013)



Soft Q EoS $\rightarrow$ Hybrid star



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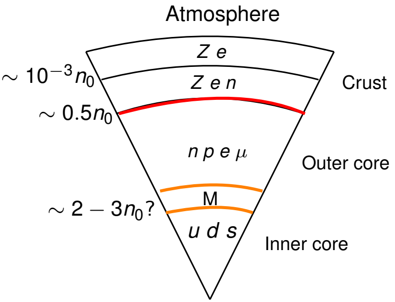
Quark core?



Deconfined *u*, *d* and *s* quarks.

Formation

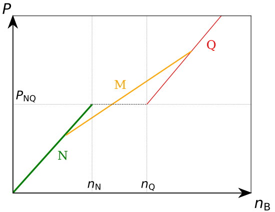
- NS slowing down due to the emission of electromagnetic or GW radiation
- NS spinning up due to the matter accretion from a companion star

A number of models for hybrid stars are consistent with 2 $M_\odot$ NSs.

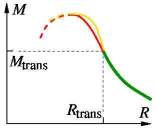
Phase transition

Generally assumed to be a first order phase transition.

Global charge neutrality but not local:



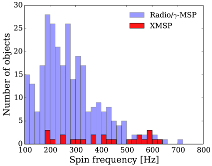
N: normal phase; Q: quark phase; M: mixed phase



Rotation

Rotation

Observations



Haensel+ EPJA (2016)

- $\sim$ 10% of PSR with a spin frequency $f > 100$ Hz.
- fastest rotating NS: PSR J1748-2446a with $f_{\rm obs}^{\rm max} = 716$ Hz.

Keplerian frequency $f_{\rm K}$

= frequency beyond which the star is destroyed by rotational forces: "mass-shedding limit"

- Softer EoS: smaller $f_{\rm K}$ compared to a stiffer EoS.
- if $f_{\rm K}$[EOS] $< f_{\rm obs}^{\rm max}$, then EoS ruled out
- $f_{\rm K}$[EOS] $\sim 1.6 - 2.0$ kHz...

NSs are uniformly rotating

- born differentially rotating
- Shear viscosity and possibly convective and turbulent motions acting against differential rotation on a time scale of days to few years.

Slow-rotation approximation

- Hartle,...: rotation as a small perturbation of the spherically symmetric TOV solution to different orders in $\Omega = 2\pi f$
- suitable for most NSs, with $f \ll f_{\rm K}$, but NOT for rapidly rotating stars in particular near the mass-shedding limit.

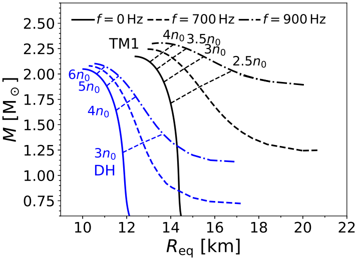
Arbitrary rotating

Einstein equation:

- still the stress-energy tensor of a perfect fluid
- but now metric for a stationary and axisymmetric star

Nrostar (LORENE), RNS,... codes

Rotation



NSs are uniformly rotating

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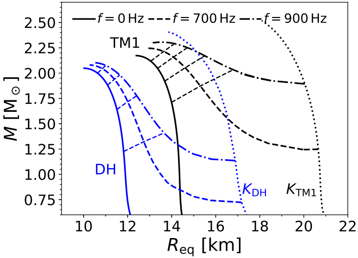
Effects of rotation

- for a given n_c increase of the equatorial radius
- increase of the mass

Rotation



K lines: mass-shedding configurations.

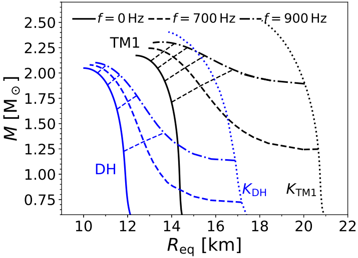
NSs are uniformly rotating

- born differentially rotating
- Shear viscosity and possibly convective and turbulent motions acting against differential rotation on a time scale of days to few years.

Effects of rotation

- for a given $n_{\rm c}$ increase of the equatorial radius
- increase of the mass
- $M^{\rm K}_{\rm max} \simeq 1.2 M^{f=0}_{\rm max}$
- $R(M^{\rm K}_{\rm max}) \simeq 1.4 R(M^{f=0}_{\rm max})$

Rotation



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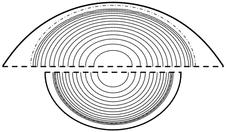

Haensel+ (2016)
$M = 1.4\,M_\odot$ at $f = 716$ Hz
upper: TM1 EoS - mass-shedding (cusp),
lower: DH EoS.

NSs are uniformly rotating

- born differentially rotating
- Shear viscosity and possibly convective and turbulent motions acting against differential rotation on a time scale of days to few years.

Effects of rotation

- for a given n_c increase of the equatorial radius
- increase of the mass
- $M_{\rm max}^{\rm K} \simeq 1.2 M_{\rm max}^{f=0}$
- $R(M_{\rm max}^{\rm K}) \simeq 1.4 R(M_{\rm max}^{f=0})$

Proper modeling of the properties of rotating NSs is important!

Compose

`compose.obspm.fr`

Conclusions

- Goal: constrain the properties of the nuclear interaction and of matter inside NSs with astrophysical observations and nuclear experiments.
- Currently: only real constraint is from mass measurements;
- More to come in the next few years thanks to new instruments (in particular radius with NICER, Athena);
- GW detections from NS binary systems will most likely offer complementary constraints. . .
- More constraints thanks to nuclear experiments (in particular PREX-II, CREX).

Exciting times ahead!!!

Further Reading I

Introduction:

- Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects
  S. L. Shapiro & S. A. Teukolsky
  Wiley
- Compact Stars - Nuclear Physics, Particle Physics, and General Relativity
  N. K. Glendenning
  Springer

More advanced:

- Neutron Stars 1 : Equation of State and Structure
  P. Haensel, A.Y. Potekhin, & D.G. Yakovlev
  Springer
- The Physics and Astrophysics of Neutron Stars *(specifically chapters 5, 6, & 7)*
  L. Rezzolla, P. Pizzochero, D. I. Jones, N. Rea & I. Vidana (eds)
  Springer
  arXiv:1806.02833, 1804.03020, and 1803.01836
- Equations of state for supernovae and compact stars
  M. Oertel, M. Hempel, T. Klähn, & S. Typel
  Reviews of Modern Physics 89 (2017); arXiv:1610.03361
- Physics of Neutron Star Crusts
  N. Chamel & P. Haensel
  Living Reviews in Relativity (2008); arXiv:0812.3955

Further Reading II

- Masses, Radii, and the Equation of State of Neutron Stars
  F. Özel & P. Freire
  Annual Review of Astronomy and Astrophysics 54 (2016); arXiv:1603.02698
- Observational constraints on neutron star masses and radii
  M. Miller & F. Lamb
  European Physical Journal A 52 (2016); arXiv:1604.03894
- Rotating neutron stars with exotic cores: masses, radii, stability
  P. Haensel, M. Bejger, M. Fortin, & J. L. Zdunik
  European Physical Journal A 52 (2016); arXiv:1601.05368
- From hadrons to quarks in neutron stars: a review
  G. Baym, T. Hatsuda, T. Kojo et al.
  Reports on Progress in Physics 81 (2018); arXiv:1707.04966,
- Rotating stars in relativity
  V. Paschalidis & N. Stergioulas
  Living Reviews in Relativity (2017); arXiv:1612.03050