Single-boson exchange functional renormalization group application to the two-dimensional Hubbard model at weak coupling

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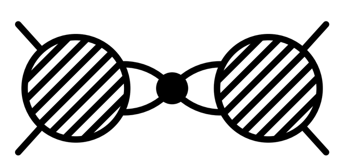
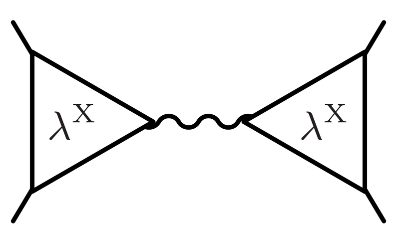
Functional renormalization group and single-boson exchange decomposition

- The **single-boson exchange (SBE)** decomposition [1] relies on a splitting of the 2-particle vertex between its *U-irreducible* and its *U-reducible* parts:

$$V = \mathcal{I}^{U_{\rm irr}} +$$ 

$$\Rightarrow V(k_1,k_2,k_3) = \mathcal{I}^{U_{\rm irr}}(k_1,k_2,k_3) + \nabla^{\rm M}_{k_1k_3}(k_2-k_3) + \frac{1}{2}\left[\nabla^{\rm M}_{k_1k_4}(k_3-k_1) + \nabla^{\rm D}_{k_1k_4}(k_3-k_1)\right] + \nabla^{\rm SC}_{k_1k_3}(k_1+k_2) - 2U$$

with $\nabla^{\rm X}_{kk'}(Q) \equiv$ 

$= \lambda^{\rm X}_k(Q)\, w^{\rm X}(Q)\, \lambda^{\rm X}_{k'}(Q)$

$$\mathcal{I}^{U_{\rm irr}} = \mathcal{I}^{\rm 2PI} - U + M^{\rm M} + M^{\rm D} + M^{\rm SC}$$

- The many-body method used in this study is a **functional renormalization group (fRG)** approach based on the SBE decomposition, relying on a set of coupled differential equations for the Yukawa couplings $\lambda^{\rm X}$, the bosonic propagators $w^{\rm X}$ and the rest functions $M^{\rm X}$ [2]
- Comparisons will also be performed with the conventional fermionic fRG based on a decomposition of the 2-particle-reducible function $\phi = V - \mathcal{I}^{\rm 2PI}$ in terms of high-frequency asymptotics [3]:

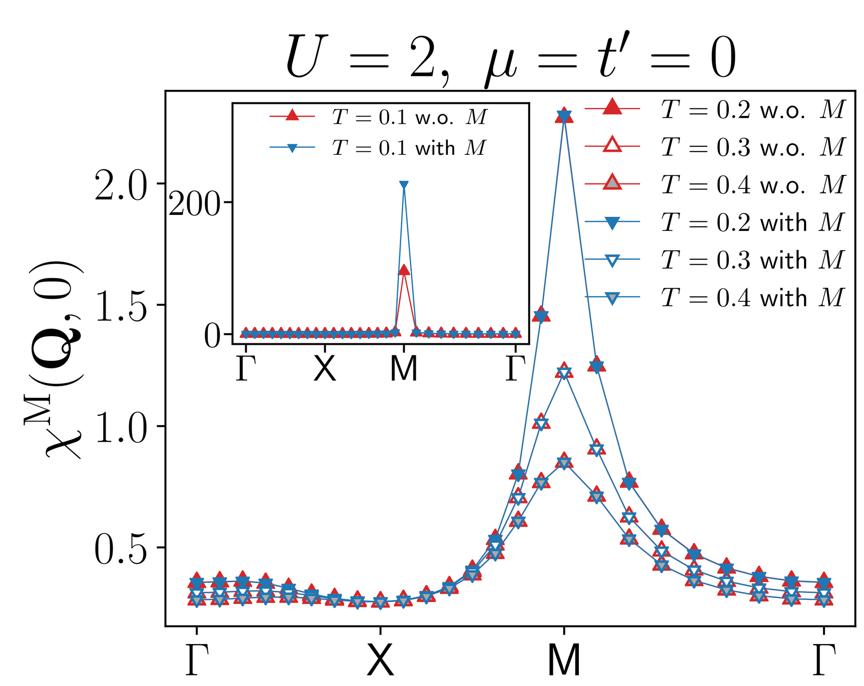
$$\phi^{\rm X}_{kk'}(Q) = \mathcal{K}^{(1){\rm X}}_{kk'}(Q) + \mathcal{K}^{(2){\rm X}}_{k}(Q) + \mathcal{K}^{(2){\rm X}}_{k'}(Q) + \mathcal{R}^{\rm X}_{kk'}(Q)$$

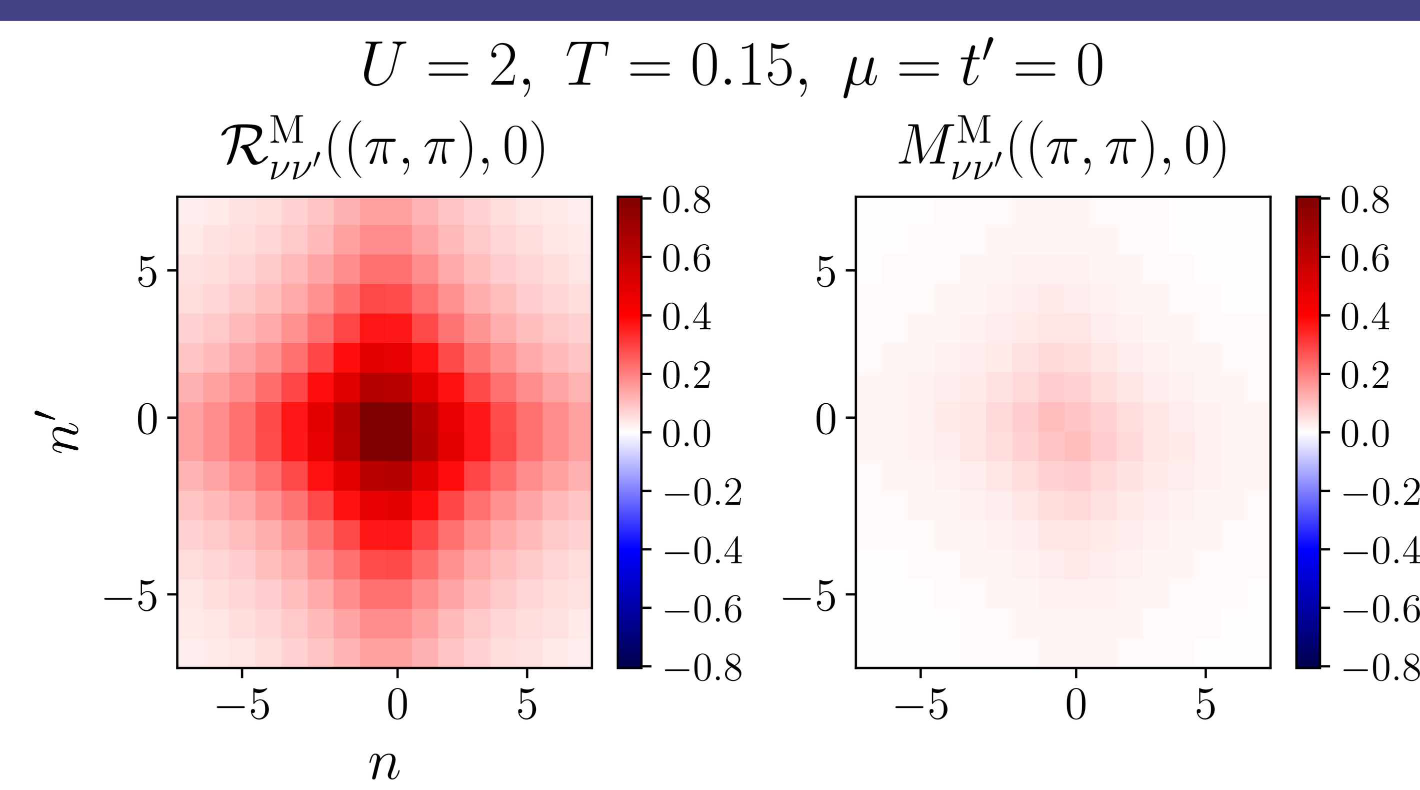
$$\text{with } \mathcal{K}^{(1){\rm X}}(Q) = \lim_{\nu,\nu'\to\infty} \phi^{\rm X}_{(\mathbf{k},\nu),(\mathbf{k}',\nu')}(Q)$$

$$\mathcal{K}^{(2){\rm X}}_{k}(Q) = \lim_{\nu'\to\infty} \phi^{\rm X}_{k,(\mathbf{k}',\nu')}(Q) - \mathcal{K}^{(1){\rm X}}(Q)$$

Results

- Pseudo-critical *transition* observed in the magnetic channel at half filling

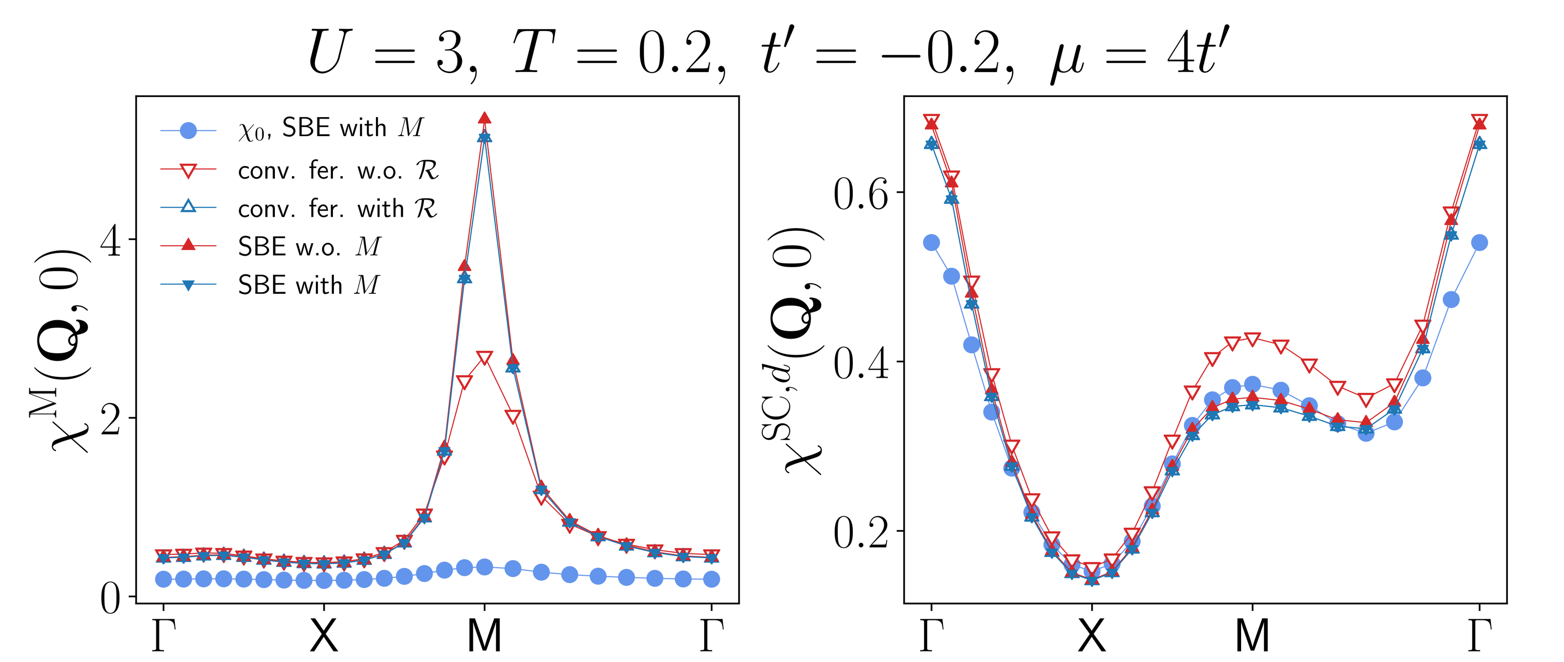




- *Divergence* of $\mathcal{R}^{\rm M}$ near the transition *absorbed* by the bosonic propagator $w^{\rm M}$ so that $M^{\rm M}$ remains residual even fairly close to the transition:

$$M^{\rm M}_{kk'}(Q) = \mathcal{R}^{\rm M}_{kk'}(Q) - [\lambda^{\rm M}_k(Q)-1]w^{\rm M}(Q)[\lambda^{\rm M}_{k'}(Q)-1]$$

- $M^{\rm X}$ negligible in most situations (contrary to $\mathcal{R}^{\rm X}$) and *run times significantly reduced* by discarding the flow of $M^{\rm X}$
- Vertex corrections of the bare bubble contributions $\chi^{\rm X}_0$ to the susceptibilities nicely captured by the SBE diagrams



↪ Relative difference between SBE (conv. fer.) results with and without rest function always less than 4% (larger than 22%) at the physical values for $\mathbf{Q}$

Conclusions and outlooks

- Main conclusions:
  - Contribution of the SBE rest functions $M^{\rm X}$ always **negligible**, except in the vicinity of the pseudo-critical transition ⇒ SBE scheme yields the *natural decomposition* of the two-particle vertex
  - Contrary to the conventional fermionic fRG, the rest function can be **neglected**, which allows for a significant *reduction of the numerical cost*
- Outlooks for the SBE-based fRG:
  - Calculation of **multiloop** corrections
  - Formulation for **non-local** interactions

References

[1] F. Krien et al. Single-boson exchange decomposition of the vertex function. *Phys. Rev. B*, 100:155149, 2019.

[2] P. M. Bonetti et al. Single-boson exchange representation of the functional renormalization group for strongly interacting many-electron systems. *Phys. Rev. Research*, 4:013034, 2022.

[3] N. Wentzell et al. High-frequency asymptotics of the vertex function: Diagrammatic parametrization and algorithmic implementation. *Phys. Rev. B*, 102:085106, 2020.