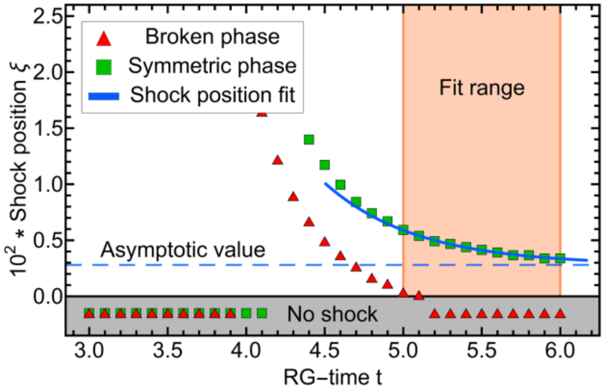
Computational fluid dynamics and the fRG



Nicolas Wink

Based on

Grossi, NW, arxiv:1903.09503

Grossi, Ihssen, Pawlowski, NW, PRD 104 (2021)

Koenigstein, Steil, NW, Grossi, Braun, Buballa, Rischke, arxiv:2108.02504 (to appear in PRD)

Koenigstein, Steil, NW, Grossi, Braun, arxiv:2108.10085 (to appear in PRD)

Steil, Koenigstein, arxiv:2108.04037 (to appear in PRD)

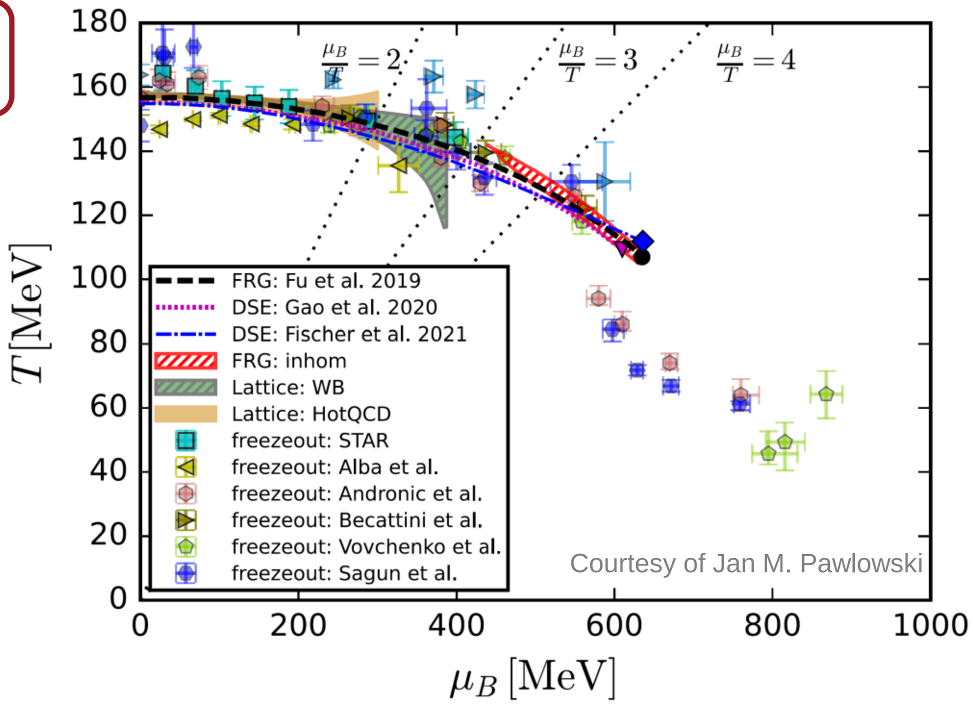
Stoll, Zorbach, Koenigstein, Steil, Rechenberger arxiv: 2108.10616

Ihssen, Pawlowski, arxiv:2207.10057

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

(Our) Motivation

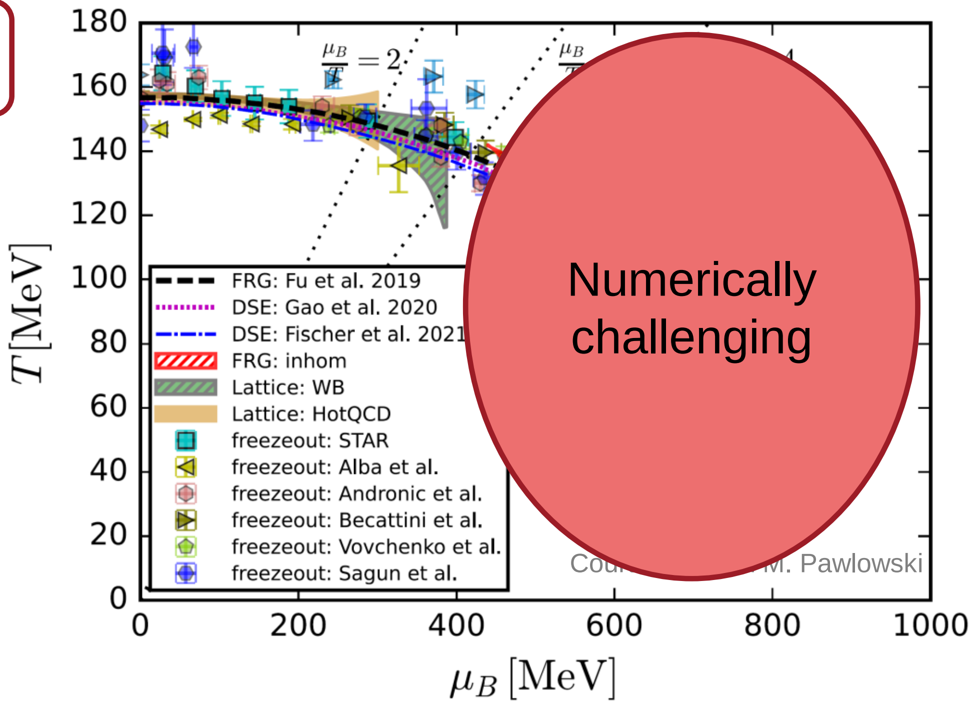
QCD phase diagram



See talk by Jens Braun

(Our) Motivation

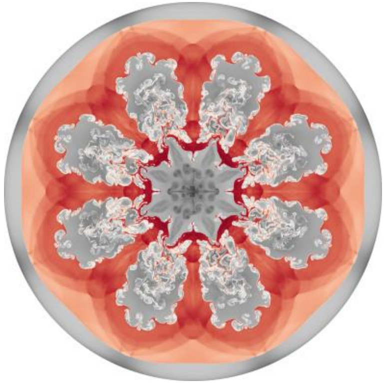
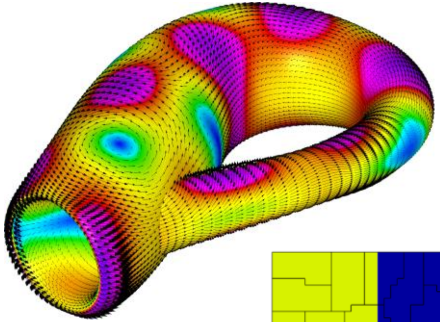
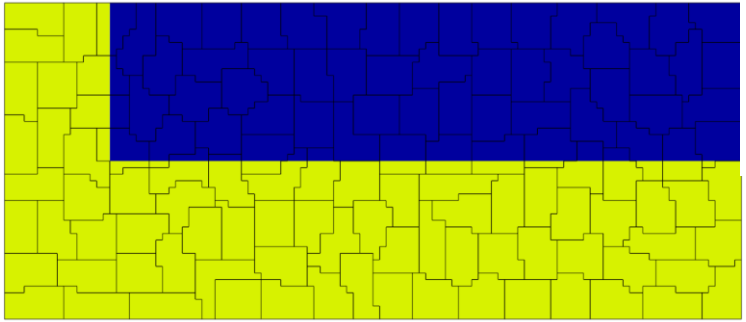
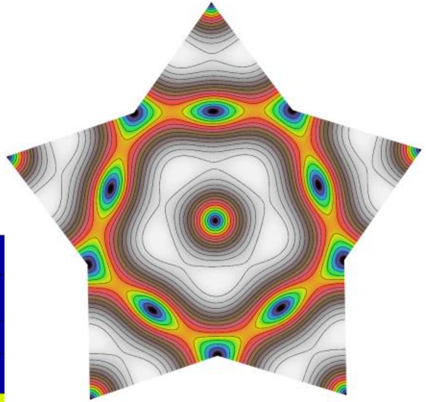
QCD phase diagram



See talk by Jens Braun

Discontinuous Galerkin/Finite Volume methods

& the fRG

A simple example

N-component scalar field in LPA

$$\Gamma_k = \int_x \left\{ \frac{1}{2}(\partial_\mu \phi_a)^2 + V(\rho) \right\}$$

Invariant:

$$\rho = \frac{1}{2}\phi_a \phi^a$$

A simple example

N-component scalar field in LPA

$$\Gamma_k = \int_x \left\{ \frac{1}{2}(\partial_\mu \phi_a)^2 + V(\rho) \right\}$$

Invariant:

$$\rho = \frac{1}{2}\phi_a \phi^a$$

RG-time:

RG-scale

$$t = -\log \frac{k}{\Lambda}$$

UV scale

Note the extra minus sign for positive time evolution

A simple example

N-component scalar field in LPA

$$\Gamma_k = \int_x \left\{ \frac{1}{2}(\partial_\mu \phi_a)^2 + V(\rho) \right\}$$

Invariant:

$$\rho = \frac{1}{2}\phi_a \phi^a$$

RG-time:

RG-scale

$$t = -\log \frac{k}{\Lambda}$$

UV scale

Note the extra minus sign for positive time evolution

Flow equation

$$\partial_t V(\rho) = -\frac{\Omega_d}{(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{d} \left(\frac{1}{(\Lambda e^{-t})^2 + V'(\rho)} + \frac{1}{N-1} \frac{1}{(\Lambda e^{-t})^2 + V'(\rho) + 2\rho V''(\rho)} \right)$$

A simple example

N-component scalar field in LPA

$$\Gamma_k = \int_x \left\{ \frac{1}{2}(\partial_\mu \phi_a)^2 + V(\rho) \right\}$$

Invariant:

$$\rho = \frac{1}{2}\phi_a \phi^a$$

RG-time:

$$t = -\log \frac{k}{\Lambda}$$

RG-scale

UV scale

Note the extra minus sign for positive time evolution

Flow equation

$$\partial_t V(\rho) = -\frac{\Omega_d}{(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{d} \left(\frac{1}{(\Lambda e^{-t})^2 + V'(\rho)} \cancel{+ \frac{1}{N-1} \frac{1}{(\Lambda e^{-t})^2 + V'(\rho) + 2\rho V''(\rho)}} \right)$$

Large N scalar field

$$\partial_t V(\rho) = - \frac{\Omega_d}{d(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + V'(\rho)}$$

Large N scalar field

Introduce new variable for the derivative

$$u(\rho) = \partial_\rho V(\rho)$$

$$\partial_t V(\rho) = -\frac{\Omega_d}{d(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + V'(\rho)}$$

Large N scalar field

Introduce new variable for the derivative

$$\partial_t V(\rho) = -\frac{\Omega_d}{d(2\pi)^d}\frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + V'(\rho)}$$

$$u(\rho) = \partial_\rho V(\rho)$$

RHS defines a flux:

$$f(t,u) = -\frac{\Omega_d}{d(2\pi)^d}\frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + u}$$

Large N scalar field

Introduce new variable for the derivative

$$u(\rho) = \partial_\rho V(\rho)$$

$$\partial_t V(\rho) = -\frac{\Omega_d}{d(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + V'(\rho)}$$

RHS defines a flux:

$$f(t,u) = -\frac{\Omega_d}{d(2\pi)^d} \frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + u}$$

$$\Rightarrow \quad \partial_t u + \partial_\rho f(t,u) = 0$$

Conservation law

Hydro methods for fRG

Natural interpretation

$$\boxed{\partial_t u + \partial_\rho f(t,u) = 0}$$

Hydro methods for fRG

Natural interpretation

$$\boxed{\partial_t u + \partial_\rho f(t,u) = 0}$$

➡ Mass of the field locally conserved

Hydro methods for fRG

Natural interpretation

$$\boxed{\partial_t u + \partial_\rho f(t,u) = 0}$$

- Mass of the field locally conserved
- Hydro analogy opens possibilities
  - Direction of flow

Hydro methods for fRG

Natural interpretation

$$\boxed{\partial_t u + \partial_\rho f(t,u) = 0}$$

- Mass of the field locally conserved
- Hydro analogy opens possibilities
  - Direction of flow
  - Boundary conditions (Classical action at infinity)

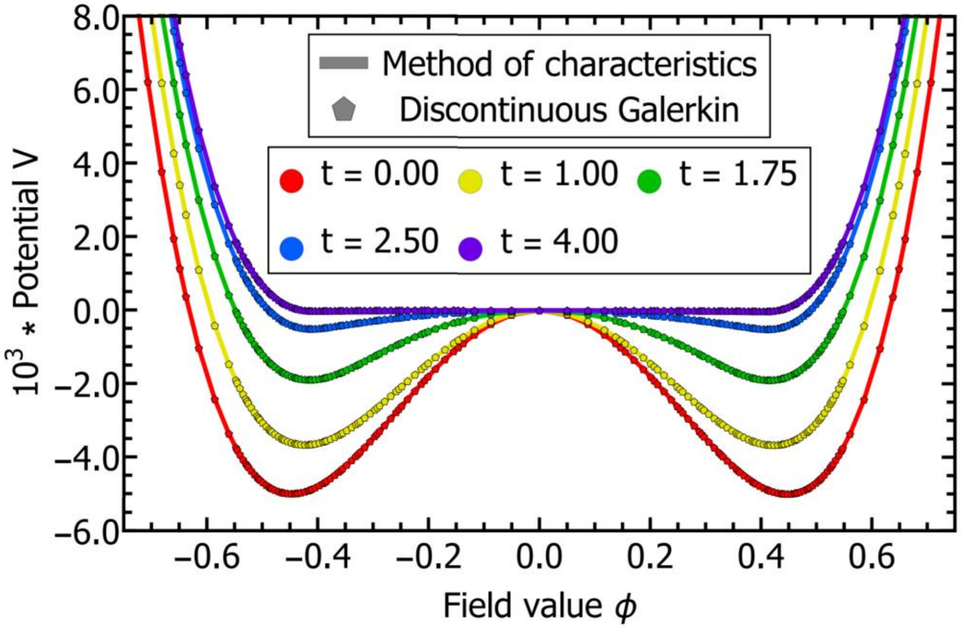
Hydro methods for fRG

Natural interpretation

$$\boxed{\partial_t u + \partial_\rho f(t,u) = 0}$$

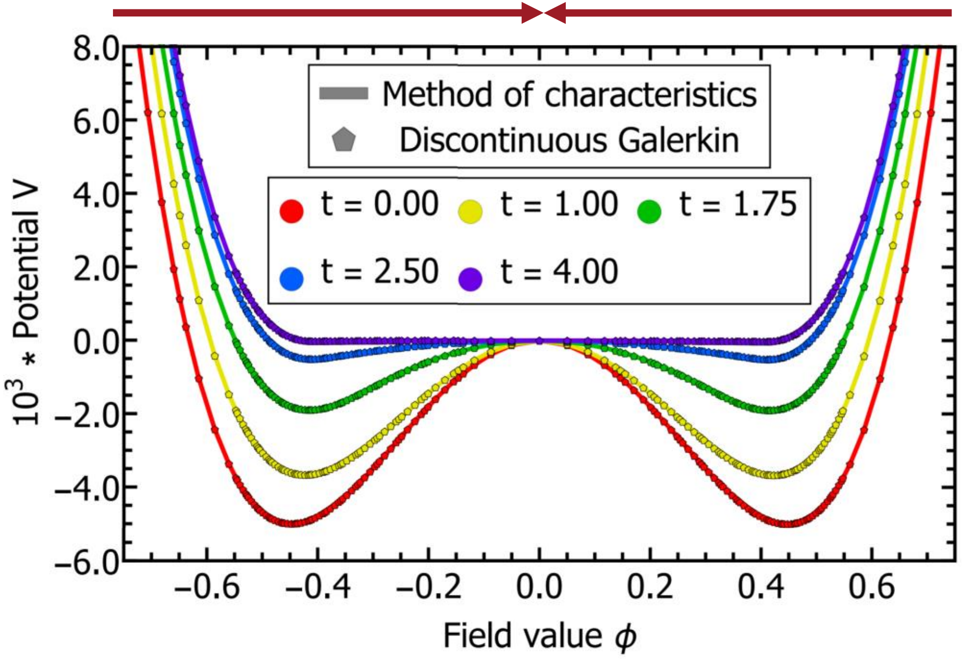
- ➡ Mass of the field locally conserved
- ➡ Hydro analogy opens possibilities
  - ➡ Direction of flow
  - ➡ Boundary conditions (Classical action at infinity)
  - ➡ Use of appropriate/existing numerical methods

Direction of the flow



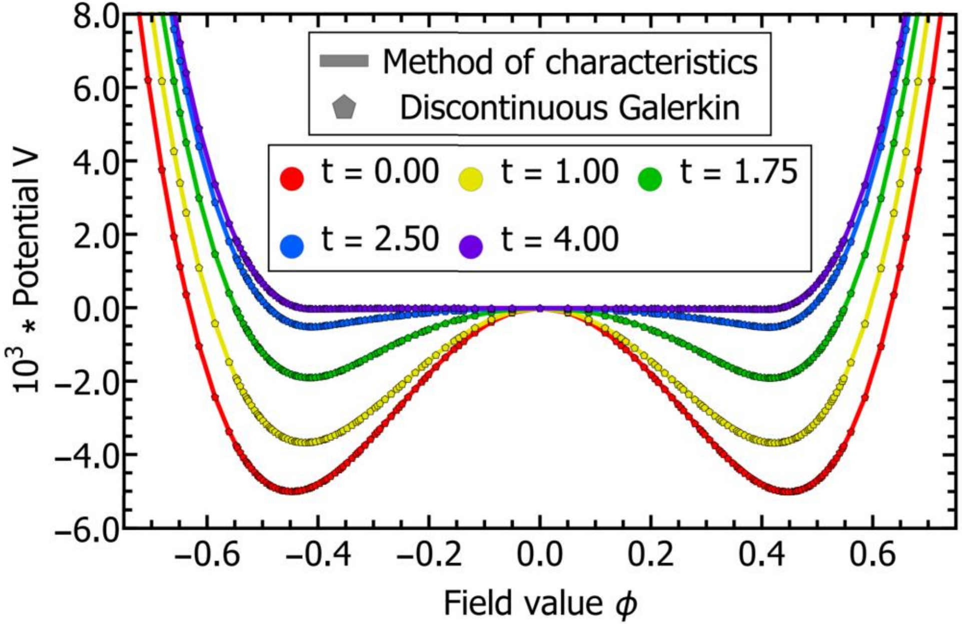
Direction of the flow

Flow towards smaller field values



Direction of the flow

Flow towards smaller field values



fRG probes theory at scale

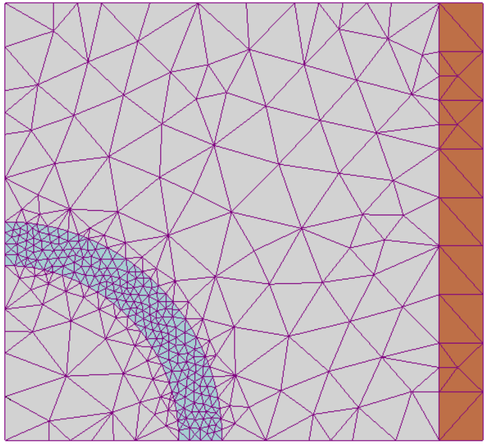
$$k \sim |\phi|$$

Theory information gets transported along length scales

Numerical methods

Solving conservation laws

$$\mathcal{R}_h = \partial_t u + \partial_\rho f(t, u) = 0$$



https://en.wikipedia.org/wiki/Finite_element_method

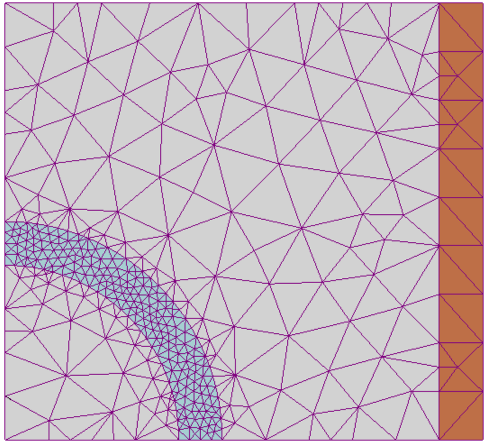
Numerical methods

Solving conservation laws

$$\mathcal{R}_h = \partial_t u + \partial_\rho f(t, u) = 0$$

Require residual to vanish in a weak sense

$$\int_\Omega \mathcal{R}_h(t, x)\psi_h(x) = 0$$



https://en.wikipedia.org/wiki/Finite_element_method

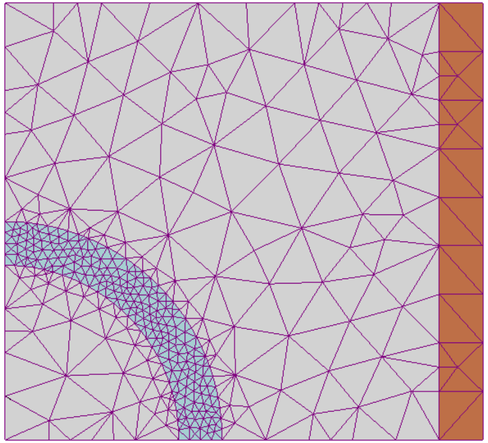
Numerical methods

Solving conservation laws

$$\mathcal{R}_h = \partial_t u + \partial_\rho f(t,u) = 0$$

Require residual to vanish in a weak sense

$$\int_\Omega \mathcal{R}_h(t,x)\psi_h(x) = 0$$



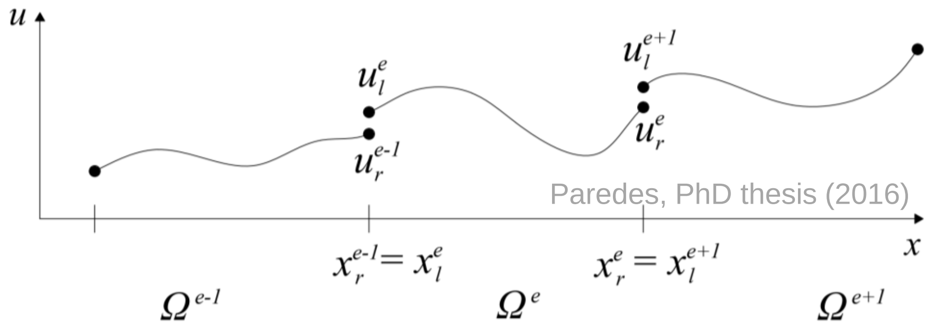
https://en.wikipedia.org/wiki/Finite_element_method

→ Solution does NOT need to be continuous

Numerical methods

Discontinuous elements

$$\int_{D^k} \left(\partial_t u_h^k + \partial_x f_h^k(u_h^k)\right)\psi_n \; \mathrm{d}x = 0$$



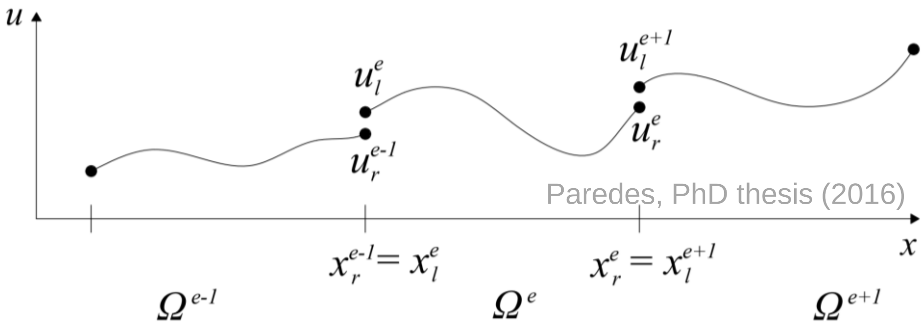
Paredes, PhD thesis (2016)

Numerical methods

Discontinuous elements

$$\int_{D^k} \left(\partial_t u_h^k + \partial_x f_h^k(u_h^k)\right)\psi_n \, \mathrm{d}x = 0$$

More dof than equations



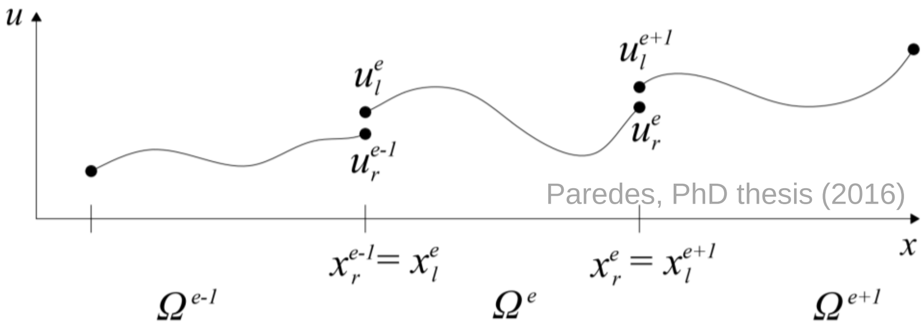
Paredes, PhD thesis (2016)

Numerical methods

Discontinuous elements

$$\int_{D^k} \left(\partial_t u_h^k + \partial_x f_h^k(u_h^k)\right)\psi_n \,\mathrm{d}x = 0$$

More dof than equations



Partial Integration → Weak form

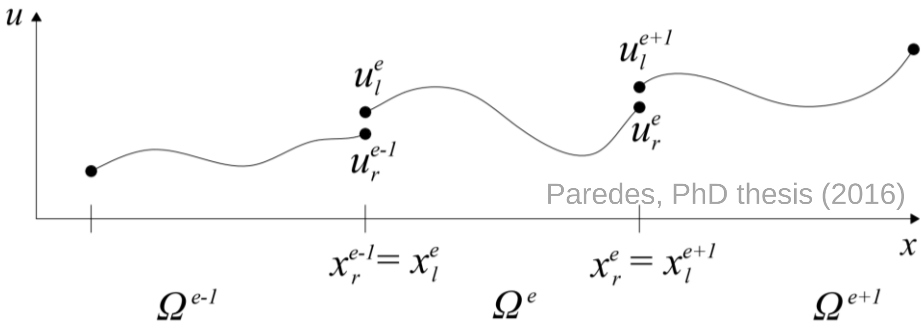
$$\int_{D^k} \left(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \right)\mathrm{d}x = -\int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \,\mathrm{d}x$$

Numerical methods

Discontinuous elements

$$\int_{D^k} \left(\partial_t u_h^k + \partial_x f_h^k(u_h^k)\right)\psi_n \, \mathrm{d}x = 0$$

More dof than equations



Paredes, PhD thesis (2016)

Partial Integration ➡ Weak form

Numerical flux

$$\int_{D^k} \left(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \right) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Discontinuous Galerkin & Finite Volume

Weak form of equation

$$\int_{D^k} \Big(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \Big) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Discontinuous Galerkin & Finite Volume

Weak form of equation

$$\int_{D^k} \Big(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \Big) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Yields family of Discontinuous Galerkin schemes

Discontinuous Galerkin & Finite Volume

Weak form of equation

$$\int_{D^k} \Big(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \Big) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Yields family of Discontinuous Galerkin schemes

➡ Variable local polynomial order

Discontinuous Galerkin & Finite Volume

Weak form of equation

$$\int_{D^k} \Big(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \Big) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Yields family of Discontinuous Galerkin schemes

- Variable local polynomial order
- Rather general conditions on numerical flux for consistent scheme

Discontinuous Galerkin & Finite Volume

Weak form of equation

$$\int_{D^k} \Big(\left(\partial_t u_h^k\right) \psi_n - f_h^k(u_h^k)\partial_x \psi_n \Big) \mathrm{d}x = - \int_{\partial D^k} \hat{\mathbf{n}} \cdot f^* \psi_n \, \mathrm{d}x$$

Yields family of Discontinuous Galerkin schemes

- Variable local polynomial order
- Rather general conditions on numerical flux for consistent scheme
- Finite Volume methods special case for order N=0 (constant elements)

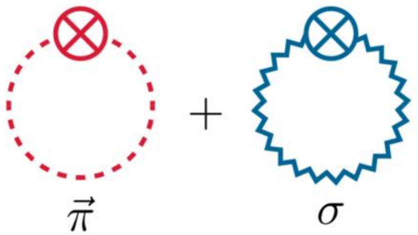
The zero-dimensional O(N)-model

The zero-dimensional O(N)-model

Why zero dimensions?

$$\partial_t U(t,\sigma) = \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{N-1}{r(t) + \frac{1}{\sigma}\,\partial_\sigma U(t,\sigma)} +$$

$$+ \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{1}{r(t) + \partial_\sigma^2 U(t,\sigma)} =$$

$=$ 

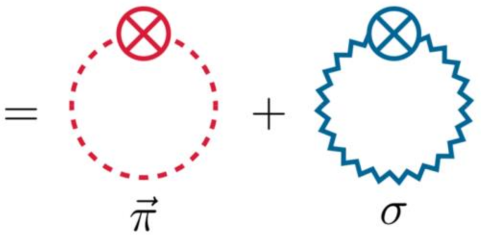
Koenigstein, Steil, NW, et al, arXiv:2108:02504

The zero-dimensional O(N)-model

Why zero dimensions?

→ Everything well defined,
reference solutions

$$\partial_t U(t,\sigma) = \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{N-1}{r(t) + \frac{1}{\sigma}\,\partial_\sigma U(t,\sigma)} + \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{1}{r(t) + \partial_\sigma^2 U(t,\sigma)} =$$



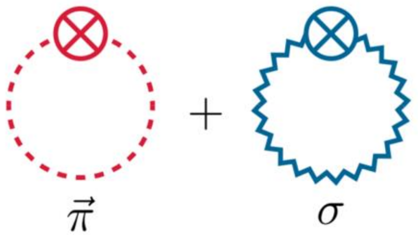
Koenigstein, Steil, NW, et al, arXiv:2108:02504

The zero-dimensional O(N)-model

Why zero dimensions?

$$\partial_t U(t,\sigma) = \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{N-1}{r(t)+\frac{1}{\sigma}\,\partial_\sigma U(t,\sigma)} + \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{1}{r(t)+\partial_\sigma^2 U(t,\sigma)} =$$

- ➡ Everything well defined, reference solutions
- ➡ Suitable to explore new methods

$=$ 

Koenigstein, Steil, NW, et al, arXiv:2108:02504

The zero-dimensional O(N)-model

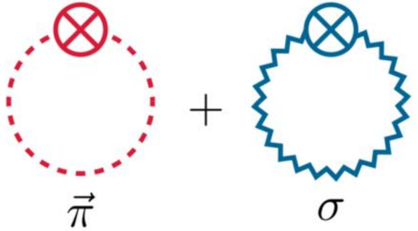
Why zero dimensions?

- Everything well defined, reference solutions
- Suitable to explore new methods

First study with Finite Volume (KT) scheme

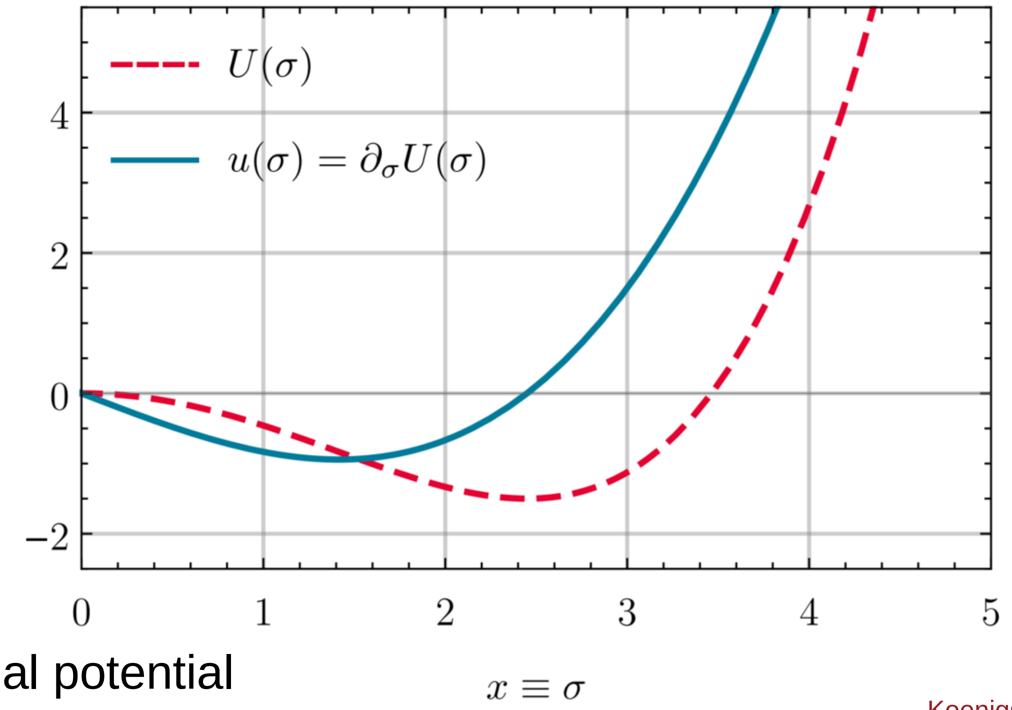
Diffusion enters the game

$$\partial_t U(t,\sigma) = \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{N-1}{r(t) + \frac{1}{\sigma}\,\partial_\sigma U(t,\sigma)} + \left[\tfrac{1}{2}\,\partial_t r(t)\right] \frac{1}{r(t) + \partial_\sigma^2 U(t,\sigma)} =$$

$=$ 

Koenigstein, Steil, NW, et al, arXiv:2108:02504

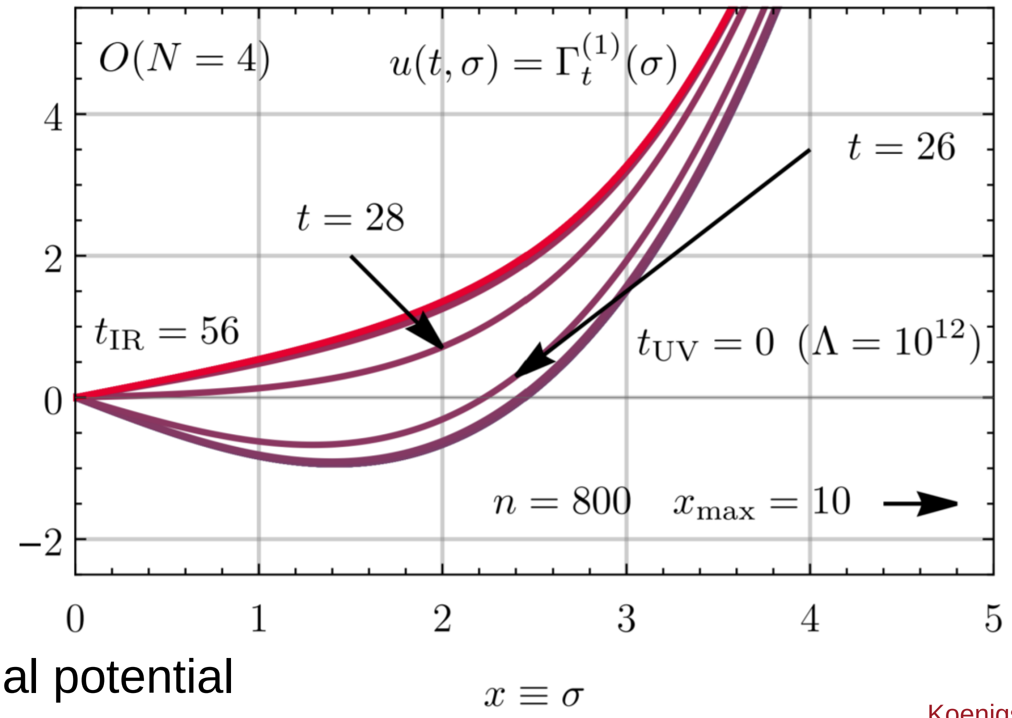
The zero-dimensional O(N)-model



Analytic initial potential

Koenigstein, Steil, NW, et al, arXiv:2108:02504

The zero-dimensional O(N)-model



Analytic initial potential

Koenigstein, Steil, NW, et al, arXiv:2108:02504

A (very) interesting corollary

Are RG flows reversible?

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

A (very) interesting corollary

Are RG flows reversible? ➡ No

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

A (very) interesting corollary

Are RG flows reversible? ➡ No

➡ Existence of a (unique) solution requires a *numerical* entropy

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

A (very) interesting corollary

Are RG flows reversible? ➡ No

➡ Existence of a (unique) solution requires a *numerical* entropy

➡ Diffusion naturally produces entropy

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

A (very) interesting corollary

Are RG flows reversible? ➡ No

➡ Existence of a (unique) solution requires a *numerical* entropy

➡ Diffusion naturally produces entropy

Shocks are another source of entropy

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

A (very) interesting corollary

Are RG flows reversible? ➡ No

➡ Existence of a (unique) solution requires a *numerical* entropy

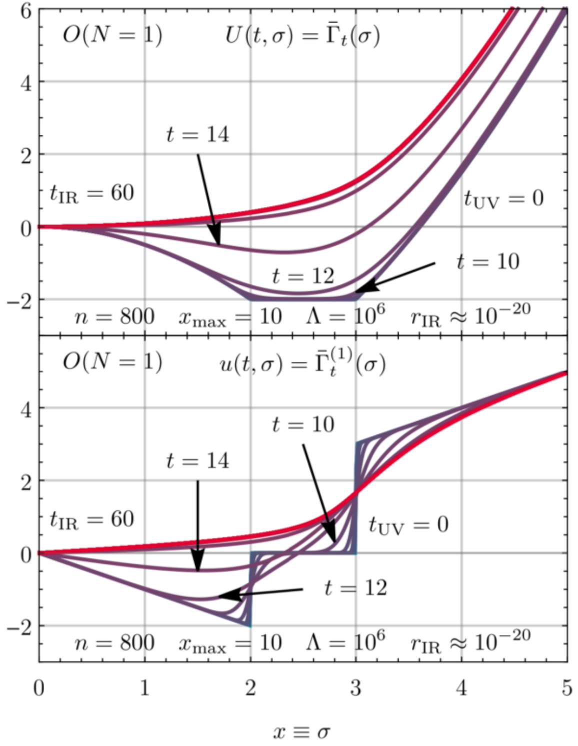
➡ Diffusion naturally produces entropy

Shocks are another source of entropy

➡ Connection to $\mathcal{C}/\mathcal{F}/\mathcal{A}$-function?

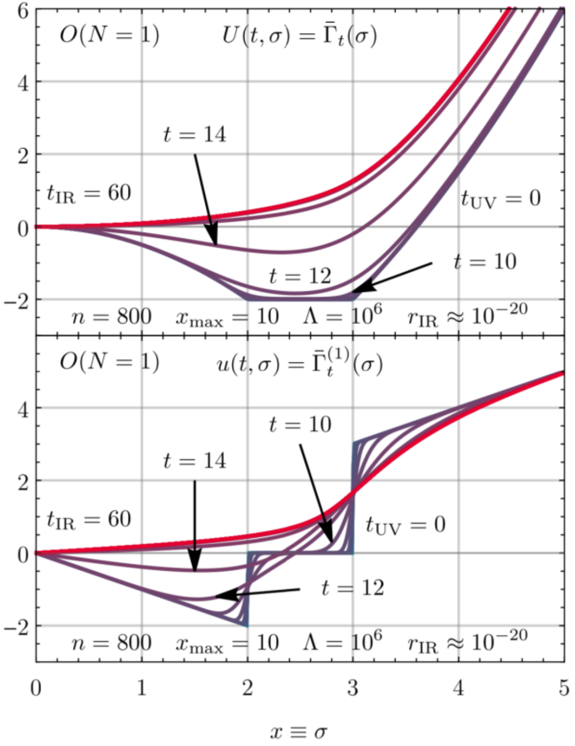
Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

The zero-dimensional O(N)-model



Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

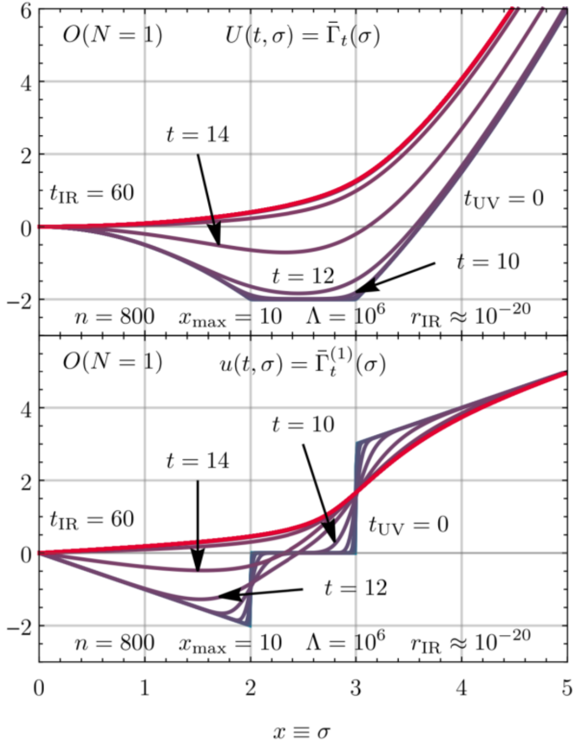
The zero-dimensional O(N)-model

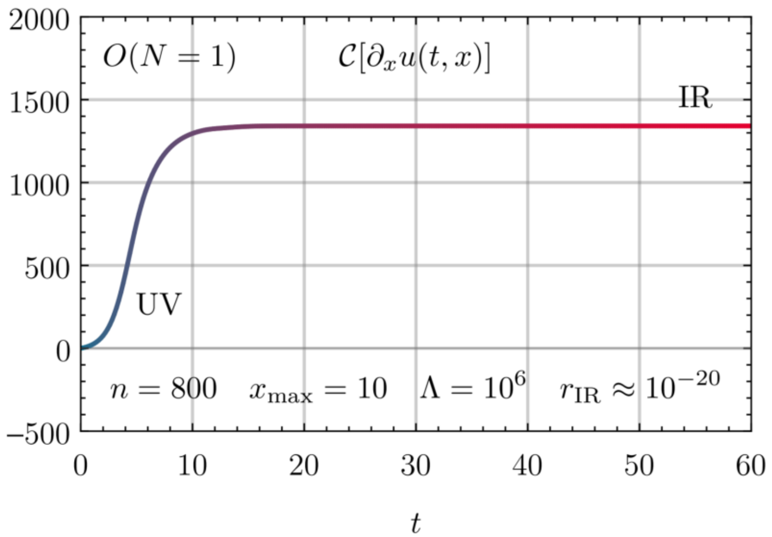


$$\mathcal{C}[\partial_x u(t,x)] = -\left(\mathcal{N}(x) - 2\int_0^\infty \mathrm{d}x\,[\partial_x u(t,x)]^2\right)$$

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

The zero-dimensional O(N)-model

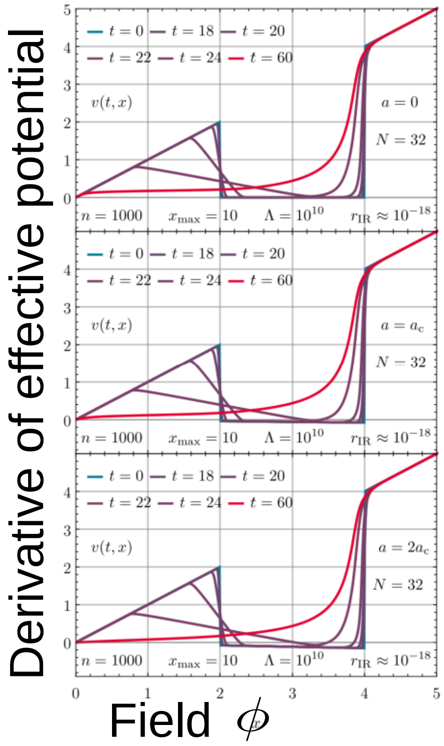




$$\mathcal{C}[\partial_x u(t,x)] = -\left(\mathcal{N}(x) - 2\int_0^\infty \mathrm{d}x\,[\partial_x u(t,x)]^2\right)$$

Koenigstein, Steil, NW, Grossi, Braun, arXiv:2108:10085

Large N vs Finite N



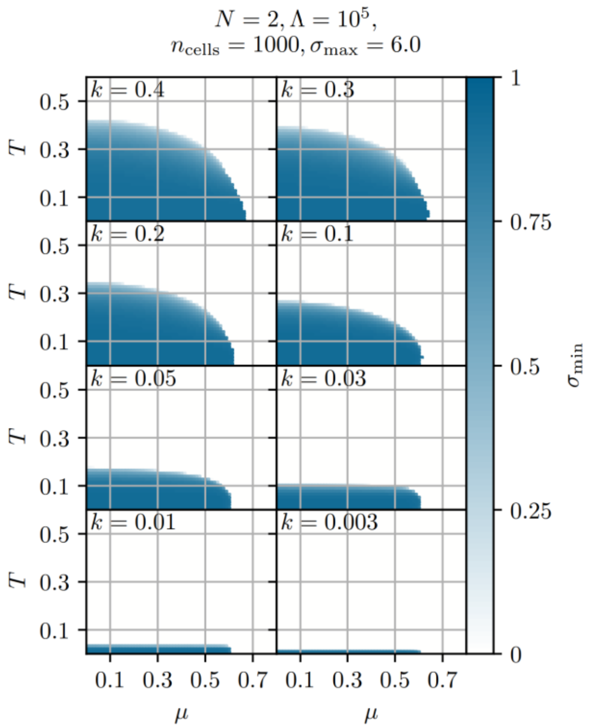
Diffusion smoothens initial non-analyticities away

Success of 1/N expansion linked to evolution of shocks and rarefaction waves

Steil, Koenigstein, arxiv:2108.04037

(1 + 1)-dimensional Gross-Neveu(-Yukawa)

(1 + 1)-dimensional Gross-Neveu(-Yukawa)



- Pre-condensation
- Condensate melts at all finite temperatures

Poster: Niklas Zorbach & Jonas Stoll

Stoll, Zorbach, Koenigstein, Steil, Rechenberger arxiv: 2108.10616

O(N) model in the large N limit

Another instructive study

O(N)-model at large N

$$\partial_t u + \partial_\rho f(t,u) = 0 \qquad u(\rho) = \partial_\rho V(\rho)$$

→ Conservation Law

$$f(t,u) = -\frac{\Omega_d}{d(2\pi)^d}\frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + u}$$

Grossi, NW, arxiv:1903.09503

Another instructive study

O(N)-model at large N

$$\partial_t u + \partial_\rho f(t,u) = 0 \qquad u(\rho) = \partial_\rho V(\rho)$$

Conservation Law

$$f(t,u) = -\frac{\Omega_d}{d(2\pi)^d}\frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + u}$$

Now d=2+1 → Spontaneous symmetry breaking

Grossi, NW, arxiv:1903.09503

Another instructive study

O(N)-model at large N

$$\partial_t u + \partial_\rho f(t,u) = 0 \qquad u(\rho) = \partial_\rho V(\rho)$$

Conservation Law

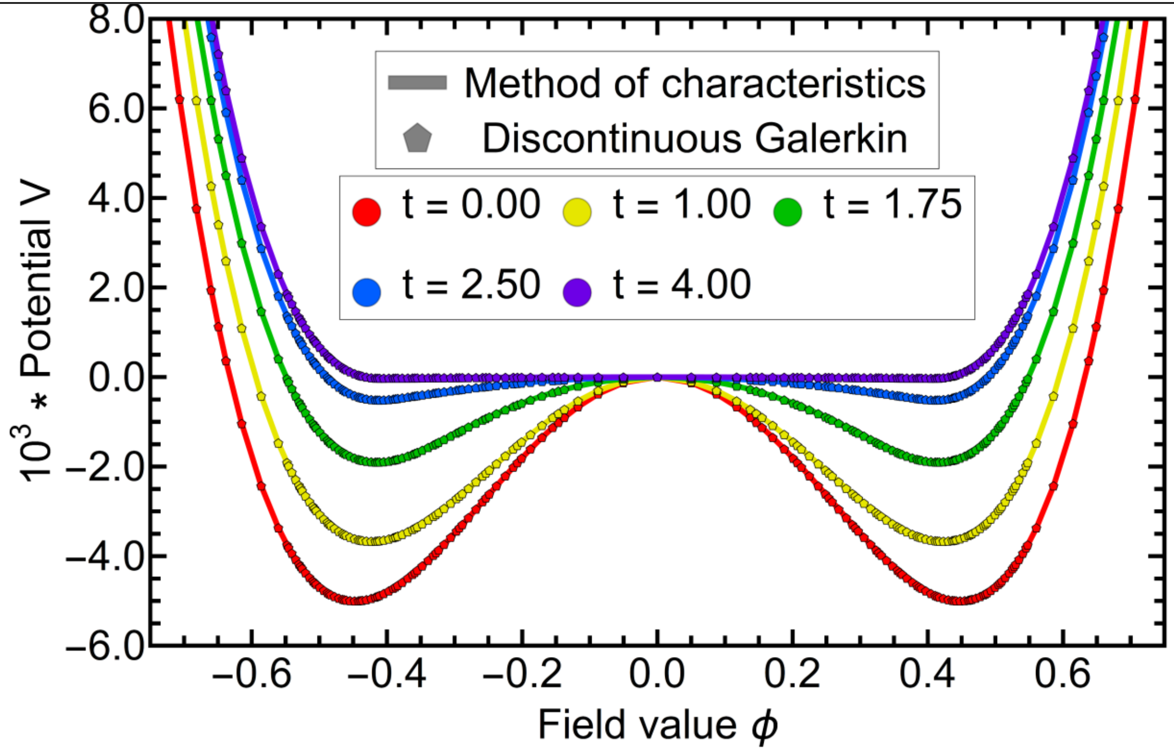
$$f(t,u) = -\frac{\Omega_d}{d(2\pi)^d}\frac{(\Lambda e^{-t})^{d+2}}{(\Lambda e^{-t})^2 + u}$$

Now d=2+1 → Spontaneous symmetry breaking

Study with Discontinuous Galerkin methods

Grossi, NW, arxiv:1903.09503

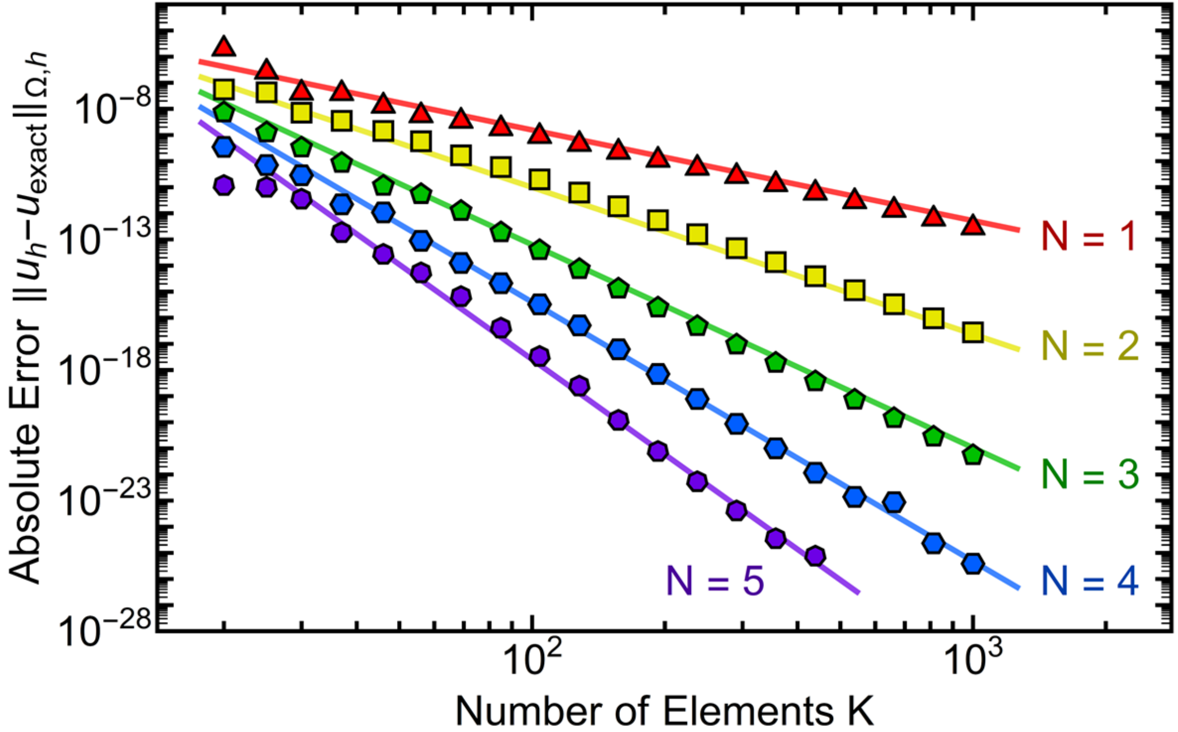
O(N) theory at large N



- Example of symmetry broken phase
- Convexity restoration nicely visible

Grossi, NW, arxiv:1903.09503

O(N) theory at large N

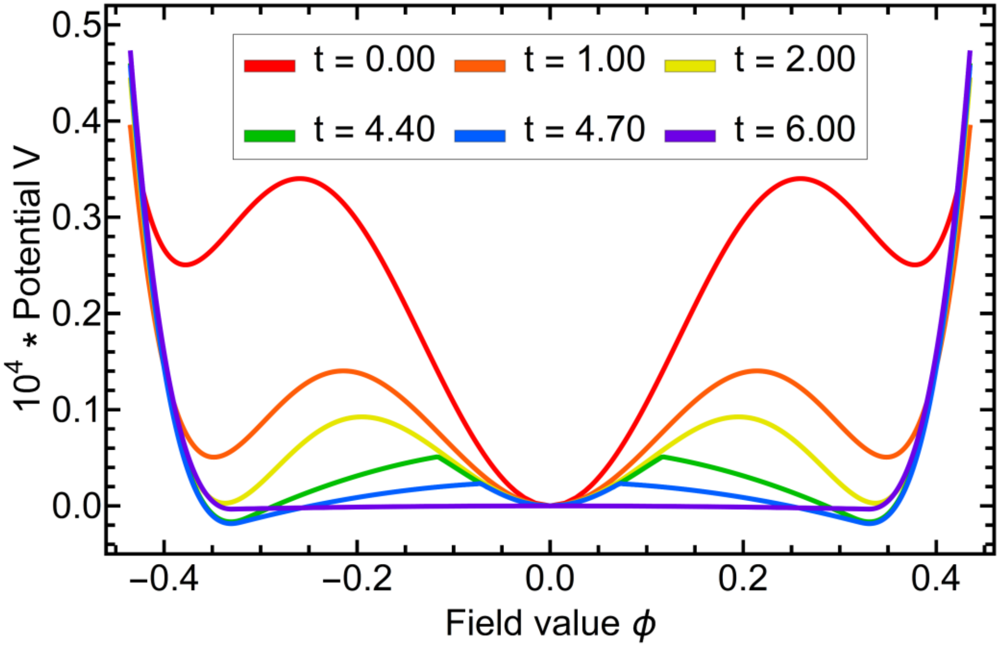


Shows expected convergence

- Power law for number of elements
- Spectral for local order of interpolation

Grossi, NW, arxiv:1903.09503

O(N) theory at large N



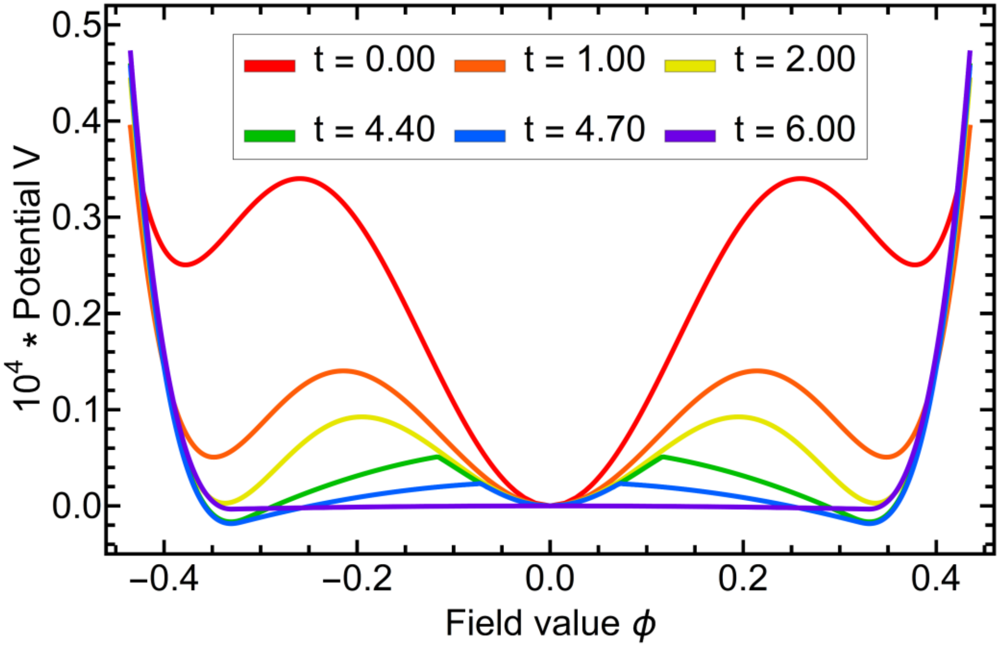
Consider initial potential with first order phase transition

$$u(t=0) = \lambda_2 + \lambda_4 \rho + \lambda_6 \rho^2$$

with $\lambda_4 < 0$

Grossi, NW, arxiv:1903.09503

O(N) theory at large N



Consider initial potential with first order phase transition

$$u(t=0) = \lambda_2 + \lambda_4 \rho + \lambda_6 \rho^2$$

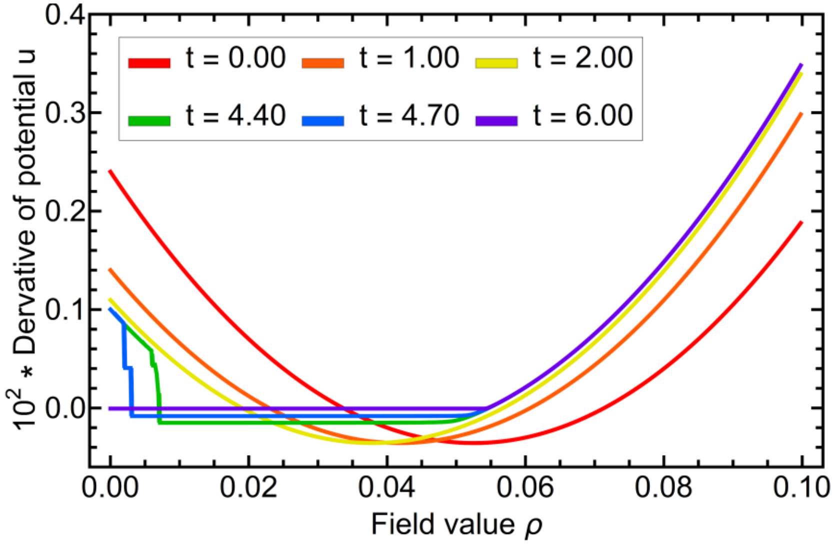
with $\lambda_4 < 0$

Non-analyticity at finite RG-scale

Grossi, NW, arxiv:1903.09503

O(N) theory at large N

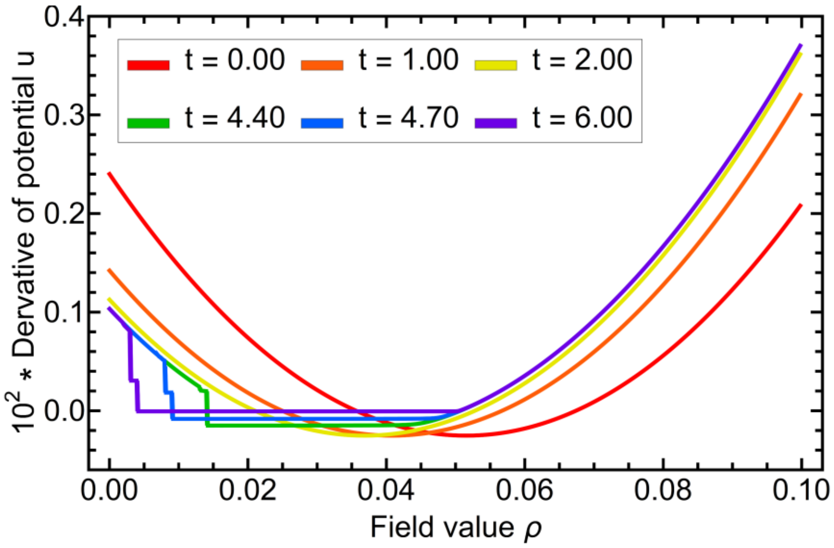
Broken phase



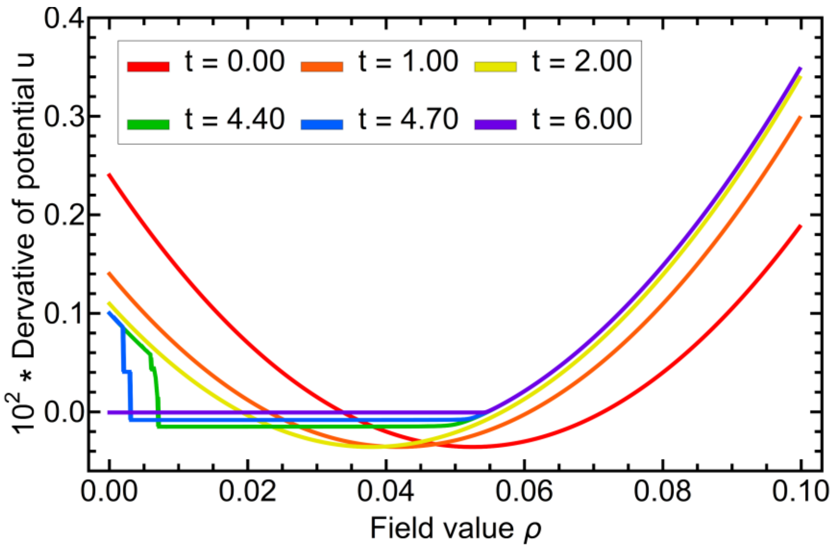
Grossi, NW, arxiv:1903.09503

O(N) theory at large N

Symmetric phase



Broken phase

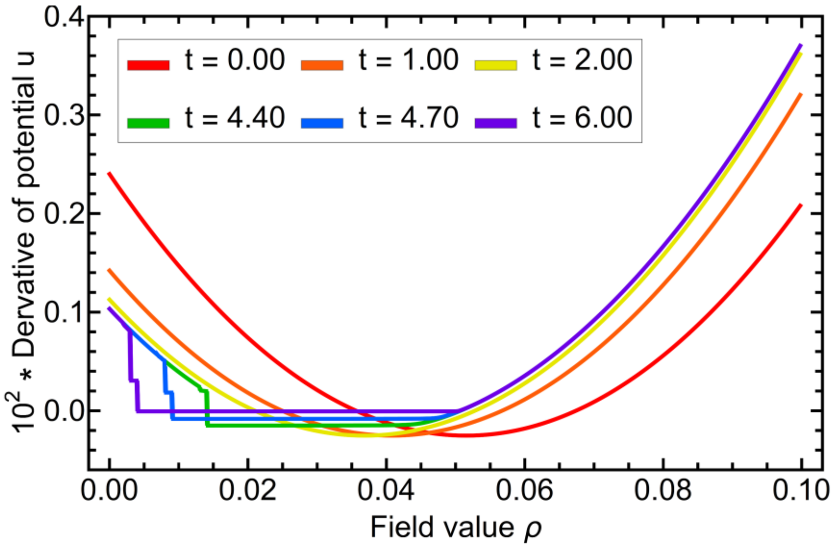


Grossi, NW, arxiv:1903.09503

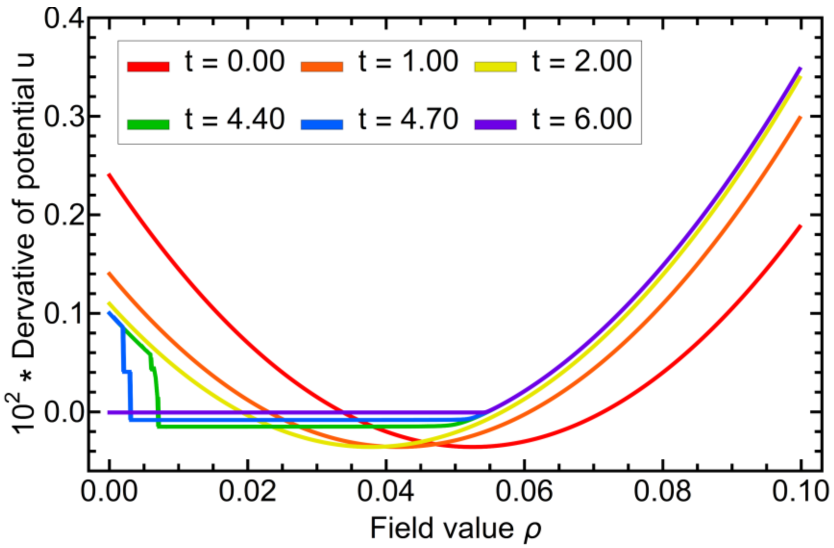
O(N) theory at large N

TECHNISCHE UNIVERSITÄT DARMSTADT

Symmetric phase



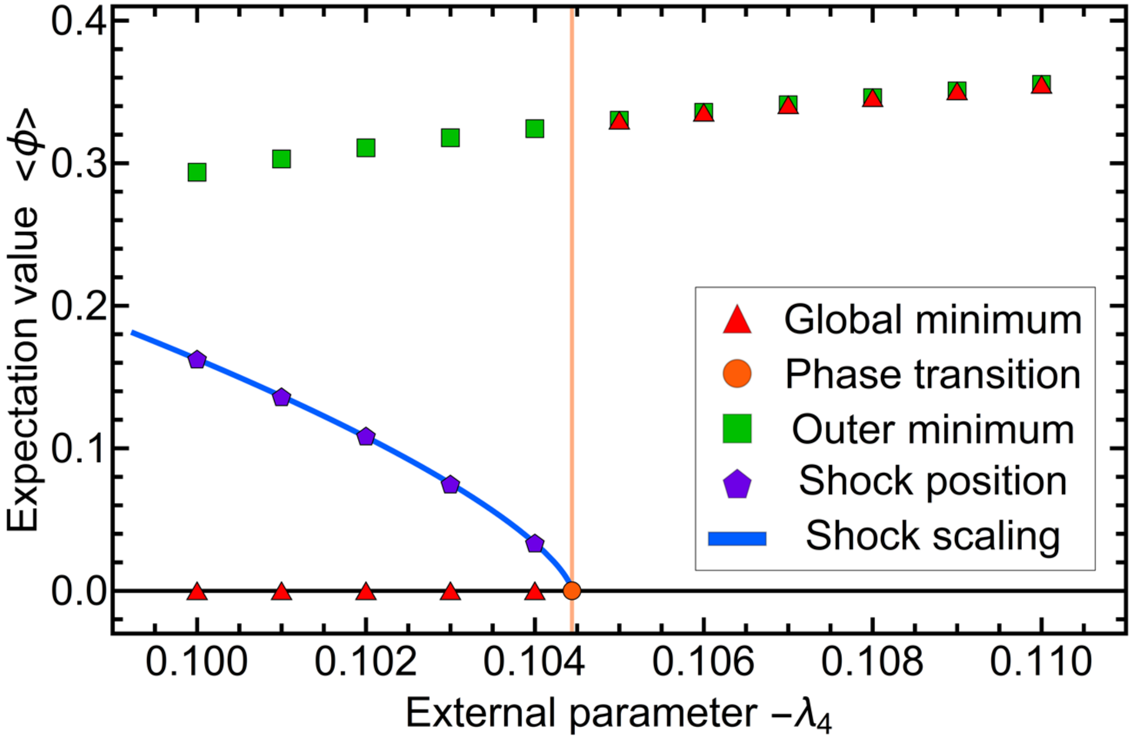
Broken phase



➡ Mechanism in terms of traveling discontinuity

Grossi, NW, arxiv:1903.09503

O(N) theory at large N



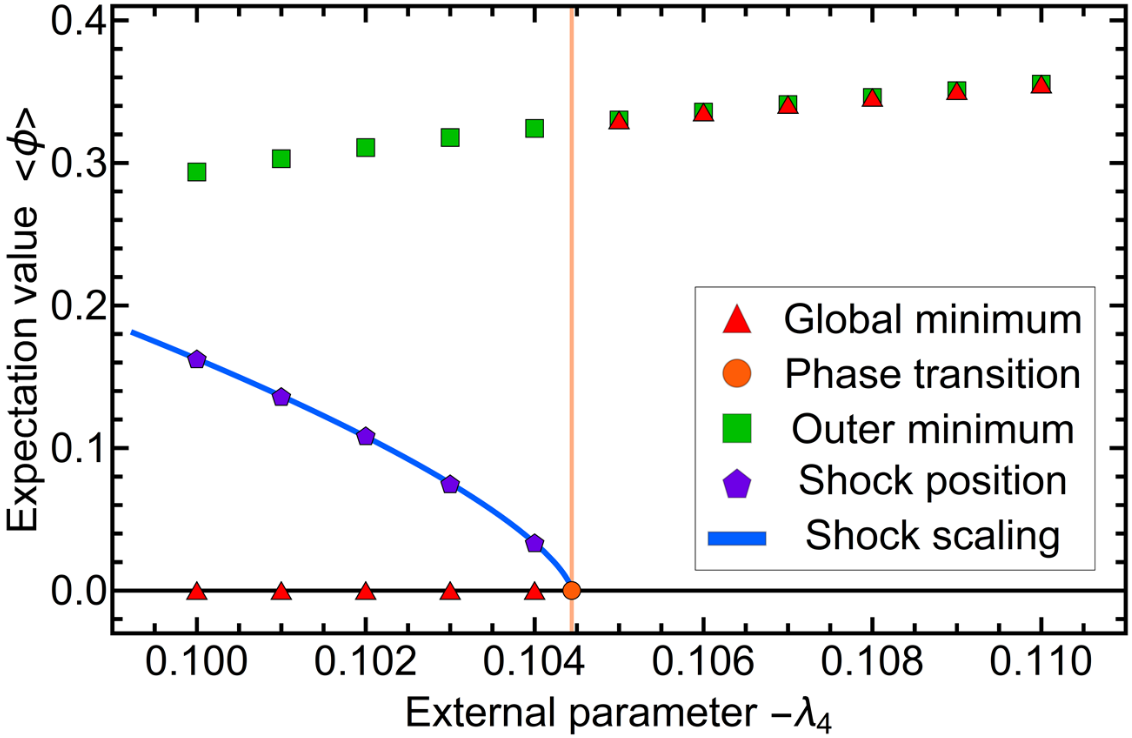
→ First order phase transition

→ Shock shows scaling at phase transition

(exponent: 0.68±0.01)

Grossi, NW, arxiv:1903.09503

O(N) theory at large N



→ First order phase transition

→ Shock shows scaling at phase transition

(exponent: 0.68±0.01)

Mechanism for first-order transitions

Grossi, NW, arxiv:1903.09503

Local Discontinuous Galerkin

Local Discontinuous Galerkin

Beyond Large N complicates the structure of the equation

$$\partial_k V(\rho) = \frac{\Omega_d}{(2\pi)^d} \frac{k^{d+2}}{d} \left(\frac{N-1}{k^2 + V'(\rho)} + \frac{1}{k^2 + V'(\rho) + 2\rho V''(\rho)} \right)$$

Radial mode

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Local Discontinuous Galerkin

Beyond Large N complicates the structure of the equation

$$\partial_k V(\rho) = \frac{\Omega_d}{(2\pi)^d}\frac{k^{d+2}}{d}\left(\frac{N-1}{k^2+V'(\rho)}+\frac{1}{k^2+V'(\rho)+2\rho V''(\rho)}\right)$$

Radial mode

➡ Take flow direction also for higher derivatives into account

Local Discontinuous Galerkin

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Local Discontinuous Galerkin

Beyond Large N complicates the structure of the equation

$$\partial_k V(\rho) = \frac{\Omega_d}{(2\pi)^d}\frac{k^{d+2}}{d}\left(\frac{N-1}{k^2+V'(\rho)}+\frac{1}{k^2+V'(\rho)+2\rho V''(\rho)}\right)$$

Radial mode

➡ Take flow direction also for higher derivatives into account

Local Discontinuous Galerkin

➡ Stabilizes numerics significantly

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Local Discontinuous Galerkin

Beyond Large N complicates the structure of the equation

$$\partial_k V(\rho) = \frac{\Omega_d}{(2\pi)^d}\frac{k^{d+2}}{d}\left(\frac{N-1}{k^2+V'(\rho)} + \frac{1}{k^2+V'(\rho)+2\rho V''(\rho)}\right)$$

Radial mode

➡ Take flow direction also for higher derivatives into account

Local Discontinuous Galerkin

On arXiv since Tuesday

➡ Stabilizes numerics significantly

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Local Discontinuous Galerkin

TECHNISCHE UNIVERSITÄT DARMSTADT

GitHub: https://github.com/satfra/dune-FRGDG

→ Implemented as module in the high performance PDE framework DUNE

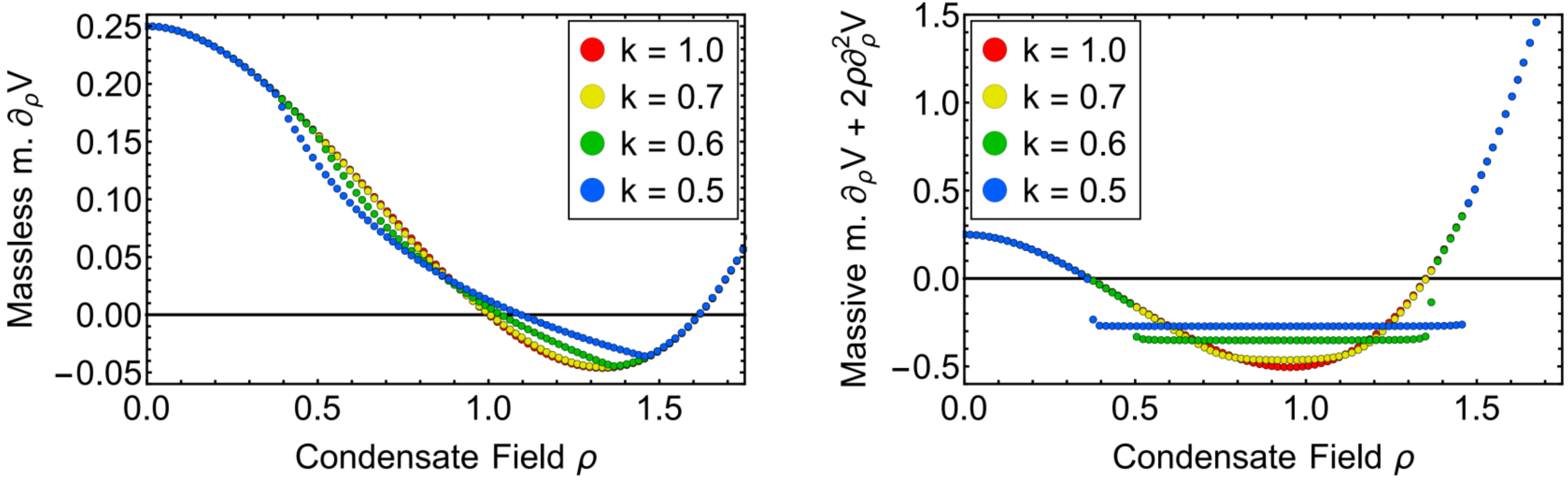
Poster: Franz Sattler

```
void flux(...) const
{
  const X xg = e.geometry().global(x);

  const RF F = f<N-1>(u[0])
    + f<1>(u[0] + 2.*xg[0]*u[1]);
  const RF G = u[1] * f<N-1,1>(u[0]);

  Flux[0][0] = F;
  Flux[1][0] = G;
}
```

Easy adaptation of other models

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Shock development



(a) RG-time dependence of the massless modes π. (b) RG-time dependence of the massive mode σ.

UV Potential

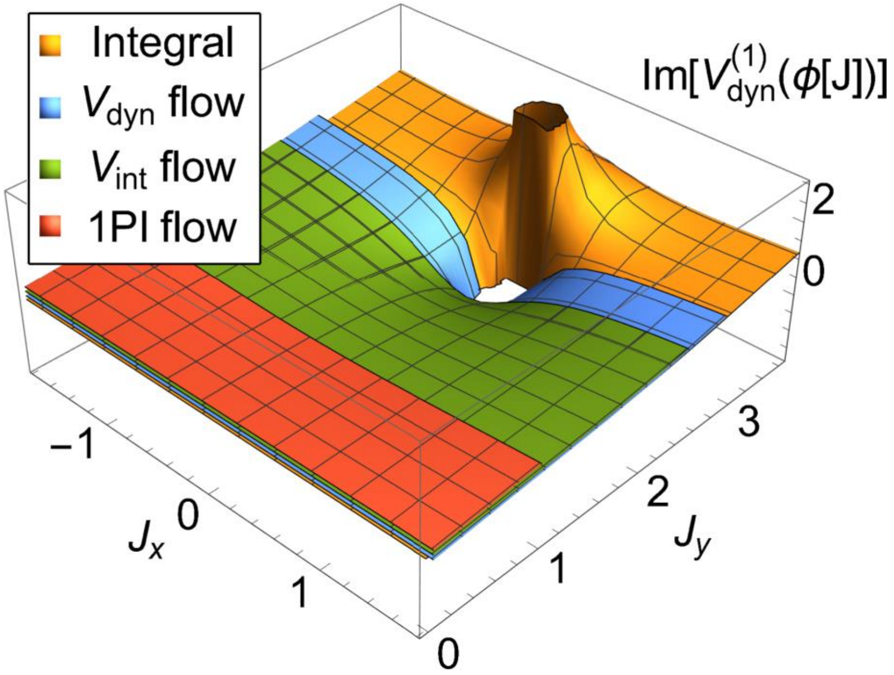
$$V_{t=0}(\rho) = \lambda_2 \rho + \lambda_4 \frac{\rho^2}{2} + \lambda_6 \frac{\rho^3}{3} + \lambda_8 \frac{\rho^4}{4}$$

Possibility for shocks in the presence of diffusion

Ihssen, Pawlowski, Sattler, NW arxiv:2207.12266

Towards the QCD phase diagram

Lee-Yang zeros



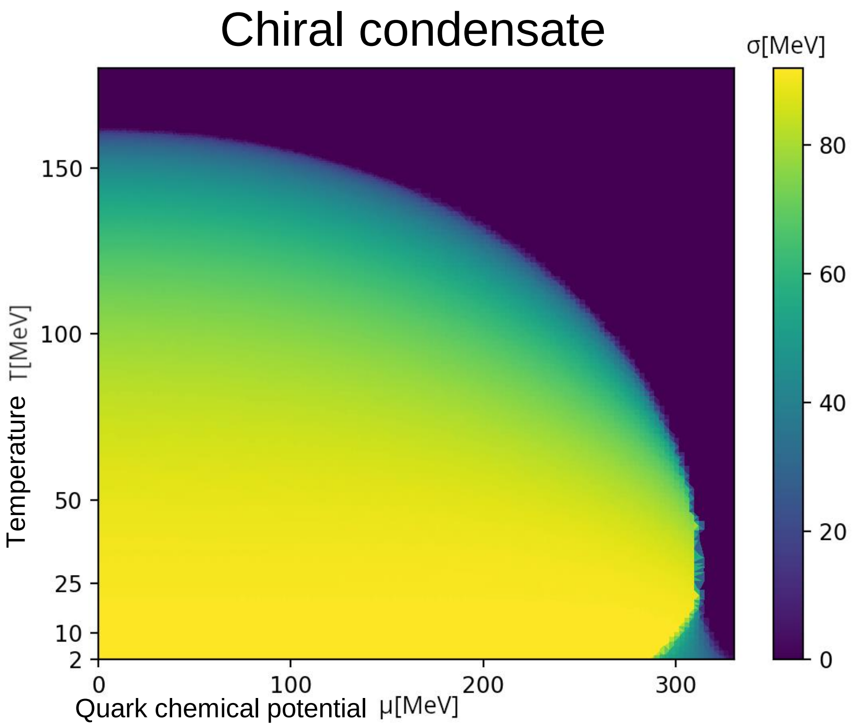
➡ Resolve complex plane

➡ Different RG equations up to the Lee-Yang zero

Talk: Friederike Ihssen

Ihssen, Pawlowski arxiv:2207.10057

Quark-Meson model phase diagram



- Two flavor & chiral limit
- Resolve shocks at large densities

Poster: Franz Sattler

Grossi, Ihssen, Pawlowski, NW, PRD 104 (2021)
Ihssen, Pawlowski, Sattler, NW in prep

Challenges

Challenges

New equations still require substantial work

➡ Understand general structure better

➡ Black box type discretization in field space

Challenges

New equations still require substantial work

➡ Understand general structure better

➡ Black box type discretization in field space

Handle flatness of potential (in broken phase)

➡ Improve/Adapt time stepping methods

➡ Change of coordinates

Summary

Novel numerical treatment of equations

First order phase transitions $\leftrightarrow$ shocks

Lots of possibilities for applications

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[\phi] + R_k} \partial_k R_k \right\}$$

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[\phi] + R_k}\,\partial_k R_k\right\}$$

Approximate momentum dependence

and keep field dependence

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[\phi] + R_k}\,\partial_k R_k\right\}$$

Approximate momentum dependence

and keep field dependence

Resulting PDE

$$\boxed{\partial_t u \;+\; \partial_x f^{(c)}(u,t,x) \;+\; \partial_x f^{(D)}(u,\partial_x u,t,x) \;=\; s(u,t,x)}$$

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[\phi] + R_k}\,\partial_k R_k\right\}$$

Approximate momentum dependence

and keep field dependence

Resulting PDE

$$\boxed{\partial_t u \;+\; \partial_x f^{(c)}(u,t,x) \;+\; \partial_x f^{(D)}(u,\partial_x u,t,x) \;=\; s(u,t,x)}$$

Conservation term
(Goldstone modes)

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[\phi] + R_k} \partial_k R_k \right\}$$

Approximate momentum dependence

and keep field dependence

Resulting PDE

$$\boxed{\partial_t u \; + \; \partial_x f^{(c)}(u, t, x) \; + \; \partial_x f^{(D)}(u, \partial_x u, t, x) \; = \; s(u, t, x)}$$

Conservation term
(Goldstone modes)

Diffusion term
(Radial mode)

Quark-Meson model

Consider simple Quark-Meson setting

$$\partial_k \Gamma_k[\Phi] = \frac{1}{2}\mathrm{Tr}\left\{\frac{1}{\Gamma^{(2)}[\phi] + R_k}\,\partial_k R_k\right\}$$

Approximate momentum dependence

and keep field dependence

Resulting PDE

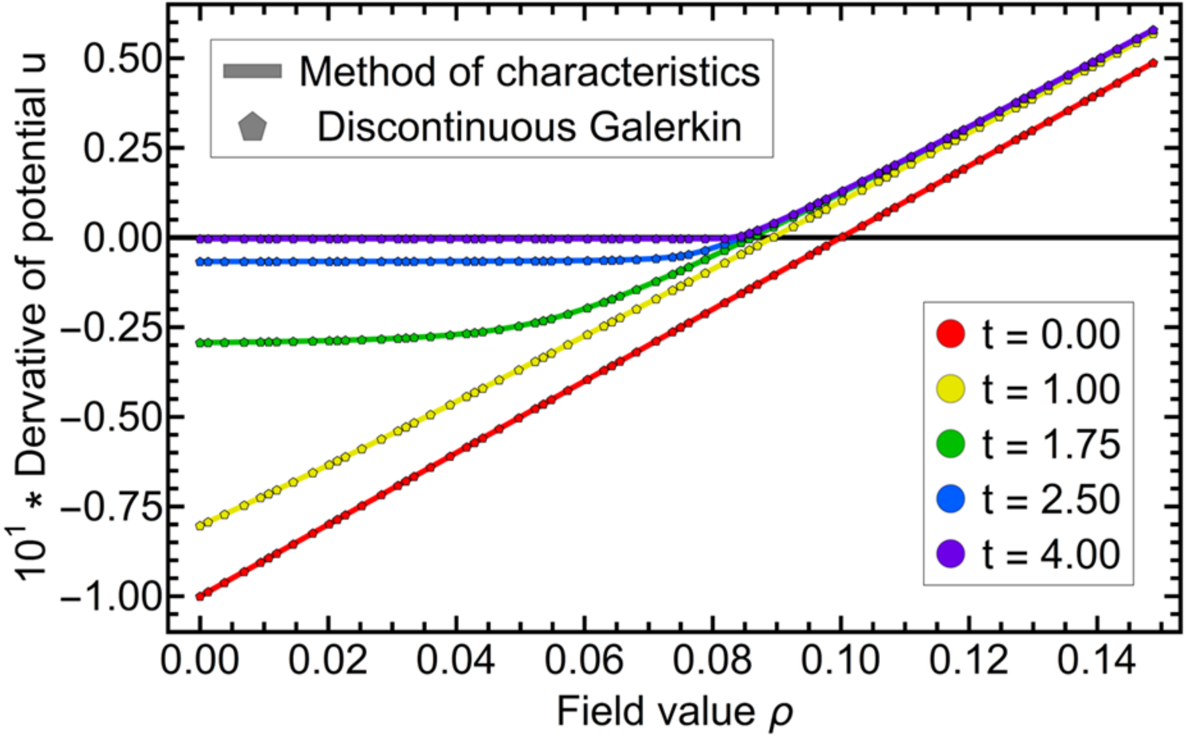
$$\boxed{\partial_t u \;+\; \partial_x f^{(c)}(u,t,x) \;+\; \partial_x f^{(D)}(u,\partial_x u,t,x) \;=\; s(u,t,x)}$$

Conservation term
(Goldstone modes)

Diffusion term
(Radial mode)

Source term
(Fermions)

O(N) theory at large N



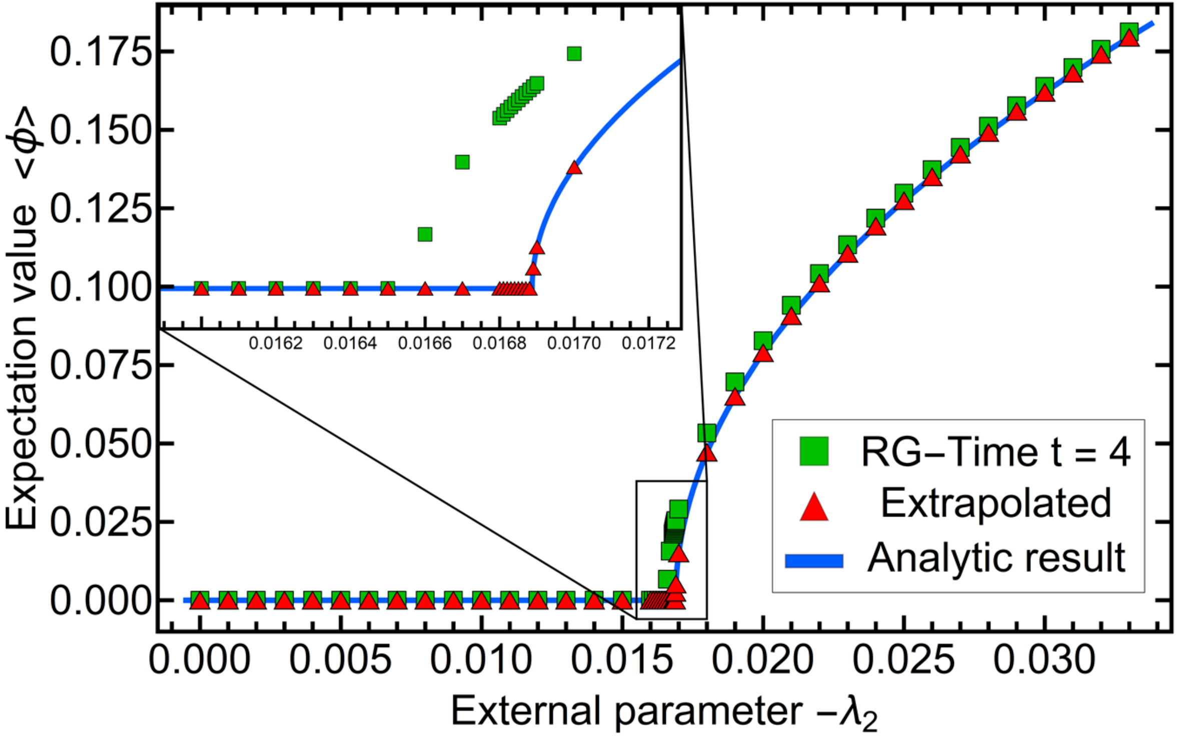
- Zero crossing signals position of minimum

Example of symmetry broken phase

- Convexity restoration nicely visible

Grossi, NW, arxiv:1903.09503

O(N) theory at large N



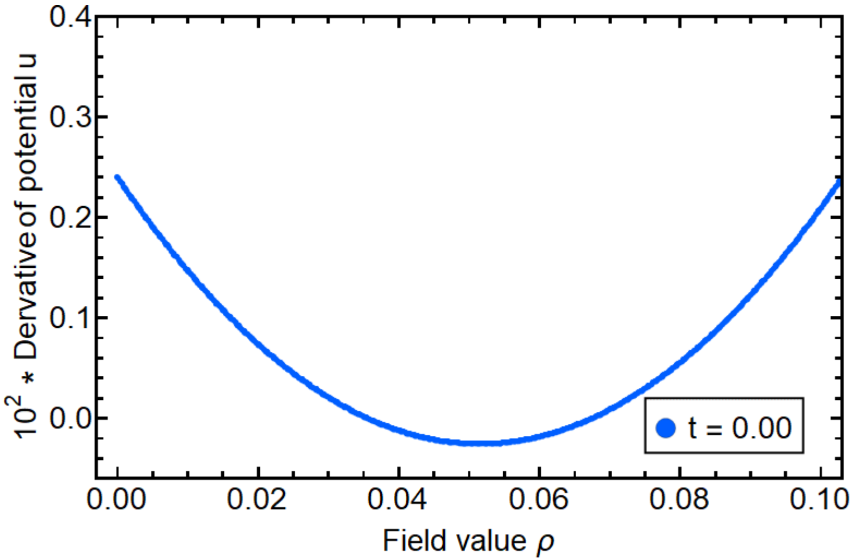
→ Expected second order phase transition

→ (Easy) extrapolation in RG-time possible

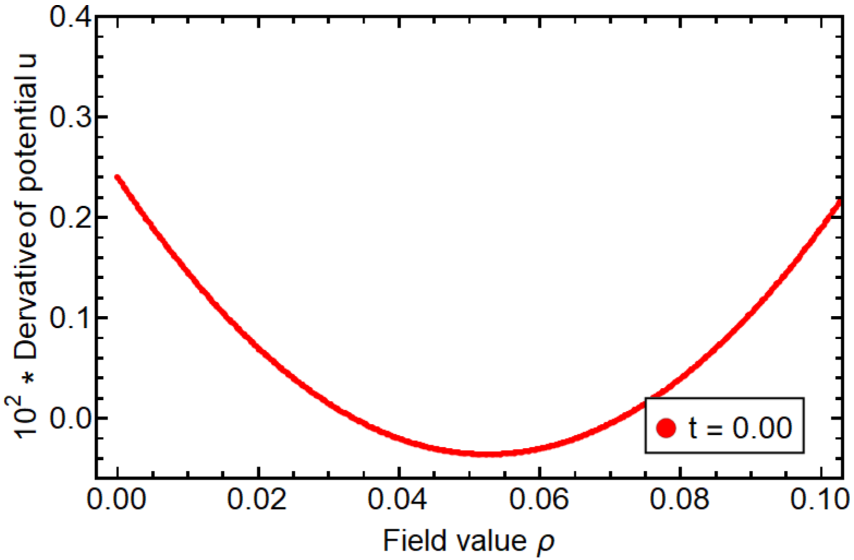
Grossi, NW, arxiv:1903.09503

O(N) theory at large N

Symmetric phase



Broken phase

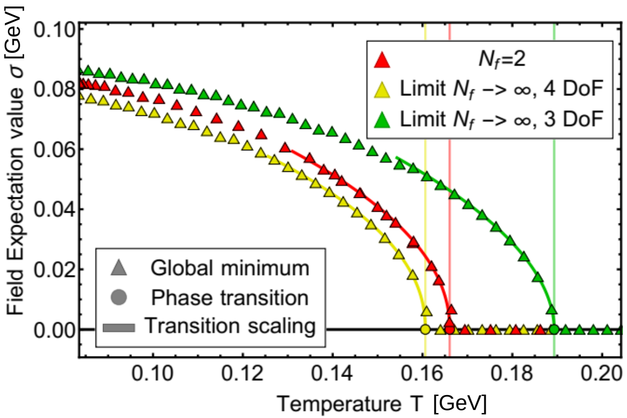


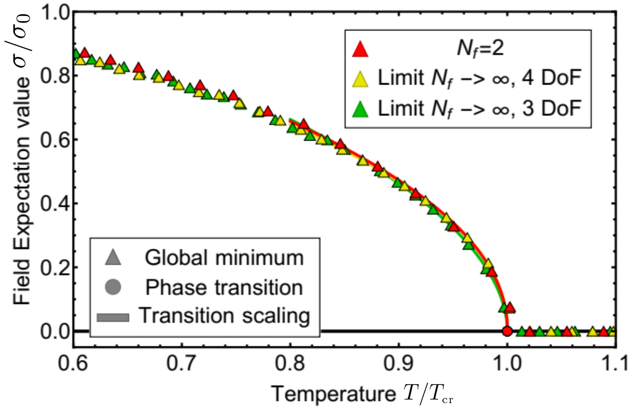
Grossi, NW, arxiv:1903.09503

Quark-Meson model

Include field dependent effective potential and Yukawa coupling

$$\Gamma_k[q,\bar q,\phi] = \int_x \Big\{ \bar q(\gamma_\mu \partial_\mu - \gamma_0 \mu_q) q + \frac{1}{2}(\partial_\mu \phi)^2 + h_k(\rho)\, \bar q(\tau_0 \sigma + \boldsymbol{\tau\pi}) q + V_k(\rho) - c_\sigma \sigma \Big\}$$

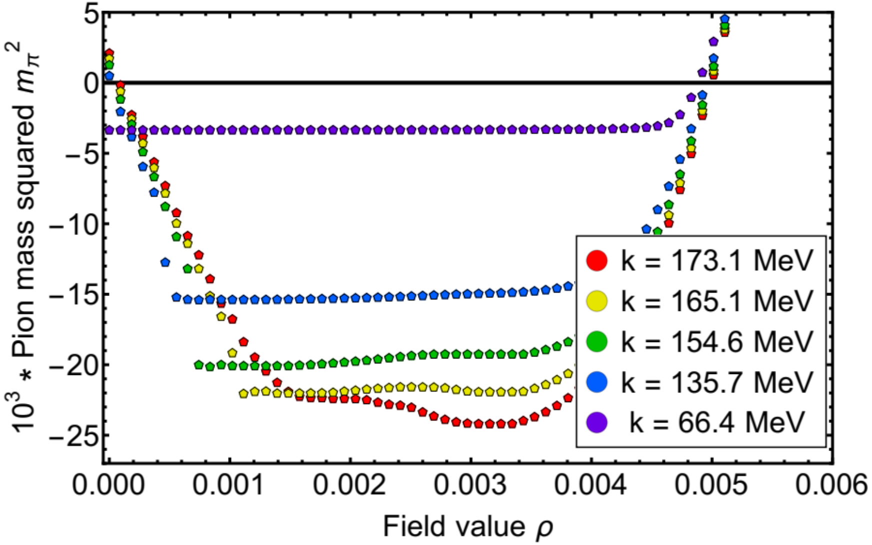




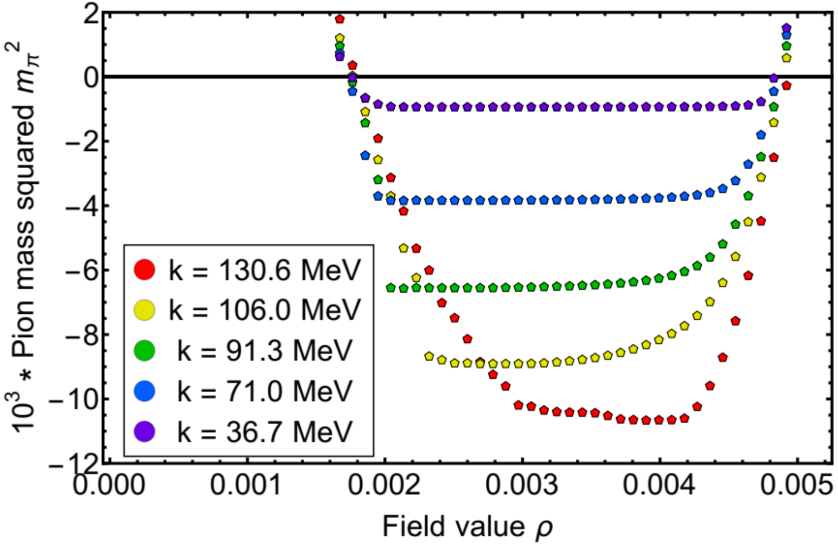
Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

Shock development at large densities

Broken phase

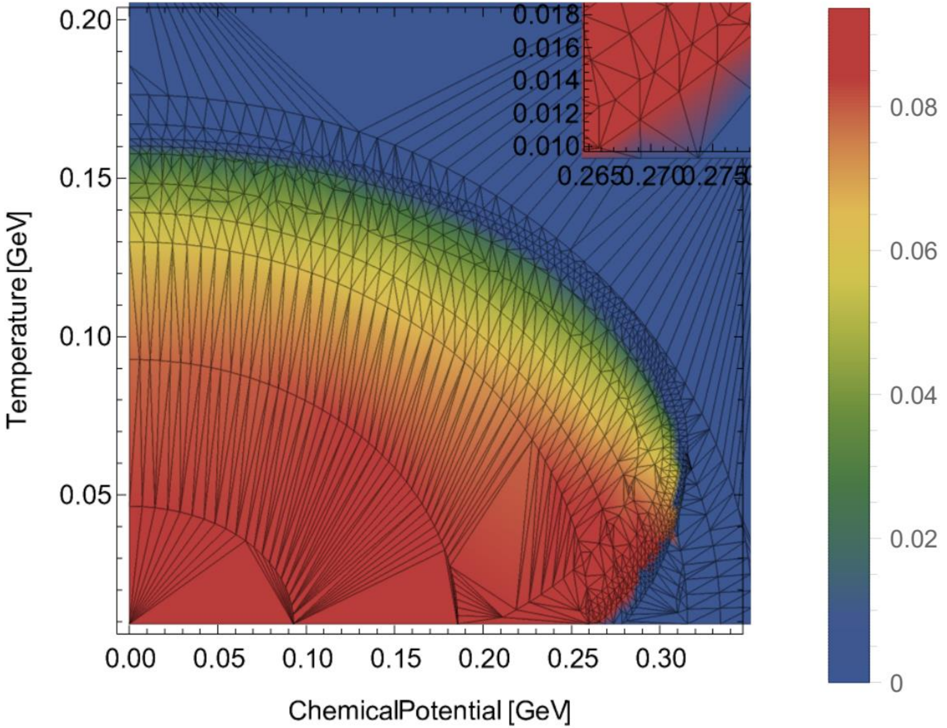


Symmetric phase



Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

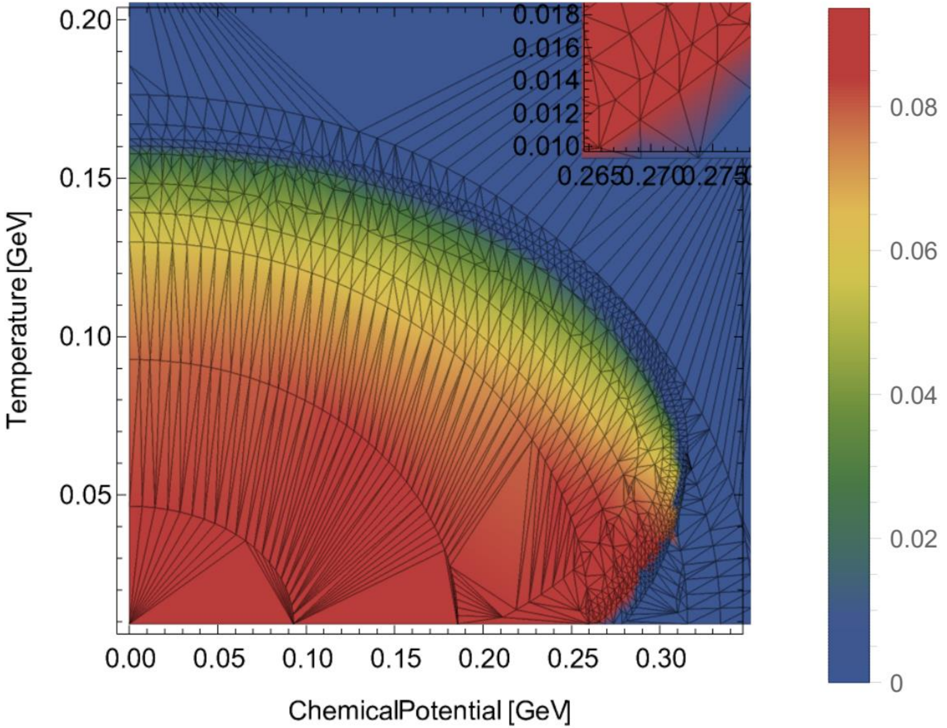
Constant Yukawa coupling



➡ Slight shock development at large densities

Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

Constant Yukawa coupling

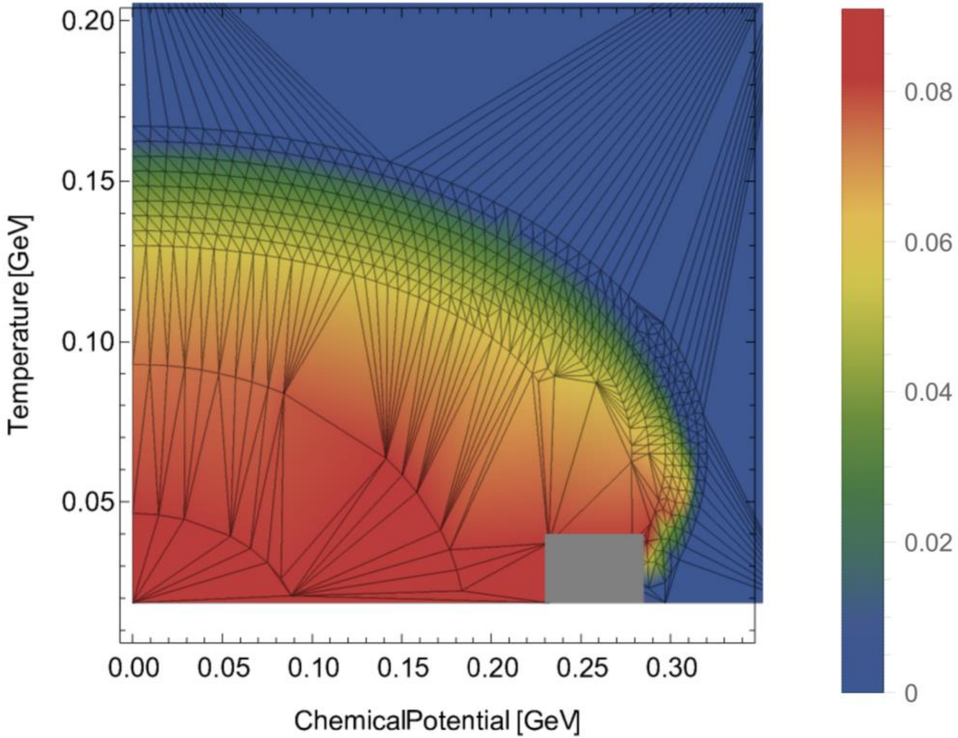


➡ Slight shock development at large densities

➡ Numerically at low temperatures “relatively” hard

Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

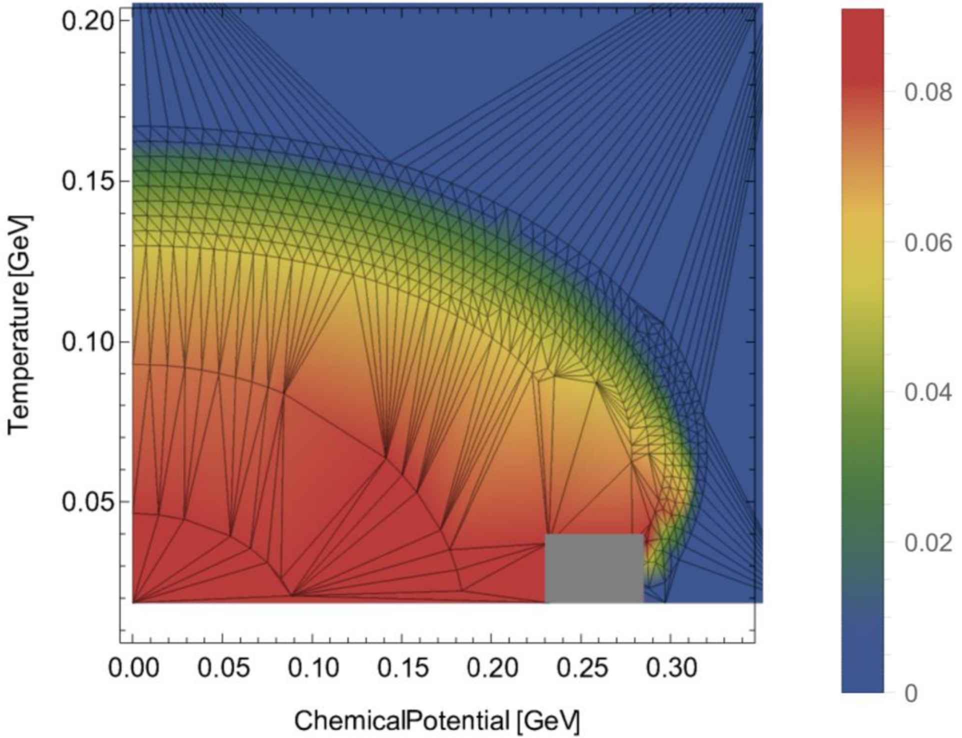
Dynamic Yukawa coupling



➡ Phase boundary shifts only slightly

Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

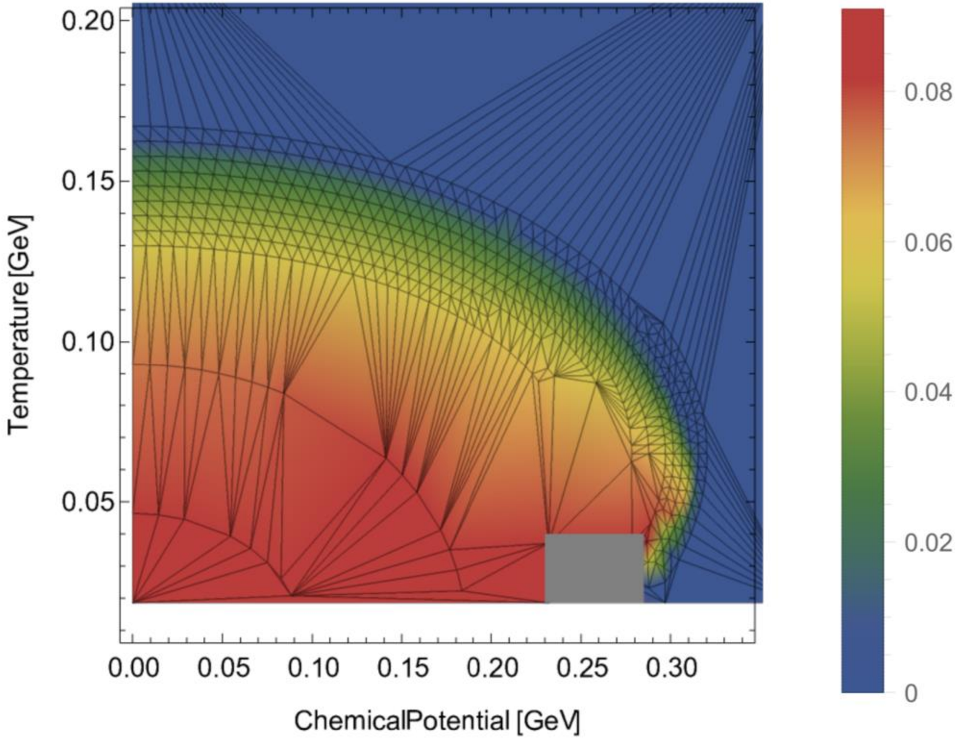
Dynamic Yukawa coupling



- Phase boundary shifts only slightly
- Crossover softened

Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)

Dynamic Yukawa coupling



- Phase boundary shifts only slightly
- Crossover softened
- Important for quantitative description

Grossi, Ihssen, Pawlowski NW, PRD 104 (2021)