Constructing CFTs from AdS flows

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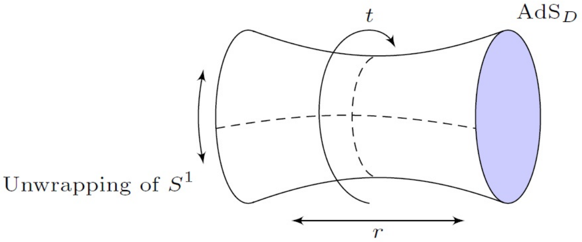
Overview of AdS/CFT

Anti-de-Sitter space



- AdS$_{d+1}$ is a maximally symmetric solution of Einstein's equations with negative cosmological constant Λ
- Symmetry group (S)O$(d,2)$ $(= (d+1)(d+2)/2$ Killing vectors)
- Closed time-like curves $\rightsquigarrow$ work on universal cover, or Euclidean AdS (= hyperbolic space)
- Poincaré patch with metric $\mathrm{d}s^2 = (\ell/z)^2(\mathrm{d}z^2 - \mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2)$ covers half of AdS but full EAdS, ℓ is the AdS radius
- Conformal boundary at $z = 0$ $(r \to \infty)$

Conformal field theory

- CFT_d is a quantum field theory with enhanced symmetry group (S)O$(d, 2)$ (Poincaré transformations + dilations D + special conformal transformations K^μ)
- Conformal symmetry fixes two- and three-point-functions of primary operators $\mathcal{O}_A$ (ϕ, ϕ^2, $\phi\overleftrightarrow{\partial_\mu}\phi$, ...)
- $\langle \mathcal{O}_A(x)\mathcal{O}_B(y)\rangle \sim \delta_{AB}|x-y|^{-2\Delta_A}$
- $\langle \mathcal{O}_A(x)\mathcal{O}_B(y)\mathcal{O}_C(z)\rangle \sim$ $\lambda_{ABC}|x-y|^{\frac{\Delta_C-\Delta_A-\Delta_B}{2}}|x-z|^{\frac{\Delta_B-\Delta_A-\Delta_C}{2}}|y-z|^{\frac{\Delta_A-\Delta_B-\Delta_C}{2}}$
- $\{\Delta_A, \lambda_{ABC}\}$ = conformal data, all higher n-point functions determined from operator product expansion
- Central charge c counts degrees of freedom (stress tensor)
- Examples: free massless bosons $c = 1$, Majorana fermions $c = 1/2$, gauge fields, $\mathcal{N} = 4$ SU(N) super-Yang–Mills theory $c \sim N(N-1)$

AdS/CFT correspondence

- Way to generate citations: Maldacena (hep-th/9711200) 17763 citations, Witten (hep-th/9802150) 11398 citations as of Tuesday

AdS/CFT correspondence

- Concrete realisation of holographic principle: theory with gravity (in the bulk) is equivalent to theory without gravity in one less dimension (on the boundary)
- Bulk scalar $\leftrightarrow$ boundary scalar, bulk gauge field $\leftrightarrow$ boundary conserved current, bulk graviton $\leftrightarrow$ boundary stress tensor
- More concrete: correlation functions of bulk and boundary map into each other under suitable identification of fields and sources
- Original example: type IIB supergravity on $AdS_5 \times S_5$ equivalent to $\mathcal{N} = 4$ SYM
- In essence: $(S)O(d,2) = (S)O(d,2)$ (or suitable superalgebras)
- Strong-weak coupling duality: small-coupling expansion in bulk describes expansion in large central charge in CFT $g \sim c^{-1/2}$

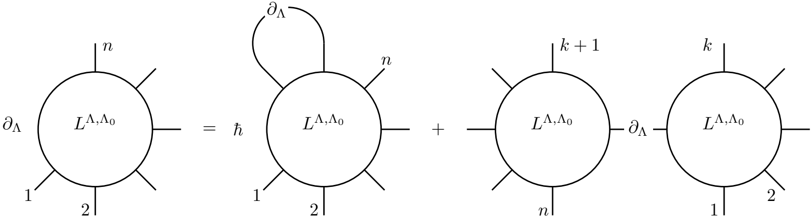
RG flows

RG flow equations on curved manifolds 1/2

- Many incarnations of RG flow equations (Stueckelberg–Petermann, Wilson, Wegner–Houghton, Polchinski, Keller–Kopper–Salmhofer, Wetterich, Morris, ...)
- Wetterich equation: flow of 1PI effective action, involves inverse of exact propagator
- Polchinski equation: flow of effective action, involves free propagator
- Keller–Kopper–Salmhofer: use both UV cutoff Λ_0 and IR cutoff Λ in Polchinski equation and flow Λ
- Useful regulator on Riemannian manifold: heat kernel $K_{d,\Delta}$ satisfying $\left[\partial_\tau - \nabla^2 + \frac{\Delta(\Delta-d)}{\ell^2}\right] K_{d,\Delta}(X,Y,\tau) = 0$, $\lim_{\tau\to 0} K_{d,\Delta}(X,Y,\tau) = \delta(X,Y)$ with $\Delta = d/2 + \sqrt{m^2\ell^2 + d^2/4}$

RG flow equations on curved manifolds 2/2

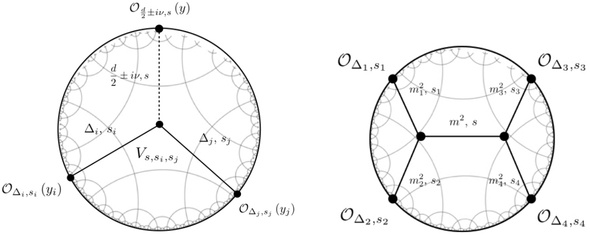
- Regulated free propagator $G_{\Delta}^{\Lambda,\Lambda_0}(X,Y) = \int_{\Lambda_0^{-2}}^{\Lambda^{-2}} K_{d,\Delta}(X,Y,\tau)\,\mathrm{d}\tau$
- RG flow equation for generating functional of connected, amputated Green's functions 2205.15247



- Boundary conditions: Bare action at $\Lambda = \Lambda_0$ gives BC for irrelevant operators, arbitrary finite BC at $\Lambda = 0$ for marginal and relevant operators

From the AdS bulk to the boundary 1/2

- Witten diagrams ≈ Feynman diagrams for AdS/CFT

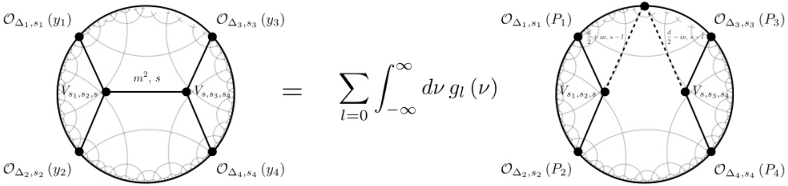


1702.08619

- External legs involve bulk-to-boundary propagator, internal legs involve full bulk propagator
- Bulk-to-boundary propagator:
  $G_{\partial\Delta}(X,P) = \lim_{z_Y\to 0}(z_Y/\ell)^{-\Delta}G_\Delta(X,Y)$

From the AdS bulk to the boundary 2/2

- Use split representation for bulk propagator 1702.08619



$$\partial_\Lambda G_\Delta^{\Lambda,\Lambda_0}(X,Y) = -\frac{2}{\ell^3\Lambda^3}\int_{-\infty}^{\infty} \exp\left(-\frac{\nu^2+(\Delta-\rho)^2}{\ell^2\Lambda^2}\right)\frac{\nu^2}{\pi}$$

$$\times \int G_{\partial(\rho+\mathrm{i}\nu)}(X,P)\,G_{\partial(\rho-\mathrm{i}\nu)}(Y,P)\,\mathrm{d}P\,\mathrm{d}\nu\ , \quad \rho = d/2$$

Flowing CFTs 1/2

- AdS SO(d, 2) symmetry is unbroken by regulator $\rightsquigarrow$ boundary correlators keep SO(d, 2) symmetry, which on them acts as conformal symmetry
- Flow equation connecting a family of different CFTs:

$$\partial_\Lambda S^{\Lambda,\Lambda_0}_{A_1\cdots A_n}(P_1,\ldots,P_n) = -\frac{1}{\ell^3\Lambda^3}\sum_B \int_{-\infty}^{\infty} \exp\left(-\frac{\nu^2+(\Delta_B-\rho)^2}{\ell^2\Lambda^2}\right)\frac{\nu^2}{\pi}$$

$$\times \int \mathcal{P}f_{Q'\to Q}\Bigg[\hbar\, S^{\Lambda,\Lambda_0}_{A_1\cdots A_n B(\rho+\mathrm{i}\nu)B(\rho-\mathrm{i}\nu)}(P_1,\ldots,P_n,Q,Q')$$

$$-\sum_{I\cup J=\{1,\ldots,n\}} S^{\Lambda,\Lambda_0}_{\{A_I\}B(\rho+\mathrm{i}\nu)}(\{P_I\},Q)\,S^{\Lambda,\Lambda_0}_{\{A_J\}B(\rho-\mathrm{i}\nu)}(\{P_J\},Q)\Bigg]\,\mathrm{d}Q\,\mathrm{d}\nu$$

Flowing CFTs 2/2

- Boundary conditions at $\Lambda = \Lambda_0$ (bare action) correspond to contact diagrams
- The flow recursively constructs Witten diagrams by sewing together lower-order diagrams over the boundary
- Explicit formulas for the flow of the conformal data (scaling dimension, three-point coefficients)
- Conformal symmetry allows to write higher n-point functions as Mellin integrals ($\cong$ conformally invariant Fourier transformation) $\rightsquigarrow$ also explicit formulas for flow of Mellin amplitudes

Summary and outlook

- We derived the RG flow connecting a family of CFTs using the AdS/CFT correspondence
- Example of results: one-loop correction to conformal dimension of ϕ_i in a model dual to the bulk O(N) vector model: $\Delta_{\phi_1}^{(1),0,\infty} = \tilde{\lambda}_{11mm}\frac{2\pi^{2\rho-1}\ell^{4\rho-2}\Gamma(\rho)}{\Gamma(2\rho)(\Delta_\phi-\rho)}\Gamma(\Delta_\phi)\Gamma(2\rho-\Delta_\phi)\sin[\pi(\Delta_\phi-\rho)]$
- Drawback: using Polchinski equation, flow does not close on any subset of correlation functions (uncontrolled truncations)
- Future work: using operator product expansion, obtain closed system of equations and apply to interesting examples ($\mathcal{N}=4$ super-Yang–Mills), maybe solve numerically

Thank you for your attention

Questions?

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