Spectral functions from renormalised flow equations

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Invitation

Many interesting non-perturbative phenomena in physics are inherently timelike, ranging from scattering processes, the formation and spectrum of bound states to the time evolution of quantum systems close and far from equilibrium. The *spectral* fRG is a genuine real time approach which uses the spectral representation of the propagator to enable the analytic evaluation of the flow equation in the full complex frequency plane. We use the framework of renormalised spectral flows, put forward in [1], and apply it to a scalar ϕ^4-theory in (2+1) dimensions. We impose onshell-renormalisation conditions to control the flow in theory space and calculate self-consistent spectral functions. The content of this poster is based on [2]

Spectral functions and spectral diagrams

- Spectral representation for the propagator

$$G(p) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda\rho(\lambda)}{\lambda^2+p^2}, \quad \rho(w) = 2\,\mathrm{Im}\, G(w_R)\,.$$

- Existence of the spectral representation is tied to causality, i.e. the absence of non-analyticities away from the real axes.
- Parametrisation of spectral function in terms of poles (asymptotic particle states) and cuts (scattering continuum):

$$\rho(\lambda) = \frac{2\pi}{Z}\,\delta(\lambda^2 - m^2_{pole}) + \tilde\rho(\lambda)\,.$$

- Insert spectral representation in diagramatic functional expressions, see [3]:

$$D[p] = g \prod_i^{N_{loops}} \int_{q_i} \prod_j^N G(l_j)$$

$$= g \prod_j^N \int_{\lambda_j} \rho(\lambda_j) I(\vec\lambda, p) \quad \text{with}$$

$$I(\vec\lambda, p) = \prod_i^{N_{loops}} \int_{q_i} \prod_j^N \frac{1}{\lambda_j^2 + l_j^2}\,.$$

- Integrate loop integrals analytically to extract the full momentum structure.

Spectral CS-regulator

- Need a causal regulator which does not spoil Lorentz invariance.
  $\Rightarrow$ Mass-like Callan-Symanzik cut-off

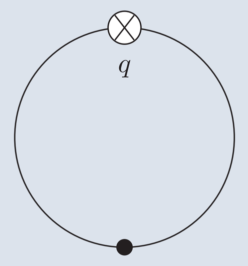
$$R_k = Z_\phi k^2\,.$$

  with the wave function renormalisation Z_ϕ.
- Suppression of infrared modes by shifting the mass:

$$S_k = \int_p \Big\{ \frac{1}{2}\phi\Big(p^2 + m_0^2 + k^2\Big)\phi + \frac{\lambda_\phi}{4!}\phi^4 \Big\}\,.$$

- Trade UV-regularisation for causality and Lorentz invariance.

Renormalised flowequation

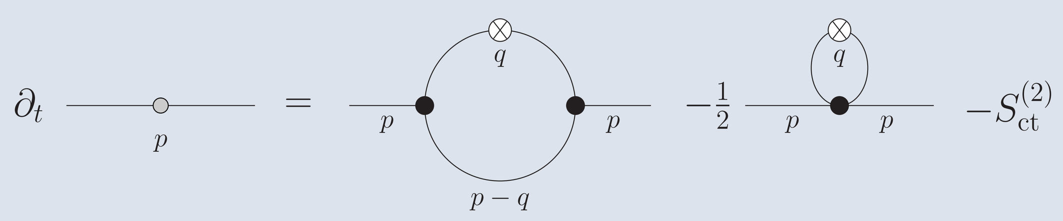
$\partial_t \Gamma_k = \frac{1}{2}$  $- \partial_t S_{\text{ct}}$

- Without UV-regularisation, Wetterich flow needs to be renormalised in $d > 2$.
  $\Rightarrow$ Renormalisation via counterterm action.
- The Callan-Symanzik flow connects infinitely massive theories in the UV with light (or massless) theories in the infrared.
  $\Rightarrow$ Every point on the solution trajectory represents a physical theory!
- CS-flow with counterterms can be derived as a limit of UV-regulated (spectral) flows.
- Counterterm-flow is fixed by an explicit (flowing) renormalisation condition.
- The real-time nature of the approach allows us to choose an on-shell renormalisation scheme:

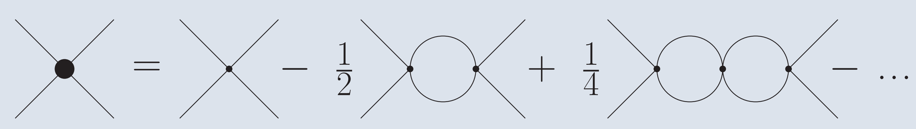
$$\partial_t \Big\{ \Gamma_k^{(2)}[\phi_0]|_{p^2=-k^2} = -k^2 \Big\}$$

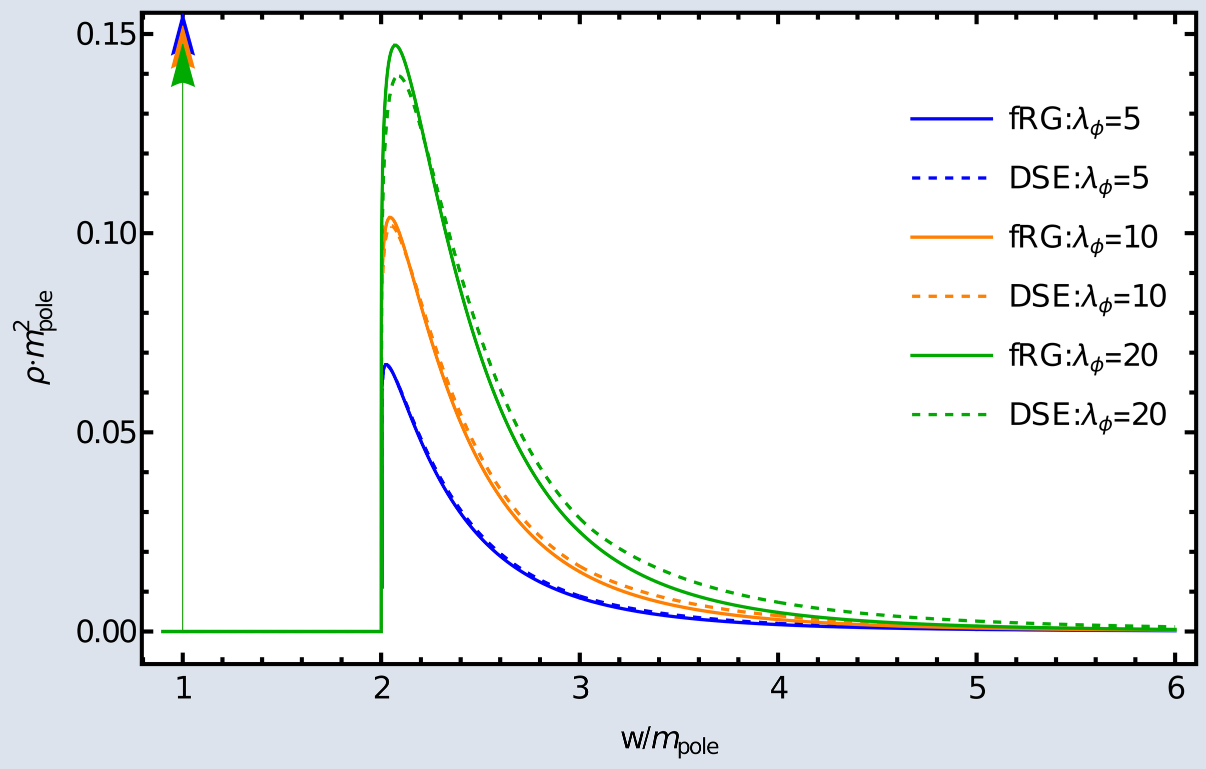
$$\Rightarrow \quad m^2_{pole}(k) = k^2\,.$$

Spectral functions and propagators

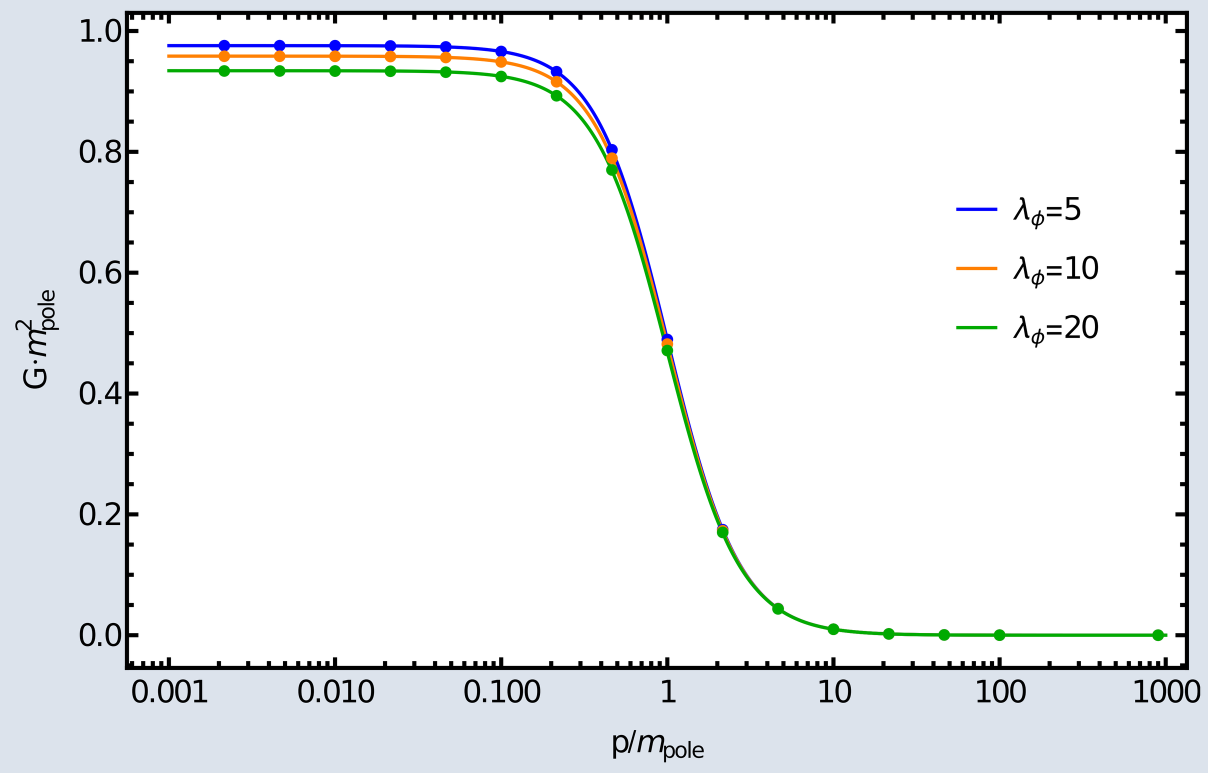


- Diagrams finite, but we use counter-term flow to control the flow in theory space.
- Flowequations evaluated in the broken phase, i.e. $\phi_0 \neq 0$.
- Non-trivial $\Gamma^{(4)}$ included via s-channel bubble resummation:





Spectral functions in comparison to the DSE results from [3] within a skeleton-expansion with the same approximation for the vertices. All quantities are measured in units of m_{pole}.



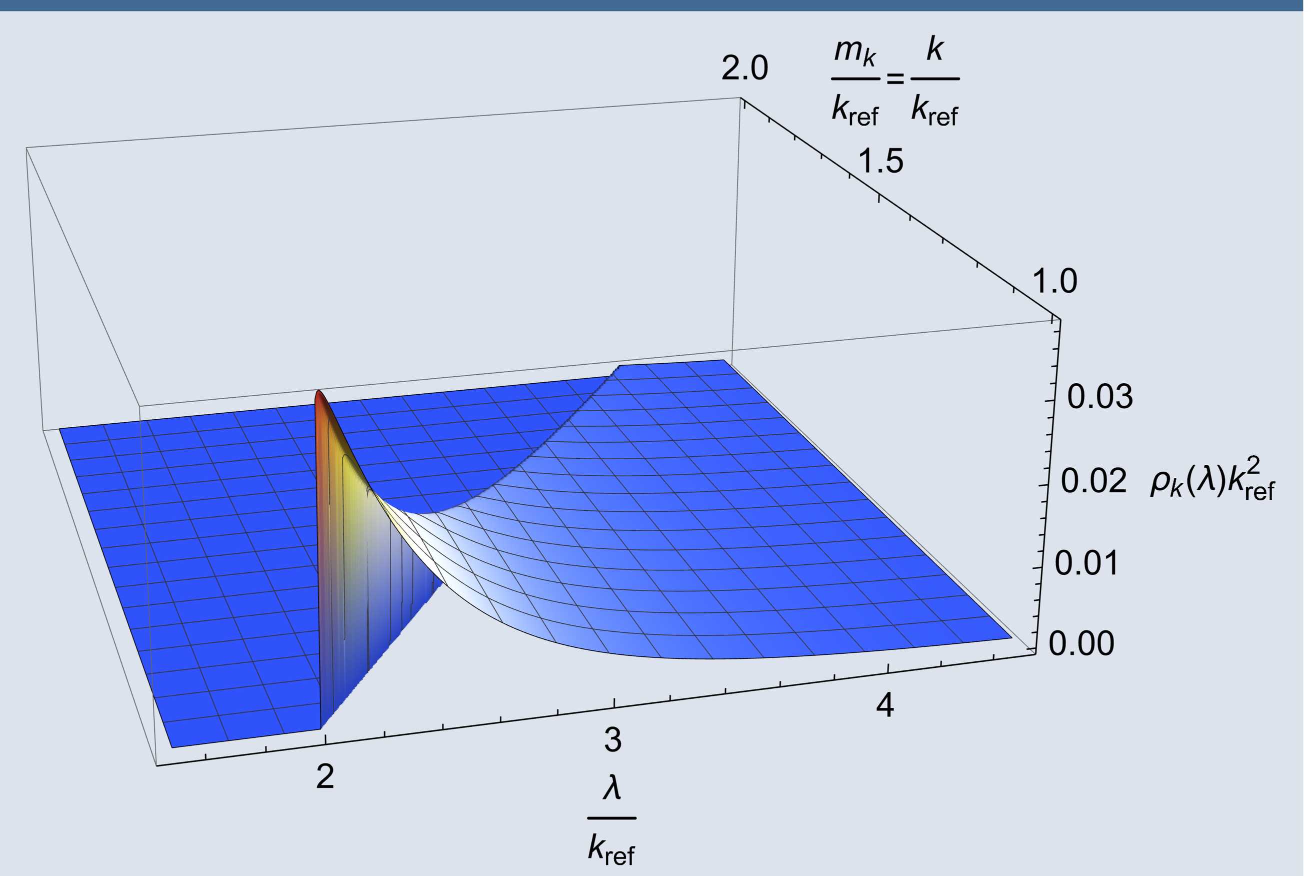
Euclidean propagators from the fRG computation: Solid lines are directly calculated from the (euclidean) flow, dots are propagator values calculated from the spectral representation.

RG-evolution of the spectral tail

- The theory has only one meaningfull dimensionless parameter:

$$\sigma(\tilde k = \frac{k}{k_{ref}}) = \frac{\lambda_\phi}{m_{pole}} = \frac{\lambda_\phi}{k}$$

- Right: RG-evolution of the scattering tail $\tilde\rho(\lambda)$ for the trajectory $\sigma(\tilde k = 2) = 2.5 \to \sigma(\tilde k = 1) = 5$.
  All quantities are measured in units of an arbitrary reference scale k_{ref}.



References

[1] J. Braun, Y.-r. Chen, W.-j. Fu, A. Geissel, J. Horak, C. Huang, F. Ihssen, J. M. Pawlowski, M. Reichert, F. Rennecke, Y.-y. Tan, S. Töpfel, J. Wessely, and N. Wink, *Renormalised spectral flows.* arXiv 2206.10232, 2022.

[2] J. Horak, F. Ihssen, J. M. Pawlowski, J. Wessely, and N. Wink, *Spectral functions from spectral flows in the ϕ^4-theory.* in preparation, 2022.

[3] J. Horak, J. M. Pawlowski, and N. Wink, *Spectral functions in the ϕ^4-theory from the spectral DSE*, 2020.