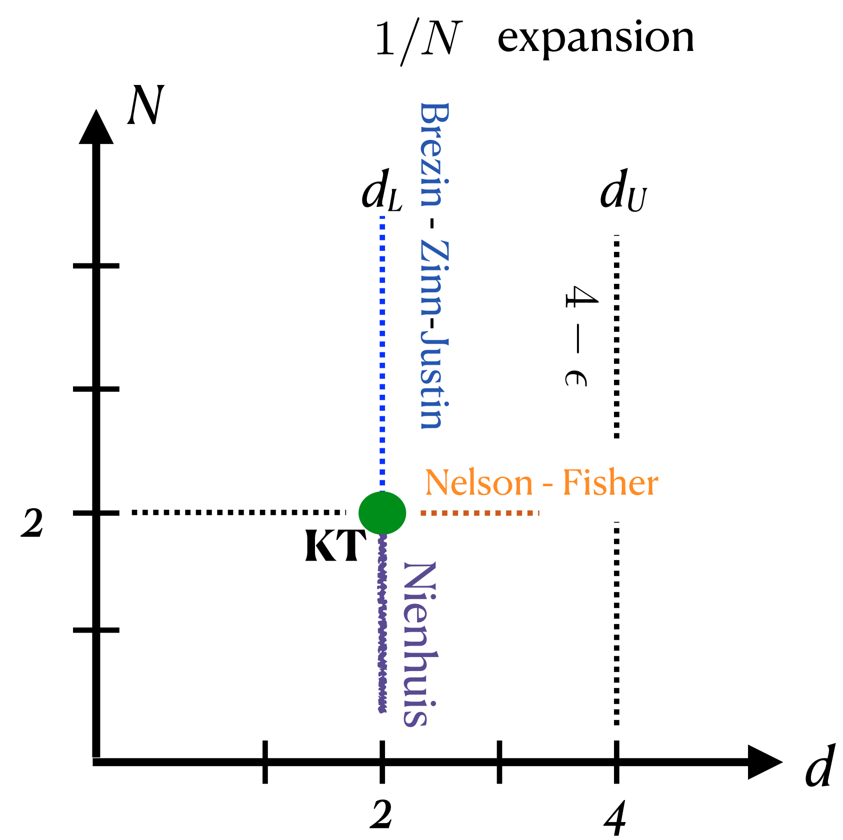
The *O(N)* models in the vicinity of *d=2*

Paweł Jakubczyk
University of Warsaw

Together with
Andrzej Chlebicki

The $O(N)$ models in the (d,N) plane

$$\mathcal{S}[\phi] = \int d^d x \left[\frac{1}{2} (\nabla \phi)^2 + \frac{\lambda}{8} (\phi^2 - \phi_0^2)^2 \right]$$



The Cardy-Hamber approach (1980)

J. L. Cardy and H. W. Hamber, *$O(n)$ Heisenberg model close to $n = d = 2$*, Phys. Rev. Lett. **45**, 499 (1980), doi:10.1103/PhysRevLett.45.499.

Assume analyticity of the RG equations (in a truncated space)

Combine Brezin - Zinn-Justin and Nelson - Fisher

$$\dot{g} = -\epsilon g + (N-2) f(g) + 4\pi^3 y^2 + \ldots$$
$$\dot{y^2} = \left(4 - \frac{2\pi}{g}\right) y^2 + \ldots$$

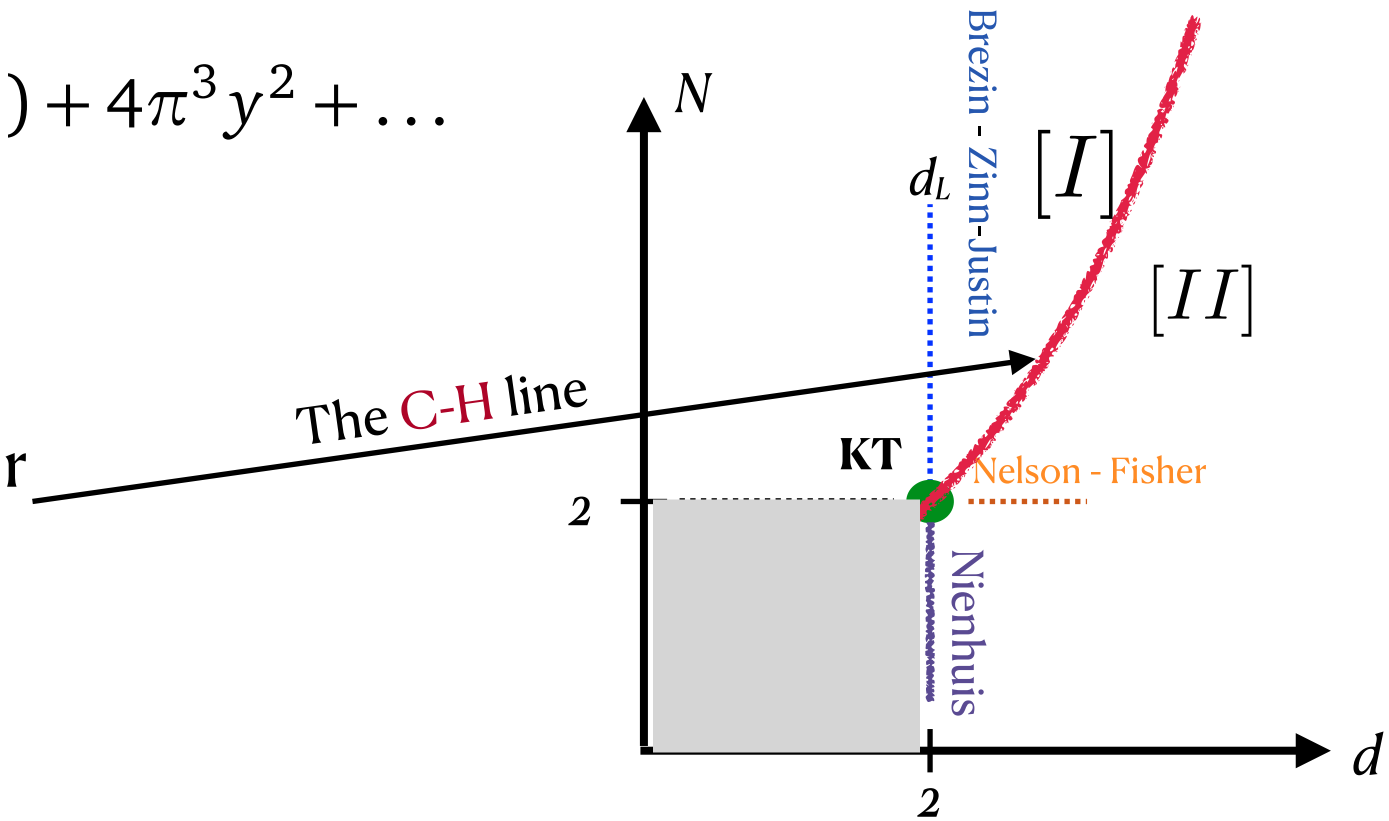
$N = 2$ recovers Nelson - Fisher

$y^2 = 0$ recovers Brezin - Zinn-Justin

The Cardy-Hamber approach (1980)

$$\dot{g} = -\epsilon g + (N-2) f(g) + 4\pi^3 y^2 + \dots$$
$$\dot{y^2} = \left(4 - \frac{2\pi}{g}\right) y^2 + \dots$$

Fixed points and their stability analysis



Present talk: $d, N \geq 2$

$d < 2$ Talk by Andrzej Chlebicki **today 16.55**

Completely different interpretation of the C-H line.

$[I]$ BZJ FP is **critical**
NF FP does not exist

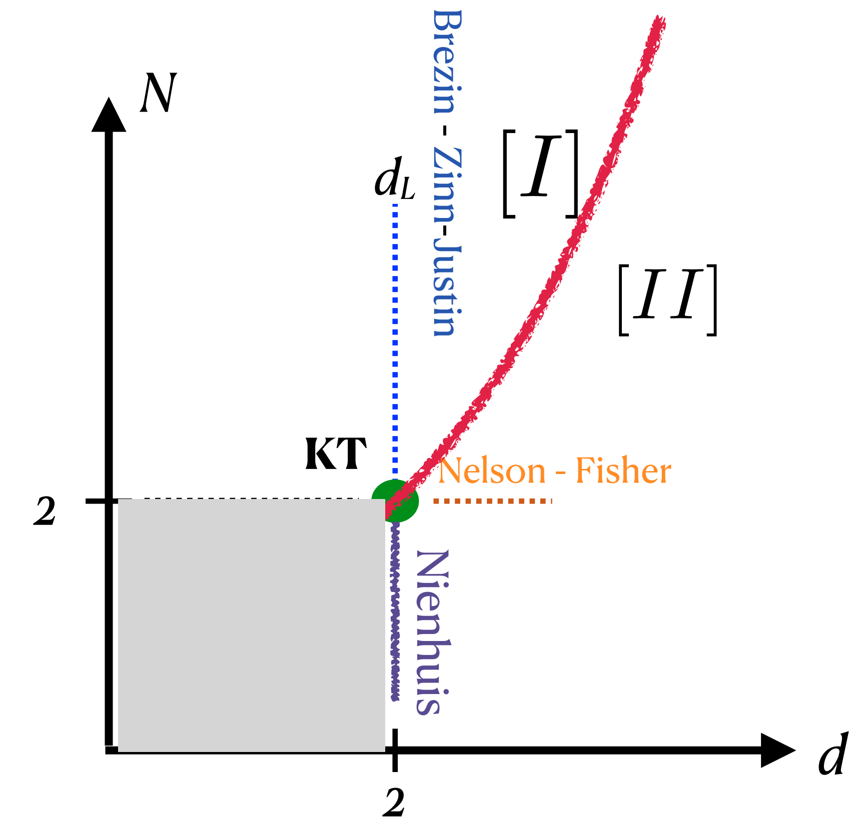
$[II]$ BZJ FP is **tricritical**
NF FP is **critical**

C-H line: BZJ and NF FPs coincide

Prediction of **non-analyticity** of critical exponents **across the C-H line**.

Discontinuous derivatives

The Cardy-Hamber approach (1980)



Remarks:

Possibly good physical reasons for the existence of the C-H line (relevance of vortex-like excitations)

Physical interpretation of *y* ambiguous for $(d, N) \neq (2, 2)$

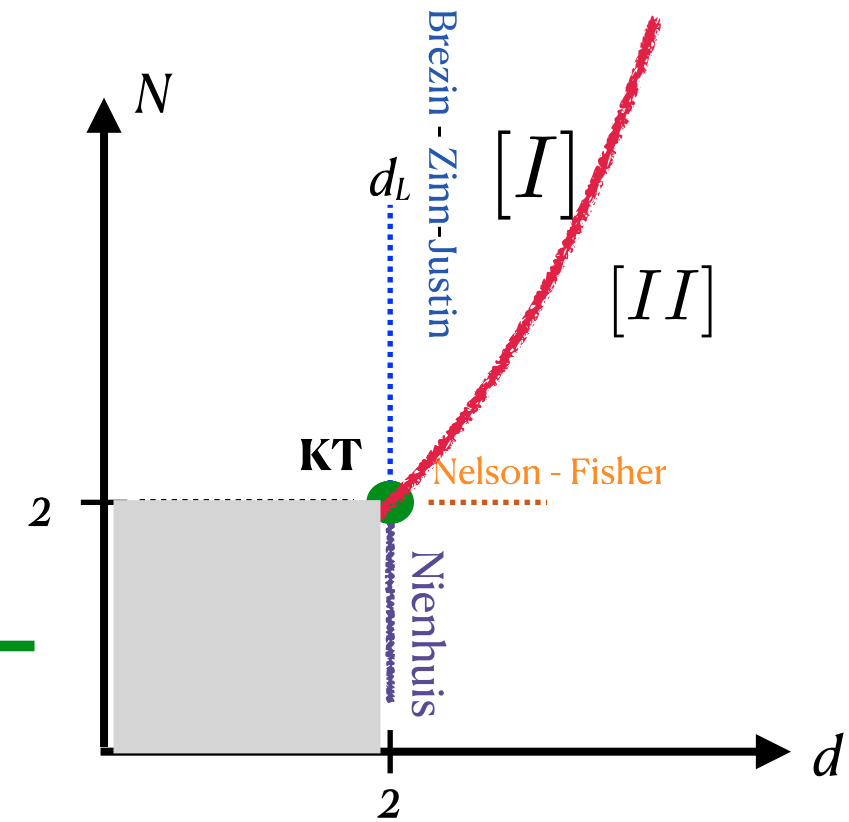
The C-H line **not** detected within other approaches (notably 1/N expansion)

The C-H line computed in linear approximation and "would appear to pass *N*=3 somewhat below *d*=3".

Can this problem be readdressed from the point of view of the DE [here at order DE2]?

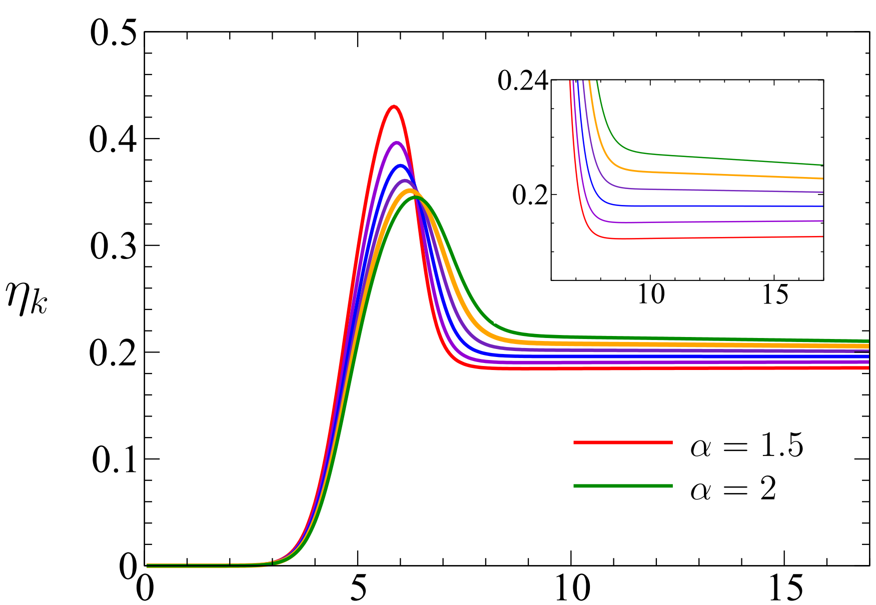
$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k \left[\Gamma_k^{(2)}[\phi] + R_k\right]^{-1}\right\}$$

$$\Gamma_k[\phi] = \int d^d x \left\{U_k(\rho) + \frac{1}{2}(Z_k(\rho) - 2\rho Y_k(\rho))(\nabla\phi)^2 + \frac{1}{4}Y_k(\rho)(\nabla\rho)^2\right\}$$



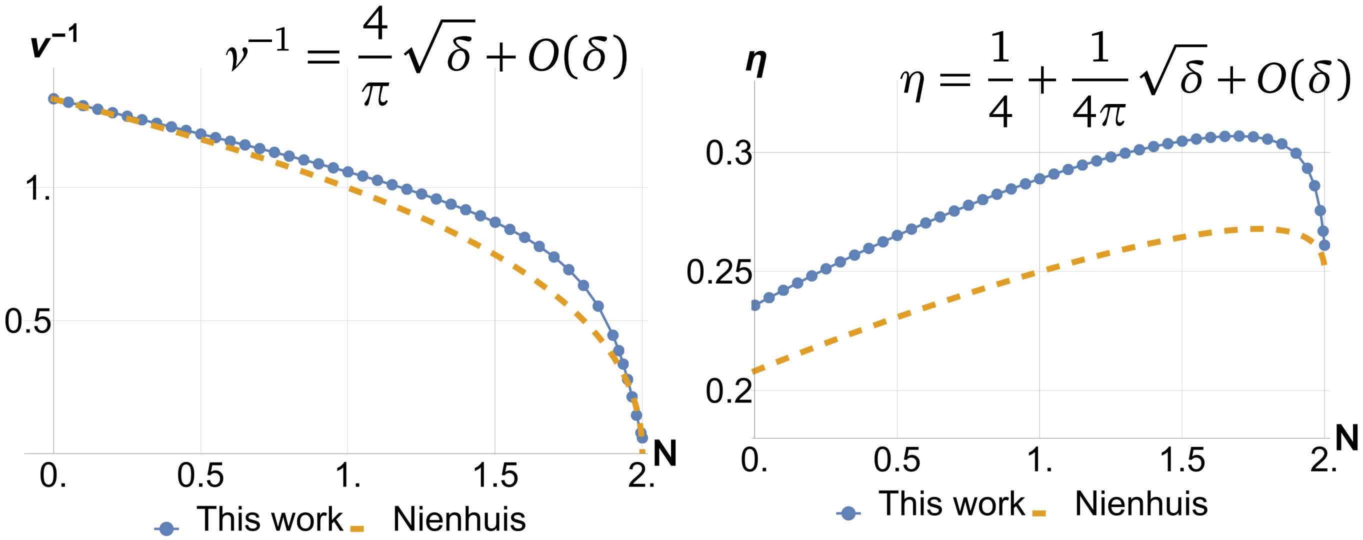
Benchmarks in *d*=2:

KT case:



Nienhuis line:

$N = 2 - \delta$

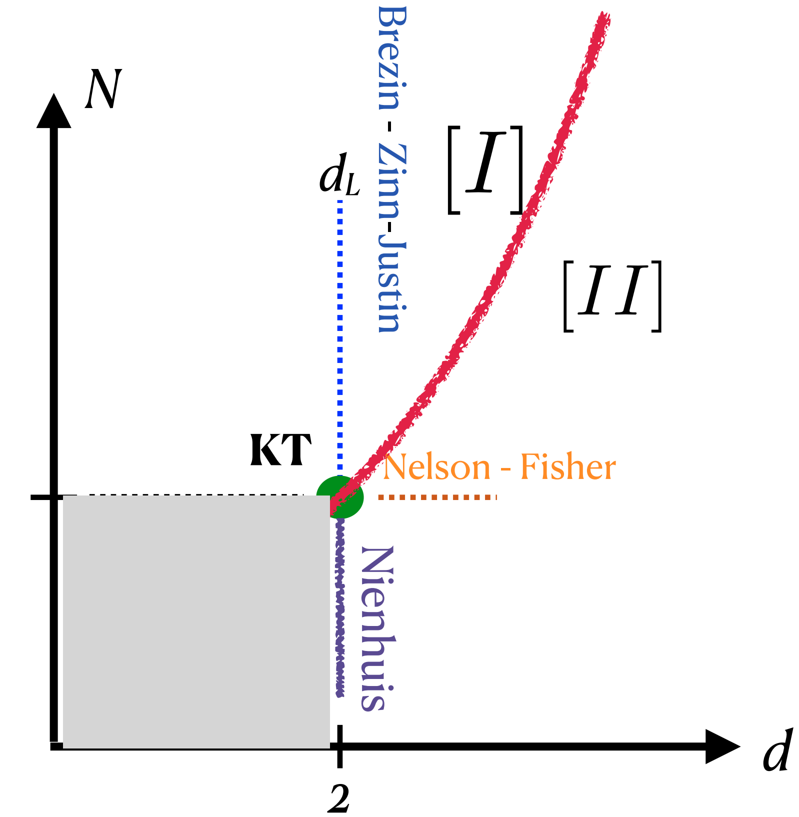


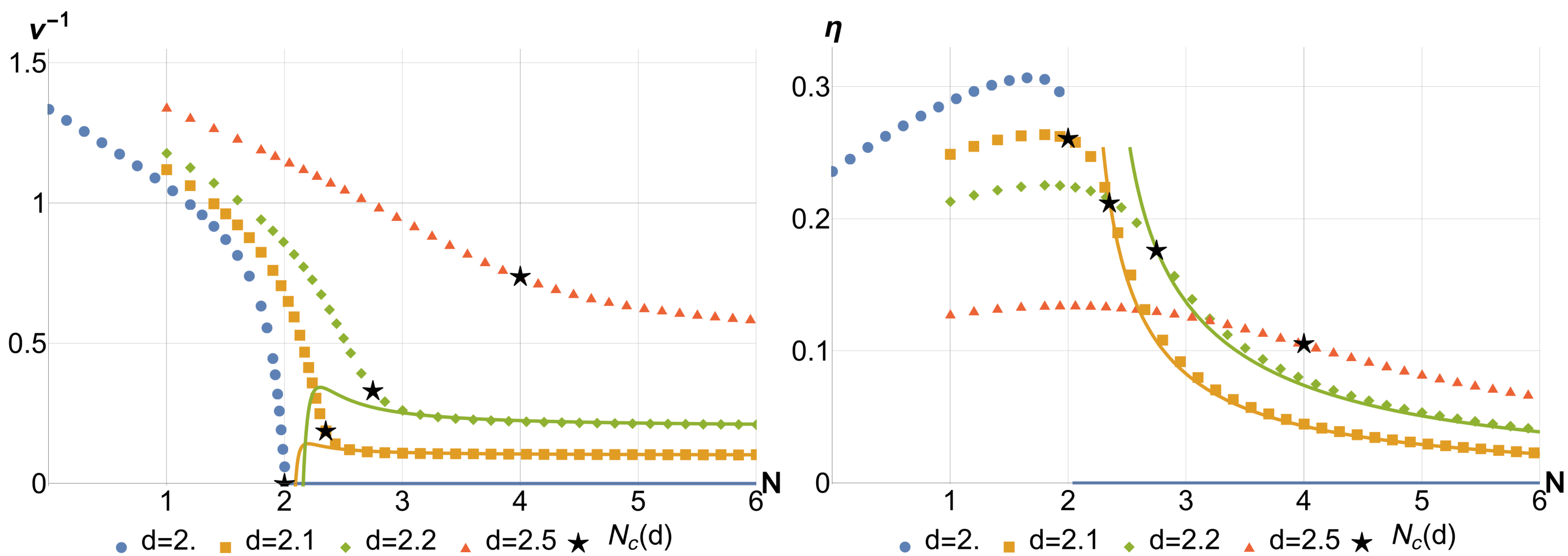
Nonanalyticity approaching (2,2) along the Nienhuis line **recovered** by DE2.

Critical exponents for *d*,*N*>2:

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\left\{\partial_k R_k \left[\Gamma_k^{(2)}[\phi] + R_k\right]^{-1}\right\}$$

$$\Gamma_k[\phi] = \int d^d x \left\{U_k(\rho) + \frac{1}{2}(Z_k(\rho) - 2\rho Y_k(\rho))(\nabla\phi)^2 + \frac{1}{4}Y_k(\rho)(\nabla\rho)^2\right\}$$



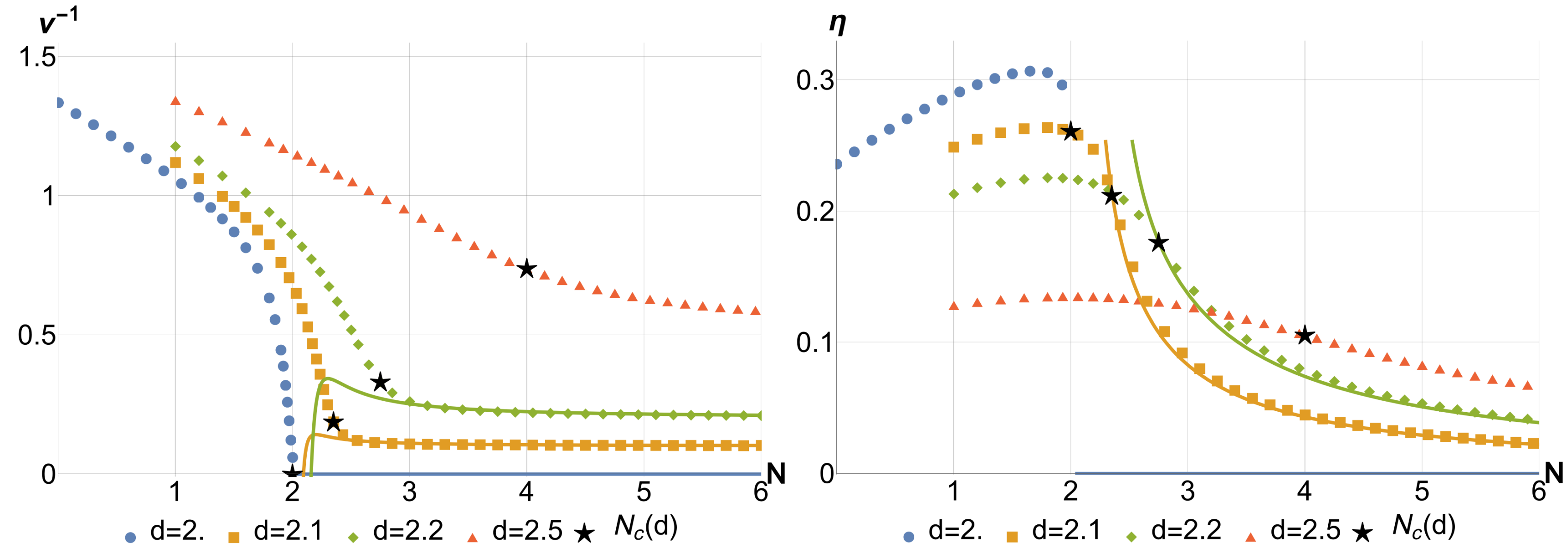


Signatures of the (hypothetical) C-H line visible only as crossover.

→ Sharpening for *d* approaching 2.

→ Very smooth for *d* large.

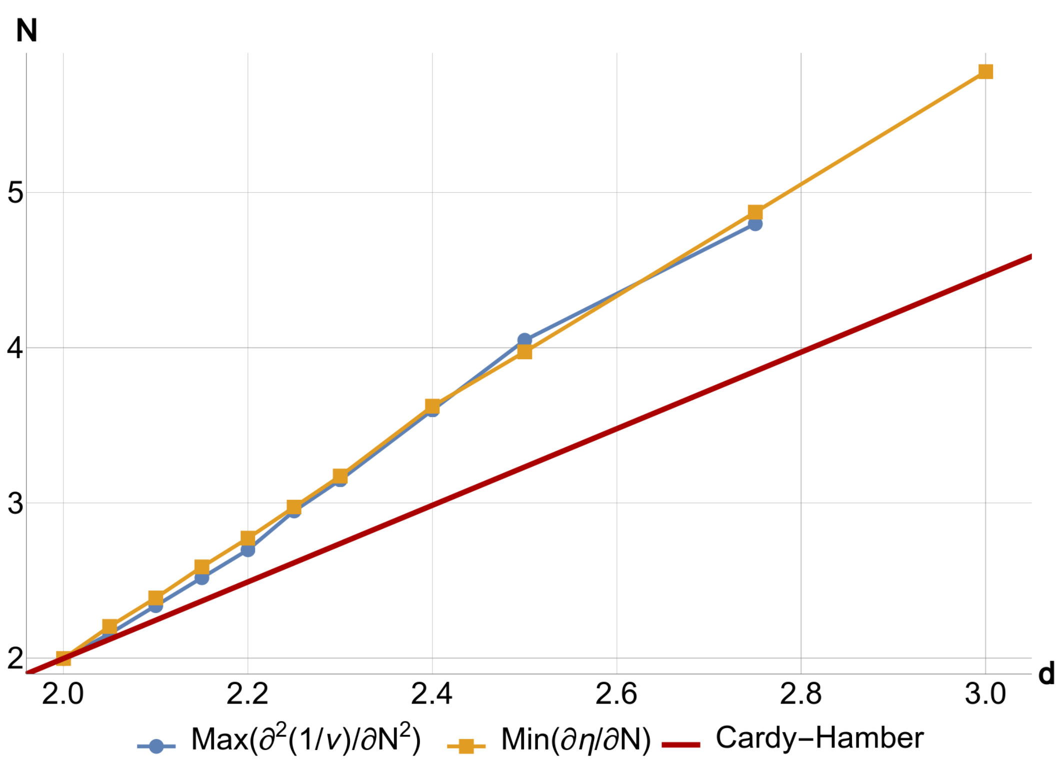
Critical exponents for *d,N*>2:



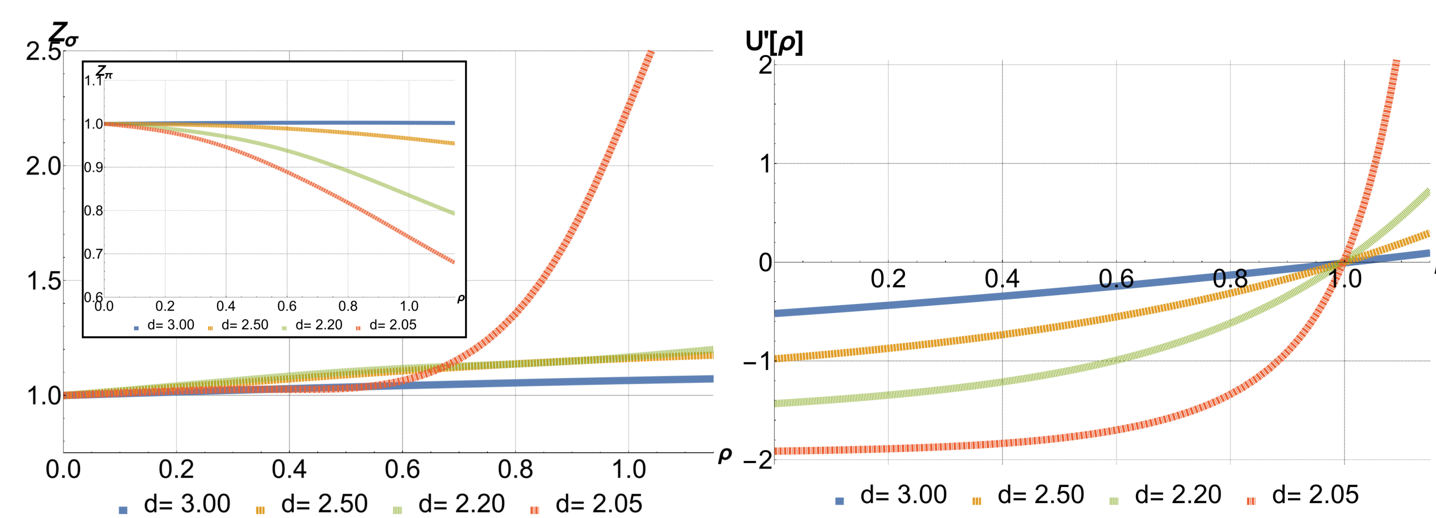
Signatures of the (hypothetical) C-H line visible only as crossover.

Sharpening for *d* approaching 2.

Very smooth for *d* large.



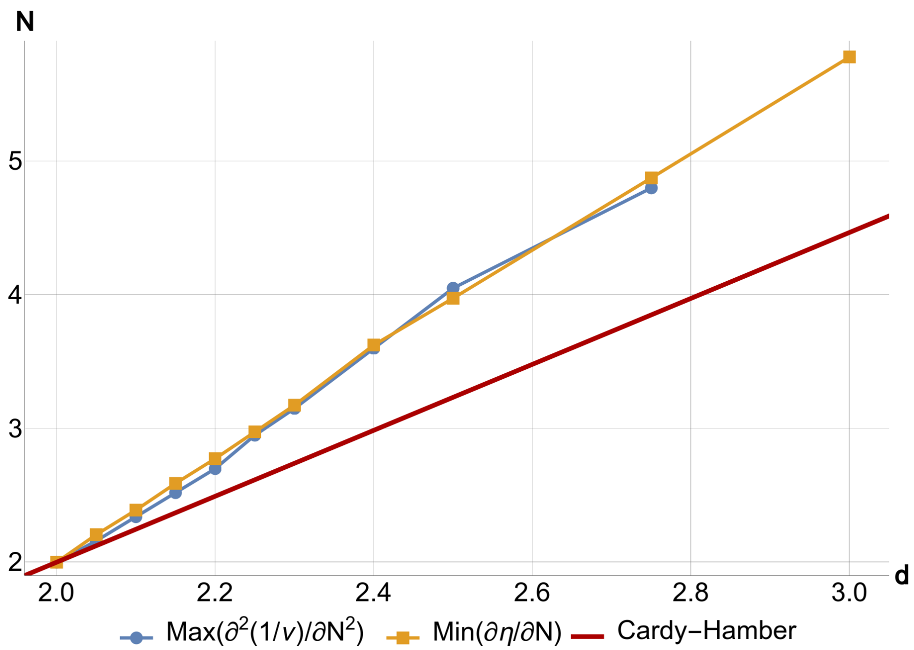
Crossover in the FP structure:



Signatures of the (hypothetical) C-H line visible only as crossover.

→ Sharpening for *d* approaching 2.

→ Very smooth for *d* large.



Contradiction with the C-H prediction!

(At least numerically at this stage.)

Possible reasons:

1. **DE** is not adequate; in particular topological aspects not captured ?

2. C-H line is an artefact of the assumed analyticity of the C-H flow eq. ?

3. … ?

Possible reasons:

1. **DE** is not adequate: topological aspects not captured ?

2. C-H line is an artefact of the assumed analyticity of the C-H flow eq. ?

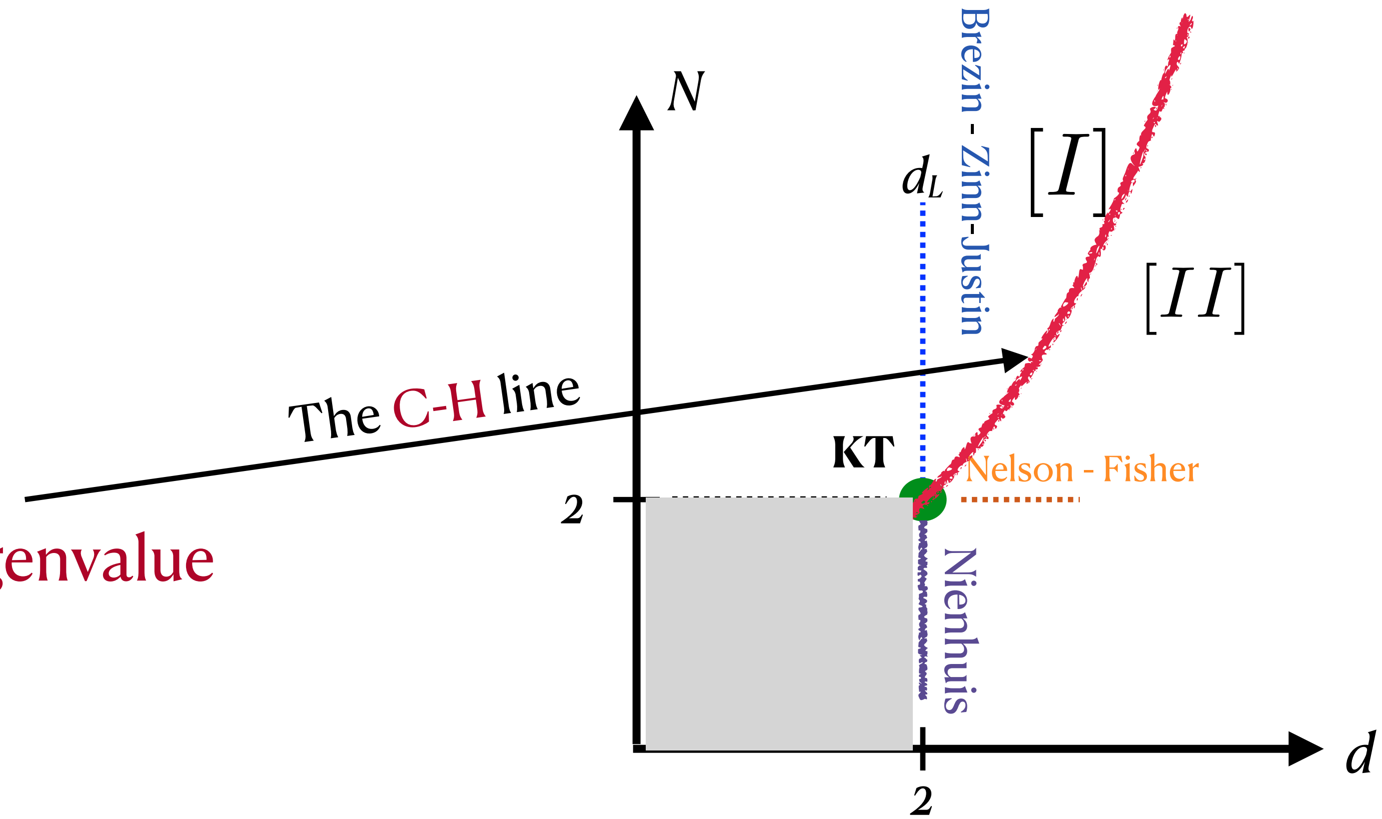
3. ... ?

Incorrect (?):

- KT
- (d,N)=(3,3)

O. I. Motrunich and A. Vishwanath, *Emergent photons and transitions in the O(3) sigma model with hedgehog suppression*, Phys. Rev. B **70**, 075104 (2004), doi:10.1103/PhysRevB.70.075104.

Another prediction of the C-H approach:



Vanishing subdominant eigenvalue

[I] BZJ FP is **critical**
NF FP does not exist

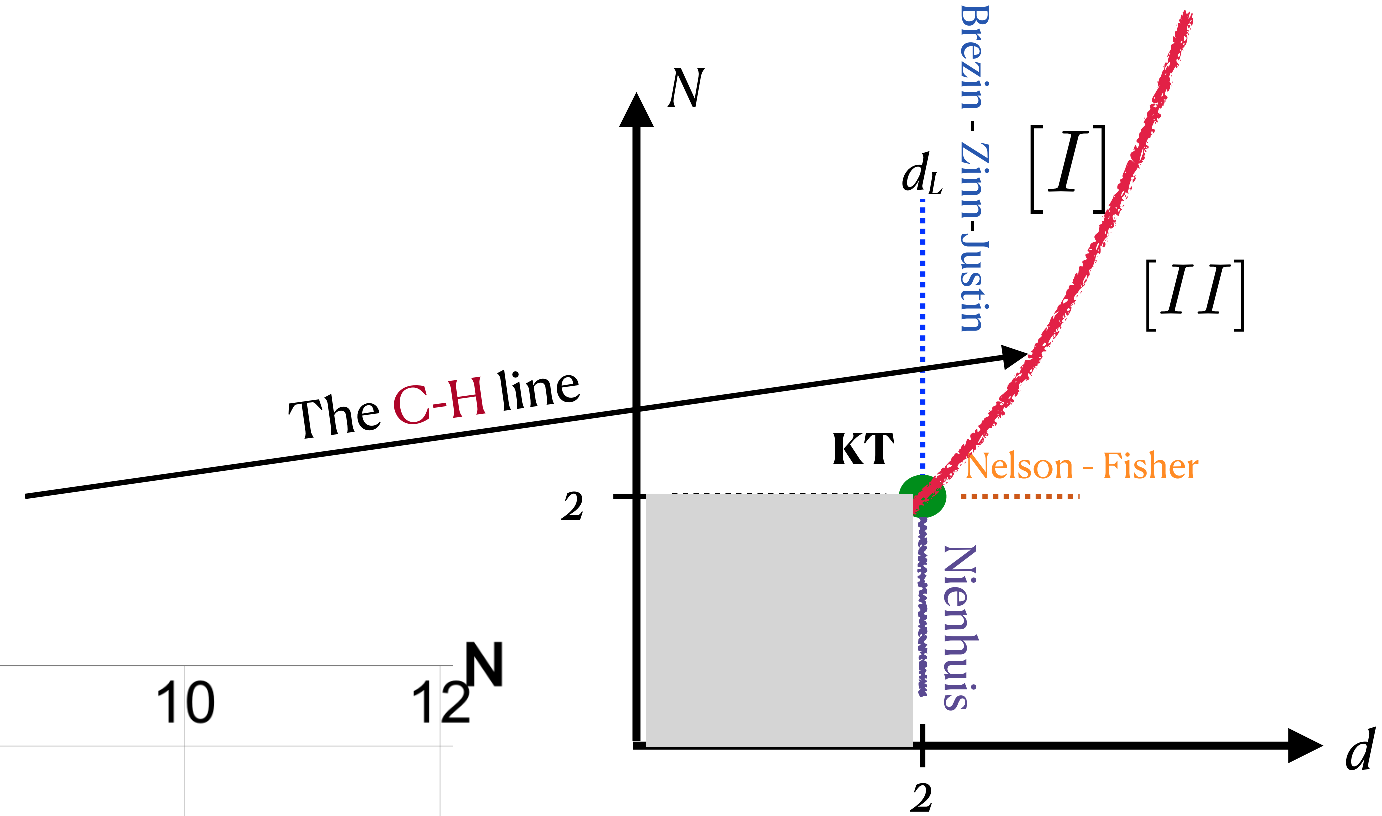
[II] BZJ FP is **tricritical**
NF FP is **critical**

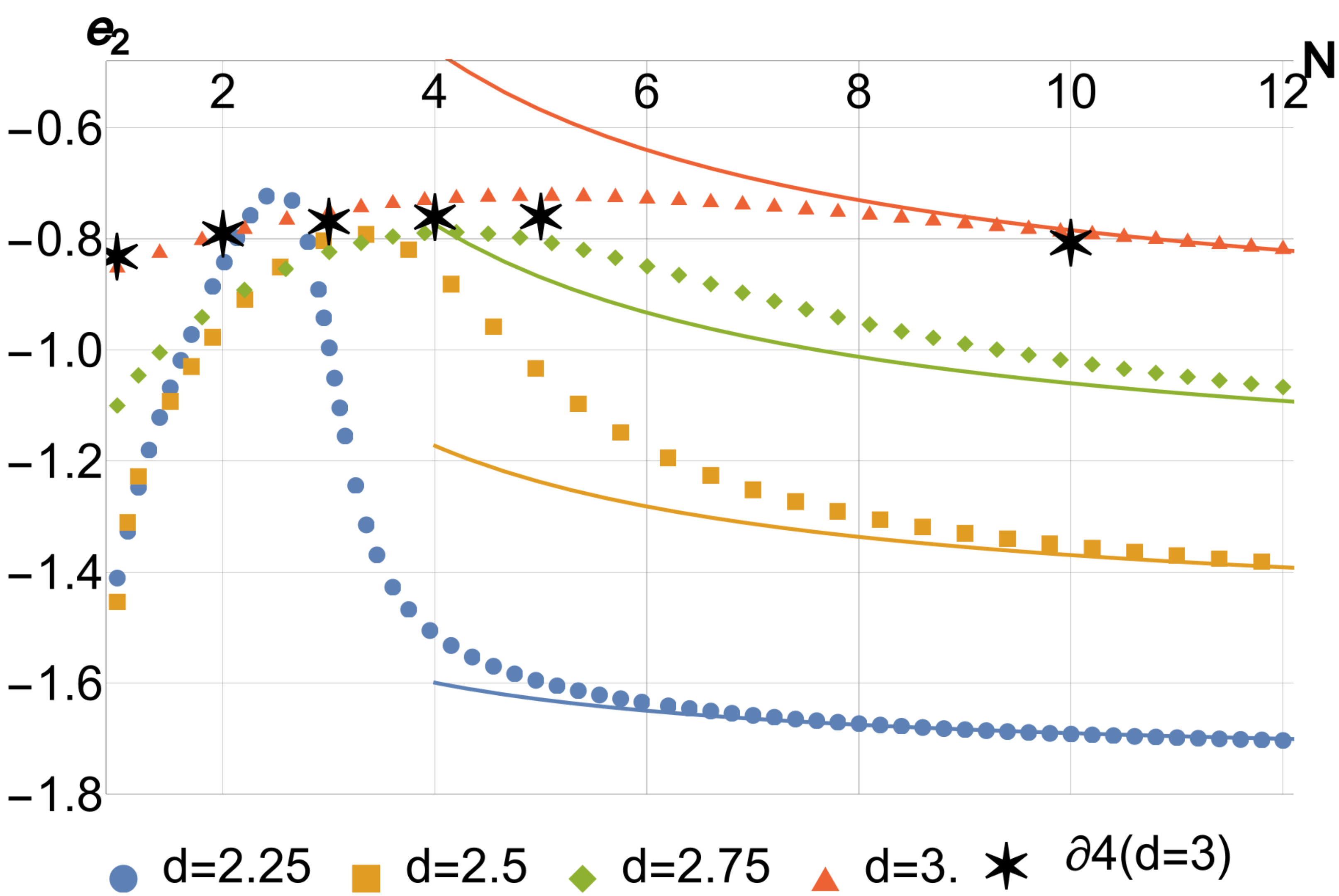
C-H line: BZJ and NF FPs coincide

In $d=3$ can be confronted against:

- DE4 → Very reliable close to $d=3$!
- MC

Vanishing subdominant eigenvalue



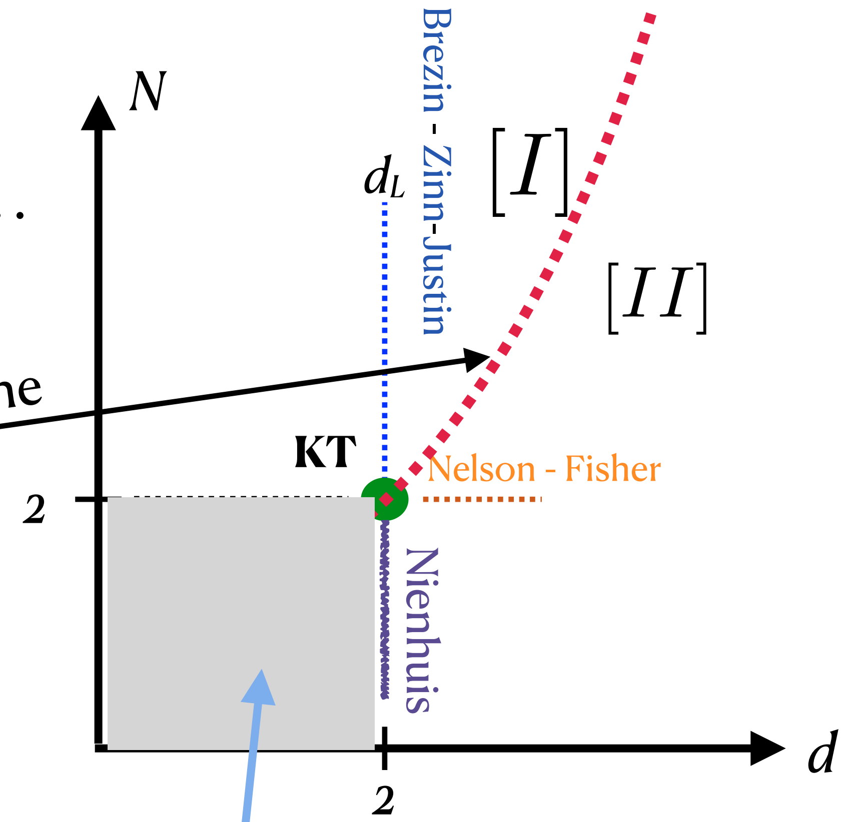


No signatures of e_2 approaching zero for d=3 (or close)

Conclusion:

$$\dot{g} = -\epsilon g + (N-2) f(g) + 4\pi^3 y^2 + \ldots$$
$$\dot{y^2} = \left(4 - \frac{2\pi}{g}\right) y^2 + \ldots$$

The C-H line



~~Vanishing subdominant eigenvalue; critical exponents' nonanalyticity.~~

DE2

Crossover, sharpening into true nonanalyticity for *d* approaching 2.

Existence of the C-H line excluded in the vicinity of *d*=3, questionable for *d* approaching 2.

A. Chlebicki & P. J. SciPost Phys. 10, 134 (2021)

(DE2 in conjunction with DE4, MC)

Part II of the C-H vs DE story (*d*<2)

See the talk by Andrzej Chlebicki