Asymptotically Safe Scalar-Tensor Theories

- and a spin(0 and 1)off -

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ERG 2022, Berlin

Report on work in collaboration with

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Outline

- Brief overview of the asymptotic safety scenario for Quantum Gravity
- Gravity-Matter systems: The role of Symmetries
- Shift-symmetric Scalar-Tensor theories: fixed-point structure
- Summary & Outlook
- Spinoff: Turning off gravity: new (?) universality class (?)

ASQG

- **Einstein-Hilbert action is not perturbatively renormalizable**
- **A predictive and well-defined theory might exist non perturbatively**
- **The problem now is: how to make progress within the non perturbative realm?**
- **Functional techniques, lattice simulations,…** **(I am going Euclidean now)**
- **Functional renormalization group**

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

sets the notion of momentum scale

definition of background Laplacian

Introduction of a regulator term to the Boltzmann weight

$$h_{\mu\nu} \, R_k(\bar{\nabla}^2)^{\mu\nu,\alpha\beta} \, h_{\alpha\beta}$$

Field modes are organized and integrated out according to the Eigenvalues of the background Laplacian

Gauge-fixing term next to Faddeev-Popov ghosts are added

Mechanism for the fixed point solution

$$\bar{g}_i = k^{d_i} g_i$$

$$k\,\partial_k \bar{g}_i \equiv \partial_t \bar{g}_i = d_i\, k^{d_i}\, g_i + k^{d_i} \beta_i$$

$$\beta_i = -\,d_i\, g_i \,+\, F_i(g)$$

canonical (“classical”) scaling   quantum scaling

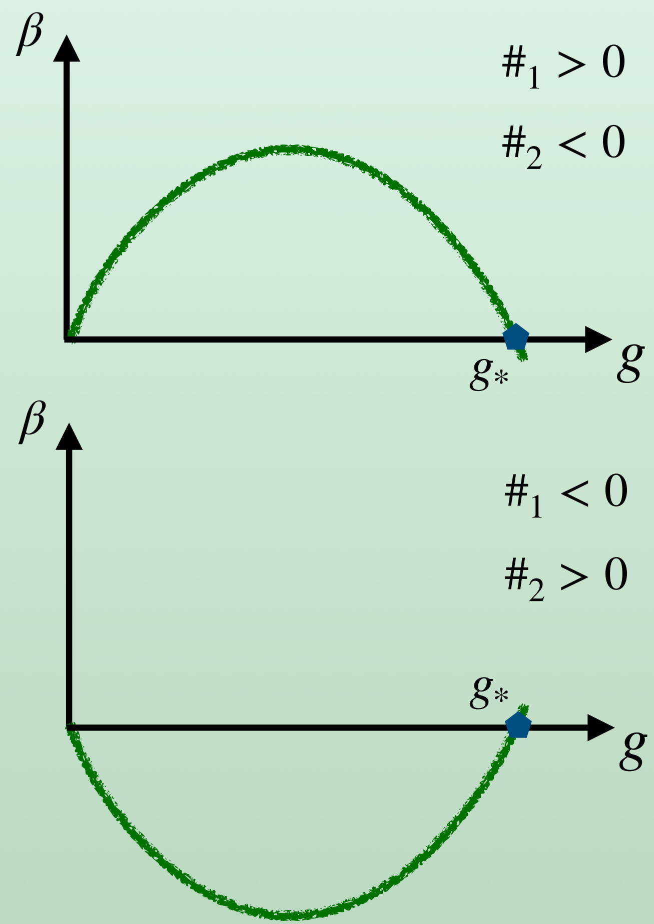
Fixed point is generated by a balancing mechanism between classical and quantum scalings

Predictivity is ensured by finitely many relevant directions.

Pure gravity as asymptotically safe QFT (?)

many encouraging results in this direction!

$$\beta = \#_1\, g + \#_2\, g^2$$

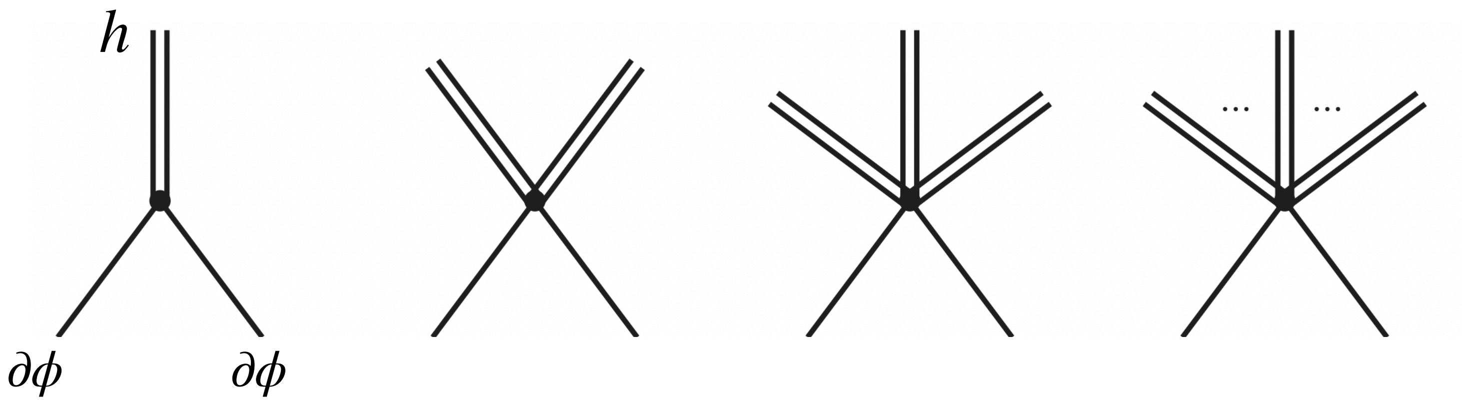


Gravity-Matter Systems & Symmetries: the scalar case

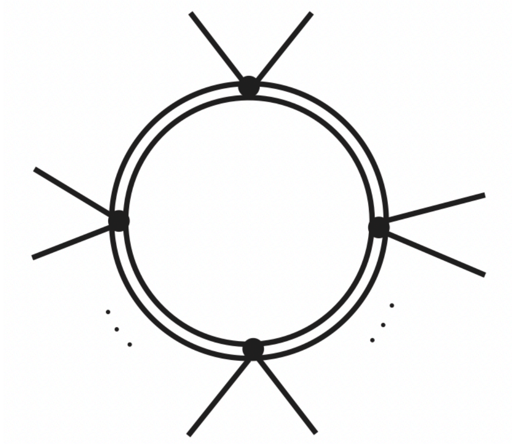
- **Basic question in this program: Does the would-be QG fixed point extends to gravity-matter systems?**
- **Coupling matter to gravity: generation of infinitely many matter-graviton vertices already from the matter-field kinetic term.**

$$S_\phi = \frac{1}{2}\int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$g^{-1} = \bar{g}^{-1} - h + hh + \ldots$$



This generates induced matter-self-interactions as well as non-minimal interactions.



induced self-interactions

The induced interactions share the same symmetries of the kinetic term. In particular,

$$\phi \to \phi + c$$

shift symmetry

$$\phi \to -\phi$$

Z_2 - symmetry

$$\mathcal{I}_1 = \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$\mathcal{I}_2 = \int \mathrm{d}^d x \sqrt{g}\, R\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$\mathcal{I}_3 = \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$\mathcal{I}_\# = \int \mathrm{d}^d x \sqrt{g}\, G(R, \mathbf{Ric}, \mathbf{Riem})^{\mu\nu} K_{\mu\nu}(X)$$

$$X_{\alpha\beta} \sim (\partial_\alpha \phi)(\partial_\beta \phi)$$

Apart from the induced interactions, one can still introduce a "standard" potential for the scalar fields in the effective action.

$$V(\phi^2) = \sum_i \bar{g}_i \phi^{2i}$$

breaks shift symmetry

$$\phi \to \phi + c$$

$$\Gamma_k = \Gamma_k^{\text{grav}} + \Gamma_k^{\phi}$$

$$\Gamma_k^{\phi} = \Gamma_k^{\text{ss}} + \Gamma_k^{\text{nss}}$$

Several results in the literature support that *nss* interactions feature just a Gaussian fixed point - the so-called Gaussian-Matter fixed point

$$\text{NGFP}^{\text{ASQGM}} = \text{NGFP}^{\text{ASQG}} \otimes \text{GFP}^{\text{matter}}$$

[see, e.g., Percacci The Great & Narain '09]

However…

shift-symmetric interactions *cannot* feature a GFP

[Eichhorn' 12, …]

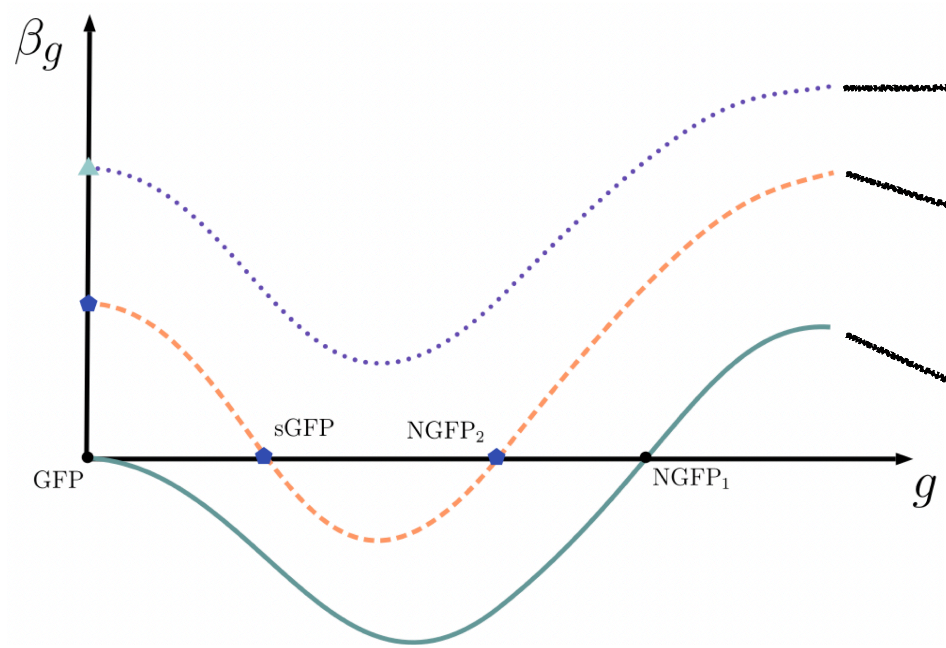
$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2} \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$+ Z_\phi^2 C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

$$\beta_c = a_0 + a_1\, c + a_2\, c^2$$

- a_0: contributions arising from the kinetic term - they do not depend on *c*
- $a_1\, c$: associated to canonical/ anomalous dimensionality of *c*
- $a_2\, c^2$: contributions from two-vertex diagrams

Setting *c* = *0* shows that, in general, this does not correspond to a fixed point → **QG fluctuations shift the GFP**



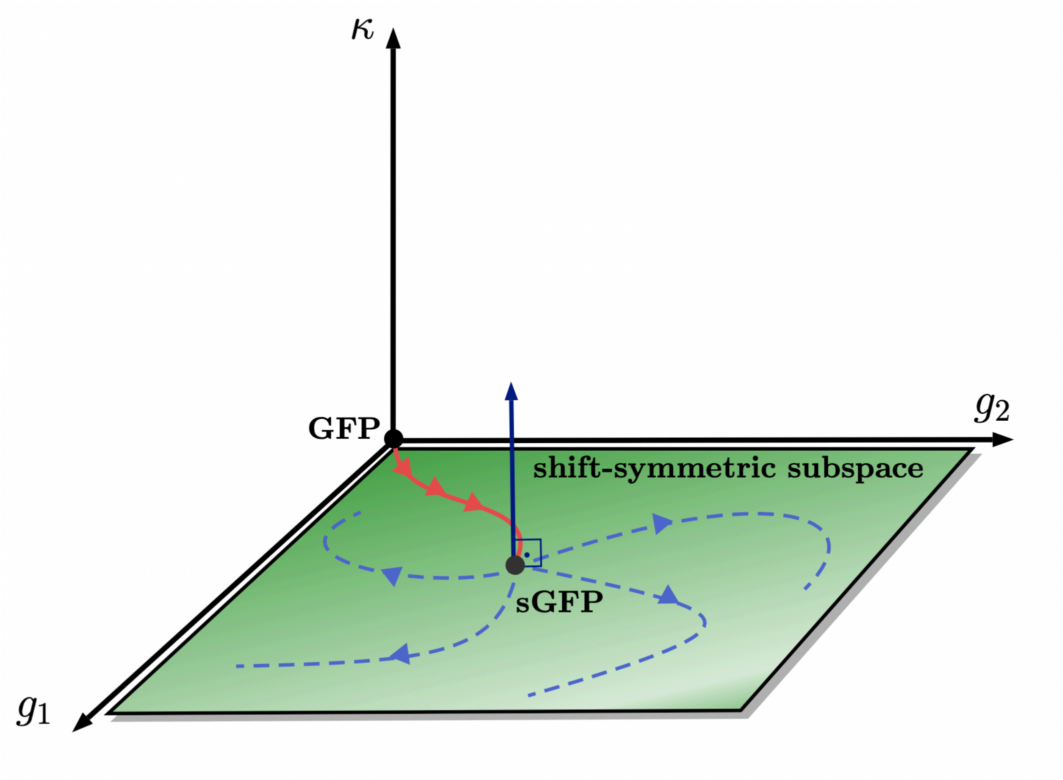
→ QG fluctuations are too strong
[Recall Marc Schiffer Talk]

→ sufficiently “small” QG fluctuations

→ no QG fluctuations

Hence, tentatively, the fixed-point structure should have the form

$$\mathrm{NGFP}^{\mathrm{ASQGM}} = \mathrm{NGFP}^{\mathrm{shift-symmetric}} \otimes \mathrm{GFP}^{\mathrm{non\ shift-symmetric}}$$



$$\int_x \frac{\delta\Gamma_k}{\delta\phi(x)} = 0$$

Shift-symmetry is preserved along the flow: shift-symmetric subspace is closed under the RG

Shift-symmetric Scalar-Tensor theories: fixed-point structure

Laporte, ADP, Saueressig, Wang, *JHEP* 12 (2021) 001

$$\Gamma_k = \Gamma_k^{\text{grav}} + \Gamma_k^{\text{ss}} + \Gamma_k^{\text{gf}} + \Gamma_k^{\text{ghost}}$$

$$\Gamma_k^{\text{grav}} = \frac{1}{16\pi G_k}\int \mathrm{d}^d x\sqrt{g}\left(2\Lambda_k - R\right)$$

$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2}\int \mathrm{d}^d x\sqrt{g}\, g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi)$$

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Eichhorn '12; de Brito, Eichhorn, dos Santos '21

$$\Gamma_k^{\text{ss}} \sim \frac{Z_\phi}{2} \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) + Z_\phi^2 C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

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Eichhorn, Lippoldt, Skrinjar '17

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& Feynman-de Donder gauge

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We employed the background approximation

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Employ a generic background to disentangle the tensor structures

We employed the background approximation

We can get some intuition about the behavior of the system of beta functions when the anomalous dimension of the scalar field is set to zero

The beta function of the induced quartic matter self-interaction has a structure of a polynomial of even degree on the coupling *c*

↓

It is possible to have multiple, 1 or no real solution!

↓

QG fluctuations might just be too strong and remove real fixed point solutions

On the other hand, the beta functions associated to the non-minimal interactions have the structure of polynomials which are odd on the non-minimal couplings

↓

There is always a real solution

↓

QG fluctuations are compatible with NGFP

$$\boxed{Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)}$$

→ **No fixed point!**

$$Z_\phi\, \tilde{C}_k \int \mathrm{d}^d x \sqrt{g}\, R^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi\, D_k \int \mathrm{d}^d x \sqrt{g}\, R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

$$Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

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fixed point!

$$Z_\phi\, D_k \int \mathrm{d}^d x \sqrt{g}\, R g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi)$$

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→ **Suitable fixed point!**

2 relevant directions

+

3 irrelevant ones

Non-minimal interactions play a crucial role in the fixed-point structure

$$Z_\phi^2\, C_k \int \mathrm{d}^d x \sqrt{g}\, g^{\mu\alpha} g^{\nu\beta} (\partial_\mu \phi)(\partial_\alpha \phi)(\partial_\nu \phi)(\partial_\beta \phi)$$

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Suitable fixed point!

ections

ones

Upon inclusion of the anomalous dimension, two fixed points are found. The second one has another relevant direction.

Non-minimal interactions play a crucial role in the fixed-point structure

Summary & Outlook

- **The induced interactions must have a NGFP**
- **Inclusion of non-minimal couplings was crucial to stabilize the fixed-point solutions**
- **The same scenario was investigated within the Essential RG scheme in a truncation involving all terms up to four derivatives [B. Knorr '22]. The results are compatible with our findings.**
- **Since the induced-matter couplings are irrelevant, they come as predictions in this framework: phenomenological implications?**
- **The toolbox developed and employed in the present analysis can be used to more sophisticated truncations.**

Spinoff: Evidence for a new shift-symmetric universality class?

Laporte, Locht, ADP, Saueressig - 2207.06749

Turning off gravity

$$\Gamma_k^{\rm ss} \sim \frac{Z_\phi}{2}\int \mathrm{d}^d x\, \delta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + Z_\phi^2\, C_k \int \mathrm{d}^d x\, \delta^{\mu\alpha}\delta^{\nu\beta}(\partial_\mu\phi)(\partial_\alpha\phi)(\partial_\nu\phi)(\partial_\beta\phi)$$

$$\beta_c\Big|_{\eta=0} = 4c + \frac{5}{8\pi^2}c^2 \longrightarrow \quad c^* = 0 \qquad c^* = -\frac{32\pi^2}{5}$$

$$\theta = -4 \qquad \theta = 4$$

Setting up a truncation of the form:

$$\Gamma_k = \int \mathrm{d}^d x\, f_k(X)$$

"(Non-)Gaussian" fixed point with a highly non-standard spectrum.

Interpretation?

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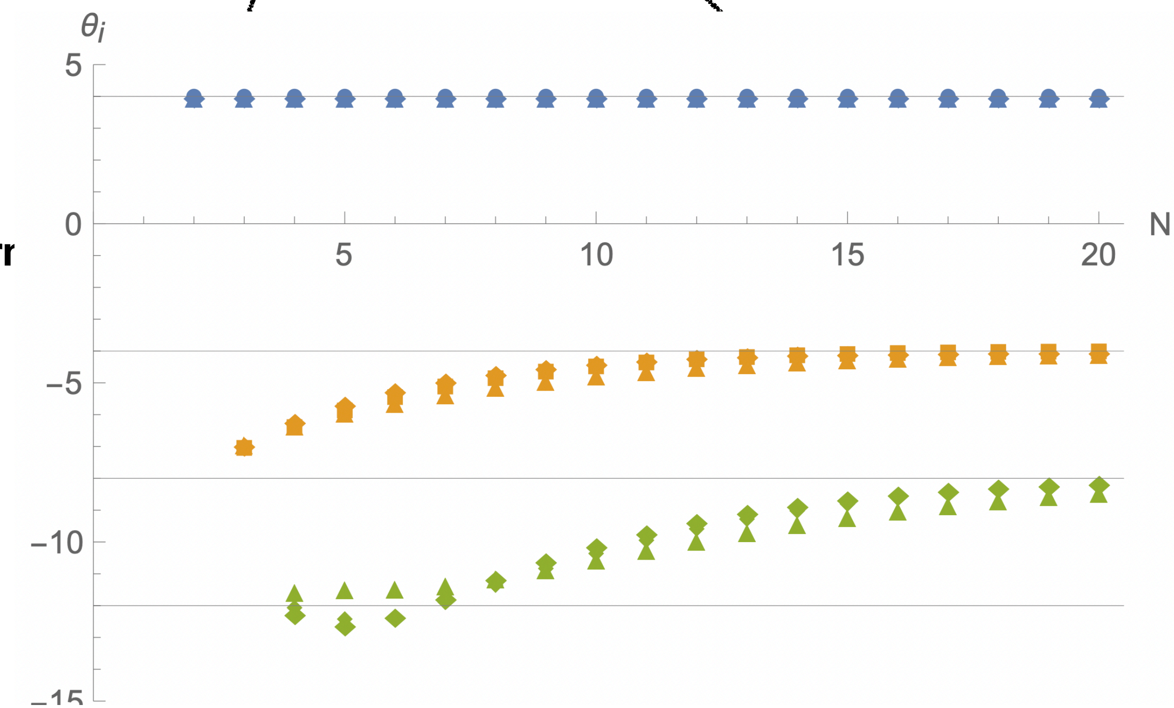

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Thank you