Asymptotically safe Einstein-Palatini gravity

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Holger Gies and Abdol Sabor Salek

Institute for Theoretical Physics, Friedrich Schiller University Jena, Germany

Email: abdol.sabor.salek@uni-jena.de

$R_{\mu\nu}\ \mathbb{T}\ G_{(x_{\mathrm{Jena}},\, x_{\mathrm{Leipzig}})}$ 2522

Funded by DFG Deutsche Forschungsgemeinschaft

Motivation

- Palatini formalism: treat metric g and connection $\tilde{\Gamma}$ as independent degrees of freedom [1]
- Analogy to $SU(N)$-symmetric gauge theories where the gauge field $\mathcal{A}_\mu$ and the metric g are independent
- UV-completion within FRG/ Asymptotic Safety: first approach with vielbeins [2, 3]; not further pursued
- New approach via "on-shell reduction": reduce fluctuation degrees of freedom $\gamma^\alpha_{\mu\nu} \to a_\mu$ through EoM to obtain treatable gauge symmetry

General connection $\tilde{\Gamma}$ and Palatini formulation of gravity

- Covariant derivative $\tilde{\nabla}$ with general connection $\tilde{\Gamma}$ (not a priory symmetric in lower two indices)

$$\begin{aligned}\tilde{\nabla} &= \partial + \tilde{\Gamma}\\ &= \partial + \Gamma + K + N\\ &= \nabla + K + N \end{aligned} \tag{1}$$

- Levi-Cevita Γ: $\Gamma^\alpha_{\mu\nu} \neq 0 \to$ curvature
- Contorsion K: $T^\alpha_{\mu\nu} = \tilde{\Gamma}^\alpha_{\mu\nu} - \tilde{\Gamma}^\alpha_{\nu\mu} \neq 0 \to$ torsion
- Non-metricity N: $\tilde{\nabla}_\mu g_{\alpha\beta} = -2 \cdot N_{\mu\alpha\beta} \neq 0 \to$ non-conversation of measures

- Curvature tensors in Palatini formulation defined accordingly (with less symmetries in indices)

$$\tilde{R}^\rho_{\sigma\mu\nu} = \partial_\mu \tilde{\Gamma}^\rho_{\nu\sigma} - \partial_\nu \tilde{\Gamma}^\rho_{\mu\sigma} + \tilde{\Gamma}^\rho_{\mu\lambda}\tilde{\Gamma}^\lambda_{\nu\sigma} - \tilde{\Gamma}^\rho_{\nu\lambda}\tilde{\Gamma}^\lambda_{\mu\sigma}, \tag{2}$$

$$\tilde{R}_{\sigma\nu} = \tilde{R}^\rho_{\sigma\mu\nu}\, g^\mu_\rho, \tag{3}$$

$$\tilde{R} = \tilde{R}_{\sigma\nu} g^{\sigma\nu} \tag{4}$$

- Treatment of gauge degrees of freedom of arbitrary connection $\tilde{\Gamma}$ not straightforward
- Approach via suitable background connection as solution of equations of motion

Einstein-Hilbert-Palatini action

- Einstein-Hilbert action in Palatini formulation

$$S[g,\tilde{\Gamma}] = \int d^4x \sqrt{g} \frac{1}{16\pi G}\left(\Lambda - 2\tilde{R}\right). \tag{5}$$

- Equations of motion (EoMs) to find the (on-shell) background connection

$$\frac{\delta S}{\delta g_{\mu\nu}} \overset{!}{=} 0 \quad \text{(differential)} \tag{6}$$

$$\frac{\delta S}{\delta \tilde{\Gamma}^\alpha_{\mu\nu}} \overset{!}{=} 0 \quad \text{(algebraic)} \tag{7}$$

- Analytic solution for algebraic set of EoMs with new degrees of freedom A_μ in Palatini formulation

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} + A_\mu \delta^\alpha_\nu, \tag{8}$$

- Additional degree of freedom A_μ inherits $U(1)$ symmetry through its field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{9}$$

- Expressing Palatini curvature tensors through ordinary ones reveals $U(1)$-symmetry of A_μ

$$\tilde{R}_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} + g_{\rho\sigma}F_{\mu\nu} \tag{10}$$

$$\tilde{R}_{\sigma\nu} = R_{\sigma\nu} + F_{\sigma\nu} \tag{11}$$

$$\tilde{R} = R \tag{12}$$

- No new physical information obtained via Palatini formulation on Einstein-Hilbert level ($\mathbb{R}^4$-symmetry)

Higher order Einstein-Palatini gravity

- Palatini formulation allows for second rank 2 tensor through different contraction ("Licci-tensor")

$$\tilde{L}_{\sigma\nu} = g^{\rho\mu}\tilde{R}_{\sigma\rho\nu\mu} = R_{\sigma\nu} - F_{\sigma\nu}. \tag{13}$$

- Two possible curvature invariants for Ricci-like tensors in Palatini formulation

$$\tilde{R}_{\sigma\nu}\tilde{R}^{\sigma\nu} = R_{\sigma\nu}R^{\sigma\nu} + F_{\sigma\nu}F^{\sigma\nu}, \tag{14}$$

$$\tilde{R}_{\sigma\nu}\tilde{L}^{\sigma\nu} = R_{\sigma\nu}R^{\sigma\nu} - F_{\sigma\nu}F^{\sigma\nu}, \tag{15}$$

- Simplest higher order action S in Palatini formulation

$$S[g,\tilde{\Gamma}] = \int d^4x\sqrt{g}\frac{1}{16\pi G}\left[\Lambda - 2\tilde{R} + \sigma^1 \tilde{R}^{\mu\nu}\tilde{R}_{\mu\nu} + \sigma^2 \tilde{L}^{\mu\nu}\tilde{R}_{\mu\nu}\right] \tag{16}$$

- Using on-shell reduction of previous section to obtain ($\sigma^{R/F} = \sigma^1 \pm \sigma^2$)

$$S[g,A] = \int d^4x\sqrt{g}\frac{1}{16\pi G}\left[\Lambda - 2R + \sigma^R R^{\mu\nu}R_{\mu\nu} + \sigma^F F^{\mu\nu}F_{\mu\nu}\right]. \tag{17}$$

- Extension to Einstein Palatini gravity expressed through ordinary Einstein gravity and a $U(1)$-symmetric gauge field
- Treatment of gauge degrees of freedom within Asymptotic Safety scenario of quantum gravity and matter known [4]

Asymptotic safety approach

- Scale-dependent analysis of quadratic order truncation in (Palatini-) gravitational sector

$$\Gamma_{k,gr}[g,A] = \int d^4x\sqrt{g}\tfrac{1}{16\pi \bar{G}_k}\left[\bar{\Lambda}_k - 2R + \bar{\sigma}^R_k R^{\mu\nu}R_{\mu\nu} + \bar{\sigma}^F_k F^{\mu\nu}F_{\mu\nu}\right] \tag{18}$$

- Dimensional running couplings $\bar{G}_k$, $\bar{\Lambda}_k$ and $\bar{\sigma}^R_k$ and inessential coupling $Z^A_k = \frac{\bar{\sigma}^F_k}{4\pi \bar{G}_k}$
- Wetterich equation in background field formalism of $\Gamma_k = \Gamma_{k,gr} + \Gamma_{k,gf} + \Gamma_{k,gh}$ to obtain flow equations for dimensionless couplings G_k, Λ_k, σ^R_k and anomalous dimension η_A

$$G_k = k^2\bar{G}_k \qquad \Lambda_k = \frac{1}{k^2}\bar{\Lambda}_k \qquad \sigma^R_k = k^2\bar{\sigma}^R_k \qquad \eta_A = -\frac{k\partial_k Z^A_k}{Z^A_k}. \tag{19}$$

- UV-attractive fixed point

$$G_\star = 1.132, \qquad \Lambda_\star = 0.2141, \qquad \sigma_\star = 0.326 \tag{20}$$

- Critical exponents

$$\theta_{1,2} = 2.033 \pm 3.169 \cdot \mathrm{i}, \qquad \theta_3 = 12.784 \tag{21}$$

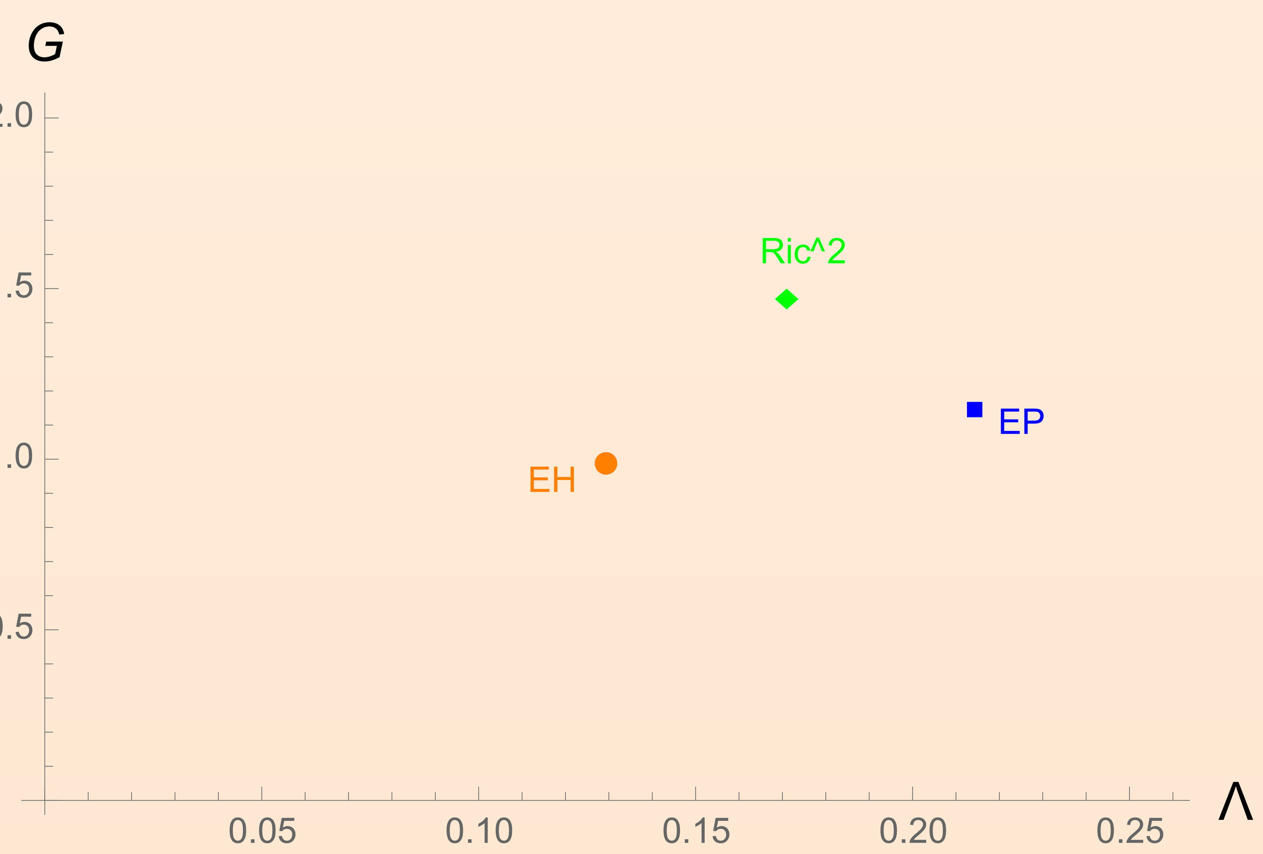

Figure 1: Diagrammatic representation of the fixed points in theory space spanned by the couplings Λ and G for various truncations. The orange circle represents the Einstein-Hilbert truncation (EH), the green diamond represents its extension to quadratic order in the Ricci tensor (Ric^2) and the blue square represents its extension to Einstein-Palatini gravity (EP).

Conclusions

- Simplest non-trivial extension of Einstein to Einstein-Palatini gravity using on-shell reduction
- Both non-metricity tensor N and contorsion tensor K are relevant expressions in this extension
- First UV-complete theory of Einstein-Palatini gravity
- Outlook: Asymptotically safe Einstein-Palatini gravity and matter with potential χSB

References

[1] Antonio N. Bernal, Bert Janssen, Alejandro Jimenez-Cano, Jose Alberto Orejuela, Miguel Sanchez, and Pablo Sanchez-Moreno. On the (non-)uniqueness of the Levi-Civita solution in the Einstein–Hilbert–Palatini formalism. Phys. Lett. B, 768:280–287, 2017.

[2] Ulrich Harst and Martin Reuter. A new functional flow equation for Einstein–Cartan quantum gravity. Annals Phys., 354:637–704, 2015.

[3] J. E. Daum and M. Reuter. Renormalization Group Flow of the Holst Action. Phys. Lett. B, 710:215–218, 2012.

[4] Pietro Donà, Astrid Eichhorn, and Roberto Percacci. Matter matters in asymptotically safe quantum gravity. Phys. Rev., D89(8):084035, 2014.