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Symmetry constraints for Callan-Symanzik flows in chiral models

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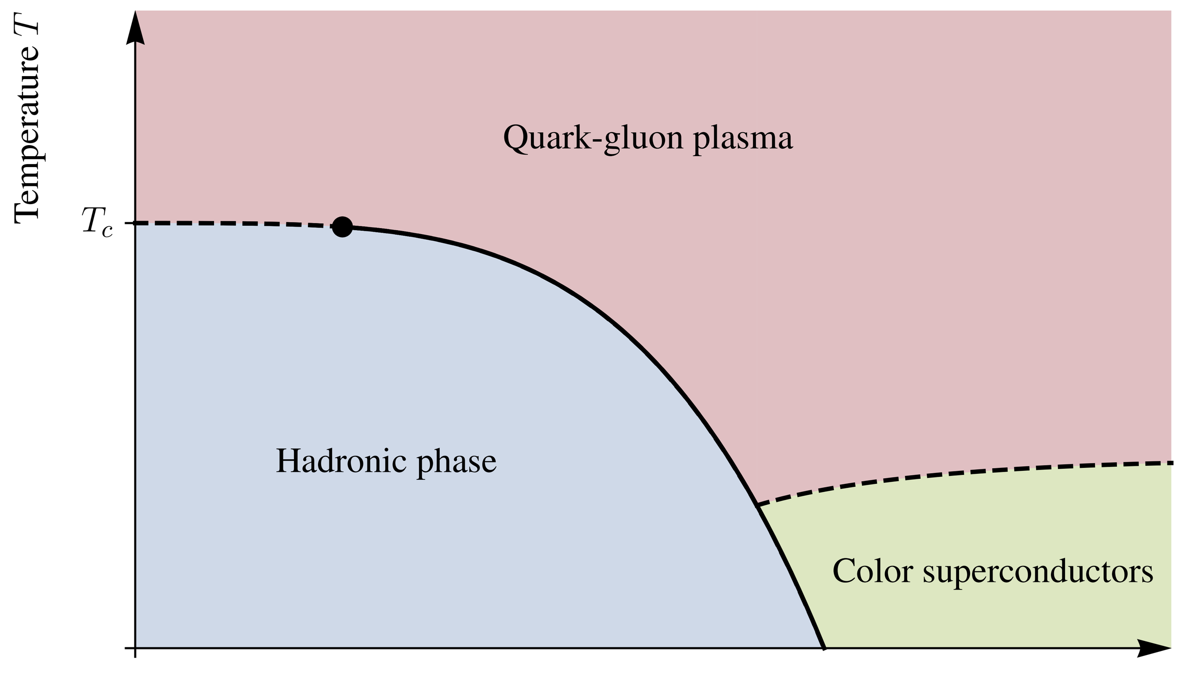
Abstract

We use the functional renormalization group (fRG) approach [1] to study the phase diagram of strong-interaction matter along the line of zero quark chemical potential. Our formulation of the fRG includes a mass-like Callan-Symanzik (CS) regulator function, which respects the Lorentz symmetry of the underlying relativistic quantum field theory. However, along with many advantages of the CS regulator [2, 3], there comes a downside of an artificial breaking of chiral symmetry. For a chirally symmetric low-energy effective model for quantum chromodynamics we present concrete symmetry constraints that have to be considered in order to obtain physical results. We demonstrate the influence of these symmetry constraints compared to a naive usage of the CS regulator based on the screening mass for the scalar meson and the critical temperature of the chiral phase transition.

Motivation

- Much of our knowledge about QCD matter at intermediate densities and beyond relies on low-energy effective models that share important symmetries with QCD. In particular, NJL-type models have proven to be a valuable tool for low-energy hadron physics.
- When extracting IR observables with non-pertubative methods such as the fRG, Lorentz symmetry is often broken artificially in favor a more accessible calculation.
- Investigation of the QCD phase diagram in a Lorentz-symmetric scheme is of high interest.



Approach

- Starting point of our studies is a two-flavor quark-meson model in the chiral limit, defined by the classical action

$$S = \int \mathrm{d}^4x \left\{ \bar{\psi}\left[\mathrm{i}\partial\!\!\!/ + \mathrm{i}\bar{h}\left(\sigma + \mathrm{i}\gamma^5\tau^j\pi_j\right)\right]\psi + \frac{\bar{m}^2}{2}\left(\sigma^2 + \vec{\pi}^2\right) \right\} .$$

- We use the fRG method, where the Wetterich equation [1] allows to interpolate smoothly between the UV physics, given by S, and the IR. Our truncation scheme involves a one-loop approximation and taking into account only fermionic loops:

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left[\Gamma_k^{(2)} + R_k\right]^{-1} \cdot \partial_k R_k\right\}$$
$$\xrightarrow[\text{ferm. loops}]{\text{1-loop approx.}} -\mathrm{Tr}\left\{\left[S^{(2)} + R_k^{\psi}\right]^{-1} \cdot \partial_k R_k^{\psi}\right\} .$$

- The cutoff function R_k is a key ingredient as it regularizes the IR by suppressing fluctuations below the cutoff scale k. Aiming for a simple Lorentz-symmetric regularization scheme, we implement the mass-like CS regulator

$$R_k^{\psi}(p,q) = \mathrm{i}k\,(2\pi)^4 \delta^{(4)}(p-q) .$$

- The RG scale k in this regulator behaves as a mass scale. Therefore, the RG flow is not a flow through momentum shells of a single theory but a flow through theories with quark mass k, where all loop momenta contribute.
- To render the CS flow finite, an additional UV regularization is required [2]. Our implementation of UV regularization relies on counter terms.

Symmetry constraints

- Due to its mass-like nature, the CS regulator couples to the σ-direction in field space and destroys the chiral symmetry of the theory. Note that the π-directions are not affected by the regulator. To cure the regulator-induced symmetry breaking, additional operations have to be performed.
- In bosonic field space, chiral symmetry manifests itself as an $O(4)$ symmetry. The reflection or $Z(2)$ symmetry is then restored by demanding

$$\Gamma_k^{(\mathrm{refl})}(\sigma,\vec{\pi}) = \hat{\mathcal{S}}\left[\Gamma_k\right](\sigma,\vec{\pi}) = \frac{1}{2}\left(\Gamma_k(\sigma,\vec{\pi}) + \Gamma_k(-\sigma,\vec{\pi})\right)$$

for the average effective action at all temperatures. This is equivalent to implementing temperature-dependent boundary conditions.
- The rotational symmetry is restored by demanding

$$\forall i \in \{1,2,3\}: \qquad \frac{1}{\sigma}\frac{\partial}{\partial\sigma}\Gamma_k^{(\mathrm{refl})} \stackrel{!}{=} \frac{1}{\pi_i}\frac{\partial}{\partial\pi_i}\Gamma_k^{(\mathrm{refl})} ,$$

which is basically a Ward-Takahashi identity for the $SO(4)$ symmetry. This has to be implemented such that the π-directions remain unaltered.
- Note that these symmetry constraints directly affect the masses of the bosonic fields. In case of the σ-meson, we have

$$\frac{\partial^2}{\partial\sigma^2}\Gamma_k \rightarrow \frac{\partial}{\partial\sigma}\left(\frac{\sigma}{\pi_i}\frac{\partial}{\partial\pi_i}\Gamma_k^{(\mathrm{refl})}\right) .$$

RG consistency

- In the scale fixing procedure, the UV reference scale $\Lambda = \Lambda_0$, at which the effective action is supposed to be known, represents a natural upper limit for all energy scales present in the vacuum theory.
- Once the IR physics is fixed by the initial condition Γ_{Λ_0}, the RG cutoff scale Λ should be increased such that $m_{\mathrm{ext}}/\Lambda \ll 1$ for all external parameters of interest:

$$\Gamma_{k_{\mathrm{IR}}} = \Gamma_{\Lambda_0} + \int_{\Lambda_0}^{k_{\mathrm{IR}}} \mathrm{d}k'\,\partial_{k'}\Gamma_{k'}$$
$$\stackrel{!}{=} \Gamma_{\Lambda}(\Lambda_0) + \int_{\Lambda}^{k_{\mathrm{IR}}} \mathrm{d}k'\,\partial_{k'}\Gamma_{k'} .$$

- For reasons of consistency, the IR physics should not depend on the chosen UV cutoff Λ,

$$\forall\Lambda \neq \Lambda_0: \qquad \Lambda\frac{\mathrm{d}}{\mathrm{d}\Lambda}\Gamma_{k_{\mathrm{IR}}} \stackrel{!}{=} 0 .$$

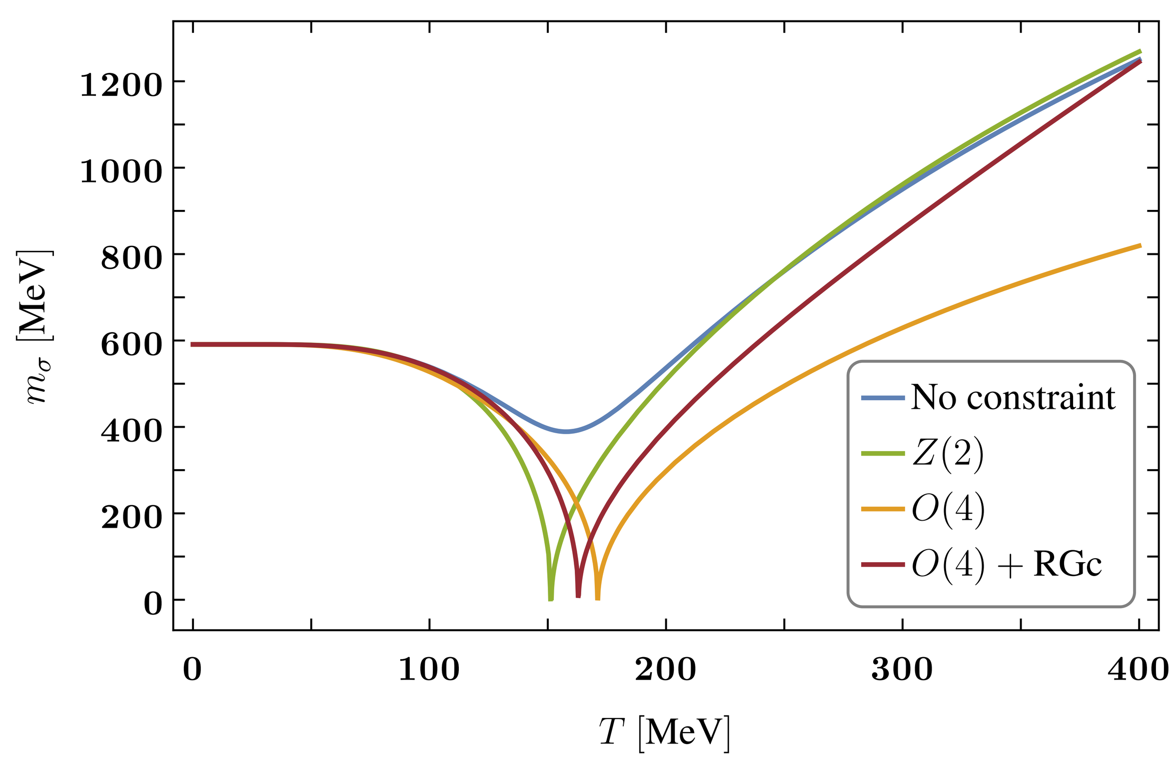
- Based on this principle, an RG-consistent UV completion [4] can be done as follows:

$$\Gamma_{\Lambda}(\Lambda_0) = \Gamma_{\Lambda_0} + \int_{\Lambda_0}^{\Lambda} \mathrm{d}k\,\partial_k\Gamma_k .$$

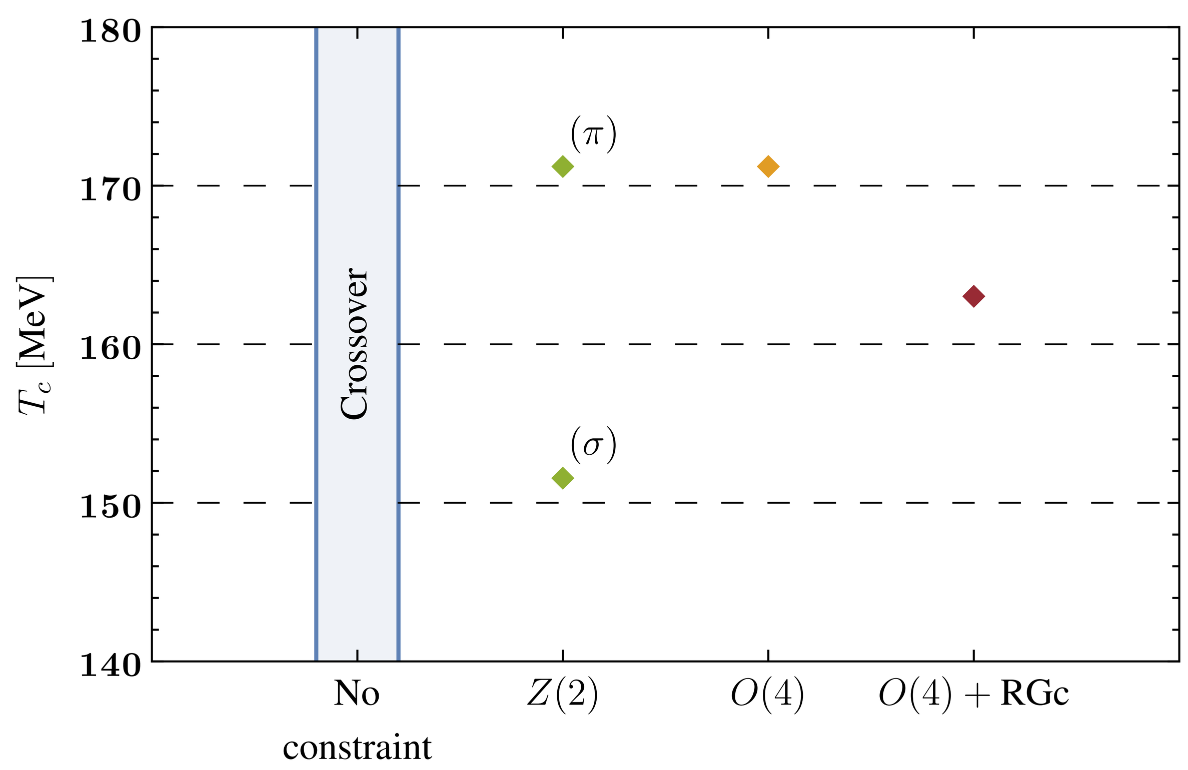
Then, $\Gamma_{\Lambda}(\Lambda_0)$ serves as the new vacuum initial condition.

Results

- For now, concentrating on the case $\mu = 0$, we present the influence of symmetry constraints on the sigma screening mass m_σ and the critical temperature T_c.
- If no corrections to the CS flow are implemented, there is no chiral phase transition but only a crossover due to a regulator-induced breaking of chiral symmetry. The effect of physical explicit symmetry breaking is hence expected to be "overshadowed" by the regulator.
- Constraints beyond reflection symmetry do not give rise to a change of the nature of the transition but only yield corrections to the critical temperature and the temperature-dependent behavior.



- If only $Z(2)$ symmetry is implemented, chiral symmetry is not fully restored causing the effective potential to still evolve differently in different directions in field space. Therefore, one can extract a critical temperature for the σ-direction as well as for the π-directions.
- Once the $O(4)$ symmetry is restored, the evolution of the effective potential along the σ-direction is in accordance with the π-directions leading to a single T_c. This value agrees identically with the critical temperature along the π-directions from before.
- An RG-consistent UV completion moreover minimizes the influence of the RG cutoff in the finite-temperature calculations. As a result, the value for the critical temperature decreases and converges to $T_c \simeq 163\,\mathrm{MeV}$.



Future research plans

- Incorporation of explicit symmetry breaking to realize the pseudo-Goldstone nature of the pions.
- Extension of calculations to finite quark chemical potential for a more detailed investigation of the QCD phase diagram.
- Extension of calculations to finite external momenta in order to extract spectral properties of the scalar and pseudo-scalar mesons.

References

[1] C. Wetterich, "Exact evolution equation for the effective potential", Phys. Lett. B 301, (1993), 90–94.

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[4] J. Braun, M. Leonhardt, J. M. Pawlowski, "Renormalization group consistency and low-energy effective theories", SciPost Phys. 6, 056 (2019), arXiv:1806.04432 [hep-ph].

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