Phase Transitions in a Yukawa-QCD Model

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Motivation

- Observe large scale separation of the Fermi scale Λ_{F} and Λ, a high energy scale up to which the standard model is thought to be valid
- Λ_{F} is intimately tied to the mass parameter m^2 in the scalar potential
- m^2 receives radiative corrections $\propto \Lambda^2$
- Initial parameters of the model need to be fine tuned to a very high precision
  $\Rightarrow$ What is the influence of the standard model parameters on the fine tuning necessary?

Functional Method: The FRG

We analyse our model with the functional renormalisation group (FRG) using the Wetterich equation [Wetterich '93]

$$\partial_t \Gamma_k = \frac{1}{2}\mathsf{STr}\left[\frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}\right], \qquad t = \ln\frac{k}{\Lambda},$$

a differential equation for the effective average action Γ_k, interpolating between S for $k \to \Lambda$ and the full quantum effective action Γ for $k \to 0$.
R_k denotes the regulator, implementing a Wilsonian idea of renormalisation

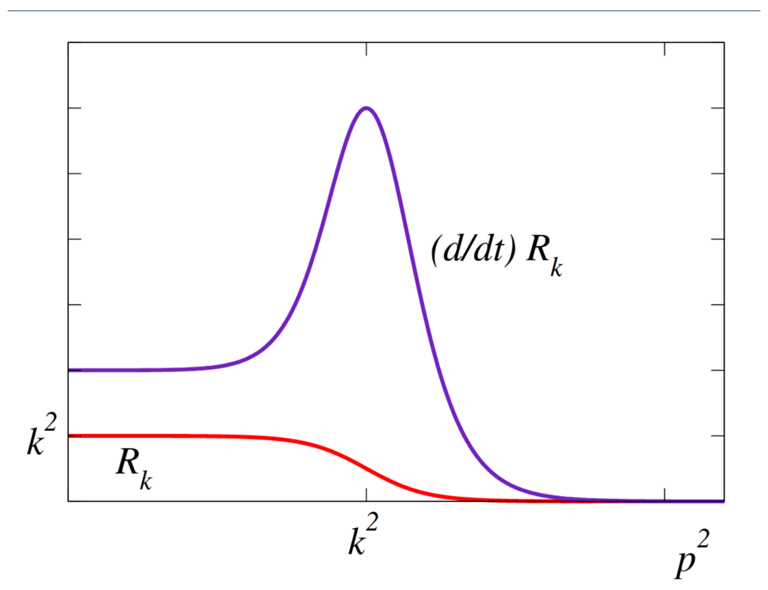


Fig. 1: Typical shape of a regulator function

Effective Action: The Yukawa-QCD Model

Starting point: QCD effective action with induced 4-fermion terms

$$\Gamma = \int_x \bar\psi_i^a \mathsf{i}\not{D}_{ij}\psi_j^a + \frac{1}{4}F^z_{\mu\nu}F^{\mu\nu}_z + \frac{(\partial_\mu A_z^\mu)^2}{2\xi} + \frac{1}{2}\bar\lambda_\sigma\left[(\bar\psi_i^a\psi_i^b)^2 - \left(\bar\psi_i^a\gamma_5\psi_i^b\right)^2\right].$$

By means of the Hubbard-Stratonovic transformation (partial bosonization) [Gies '06] we translate the four fermion channel into an effective quark meson Yukawa interaction

$$\varphi^{ab} = -\mathsf{i}\frac{\bar h}{m^2}\bar\psi^b P_{\mathsf{L}}\psi^a, \qquad \varphi^{*ab} = -\mathsf{i}\frac{\bar h}{m^2}\bar\psi^a P_{\mathsf{R}}\psi^b,$$

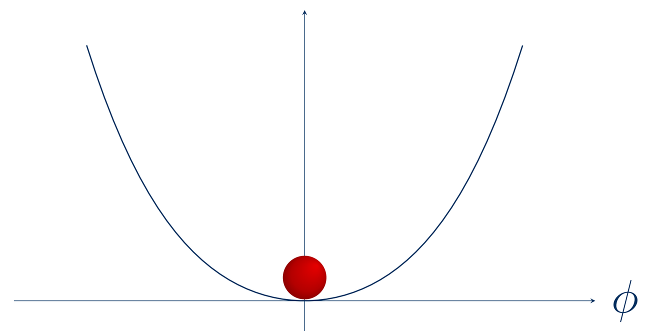
and reparametrise our newly obtained scalar field including top/bottom-type couplings to arrive at

$$\Gamma = \int_x L_{\mathsf{Kin+Gauge}} + U(\rho,\tilde\rho) + \frac{\mathsf{i}\tilde h_b}{\sqrt2}\left(\bar\psi^a_{\mathsf{L},i}\Phi^a b_{\mathsf{R},i} + \bar b_{\mathsf{R},i}\Phi^{*a}\psi^a_{\mathsf{L},i}\right) + \frac{\mathsf{i}\tilde h_t}{\sqrt2}\left(\bar\psi^a_{\mathsf{L},i}\Phi^a_{\mathcal{C}}t_{\mathsf{R},i} + \bar t_{\mathsf{R},i}\Phi^{*a}_{\mathcal{C}}\psi^a_{\mathsf{L},i}\right) + \frac{\mathsf{i}h}{\sqrt2}\left(\bar\psi^a_{\mathsf{R},i}\tilde\Phi^a b_{\mathsf{L},i} + \bar\psi^a_{\mathsf{R},i}\tilde\Phi^a_{\mathcal{C}}t_{\mathsf{L},i} + \mathsf{h.c.}\right).$$

Symmetry Breaking in the Model

Symmetry breaking in this model is happening when the scalar potential $U(\rho,\tilde\rho)$ develops a non-vanishing minimum. For a (simple) ϕ^4 potential this means

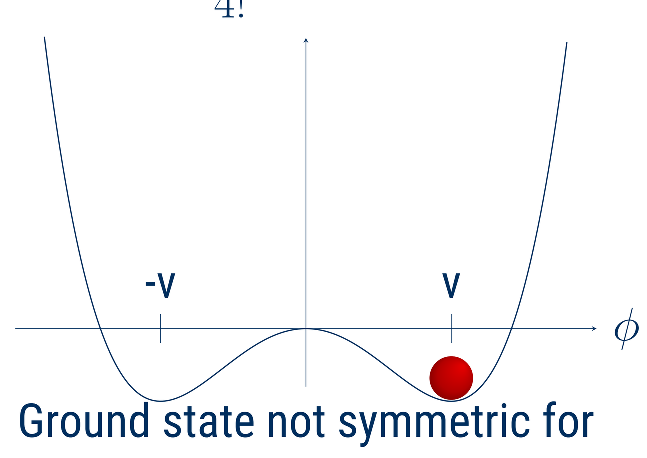
$$U = \epsilon\phi^2 + \frac{\lambda_1}{4!}\phi^4, \quad U(\phi) = U(-\phi)$$



Ground state symmetric for $\phi \to -\phi$

$\Rightarrow$ UV$\to$IR

$$U = -\epsilon\phi^2 + \frac{\lambda_1}{4!}\phi^4, \quad U(\phi) = U(-\phi)$$



Ground state not symmetric for $\phi \to -\phi$

This transition happens when the mass parameter becomes negative during our flow from the UV to the IR. In our model we use a Φ^4 potential $U(\rho,\tilde\rho) = m^2(\rho+\tilde\rho) + \frac{\lambda_1}{2}(\rho+\tilde\rho)^2 + \lambda_2\rho\tilde\rho$,

where $\rho = \Phi^{*a}\Phi^a$ and $\tilde\rho = \tilde\Phi^{*a}\tilde\Phi^a$

Gauged vs Ungauged Model

In the ungauged case our model coincides with a chiral Higgs-top-bottom model. In this model there are two phases, the symmetric regime in the right half plane and the broken (deep Higgs) regime in the left half plane, characterized by a non-vanishing vacuum expectation value v. The transition between phases is described by a second order quantum phase transition $v \propto |\delta\epsilon|^\beta$ [Kleinert et al '01]. In the gauged case we end up in the broken regime even in the right half plane caused by gauge interactions coming from QCD (deep QCD). However we observe an approximate a power-law behaviour for the vev going from the deep QCD to the deep Higgs regime for this analysis.

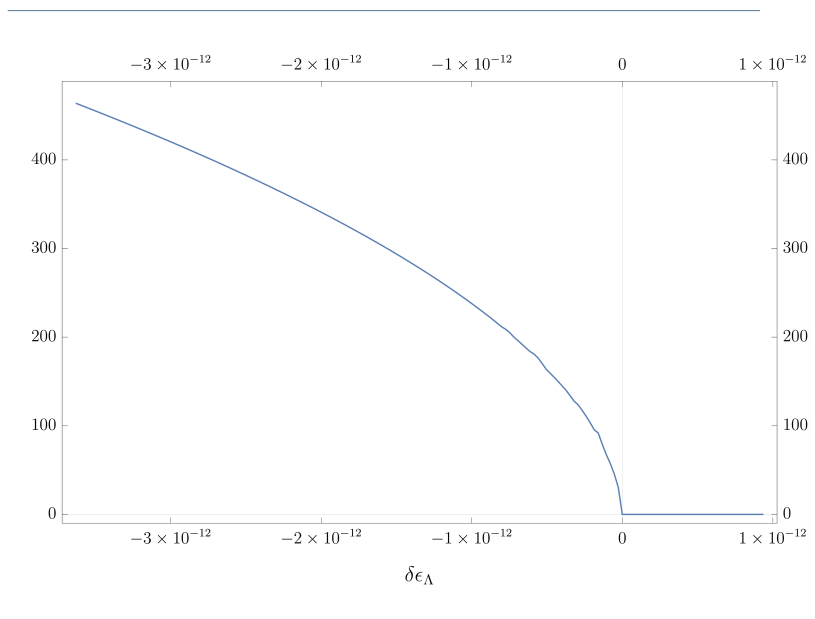


Fig. 2: Phase transition in the ungauged model

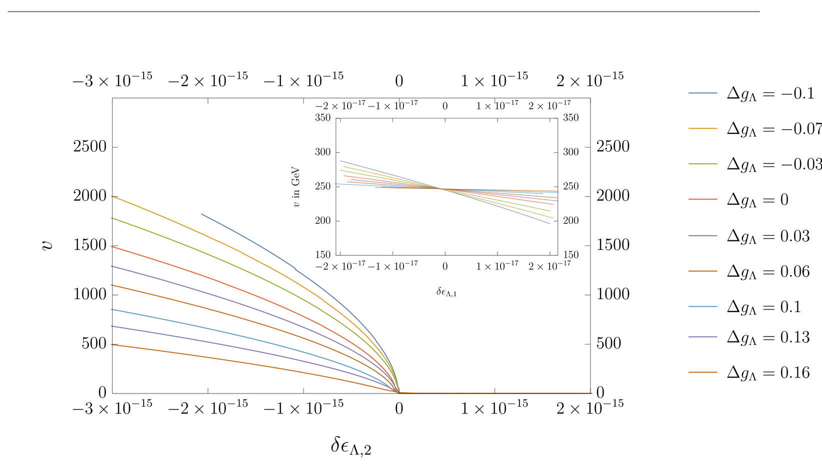


Fig. 3: Phase transition in the gauged model at different g^2_Λ

Phase Transitions

We generate data for different values of the couplings in the UV. For the analysis of the fine tuning in one parameter we keep all values for the other parameters fixed at the initialization scale Λ. Shown is the critical exponent $\eta = 2\beta - 1 (= 2 - \Theta)$, measuring the deviation of the RG exponent Θ from 2.

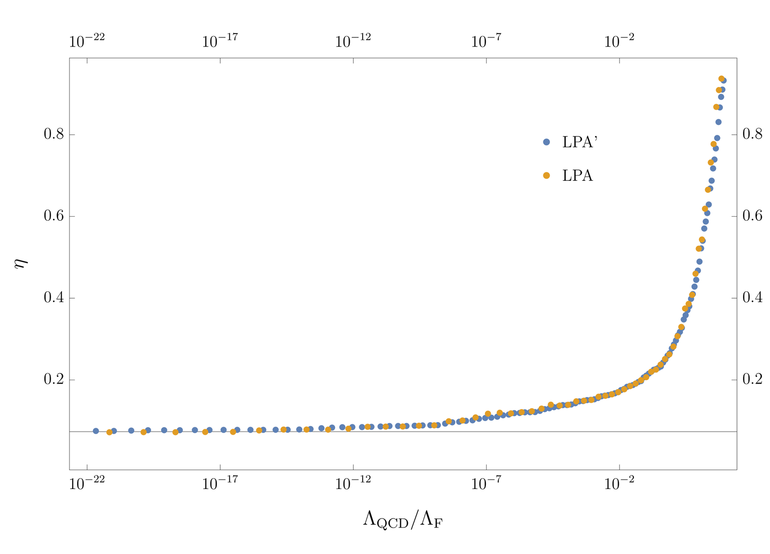


Fig. 4: Critical Exponent for the gauge coupling

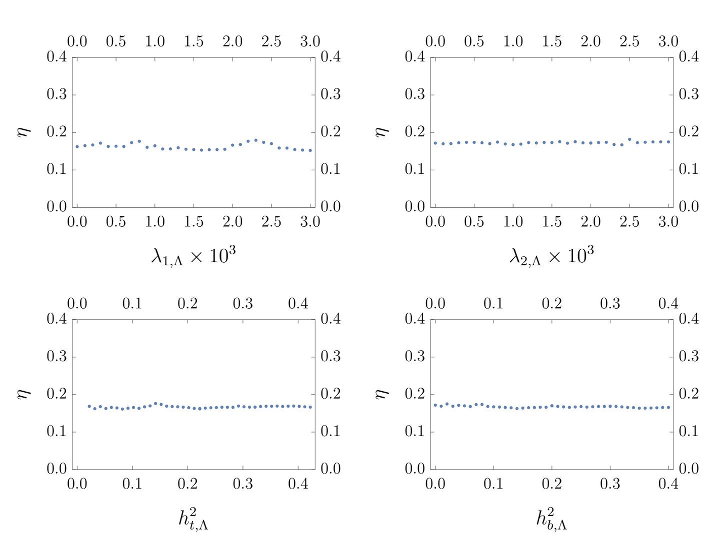


Fig. 5: For the Yukawa and scalar interactions

Scaling behaviour close to the transition

Close to Λ_{QCD} and still being in the symmetric regime, the meson Yukawa coupling h^2 dominates over $h^2_{\mathsf{t/b}}$ and we can assume $h^2 \gg h^2_{\mathsf{t/b}}$. Furthermore the scalar masses can be neglected close to the transition. In that limit our model exhibits scaling solutions, directly connecting the gauge coupling g^2 to the Yukawa coupling and scalar self interaction $h^2 \sim g^2$ and $\lambda_1 \sim g^2$ (quasi fixed-point in $\frac{h^2}{g^2}$ and $\frac{\lambda_1}{h^2}$, c.f. [Gies et al '19]). This strongly influences the running of the mass parameter through the scalar anomalous dimension $\eta_\Phi \sim h^2 \sim g^2$ and alleviates the fine tuning necessary. This can be seen in η for $\Lambda_{\mathsf{QCD}}/\Lambda_{\mathsf{F}} \to 1$, since then this scaling behaviour is still expected to occur in our flows.

Conclusions

- Have seen that Yukawa and scalar couplings have little influence on the fine-tuning problem
- Strong influence of the gauge coupling on the transition
  $\to$ Can be traced back to a quasi fixed-point in the meson sector of the model
- At the physical point $\frac{\Lambda_{\mathsf{QCD}}}{\Lambda_{\mathsf{F}}} \approx 10^{-4}$, QCD softens the fine-tuning problem with $\eta \approx 0.16$, compared to $\eta \approx 0.07$ in the ungauged model.

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