Asymptotic safety and freedom in higher derivative gravity.

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Abstract

We compute non-perturbative flow equations for the couplings of quantum gravity in fourth order of a derivative expansion also called as higher derivative gravity. The functional flow equation for arbitrary background metrics allows us to extract β-functions for all couplings. In our truncation we find two fixed points. One corresponds to asymptotically free higher derivative gravity, the other is an extension of the asymptotically safe fixed point in the Einstein-Hilbert truncation or extensions thereof. There may exist a critical trajectory between the two fixed points, starting in the extreme ultraviolet from asymptotic freedom and flowing to infra-red (IR) passing through the Asymptotically safe fixed points. We compute critical exponents and determine the number of relevant parameters for the two fixed points. We also find some critical trajectories connecting the ultra-violet (UV) and IR in simplified sub-truncations of higher derivative gravity.

Introduction

Gravitational interactions at macroscopic scales is best described by general relativity. In 4D this is characterized by the Einstein-Hilbert (E-H) action with metric as the dynamical field

$$S_{EH} = \int \sqrt{g} d^4x (U - \frac{F}{2} R).$$

Conventionally the cosmological constant is denoted by $U = \Lambda$ and Ricci scalar coupling by $F = \frac{1}{8\pi G}$ where G is the Newton constant. When we try to upgrade general relativity to a quantum field theory it is perturbatively non-renormalizable as the canonical dimension of Newton's constant is$[G] = -2$. Trends show that gravity may admit non-perturbative fixed point and be asymptotically safe in the framework of FRG which is explained in later sections of this poster [2].

The critical exponents at the fixed points are found out to be $\theta_{1,2} = 2.63 \pm i1.39$. [2]

If we introduce higher curvature terms

$$S_{HD} = \int d^4x \sqrt{g} \left[U - \frac{F}{2} R - \left(\frac{C}{2}\right) R^2 + D R_{\mu\nu} R^{\mu\nu} + \text{Topological Terms} \right],$$

the higher derivative action is asymptotically free and admits a Gaussian fixed-point [1].

Does gravity admit both Asymptotic safety and freedom ?

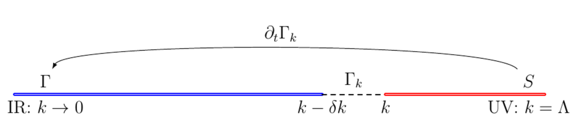
The functional renormalization group equation

We try to answer the question in the framework of functional renormalization group (FRG). This is implemented by given by the functional renormalization group equation (FRGE)

$$\partial_t \Gamma_k = \frac{1}{2} \text{sTr} \left[\frac{\partial_t \mathscr{R}_k}{(\Gamma_k^{(2)} + \mathscr{R}_k)} \right].$$

The central quantity is the effective average equation Γ_k with momentum modes $p^2 < k^2$ suppressed by an infrared regulator $\mathscr{R}_k$. The functional renormalization group equation smoothly interpolates microscopic physics embodied by bare classical action S and macroscopic physics given by the effective action at IR Γ.

This is schematically presented in the following diagram :-



To apply FRGE to gravity we work in background field formalism decomposing $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ into background and fluctuation field respectively. This allows us to implement background independence as the choice of background is left arbitrary. Additionally this allows us to introduce a coherent notion of momentum suppression. We start with $\Gamma[\bar{g}, h] = \sum_i \int d^4x \sqrt{\bar{g}} \lambda_i(k) \mathbb{O}^i(R, R_{\mu\nu}R^{\mu\nu}, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma})$ as a background diffeomorphism symmetric action [4]. Where $\mathbb{O}$ is a polynomial in curvature scalar invariants. When plugged into FRGE $\partial_t \Gamma_k = \frac{1}{2} \text{sTr} \left[\frac{\partial_t \mathscr{R}_k}{(\Gamma_k^{(2)} + \mathscr{R}_k)} \right]$ this is a set of infinitely coupled integro-differential equations.

To perform practical calculations we truncate $\Gamma[\bar{g}, h]$ upto some order in curvature.

Does gravity admit both Asymptotic safety and Freedom?

Minimum truncation to answer the question the Higher derivative action is

$$\Gamma_{\text{grav}} = \int d^4x \sqrt{g} \left[U - \frac{F}{2} R - \left(\frac{C}{2}\right) R^2 + D R_{\mu\nu} R^{\mu\nu} + \text{Topological Terms} \right].$$

To distinguish between all couplings we chose ***arbitrary but fixed backgrounds***. Arbitrary backgrounds make the computations technically more challenging. [3]

In addition we need to gauge fix for action , we compute RG flow for $\Gamma[\bar{g}, h] = \Gamma_{\text{grav}} + \Gamma_{\text{gf}} + \Gamma_{\text{ghost}}$ and we have,

$$\Gamma_{\text{gf}} = \frac{1}{2\alpha} \int d^4x \sqrt{g}\, \bar{g}^{\mu\nu} \Sigma_\mu \Sigma_\nu \ , \quad \text{and} \quad \Sigma_\mu = \bar{D}^\nu h_{\nu\mu} - \frac{\beta+1}{4} \bar{D}_\mu,$$

and corresponding ghost action.

β-functions and fixed points

We compute the β-functions for dimensionless couplings and define some useful parameters to understand our RG flows with respect to an arbitrary scale k. We define $w = \frac{Fk^{-2}}{2}$, dimensionless newton constant $g = \frac{1}{16\pi w}$ and $u = Uk^{-4}$ and we define the dimensionless spin-2 mass $m_t^2 = \frac{D-u}{w}$ and spin-0 mass $\frac{3C-u}{w}$. We found out

- **Asymptotically safe fixed point -**

	u_*	w_*	C_*	D_*	m^2_{t*}	$m^2_{\sigma*}$	θ_1	θ_2	θ_3	θ_4
FP	0.0218	0.000281	0.204	-0.0132	-0.618	28.0	3.1	2.4	10.9	-88.1

We found 3 Relevant and 1 irrelevant direction. The critical exponents for θ_1 and θ_2 are close to the ones found in Einstein-Hilbert action.

- **Asymptotically free fixed point -**

	u_*	w_*	C_*^{-1}	D_*^{-1}	θ_1	θ_2	θ_3	θ_4
FP	0.0069	0.0127	0	0	4	2	0	0

We recovered the asymptotically safe fixed point and critical exponents are given by the canonical scaling.

One must note that "absurd" values of critical exponents are truncation artifacts, with addition of higher derivative terms they tend to tame down.
One can see similar trends in Table 2 of [6]

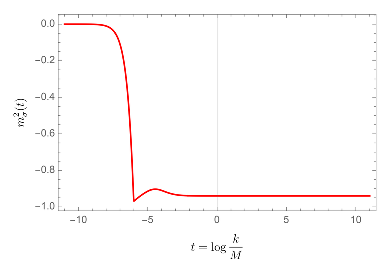
Flow trajectories

Computing flows for the full system is a technically challenging task. We have calculated the flows for some simplified sub-truncations.

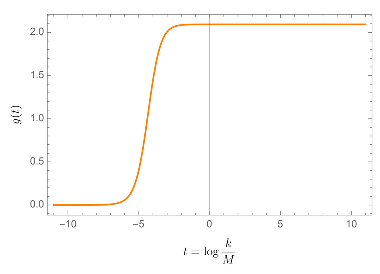
- Flow trajectories for $\Gamma_{\text{grav}} = \int d^4x \sqrt{g} \left[U - \frac{F}{2} R - \left(\frac{C}{2}\right) R^2 \right]$

Fixed points of the truncation

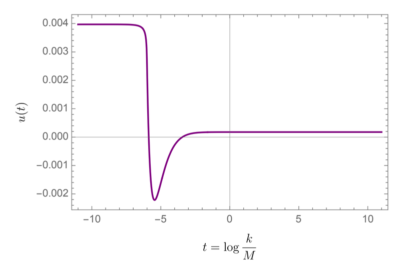
	u_*	g_*	C^*	$m^2_{\sigma*}$	θ_1	θ_2	θ_3
FP	0.0002	2.0931	0.0029	-0.9394	$2.48 + i0.342$	$2.48 - i0.342$	306.6



Flow of scalar mass in sub truncation without coupling D



Flow of dimensionless Newton constant in sub truncation without coupling D

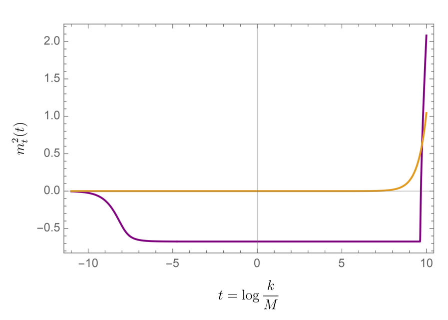


Flow of dimensionless cosmological constant in sub truncation without coupling D

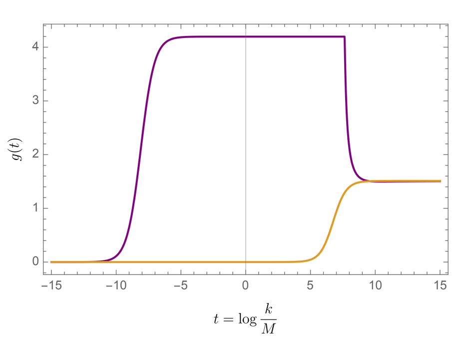
- Flow trajectories for $\Gamma_{\text{grav}} = \int d^4x \sqrt{g} \left[-\frac{F}{2} R + D R_{\mu\nu} R^{\mu\nu} \right]$

This is the simplest sub-truncation where we have both Asymptotically safe and free fixed points. Here, the purple line connects the Infrared , Asymptomatically safe and free fixed point and brown line connects the IR and the Asymptotically free fixed point directly. They correspond to two different phases of gravity.

	g_*	D_*	m_t^2	θ_1	θ_2
AF-FP	1.531	∞	∞	2	0
AS-FP	3.978	-0.003	-0.600	1.9	-390



Flow of spin-2 mass in truncation in subtraction without u and D



Flow of dimensionless Newton constant in truncation in subtraction without u and D

The computed trajectories seems to be consistent with the observed fact that in IR Newton's constant and cosmological constant is small and higher derivative couplings should be negligible.

Conclusions

- We found that gravity admits both asymptotically safe and Asymptotically free fixed point in the framework if FRG up to second order in curvature.
- We have three relevant directions for Asymptotically safe and two relevant directions for Asymptotically free fixed point.
- It is difficult to say wether such a fixed point is a truncation artifact or not.
- For a simplified sub truncation ignoring the cosmological constant and coupling of R^2 we found flow trajectories connecting the IR, Asymptotically safe and Asymptotically free fixed point.

Future outlook

- We would like to extend the FRG analysis beyond the higher derivative action and check for robustness of the Asymptotically safe and free fixed points.[4]
- With the full beta functions at hand we would like to understand more about the nature of vacuum solutions of Higher derivative gravity.
- It has been suggested that nature and properties of black hole are different at the different fixed points. Using the full beta functions we wish to take a small step toward some phenomenological predictions about nature of blackhole.[5]
- We would like to find out if critical trajectories connecting the fixed points exist in the full system containing all couplings of Higher derivative gravity.

Selected references

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