The F-theorem in the melonic limit

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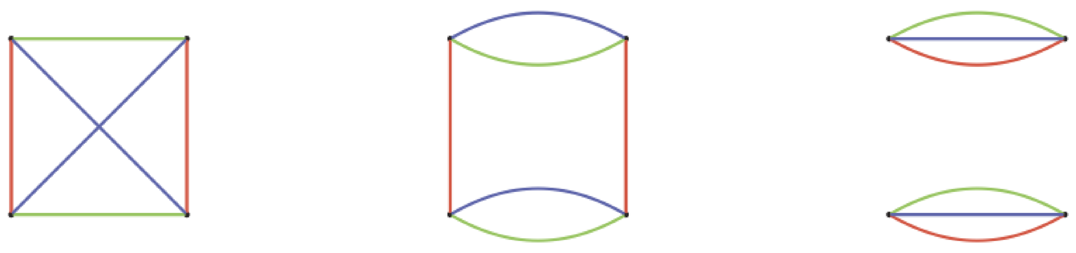
1. Introduction

Among the most intriguing features of quantum field theory in various dimensions are the so called c, a and F-theorems. These lettered theorems state that under the RG flow between various fixed points some quantities (aptly denoted c, a or F) always decrease. Intuitively, these quantities must in some way count the degrees of freedom in the theory, as the RG flow decimates the degrees of freedom when going from one fixed point to another. In $d = 3$ the decreasing quantity is F (conjectured in [5], proved in [4]): the finite part of the free energy on the sphere. We consider here the long-range bosonic $O(N)^3$ model on a spherical background, at next-to-next-to-leading order of the $1/N$ expansion. The model displays four large-N fixed points and we test and confirm the F-theorem holds in this case [2]. Our result can be viewed as further indication that the theory is unitary.

2. The $O(N)^3$ model

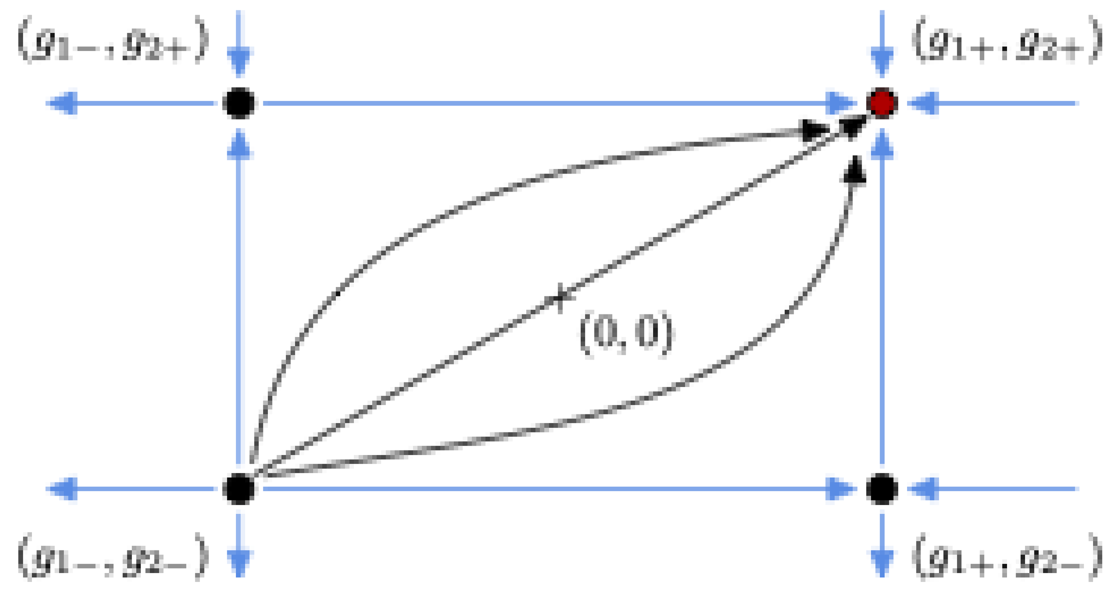
$$\mathcal{L} = \frac{1}{2}\phi_{abc}(\partial)^{2\zeta}\phi_{abc} + \mathcal{L}_{int}, \qquad \zeta = \frac{d}{4}.$$

Graphic representation of the interactions: each vertex is a field and each colored line connecting two vertices is a contraction of indices.



With renormalized couplings (left to right): g, g_1, g_2.

At large-N the theory has four lines of fixed point parameterized by g [1], which does not flow.



$$g_{1\pm} = \pm\sqrt{g^2} + \mathcal{O}(g^2), \quad g_{2\pm} = \pm\sqrt{3g^2} + \mathcal{O}(g^2).$$

3. The theory on the sphere

- $\frac{1}{|x_1-x_2|^{2\Delta}} \to \frac{\Omega(x_1)^{-\Delta}\Omega(x_2)^{-\Delta}}{|x_1-x_2|^{2\Delta}} = \frac{1}{|s(x_1,x_2)|^{2\Delta}}$

  where $s(x_1, x_2)$ is the chordal distance
- Fractional Laplacian: $\omega_n^{(\zeta)} = \frac{\Gamma(n+\frac{d}{2}+\zeta)}{a^{2\zeta}\Gamma(n+\frac{d}{2}-\zeta)}$
- On shell 2pt function

  $$G_\star(x,y) = \mathcal{Z}C(x,y) = \mathcal{Z}\frac{c(\Delta)}{|s(x,y)|^{2\Delta}}$$

  where $c(\Delta)$ and $\mathcal{Z}$ are finite numbers.

References

[1] Dario Benedetti, Razvan Gurau, and Sabine Harribey. Line of fixed points in a bosonic tensor model. *JHEP*, 06:053, 2019.

[2] Dario Benedetti, Razvan Gurau, Sabine Harribey, and Davide Lettera. The F-theorem in the melonic limit. *JHEP*, 02:147, 2022.

[3] Valentin Bonzom, Victor Nador, and Adrian Tanasa. Diagrammatics of the quartic $O(N)^3$-invariant Sachdev-Ye-Kitaev-like tensor model. *J. Math. Phys.*, 60(7):072302, 2019.

[4] H. Casini and Marina Huerta. On the RG running of the entanglement entropy of a circle. *Phys. Rev. D*, 85:125016, 2012.

[5] Daniel L. Jafferis, Igor R. Klebanov, Silviu S. Pufu, and Benjamin R. Safdi. Towards the F-Theorem: N=2 Field Theories on the Three-Sphere. *JHEP*, 06:102, 2011.

4. 2PI effective action

It is useful to work with the 2 particle irreducible effective action $\Gamma_{2PI}[G]$ (later we omit subscript the '2PI'). Note that the free energy is $F = \Gamma[G = G_\star]$.

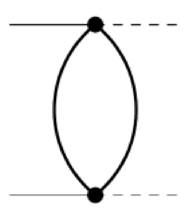
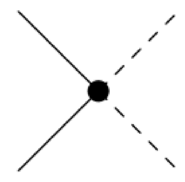
- Leading order:

  $$\mathbf{\Gamma}[G] = N^3\left(\frac{1}{2}\mathcal{Z}[C^{-1}C] + \frac{1}{2}\underbrace{[\ln(\mathcal{Z}^{-1}C^{-1})]} + \frac{m^{2\zeta}}{2}\mathcal{Z}\int_x C(x,x) + \frac{\lambda_2\mathcal{Z}^2}{4}\int_x C(x,x)^2 + \frac{\lambda^2\mathcal{Z}^4}{8}\int_{x,y} C(x,y)^4\right), \quad (1)$$

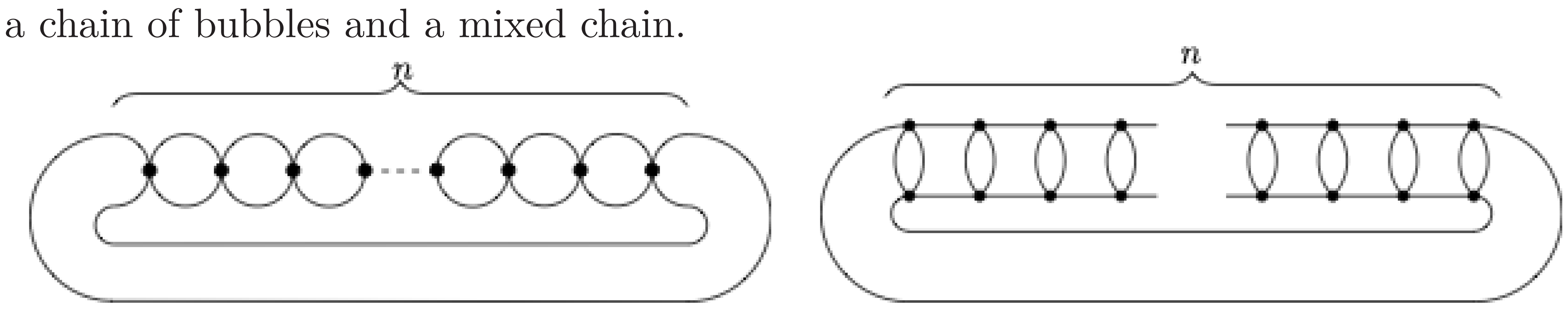
  where λ and λ_2 are bare couplings. All of the terms vanish but not the underlined one, which gives N^3 times the free energy of the free theory. The leading order has trivial free energy.

- Next-to-next-to-leading order [3]: $\Gamma_{NNLO}[G_\star] = \frac{N^2}{2}\left(\mathrm{Tr}[\ln(\mathbb{I}-K_1)] + \underbrace{\mathrm{Tr}[K_1]}_{\text{2PI condition}}\right)$,

  where K_1 has the following graphical representation:

  $K_1 = -\lambda^2$  $- \lambda_1$ 

  With the kernel K_1 we can resum 3 families of diagrams at the same time: a chain of ladders, a chain of bubbles and a mixed chain.

5. NNLO free energy

We insert the resolution of the identity in terms of partial waves

$$\mathbb{I}(x_1,x_2,x_3,x_4) = \sum_{J\in\mathbb{N}_0}\int_{\frac{d}{2}}^{\frac{d}{2}+\mathrm{i}\infty}\frac{\mathrm{d}h}{2\pi\,\mathrm{i}}\rho(h,J)\mathcal{N}^{\Delta}_{h,J}\mathcal{N}^{\widetilde{\Delta}}_{\widetilde{h},J}\Psi^{\Delta,\Delta,\widetilde{\Delta},\widetilde{\Delta}}_{h,J}(x_1,x_2,x_3,x_4), \quad (2)$$

where $\rho(h,J)$ and $\mathcal{N}^{\Delta}_{h,J}$ are known conformal quantities, $\tilde{\Delta} = d - \Delta$ and Ψ is the partial wave: $\Psi^{\Delta,\Delta,\widetilde{\Delta},\widetilde{\Delta}}_{h,J}(x_1,x_2,x_3,x_4) = \int_z \langle\phi_\Delta(x_1)\phi_\Delta(x_2)\mathcal{O}^{\mu_1\cdots\mu_J}_h(z)\rangle\langle\phi_{\widetilde{\Delta}}(x_3)\phi_{\widetilde{\Delta}}(x_4)\mathcal{O}^{\mu_1\cdots\mu_J}_{\tilde{h}}(z)\rangle$.

Two main results:

- Compute numerically: $-g\frac{\partial}{\partial g}F_{\text{NNLO}} = 7.57\times 10^{-4}\,N^2$ ($d=3$, $a=1$ and $g=1$).
- In the UV there is a non-normalizable state (dimension $h_- < \frac{d}{2}$). We must modify the resolution of the identity with an extra term. We find that

$$\begin{aligned} g\frac{\partial}{\partial g}\left(F^{UV}_{\text{NNLO}} - F^{IR}_{\text{NNLO}}\right) &= N^2\,\text{Res}\left[\rho(h,0)\frac{k(h,0)^2}{1-k(h,0)}\mathcal{N}^{\Delta}_{h,0}\mathcal{N}^{\widetilde{\Delta}}_{\widetilde{h},0}\,\text{Tr}[\Psi]\right]_{h=h_-} \\ &= 16\frac{\Gamma(-d/2)|g|^3}{2^{3d}\pi^{3d/2}\Gamma(d)}N^2 + \mathcal{O}(|g|^5). \end{aligned} \quad (3)$$