Multiloop functional renormalization group study of the Fermi polaron problem

Marcel Gievers[1,2], Richard Schmidt[2,3], Jan von Delft[1]

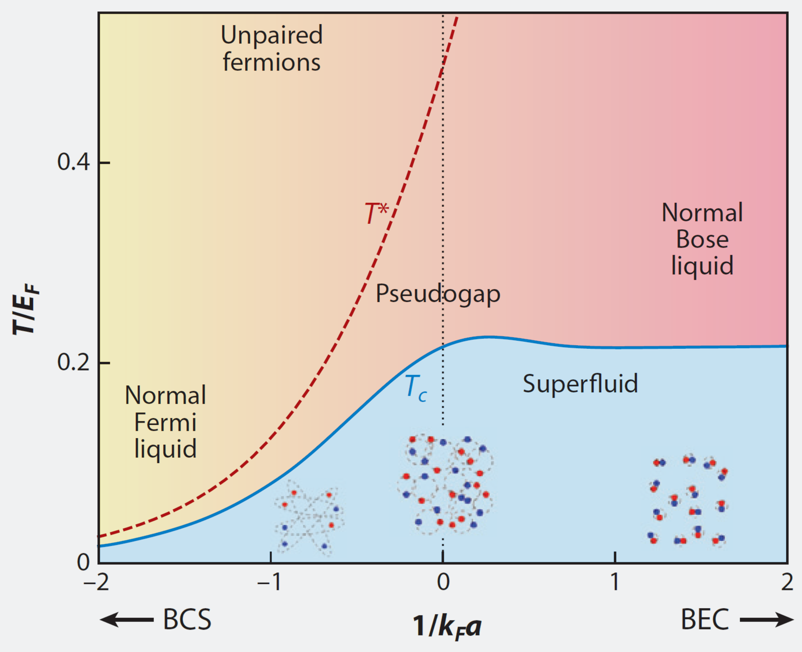
[1]Physics Department, Arnold Sommerfeld Center for Theoretical Physics, and Center for NanoScience, Ludwig-Maximilians-Universität München, 80333 Munich, Germany.
[2]Max-Planck-Institute of Quantum Optics, 85748 Garching, Germany. [3]Department of Physics and Astronomy, University of Aarhus, 8000 Aarhus C, Denmark.

Model: Two-component Fermi mixtures

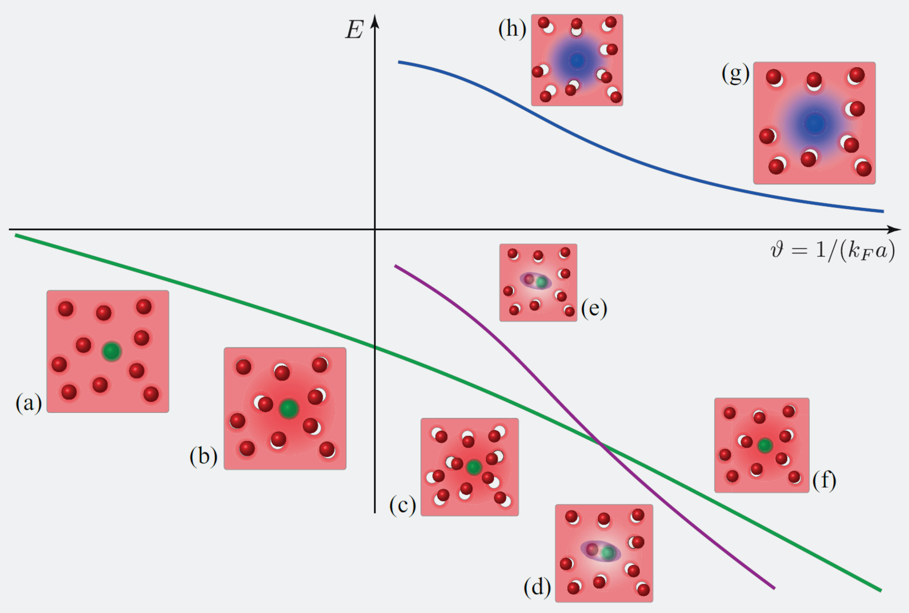
- **System:** Fermi gas of two species c and d with a local attractive interaction $g > 0$ between different species:

$$S = \int_{\tau,\boldsymbol{k}} \left[\bar{c}_{\boldsymbol{k}} (\partial_\tau - \frac{\boldsymbol{k}^2}{2m_c} - \mu_c) c_{\boldsymbol{k}} + \bar{d}_{\boldsymbol{k}} (\partial_\tau - \frac{\boldsymbol{k}^2}{2m_d} - \mu_d) d_{\boldsymbol{k}} \right] + \int_{\tau,\boldsymbol{x}} g\, \bar{c}_{\boldsymbol{x}} \bar{d}_{\boldsymbol{x}} d_{\boldsymbol{x}} c_{\boldsymbol{x}}$$

- Interaction g related to scattering length a (tunable via Feshbach resonances): $\frac{m_r}{2\pi a} = \frac{1}{g(\lambda)} + \frac{m_r}{\pi^2}\lambda$, momentum cutoff $\lambda \sim r_0^{-1}$ related to effect. interaction range
- Balanced case $n_d/n_c = 1 \Rightarrow$ **BCS-BEC crossover**
  - BCS regime: fermions c and d form Cooper pairs
  - BEC regime: bosonic bound states between c and d particles form a Bose-Einstein condensate
  - groundstate: crossover between regimes [1]



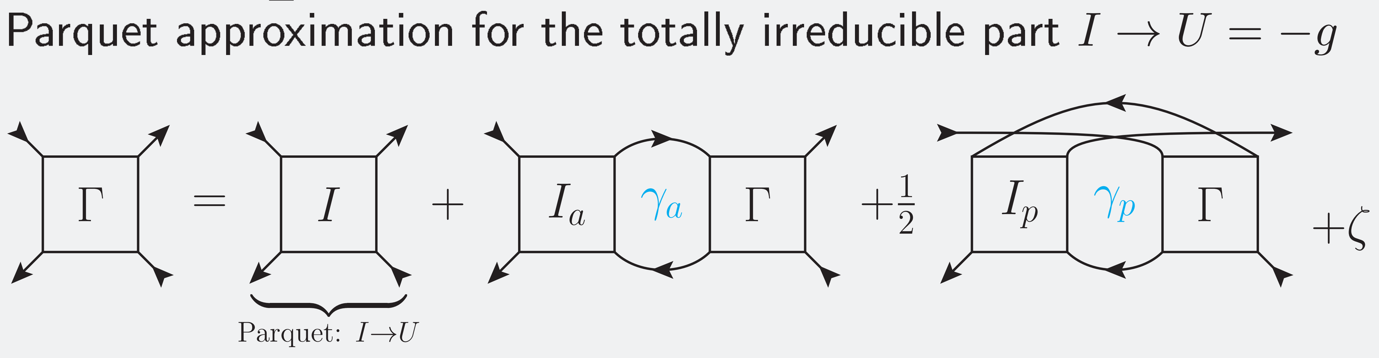
- Extreme imbalance $n_d/n_c \to 0 \Rightarrow$ **Fermi-polaron problem**
  - Attractive polaron: quasiparticle of impurity d surrounded by bath fermions c
  - Molecule: bosonic bound state between impurity d and one bath fermion c
  - Groundstate: polaron-to-molecule transition [2]
  - Repulsive polaron: metastable excitation



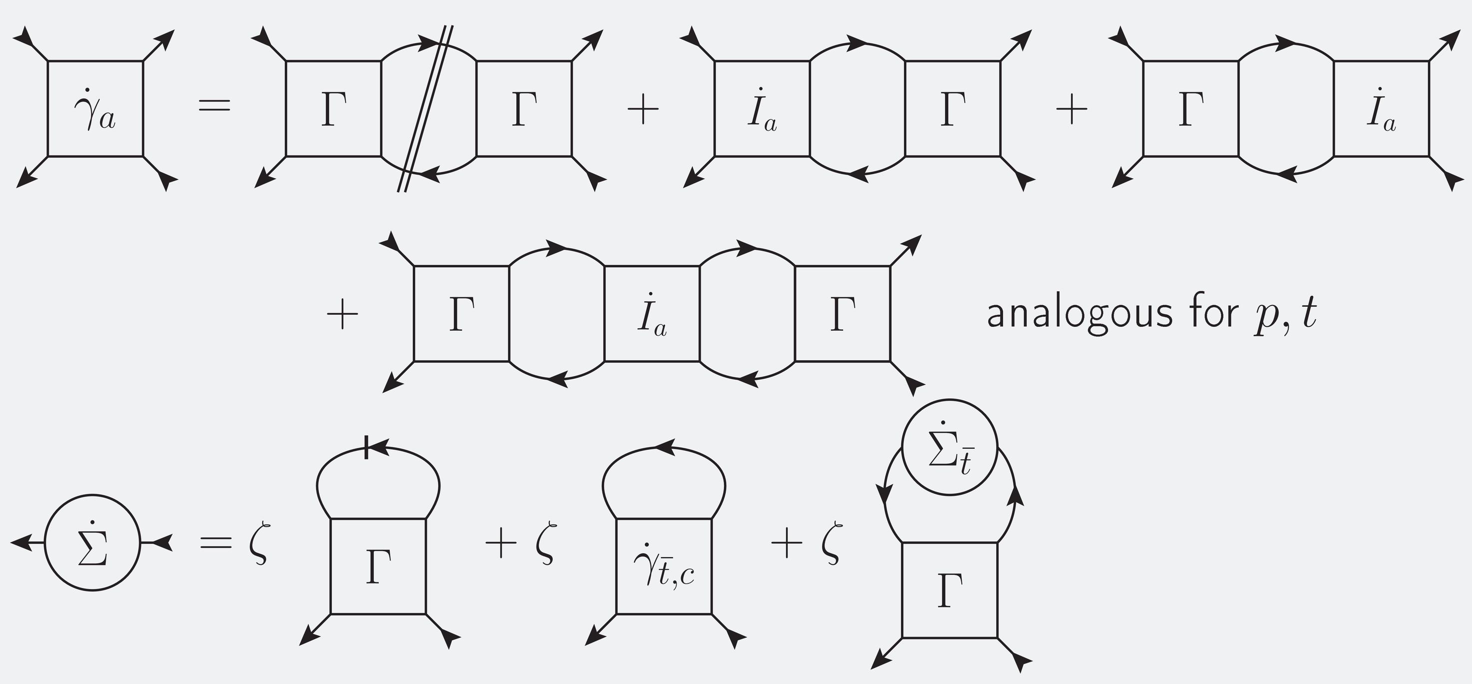
- **Theoretical methods:**
  - Variational wavefunctions, quantum Monte-Carlo methods, T-matrix approximations, functional renormalization group
- **Experimental realization:**
  - ^{40}K impurities in ^{6}Li Fermi sea $\Rightarrow$ radio-frequency spectroscopy [3]

Method: Functional renormalization group (fRG)

- **Bethe-Salpeter equations**:
  - Channel decomposition into two-particle reducible vertices γ_r, $r = a, p, t$ (antiparallel, parallel, transversal)
  - Parquet approximation for the totally irreducible part $I \to U = -g$

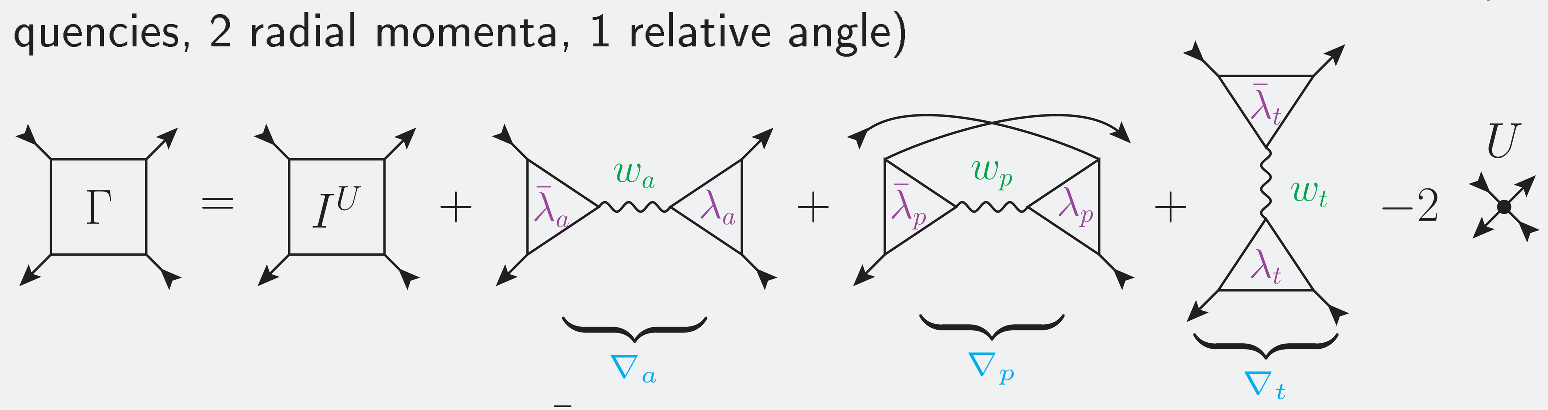


- **Multiloop functional renormalization group (mfRG)** [4]:
  - RG scale dependence Λ generates a set of hierarchical differential equations for the full vertex Γ^Λ and the self-energy Σ^Λ
  - No flow of totally irreducible vertex $\dot{I}^\Lambda = 0$, mfRG equivalent to Parquet formalism if $\Gamma^{\Lambda_i} = U$
  - Katanin substitution: single scale-propagator $S^\Lambda = \partial_\Lambda|_{\Sigma=\text{const.}} G^\Lambda$ is replaced by $\dot{G}^\Lambda = S^\Lambda + G^\Lambda \dot{\Sigma}^\Lambda G^\Lambda \Rightarrow$ self-consistency
  - Total derivative in Λ for $l \to \infty \Rightarrow$ independence of regulator



Strategy: Vertex decomposition

- Largest challenge: complex dependencies of the vertex Γ (3 frequencies, 3 radial momenta, 3 relative angles), but simpler self-energy Σ (1 frequency, 1 radial momentum)
- **Single-boson exchange decomposition (SBE)** [5-6]
  - Channel decomposition into U-reducible vertices $\nabla_r = \bar{\lambda}_r \bullet w_r \bullet \lambda_r$
  - Screened interaction w_r: propagator of an exchange boson (1 frequency, 1 radial momentum)
  - Hedin vertices $\bar{\lambda}_r, \lambda_r$: exchange process between 1 boson & 2 fermions (2 frequencies, 2 radial momenta, 1 relative angle)



$\Rightarrow$ mfRG flow equations for $w_r, \bar{\lambda}_r, \lambda_r$ with simpler frequency-momentum structure

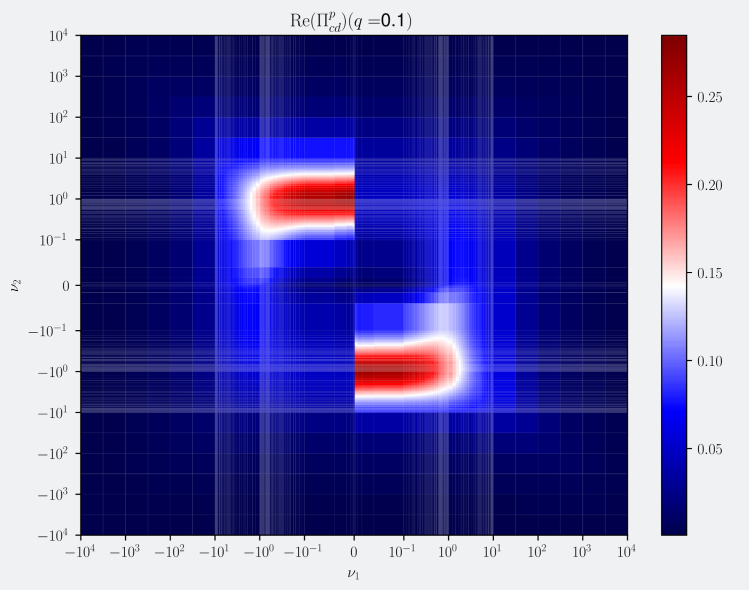
$\Rightarrow$ Successive complexity by taking into account more terms

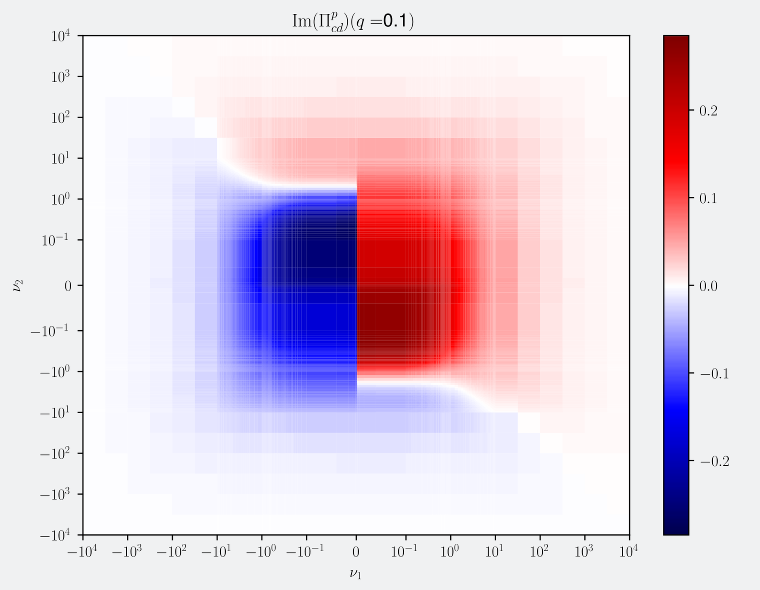
- **Angular momentum basis:**
  - Expand full vertex into spherical harmonics $\Rightarrow$ truncated unity, reduction of angular dependencies

$$(\Gamma_r)_{\boldsymbol{k}_r,\boldsymbol{k}'_r}(\boldsymbol{q}_r) = \sum_{lm} \sum_{l'm'} Y_{lm}(\Omega_r) (\Gamma_r)^{k_r,k'_r}_{lm,l'm'}(\boldsymbol{q}_r) Y^*_{l'm'}(\Omega'_r)$$

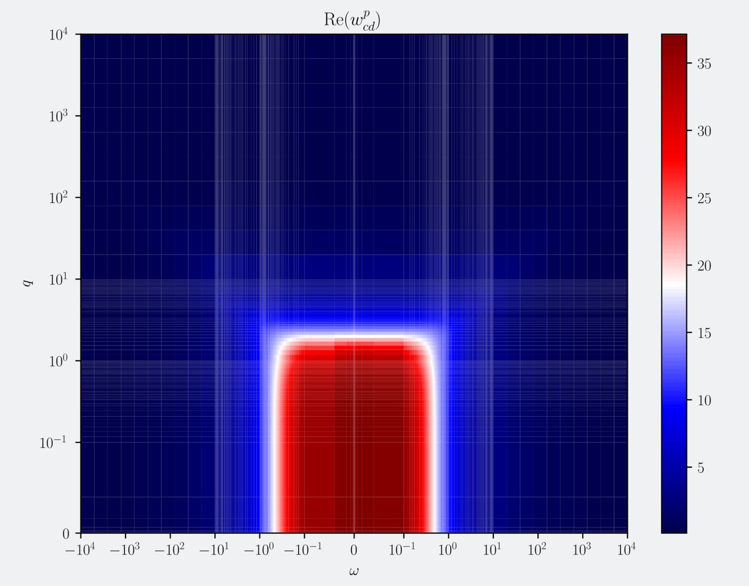
Numerical methods & benchmark results

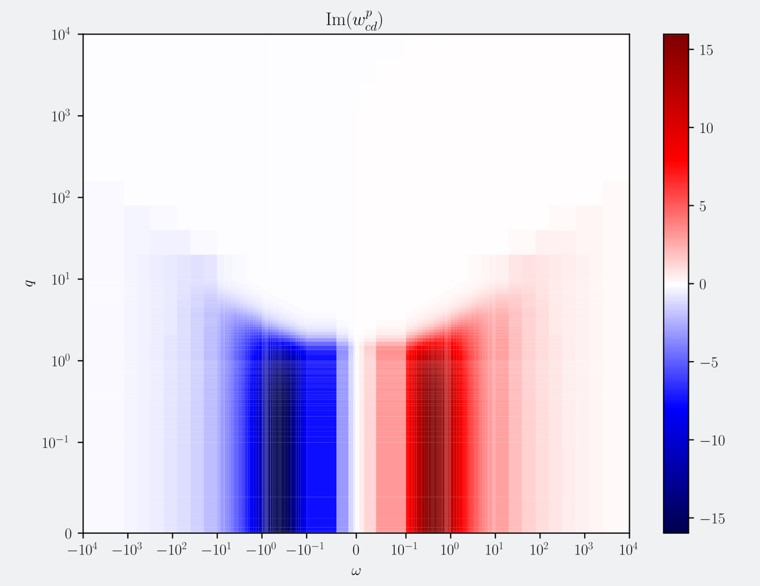
- **Parallel adaptive integrator** in higher dimensions (PAID) [7]
  - Effective calculation & high resolution of multidimensional momentum integrals, e.g., for bubble $\Pi_{cd}(\nu_1, \nu_2, |\boldsymbol{q}|) = \int_{\boldsymbol{k}} G_c(\nu_1, |\boldsymbol{k} + \frac{\boldsymbol{q}}{2}|) G_d(\nu_2, |\boldsymbol{k} - \frac{\boldsymbol{q}}{2}|)$



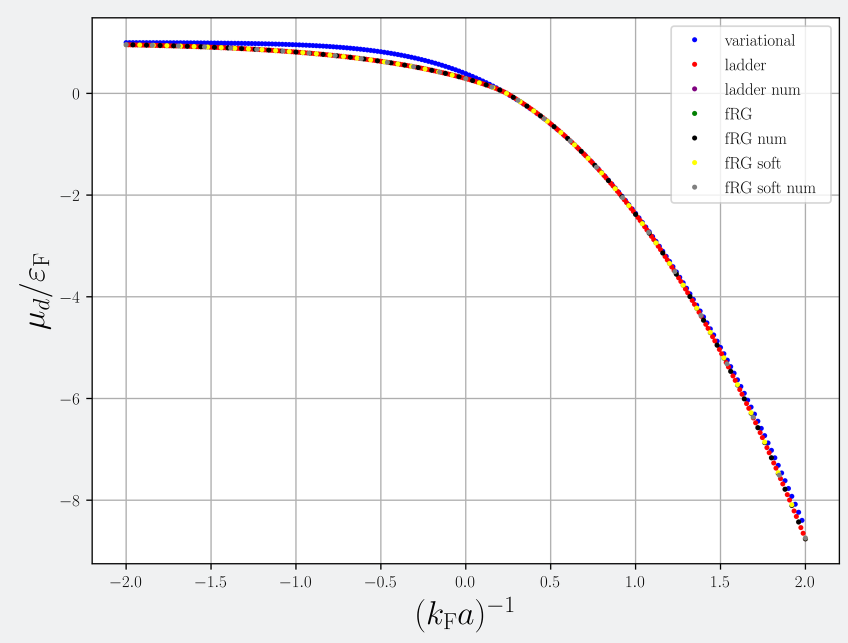


- **Adaptive ODE solver** with Cash-Karp method
  - Solving fRG flow as $\sim 10^4$ ODEs, e.g., for screened interaction $\dot{w}^p_{cd}(\omega, |\boldsymbol{q}|) = (w^p_{cd}(\omega, |\boldsymbol{q}|))^2 \int_\nu \dot{\Pi}^p_{cd}(\omega, \nu, |\boldsymbol{q}|)$





- **Benchmark results** for different methods $\Rightarrow$ starting point for higher complexity
  - Ladder diagrams as non-selfconsistent T-matrix approximation
  - Sharp and soft regulators for fRG flow
  - Numerical root-finding methods $\Rightarrow$ bound states



Outlook

- Self-energy flow $\Rightarrow$ relation to self-consistent T-matrix
- Inclusion of Hedin vertices $\Rightarrow$ form factor decomposition
- Reduced regulator dependence $\Rightarrow$ extension of physical observables (polaron lifetime, finite temperature, general mass imbalances $0 \leq n_d \leq n_c$)
- Experimental protocol to measure size of polaron cloud

[1] Randeria, et al., Annu. Rev. Cond. Mat. Phys. **5** (2014).
[2] Schmidt, Enss, Phys. Rev. A **83**, 063620 (2011).
[3] Kohstall et al., Nature **485**.7400 (2012).
[4] Kugler, von Delft, Phys. Rev. Lett. **120**, 057403 (2018),
[5] Krien et al., Phys. Rev. B **100**, 155149 (2019).
[6] Gievers, Walter, Ge, von Deflt, Kugler, Eur. Phys. J. B **95**, 108 (2022).
[7] Lichtenstein et al., Comp. Phys. Comm. **213** (2017).

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN · ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS · MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK · MCQST