Foliated asymptotically safe gravity in the fluctuation approach

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Motiviation

Consider a **Non-relativistic** (**Lorentz invariance breaking**) theory,

e. g. , $\mathcal{L}_{\phi_1\phi_2} = \frac{1}{2}\Big(\alpha\partial_t\phi_1\partial_t\phi_1 + \beta\partial_i\phi_1\partial^i\phi_1 + \gamma(\partial_t\phi_2\partial_t\phi_2 + \partial_i\phi_2\partial^i\phi_2) + V(\phi_1,\phi_2)\Big)$,

where α, β and γ are running couplings.

We have the following related questions:

- What phenomenology is related to this Lorentz violation?

  Here α/β is related to a different speed for ϕ_1, compared with ϕ_2
- What is the structure of the general (non-relativistic) theory space?

  How to recover the Lorentz invariant case with $\alpha = \beta$ technically?

  Can we find Lorentz violating fixed points for α and β?

We will study analogous questions in asymptotically safe quantum gravity.

Set up

To have a foliated structure, we do the ADM decomposition

$$\gamma_{\mu\nu} = \begin{pmatrix} N^2 + N^iN_i & N_j \\ N_i & \sigma_{ij}\end{pmatrix}$$

The analogous "time derivative" of gravity is the extrinsic curvature K^{ij}

$$K_{ij} = \frac{1}{2}\mathcal{L}_n h_{ij} = \frac{1}{2}N^{-1}(\partial_\tau\sigma_{ij} - D_iN_j - D_jN_i),$$

and corresponding "spatial derivative" and potential terms: ${}^{(3)}R$ and $\mathcal{O}^{(2)}({}^{(3)}R)$.

- This setting introduces a prefered time direction and foliation, allowing us to study corresponding issues in quantum gravity.
- This could give a **non-relativistic (kinetic term + potential)** theory, with α_1 and α_2 being the running couplings for the kinetic term

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k}\int d\tau d^{(D-1)}yN\sqrt{\sigma}\big(\underbrace{\alpha_1K^{ij}K_{ij} - \alpha_2K^2}_{\text{Kinetic terms}} - \underbrace{{}^{(3)}R + 2\Lambda_k}_{\text{Potential}}\big). \quad (1)$$

- α_1 : the relative scaling between space and time direction.

  In background approximation, $\bar{N}$ (or $\hat{N}$) has a different wavefunction normalization than σ_{ij} (or $\hat{h}_{ij}$).

  We adopt the view that the kinetic and potential terms have different scale dependences (running couplings), just as Horava gravity [1].
- α_2/α_1: difference for speeds of light of transverse traceless (TT) mode and trace mode of fluctuation.
- $\alpha_1 = \alpha_2 = 1$: Einstein Hilbert action

  If we have fixed points and RG trajectory around $\alpha_1 = \alpha_2 = 1$, we would have Lorentz invariant sub-space.

Projection rules

The RG flow is given by the Wetterich equation

$$k\partial_k\Gamma_k = \frac{1}{2}\text{STr}\left[(\Gamma^{(2)} + \mathcal{R}_k)^{-1}k\partial_k\mathcal{R}_k\right] \quad (2)$$

We will adopt the **fluctuation approach** and project Wetterich equation onto the two point vertex of the TT mode of $\hat{\sigma}_{ij}$

$$\frac{1}{16\pi G_k}\left(\frac{1}{2}(\alpha_1 q_0^2 + \vec{q}^{\,2})h^{ij}h_{ij} - \Lambda_k h^{ij}h_{ij}\right)$$

Then the flow equations would be

$$\frac{1}{32\pi}k\partial_k\left(\frac{\alpha_1}{G_k}\right) = T_{q_0}(G_k,\alpha_1,\alpha_2,\Lambda_k,\eta_N), \quad (3)$$

$$\frac{1}{32\pi}k\partial_k\left(\frac{1}{G_k}\right) = T_{\vec{q}}(G_k,\alpha_1,\alpha_2,\Lambda_k,\eta_N), \quad (4)$$

$$\frac{1}{16\pi}k\partial_k\left(\frac{\Lambda_k}{G_k}\right) = T_0(G_k,\alpha_1,\alpha_2,\Lambda_k,\eta_N). \quad (5)$$

with T_{q_0}, $T_{\vec{q}}$ and T_0 the results of trace from Eq. (2).

The computation would be done in a flat Euclidean background.

Reference

[1] P. Horava, arXiv:0901.3775 [hep-th]

[2] E. Manrique, S. Rechenberger and F. Saueressig, arXiv:1102.5012 [hep-th]; J. Biemans, A. Platania and F. Saueressig, arXiv:1702.06539 [hep-th]...

Fixed points structure

0.1 Lorentz invariant system:

$\alpha_1 = \alpha_2 = 1$, $k\partial_k\alpha_1 = k\partial_k\alpha_2 = 0$

The projection rule is not unique:

- The flow Eqs. (3) and (4) from the q_0^2 and $\vec{q}^{\,2}$ components are NOT equivalent.
- One source of this inequivalence: $\Gamma^{(hh\hat{N}\hat{N})} = h^{kl}(-q)h_{kl}(q)\hat{N}(-p)\hat{N}(p)q_0^2$ is non-relativistic.
- We have two options for projections:
  - q_0 — Volume: RG flows given by Eqs. (3) and (5),
  - $\vec{q}$ — Volume: RG flows given by Eqs. (4) and (5).

However two different projections could give qualitatively similar results:

Projection	Fixed points	Couplings		Critical Exponents	
		g^*	λ^*	θ_1	θ_2
q_0 — Volume	FP1	0.16	0.014	4.28	2.04
	FP2	0.56	1.03	53.03	5.84
	FP3	-0.33	0.26	$4.02 \pm 10.15i$	
$\vec{q}$ — Volume	FP1	0.20	0.017	4.38	2.19
	FP2	0.03	0.75	54.78	-7.19
	FP3	-0.52	0.23	$-0.76 \pm 8.82i$	

0.2 Sub-system with a relative scaling between space and time:

$\alpha_1 = \alpha_2 \neq 1$, $k\partial_k\alpha_1 = k\partial_k\alpha_2 \neq 0$

Fixed Points	Couplings			Critical Exponents		
	g^*	λ^*	α_1^*	θ_1	θ_2	θ_3
FP1	0.20	0.015	1.24	4.38	2.14	1.90
FP2	-123.61	76.56	68.88	-85.10	-5.52	2.70
FP3	29.11	-632.41	897.62	21.50	2.95	1.72

0.3 Sub-system with different speeds for different modes:

$\alpha_2 = 1$, $k\partial_k\alpha_2 = 0$

In this case the trace mode comes with a speed of 1 while the TT mode is equipped with a speed of light of $1/\sqrt{\alpha_1}$.

Fixed Points	Couplings			Critical Exponents		
	g^*	λ^*	α_1^*	θ_1	θ_2	θ_3
FP1	0.20	0.017	1.20	4.36	2.28	2.10
FP2	-0.61	0.24	1.56	-11.37	$-1.26 \pm 10.91i$	
FP3	0.057	0.75	0.64	$26.51 \pm 11.56i$		-7.13

Comparing all three sub-systems, we could see

- The Lorentz invaraint fixed point $\{g^*, \lambda^*, \alpha_1^*\} \sim \{0.20, 0.015, 1.20\}$ persists for all system.
- The fixed point of large scaling between space and time needs to be confirmed by further computation.

Infrared behavior of flows

The flows around $g_k = 0$ are

$$\begin{aligned}\beta_g &= 2g_k + \mathcal{O}[g_k^2] * f_g(\lambda_k,\alpha_1)\\ \beta_\lambda &= -2\lambda_k + \mathcal{O}[g_k] * f_\lambda(\lambda_k,\alpha_1)\\ \beta_{\alpha_1} &= \mathcal{O}[g_k] * f_{\alpha_1}(\lambda_k,\alpha_1).\end{aligned} \quad (6)$$

All the point on α_1 axis with $g_k = 0$ and $\lambda_k = 0$ could be the potential infrared limit of the theory and the relativistic one with $\alpha_1 \sim 1$ is one of them.

Conclusion and Outlook

- We give the first fluctuation computation for foliated spacetime and the results are in line with the background computation [2].
- Our theory has a high energy completion for Lorentz invariant subspace.

Correspondingly, the following works could be done in the future:

- the fluctuation computation on Lorentzian background,
- search/confirmation for Lorentz violation fixed points.