The $(1+1)$-dimensional Gross-Neveu model at non-zero μ, T and finite N

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Abstract

We investigate the Gross-Neveu model for a finite number of fermions N. The solution of the Gross-Neveu model is well-known in the large-N limit ($N \to \infty$) but unknown for finite N. We approach the finite-N case with a FRG method, more precisely the Wetterich equation. By using the local potential approximation the resulting flow equation for the scale-dependent effective potential can be transformed into a non-linear diffusion equation. This equation is solved numerically by applying a finite-volume method. No discrete chiral symmetry breaking is observed for any finite number of fermions, arbitrary chemical potentials and non-zero temperatures.

Gross-Neveu model

- The bosonized **Gross-Neveu model** in $1+1$ dimensions is defined by the action [4]

$$\mathcal{S}[\bar{\psi}, \psi, \varphi] = \int \mathrm{d}x \int_0^\beta \mathrm{d}\tau \left[\bar{\psi} \left(\not{\partial} - \mu \gamma^2 + h\, \varphi \right) \psi + N \frac{h^2}{2 g^2} \varphi^2 \right],$$

  where N is the number of fermions, μ the chemical potential and $T = 1/\beta$ the temperature.
- This action exhibits a **chiral $\mathbb{Z}_2$-symmetry**, where the interesting transformation is

$$\psi \mapsto \psi' = \gamma_{\mathrm{ch}}\, \psi, \qquad \bar{\psi} \mapsto \bar{\psi}' = -\bar{\psi}\, \gamma_{\mathrm{ch}}, \qquad \phi \mapsto \phi' = -\phi.$$

- The question we want to investigate is whether or not this symmetry is spontaneously broken (SSB) by a non-vanishing global translation-invariant minimum of the full quantum effective action.

Approach

- We use the **Functional Renormalization Group (FRG)** which is a tool to calculate the full quantum effective action by a functional differential equation, ie., the Wetterich equation [3]

$$\partial_t \bar{\Gamma}_t[\Phi] = \tfrac{1}{2} \mathrm{STr} \left[\left(\bar{\Gamma}_t^{(2)}[\Phi] + R_t \right)^{-1} \cdot \partial_t R_t \right],$$

  where R_t is the regulator, t is the RG-time defined by $t = -\ln\left(\frac{k}{\Lambda}\right)$ with the RG-scale k and the UV-cutoff Λ.
- This is an **initial value problem** with the initial condition given by the classical action $\mathcal{S}[\Phi]$ at $t = 0$. The solution $\bar{\Gamma}_t[\Phi]$ is called scale-dependent effective average action and can be regarded as a trajectory flowing towards the full quantum effective (IR-)action $\Gamma[\Phi]$ in the limit $t \to \infty$.
- Since the Wetterich equation is to complicated to be solved in full generality, we have to make a truncated ansatz for the RG-trajectory. We choose the **local potential approximation (LPA)**

$$\bar{\Gamma}_t[\bar{\psi}, \psi, \varphi] = \int \mathrm{d}x \int_0^\beta \mathrm{d}\tau \left[\bar{\psi} \left(\not{\partial} - \mu \gamma^2 + h\, \varphi \right) \psi - \tfrac{1}{2} N\, \varphi\, (\Box \varphi) + N U(t, \varphi) \right].$$

- In this truncation the **scale-dependent effective potential** $U(t, \varphi)$ is the only flowing "coupling". The kinetic term for the boson field is necessary to study the impact of their dynamics on the system.
- To obtain an RG-flow equation for the scale-dependent effective action we have to choose regulators and a proper projection procedure. In our case we use 1-d and 2-d Litim regulators and evaluate the Wetterich equation on a constant field configuration $[\varphi(\tau, x) = \sigma,\ \bar{\psi}(\tau, x) = 0,\ \psi(\tau, x) = 0]$.

LPA flow equation

- Using the 2-d Litim regulators, the LPA flow equations with infinite and finite volume as well as on a finite lattice have a similar form. They differ only in some prefactors (Ref. [6] in preparation):

$$\partial_t U_k(\sigma) = \frac{1}{V} \sum_{\boldsymbol{q} \in \tilde{V}_{\mathrm{b}}} \frac{1}{N} \frac{-\epsilon_k \Theta(1 - \epsilon^{\mathrm{b}}_{\boldsymbol{q}}/\epsilon_k)}{\epsilon_k + \partial_\sigma^2 U_k(\sigma)} - \frac{1}{V} \sum_{\boldsymbol{q} \in \tilde{V}_{\mathrm{f}}} \frac{-d_\gamma \epsilon_k \Theta(1 - \epsilon^{\mathrm{f}}_{\boldsymbol{q}}/\epsilon_k)}{\epsilon_k + \sigma^2},$$

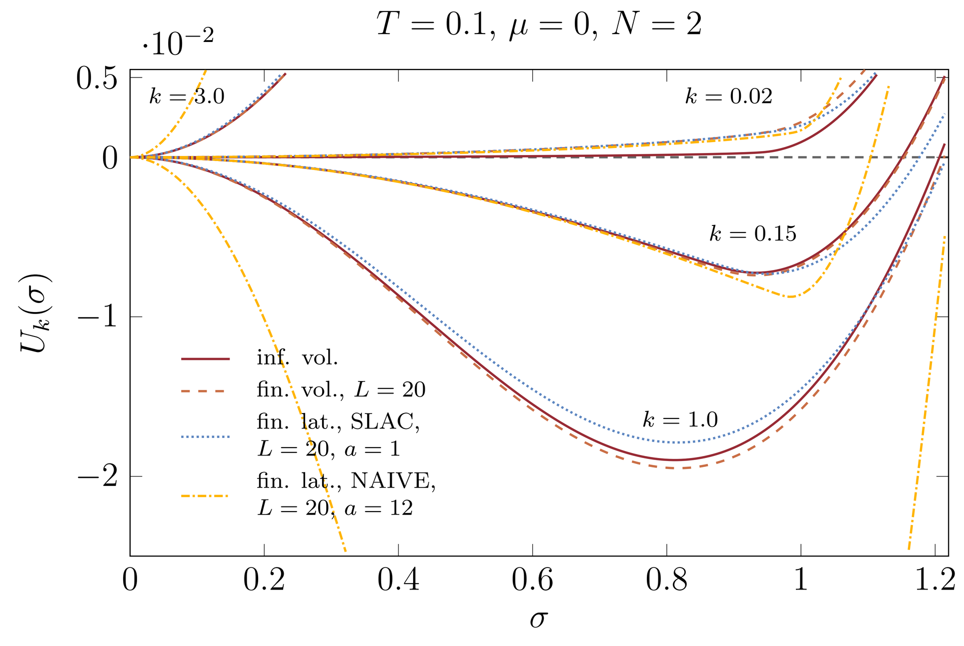
  where $\epsilon_k = k^2$ and $\tilde{V}_{\mathrm{b/f}}$ denote the bosonic and fermionic momentum space, respectively. The functions $\epsilon^{\mathrm{b/f}}_{\boldsymbol{q}}$ are the dispersion relations, which encode the discretization scheme in the case of a finite lattice.
- This flow equation can be reformulated in a conservation form, more precisely a **non-linear diffusion equation** [2, 1]

$$\partial_t u = \tfrac{\mathrm{d}}{\mathrm{d}\sigma} Q(t, \partial_\sigma u) + S(t, \sigma),$$

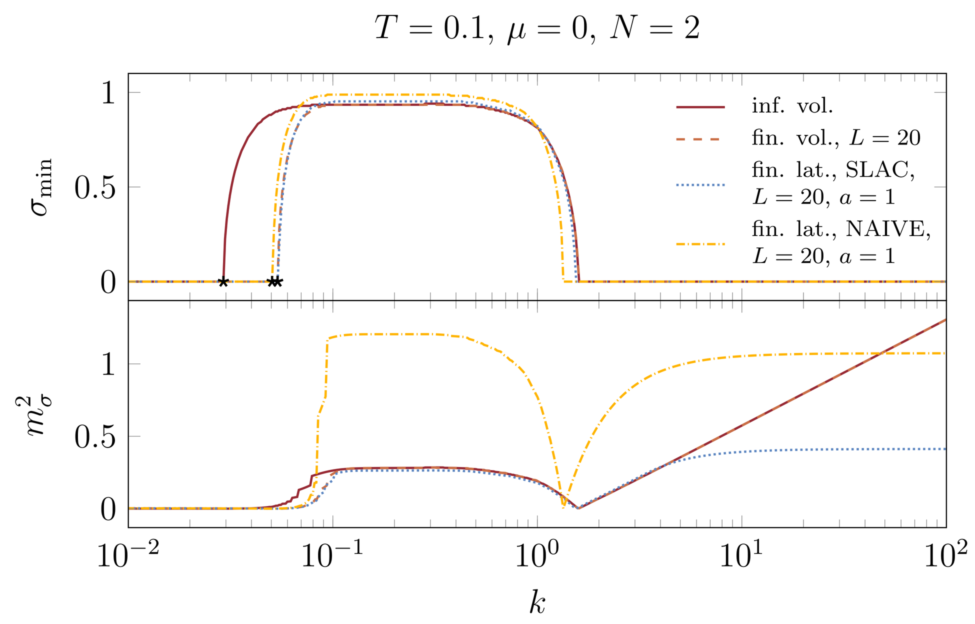
  where $u(t, \sigma) = \partial_\sigma U(t, \sigma)$ and we refer to $Q(t, \partial_\sigma u)$ as a **non-linear diffusion flux** and to $S(t, \sigma)$ as a **non-linear sink term**. We solve this PDE with a **finite volume method**.
- The mean-field (MF) approximation means that we ignore the bosonic contribution, ie., $Q = 0$.
- For all RG flows we choose the initial condition, ie., g_Λ^2 such that the MF effective potential has its global minimum at $\sigma_{\mathrm{MF},0} = 1$ for $T = \mu = 0$. This choice guarantees RG-consistency since the early RG flow is governed by the fermionic contribution.

Results

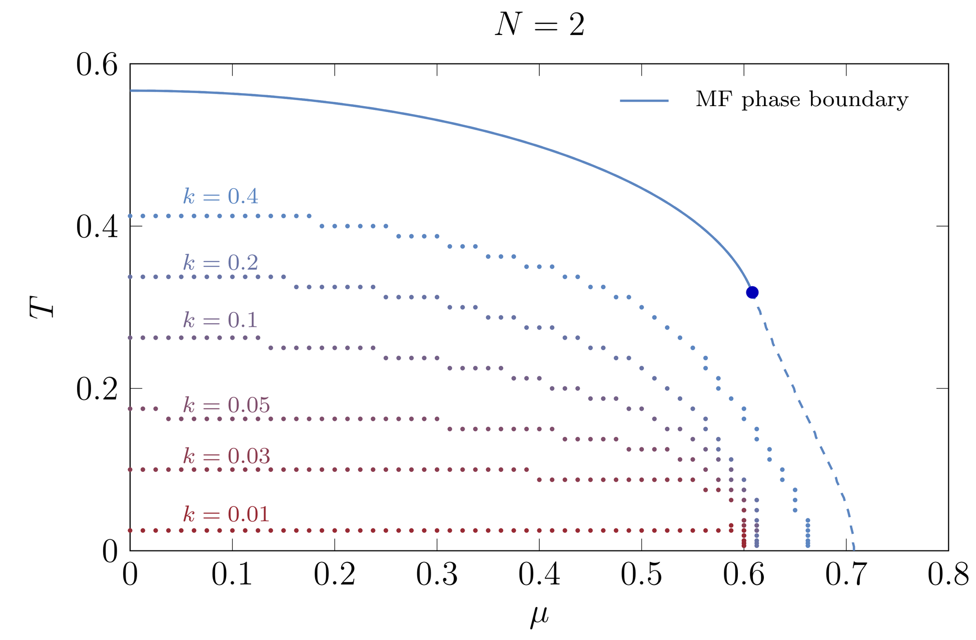
- To give a more detailed intuition about the RG flow, we have solved the diffusion equation and determined the effective potential at a single point in the phase diagram (using the 2-d Litim regulators):



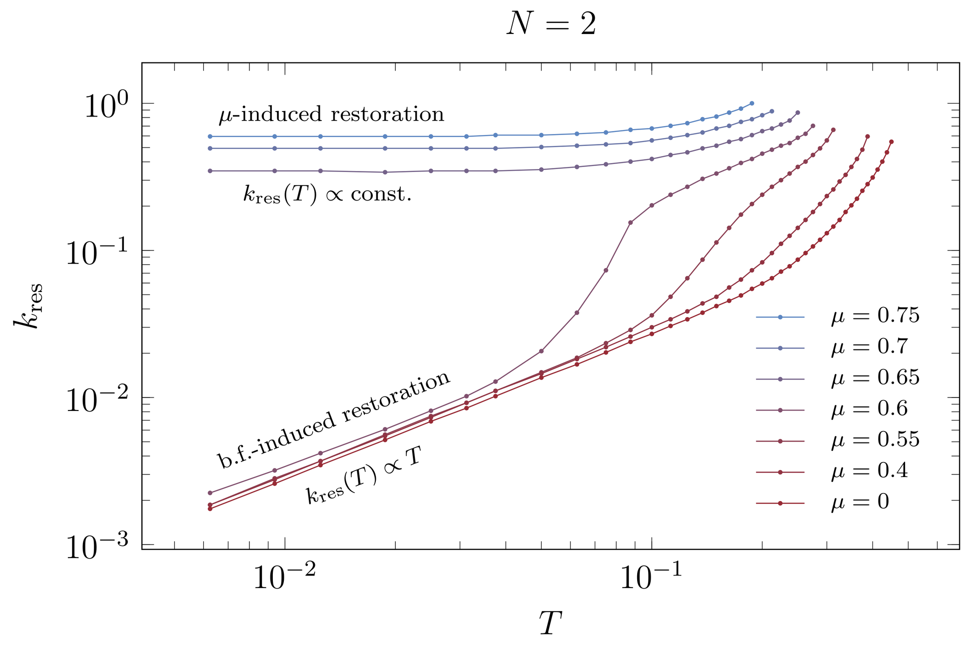
- Of special interest is the **global minimum** of $U_k(\sigma)$, σ_{min}, which serves as an order parameter for SSB in the IR:

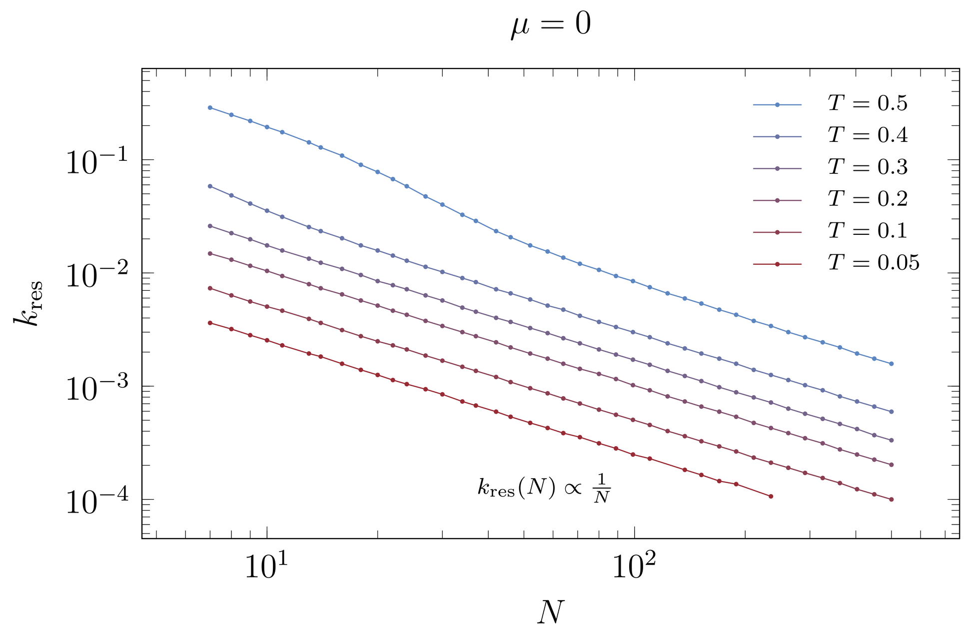


- The different scenarios share qualitatively the same behavior.
- **SSB** occurs while the RG flow but **vanishes** at some **restoration scale** k_{res} (black stars) for all tested parameter sets with $T > 0$.
- This is also the case in the infinite-volume limit using the 1-d Litim regulators, where we can also study the RG flow for $\mu \neq 0$ [1].



- For small RG-scales, the **broken phase** shrinks towards lower temperatures and **vanishes** for our test points.
- To be more quantitative, we analyze the T-, μ- and N-dependence of the restoration scale and extrapolate it w.r.t. N and T (especially for large N and low T the numerical calculations become hard):





- Our results suggest that there is **no SSB in the IR** for finite N and $T > 0$.
- For small chemical potentials and temperatures, the bosonic fluctuations (b.f.) restore the $\mathbb{Z}_2$-symmetry, while for larger chemical potentials, the chemical potential itself restores the $\mathbb{Z}_2$-symmetry.

Main result [1]

Using FRG in LPA, we find **no SSB** in the Gross-Neveu model for any finite N, non-zero T and arbitrary μ.

Together with Adrian Koenigstein, Martin J. Steil and Stefan Rechenberger. The project was supported by M. Buballa and D. H. Rischke.

References

[1] Stoll, Jonas and Zorbach, Niklas and Koenigstein, Adrian and Steil, Martin J. and Rechenberger, Stefan, "Bosonic fluctuations in the $(1+1)$-dimensional Gross-Neveu(-Yukawa) model at varying μ and T and finite N", arXiv:2108.10616.

[2] A. Koenigstein, M. J. Steil et al., "Numerical fluid dynamics for FRG flow equations: Zero-dimensional QFTs as numerical test cases – Part I & II", arXiv:2108.02504 and arXiv:2108.10085.

[3] C. Wetterich, "Exact evolution equation for the effective potential", Phys. Lett. B 301, (1993), 90–94.

[4] D. J. Gross and A. Neveu, "Dynamical Symmetry Breaking in Asymptotically Free Field Theories", Phys. Rev. D 10, (2000), 3235.

[5] M. Thies, "From relativistic quantum fields to condensed matter and back again: Updating the Gross-Neveu phase diagram", J. Phys. A 39, (2006), 12707–12734.

[6] J. Braun, L. Pannullo and N. Zorbach, in preparation.

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