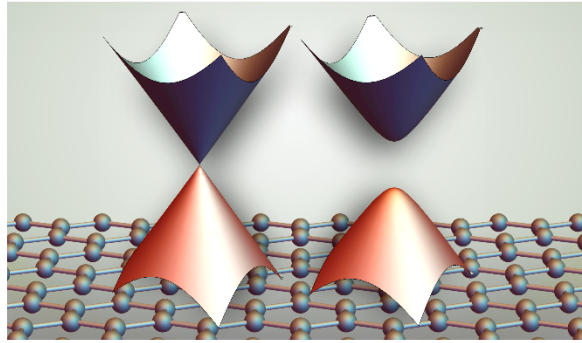




Research Group



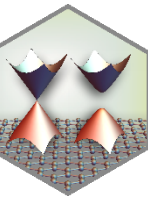
Unconventional superconductivity in moiré transition metal dichalcogenides

Laura Classen

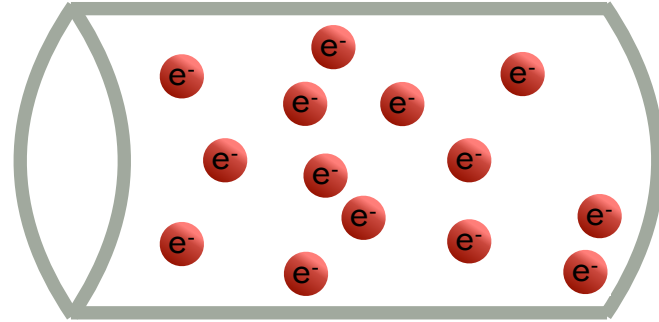
Nico Gneist, Michael Scherer (Bochum)
Lennart Klebl, Ammon Fischer, Dante Kennes (RWTH Aachen)



Correlated phases in quantum materials

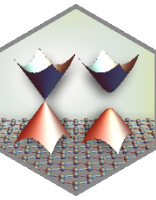


- Collective phases of interacting electron systems?

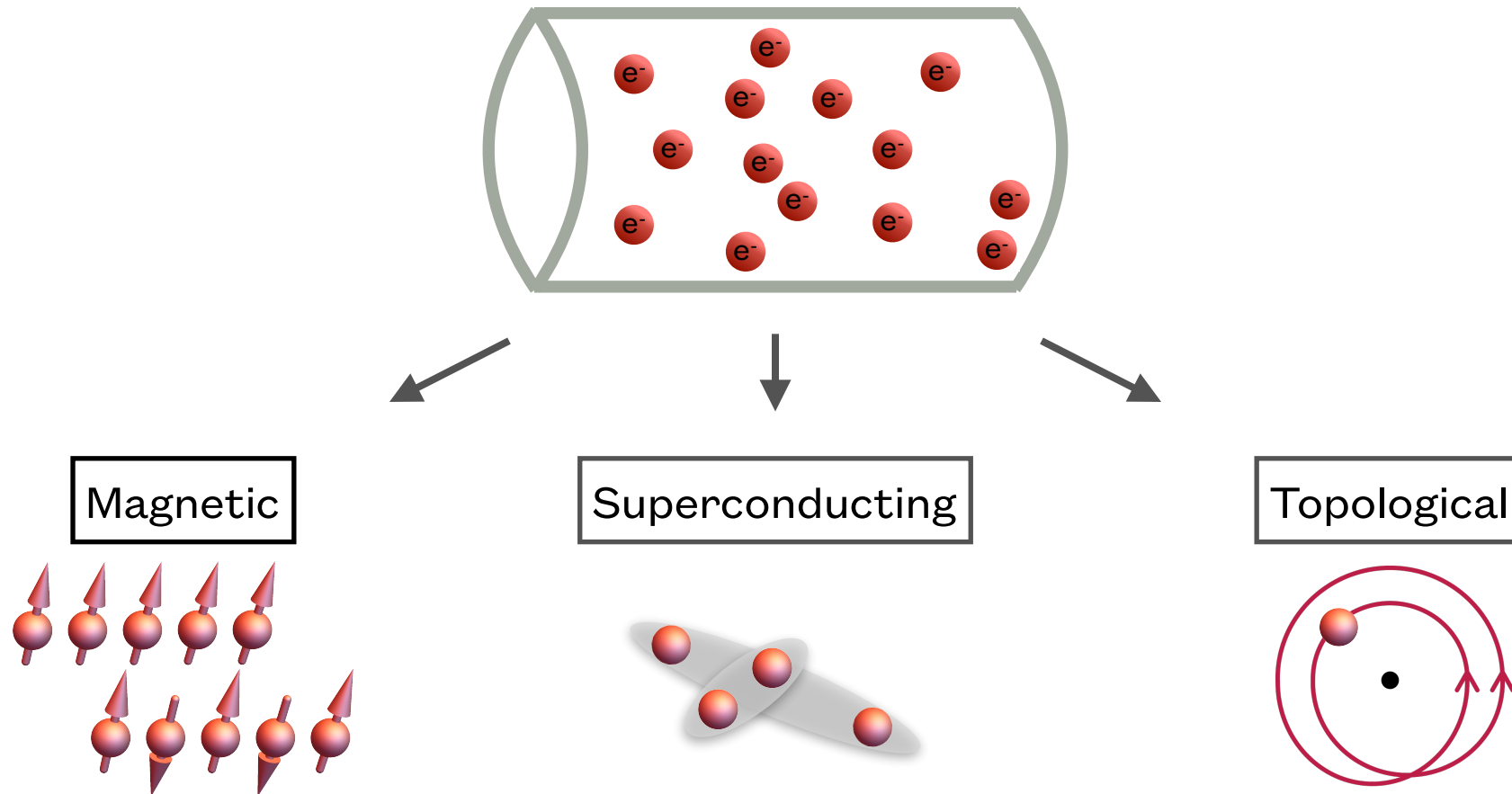




Correlated phases in quantum materials

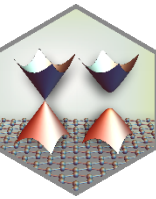


- Collective phases of interacting electron systems?

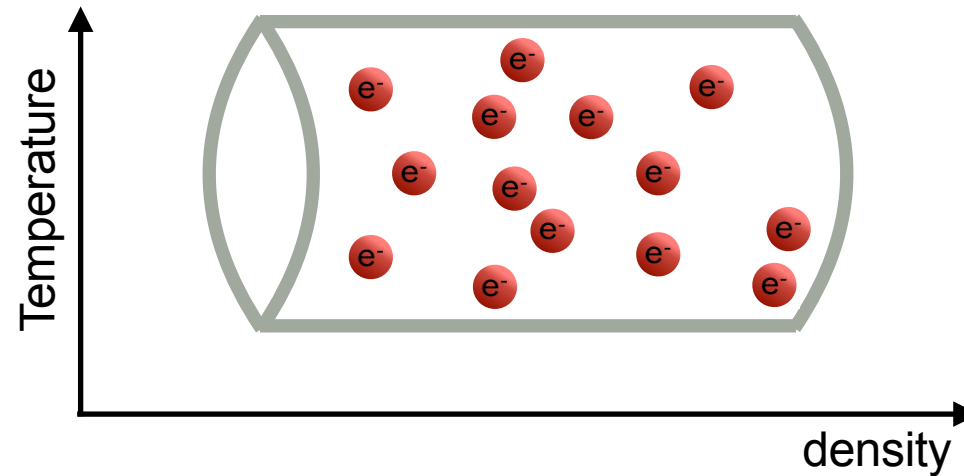




Correlated phases in quantum materials



- Collective phases of interacting electron systems?

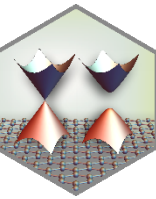


- Tuning knobs: temperature, density, external fields,

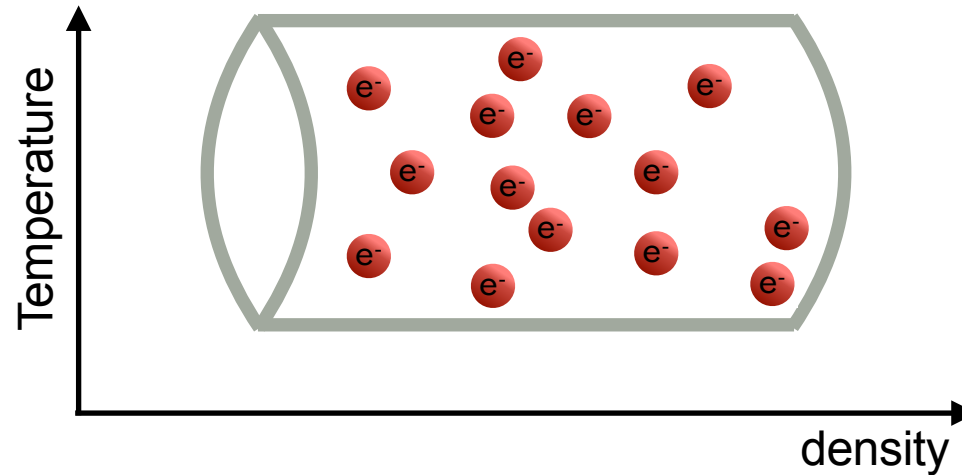




Correlated phases in quantum materials

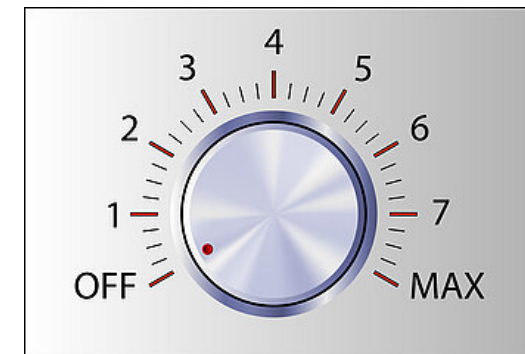


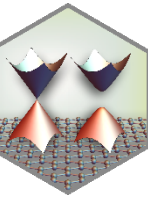
- Collective phases of interacting electron systems?



- Tuning knobs: temperature, density, external fields,
many different (classes of) materials!

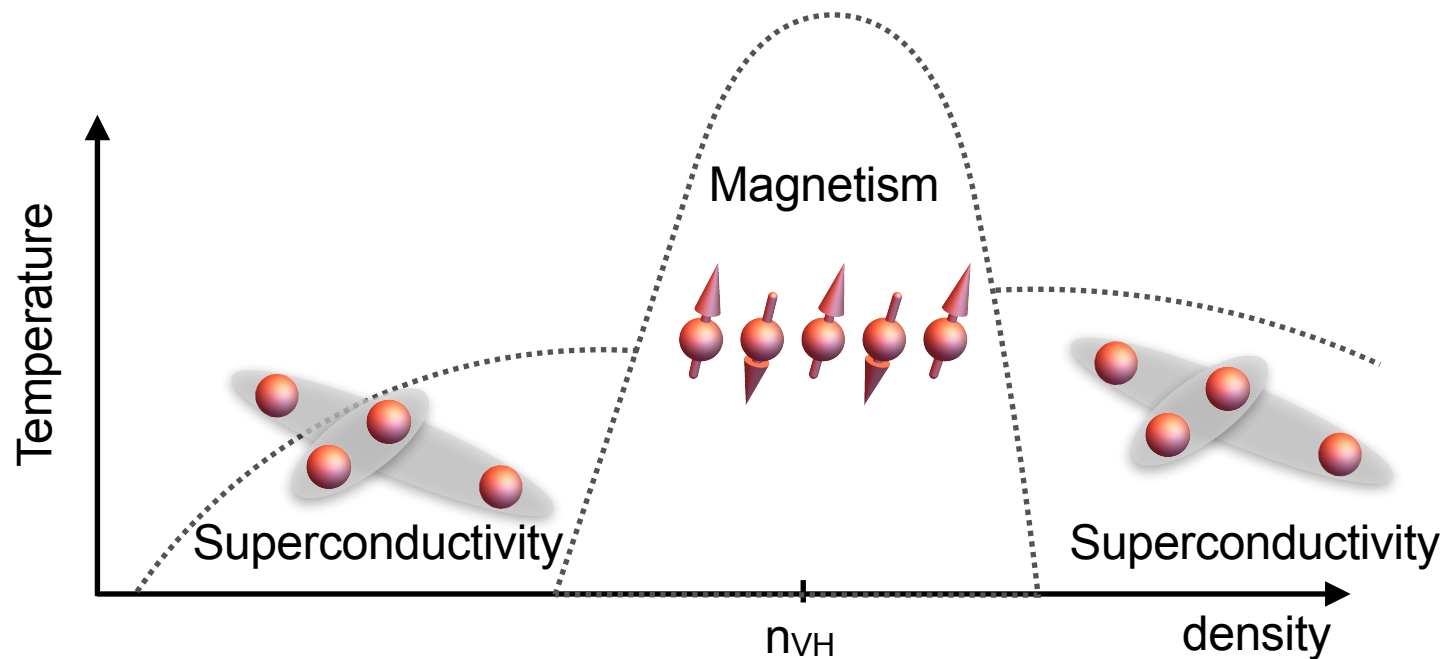
High-Tc superconductors, quantum magnets,
Dirac materials, low-dimensional systems,

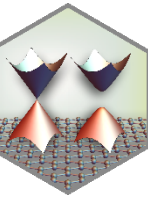




Outline

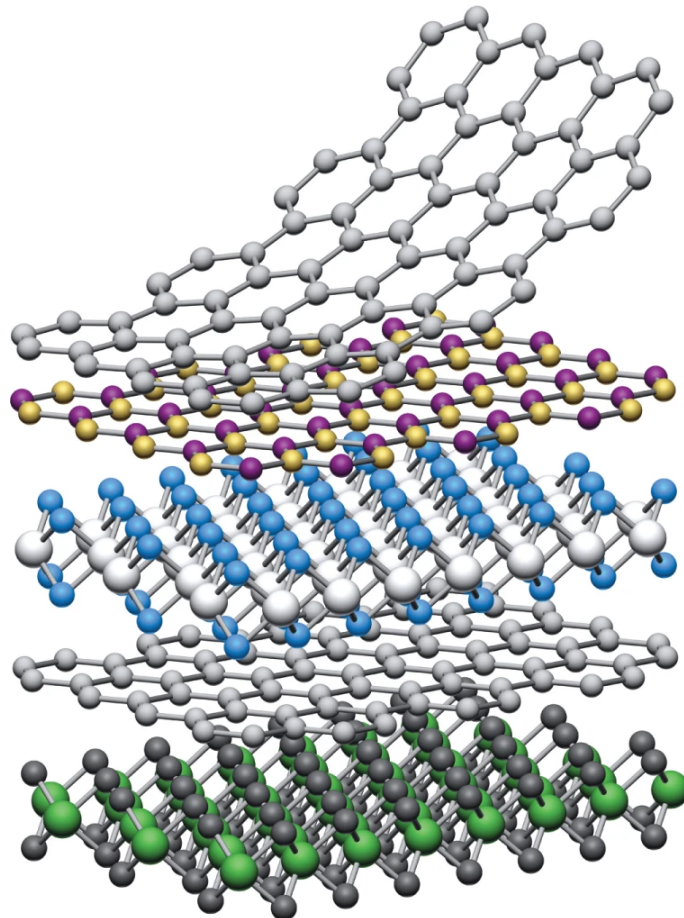
- Correlated physics in 2D moiré materials
- Simulate triangular-lattice Hubbard model
- FRG: superconductivity from repulsive interactions/interplay of orders

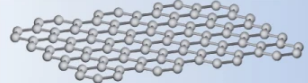

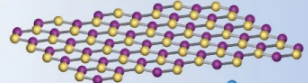

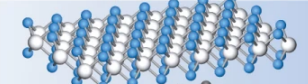

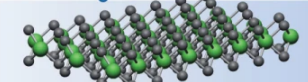

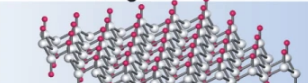



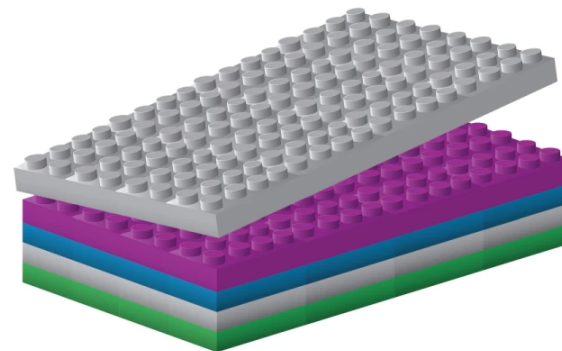


2D Materials

- Designer heterostructures from stacking and twisting 2D materials



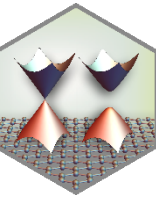
	Graphene	
	hBN	
	MoS ₂	
	WSe ₂	
	Fluorographene	



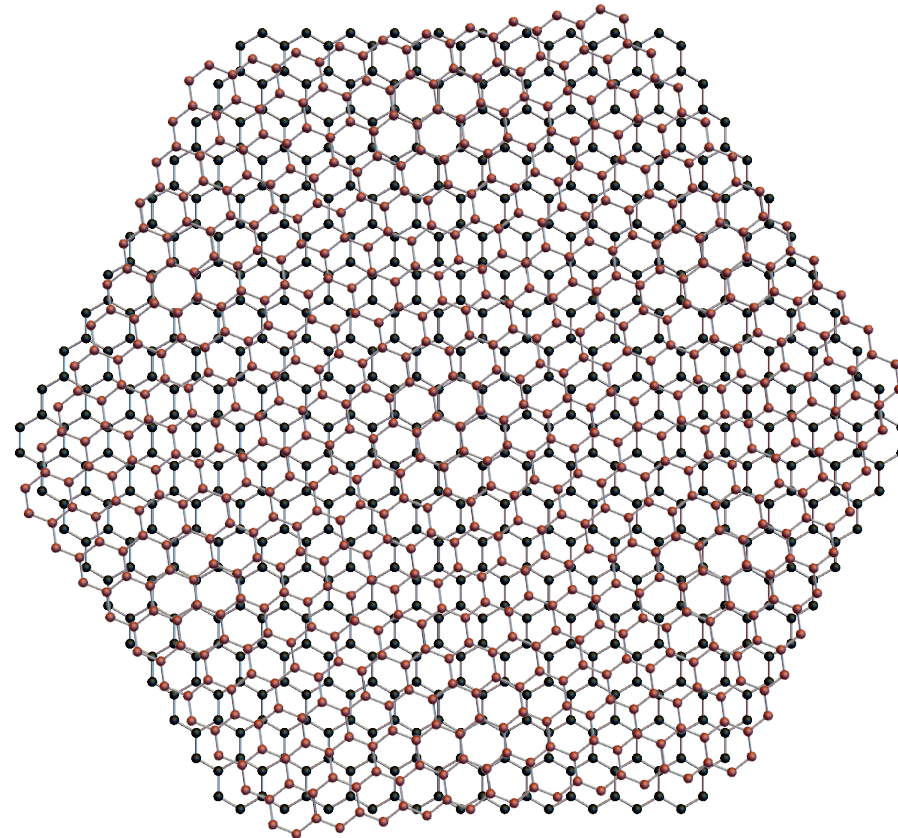
 Geim & Grigorieva, Nature (2013)



Twisted bilayer graphene

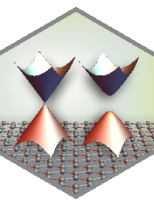


- Graphene: carbon atoms on honeycomb lattice (Dirac fermions, weakly correlated)

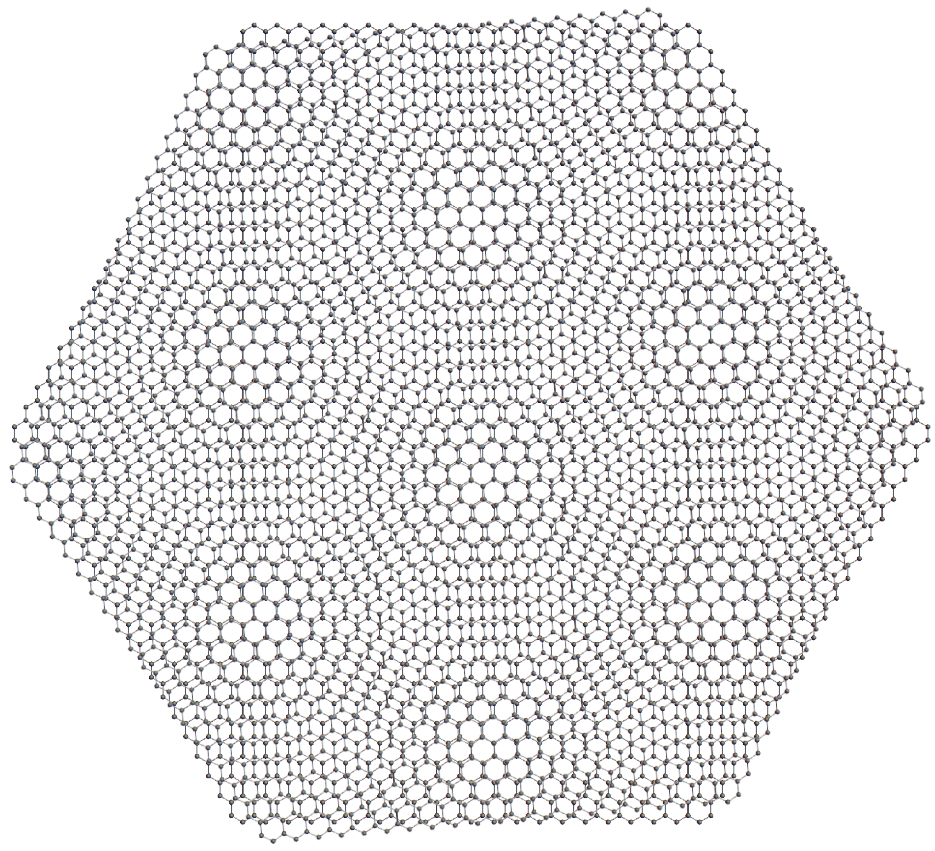




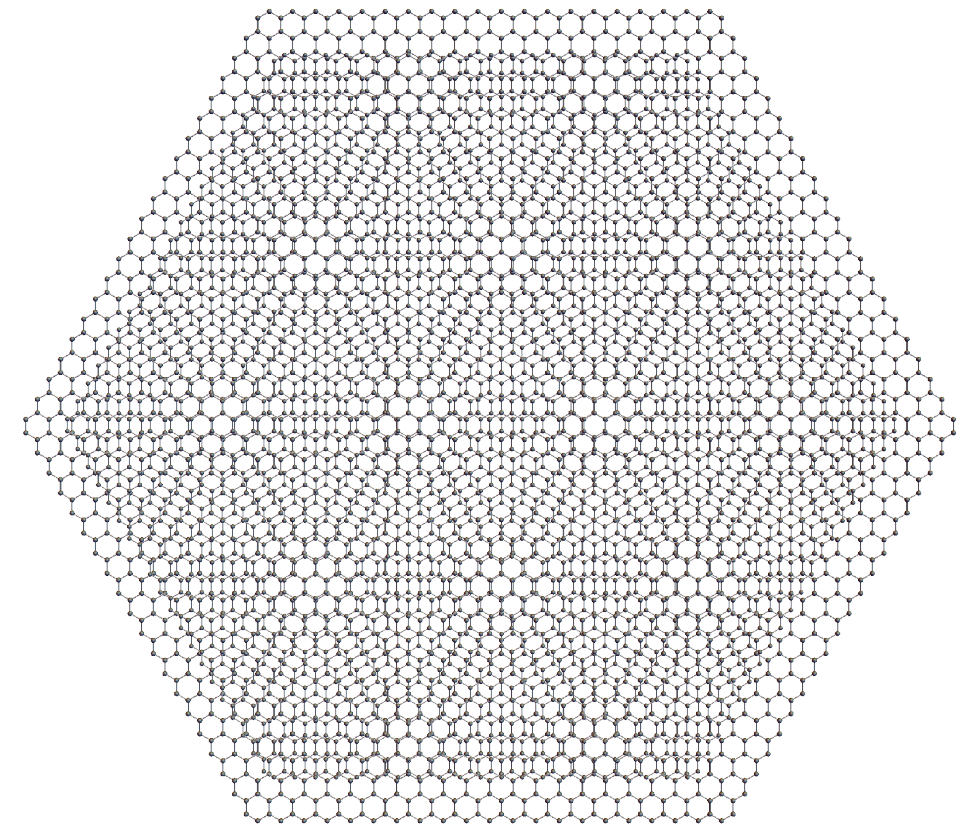
Moiré materials



- Overlay two periodic structures with small mismatch \rightarrow **moiré** pattern (superlattice)



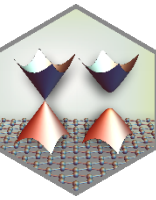
rotation by $\theta = 5^\circ$



$\theta = 0^\circ$ but different lattice constants

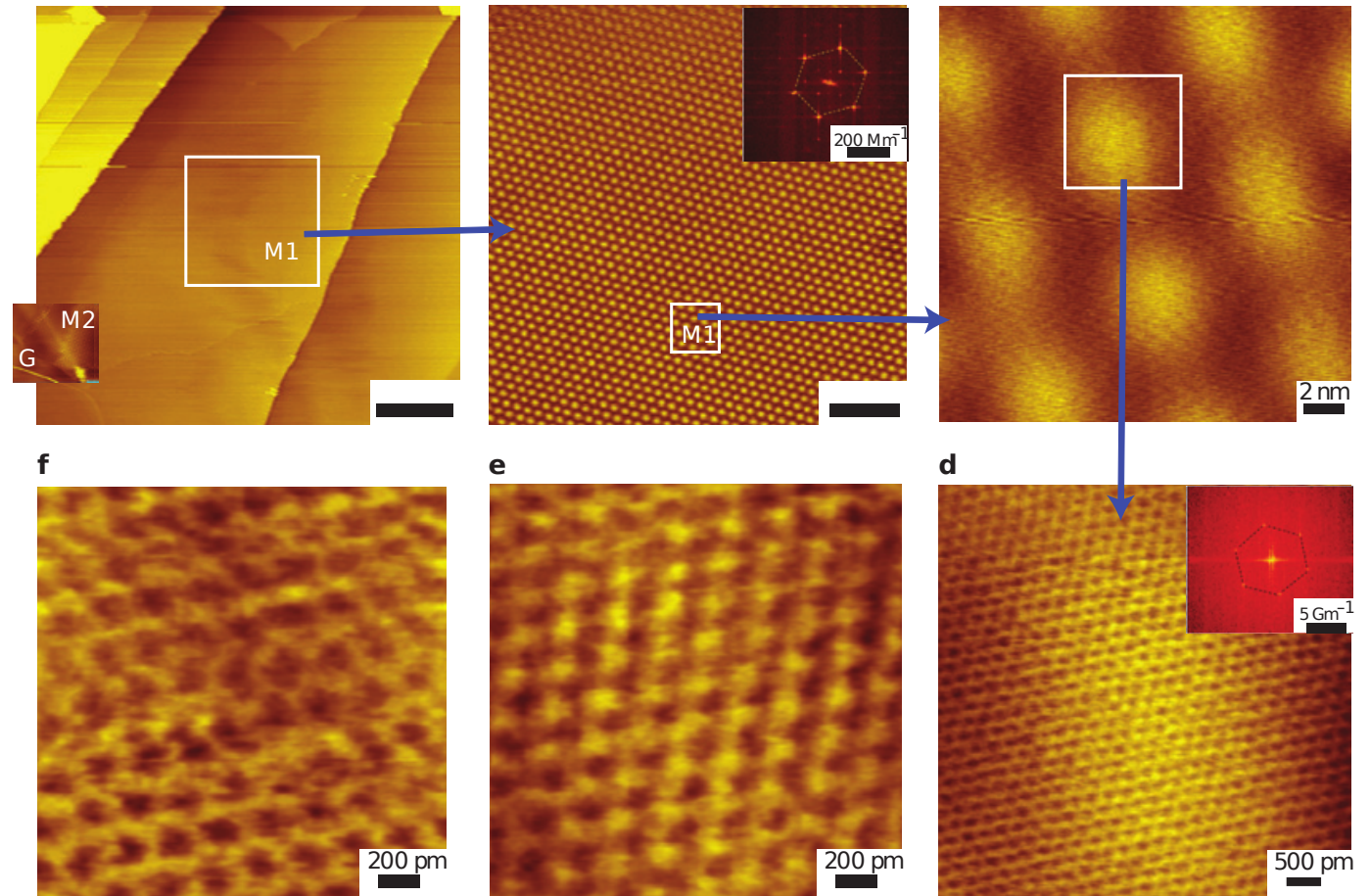


Twisted bilayer graphene



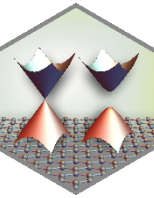
- Scanning tunneling microscopy image of two graphene layers with relative rotation of 1.8°

 Li *et al.*, Nature Physics (2010)

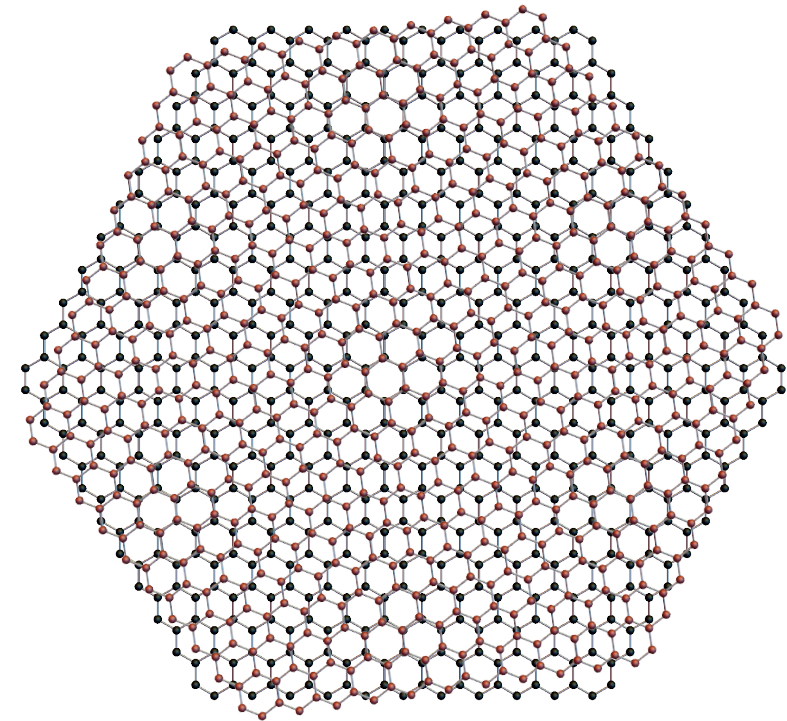
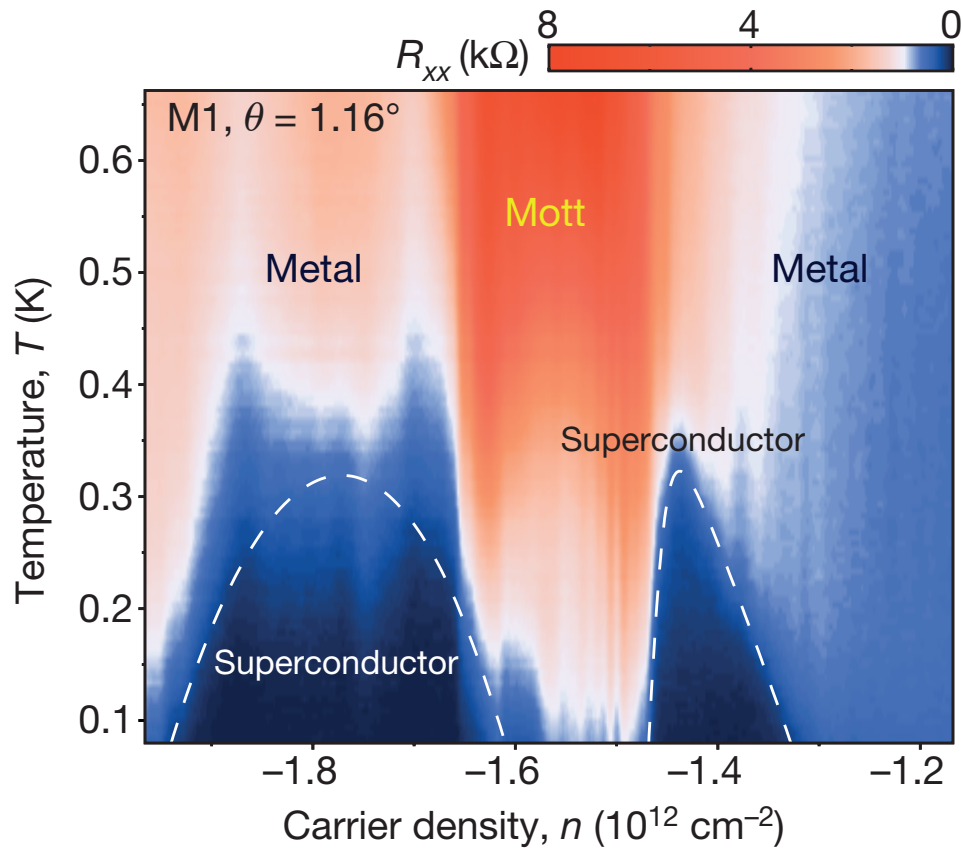




Magic angle twisted bilayer graphene



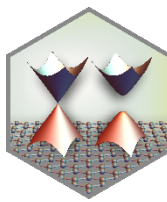
- Strong interactions arise: correlated insulators and superconductors around $\theta^\circ \sim 1$



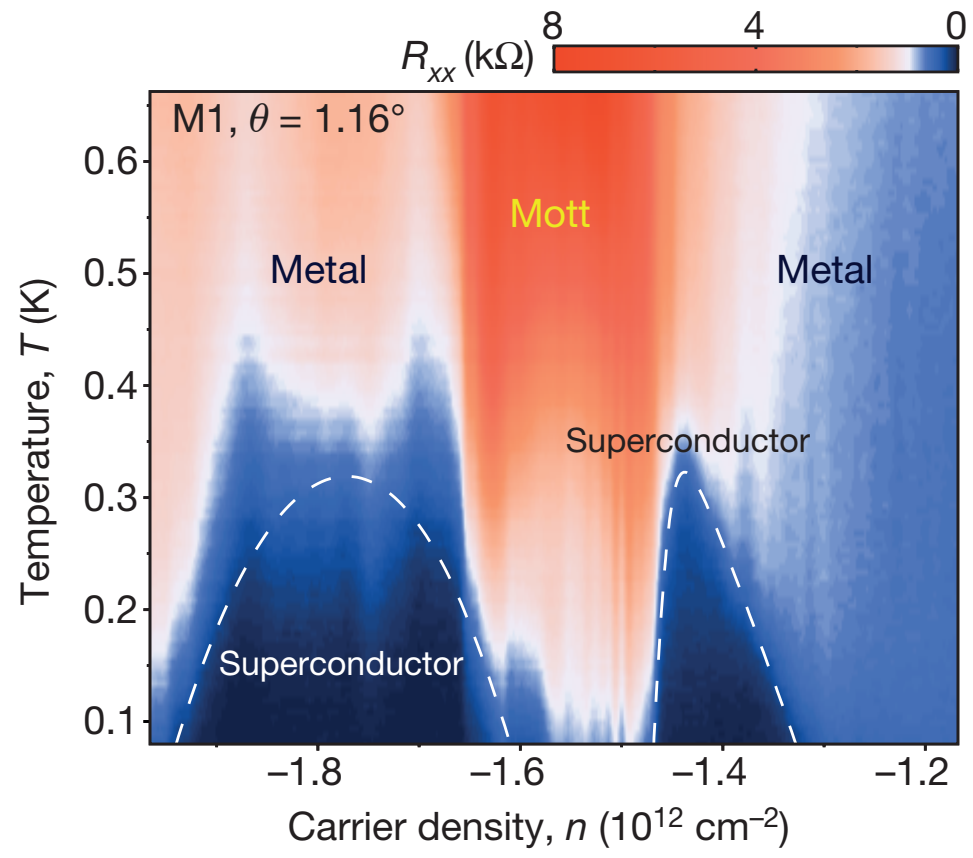
Cao *et al.*, Nature (2018)



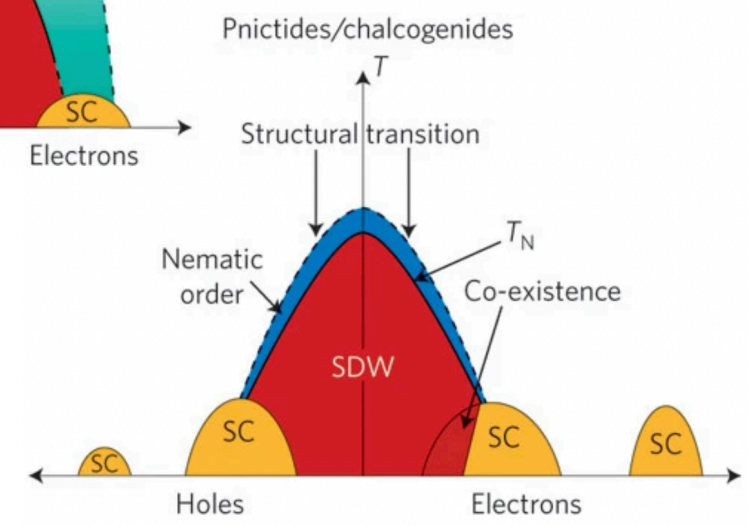
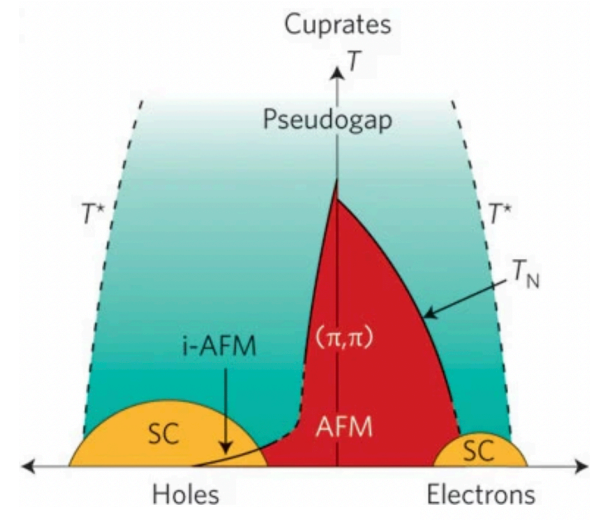
Magic angle twisted bilayer graphene



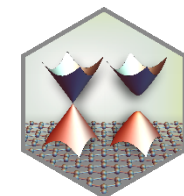
- Strong interactions arise: correlated insulators and superconductors around $\theta^\circ \sim 1$



Cao *et al.*, Nature (2018)



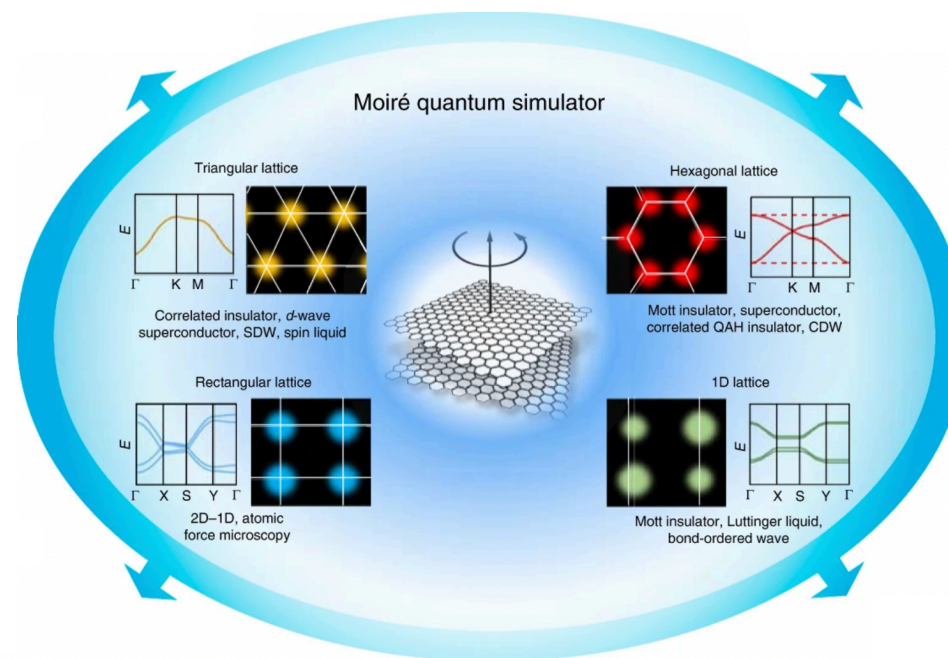
Basov, Chubukov, Nature Physics (2011)



Moiré materials as “quantum simulators”

- Correlated phases in many other 2D heterostructures
- Moiré potential quenches kinetic energy
 - New platform for study of correlated physics
- High degree of control: stacking, twisting, gating, screening layers, light-matter interaction
 - Quantum simulation between cold atoms and solid state materials

 Kennes et al, Nature Physics (2021)



Transition metal dichalcogenides

Article

Simulation of Hubbard model physics in WSe_2/WS_2 moiré superlattices

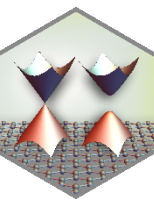
<https://doi.org/10.1038/s41586-020-2085-3>

Received: 18 August 2019

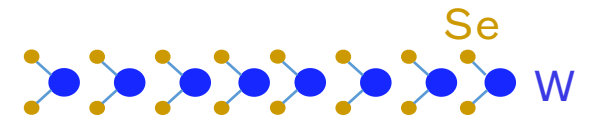
Yanhao Tang¹, Lizhong Li¹, Tingxin Li¹, Yang Xu¹, Song Liu², Katayun Barmak³, Kenji Watanabe⁴, Takashi Taniguchi⁴, Allan H. MacDonald⁵, Jie Shan^{1,6,7,8} & Kin Fai Mak^{1,6,7,8}



Transition metal dichalcogenides (TMDs)

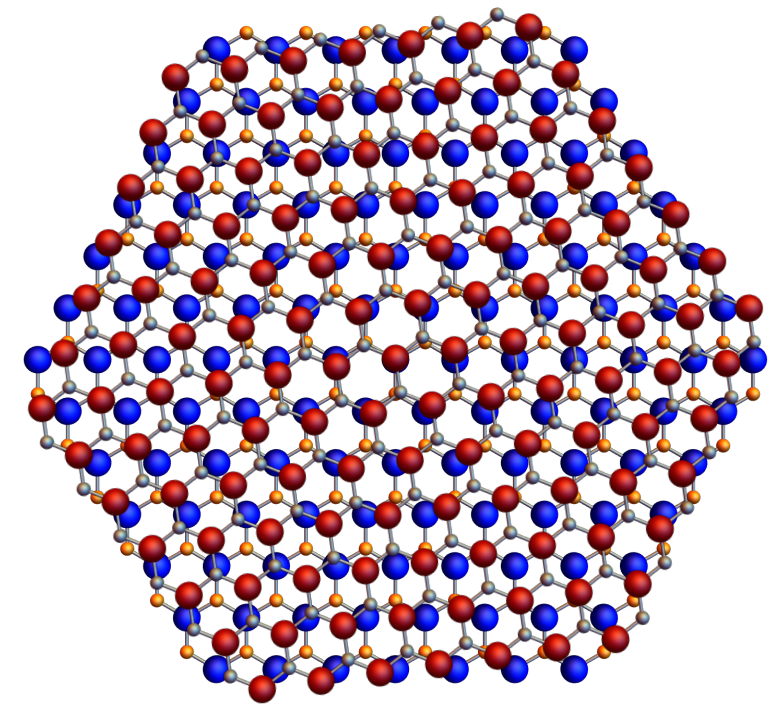


- Also form 2D materials (3 atom thick)



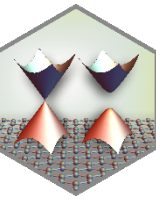
MX_2
M=Transition metal
X=Chalcogen

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg	3	4	5	6	7	8	9	10	11	12	Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac-Lr	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo

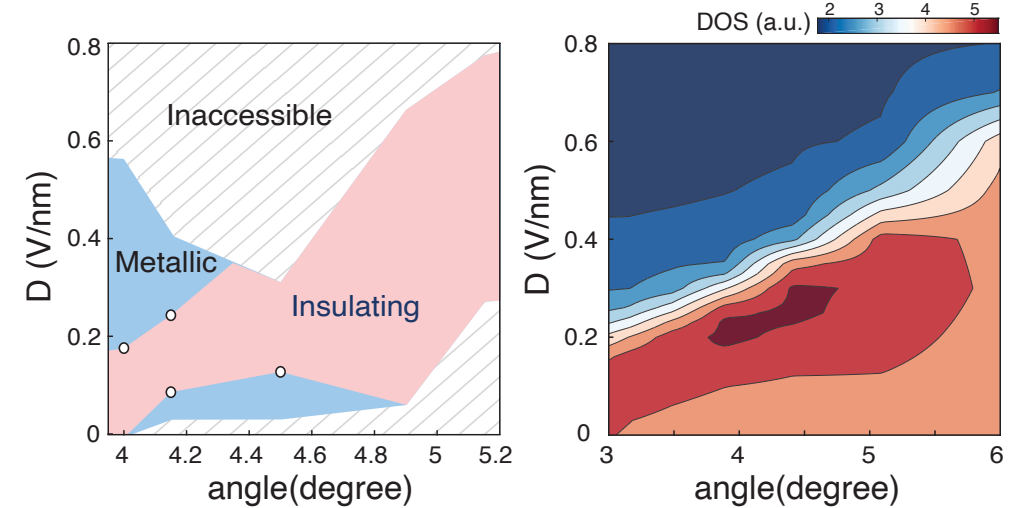




Correlated states in moiré TMDs



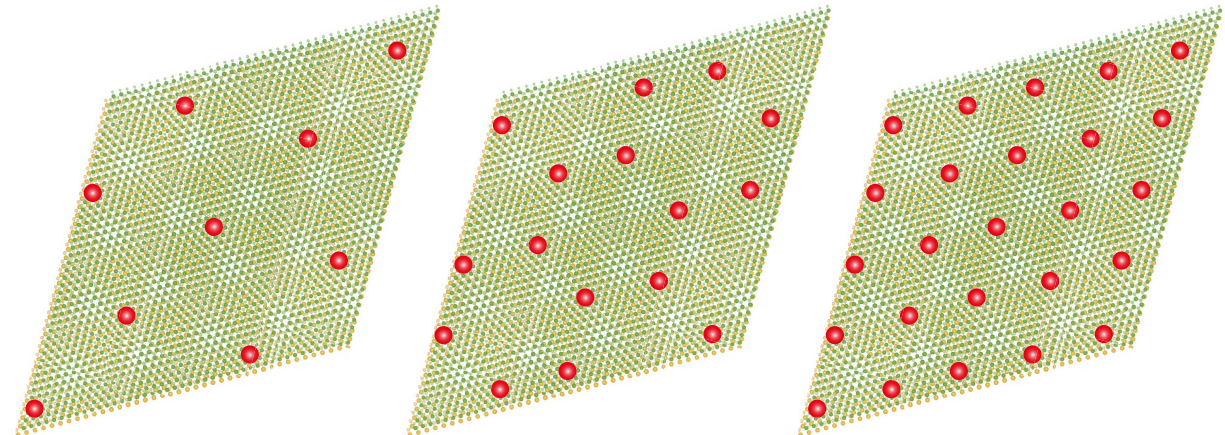
- “Magic continuum” of angles
- Also correlated insulators reported
 - “Cooperation” between insulator at 1/2 filling & (Van Hove) peak in density of states
 - Wigner crystals & stripe phases at fractional fillings → interactions of **extended** range



$$n = n_0/3$$

$$n = 2n_0/3$$

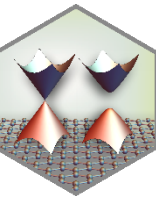
$$n = n_0$$



- Wang et al, Nature Mat. (2020)
- Regan et al., Nature (2020)
- Xu et al., Nature (2020)
- Jin et al., Nature Materials (2021)
- ...



Superconductivity (?)

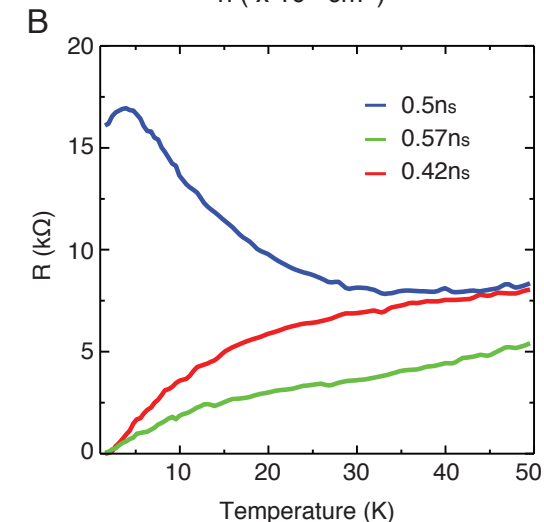
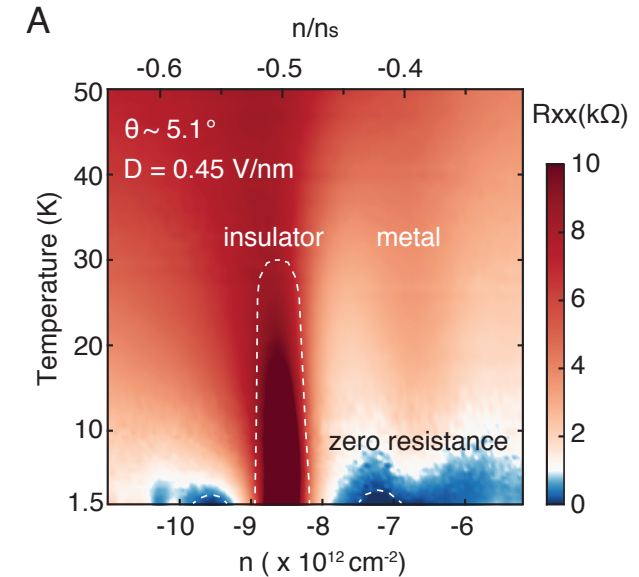


- Evidence for zero-resistance state in tWSe2

📖 Wang et al, Nature Mat. (2020)

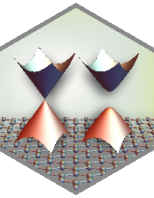
- Is superconductivity exclusive for graphene systems?
- Conventional vs. electronic mechanism?

- Theoretical viewpoint:
 - SC not exclusive [here: Van Hove scenario]
 - If unconventional: hexagonal symmetry guarantees interesting SC states
[2D irreps for p-,d- wave → nematic or topological SC]

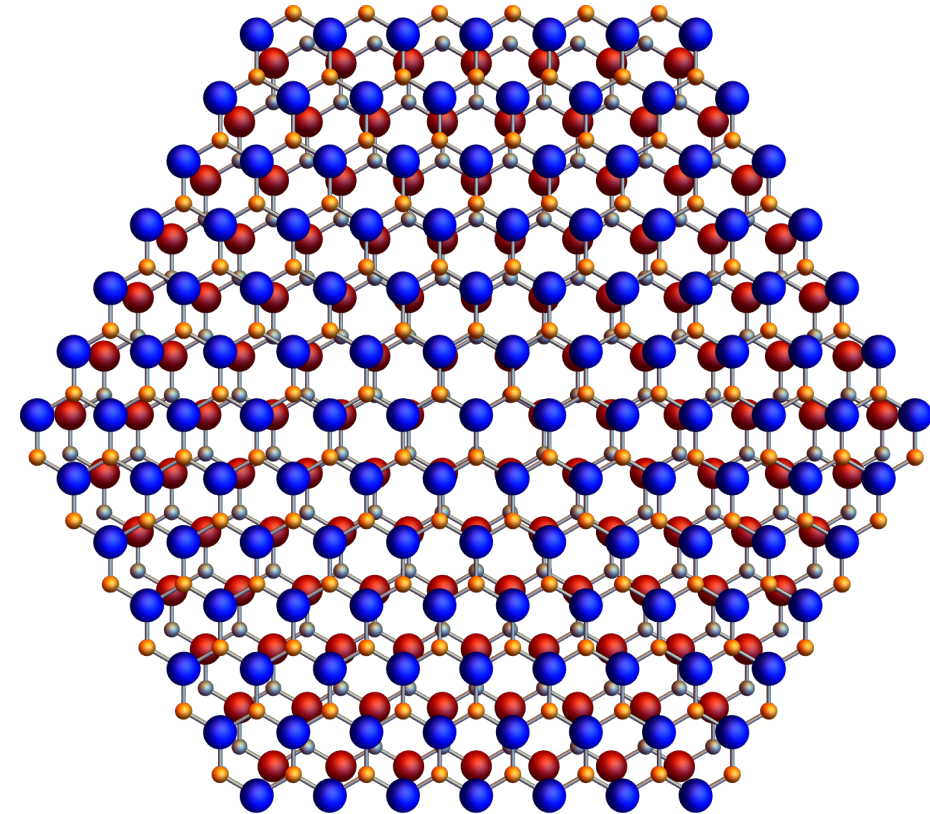
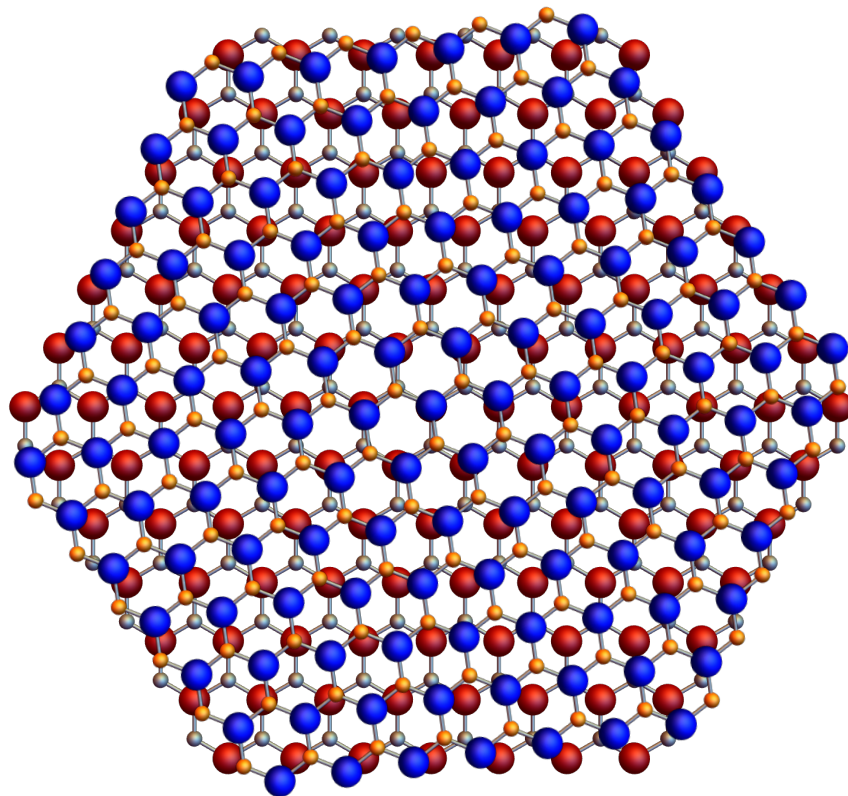
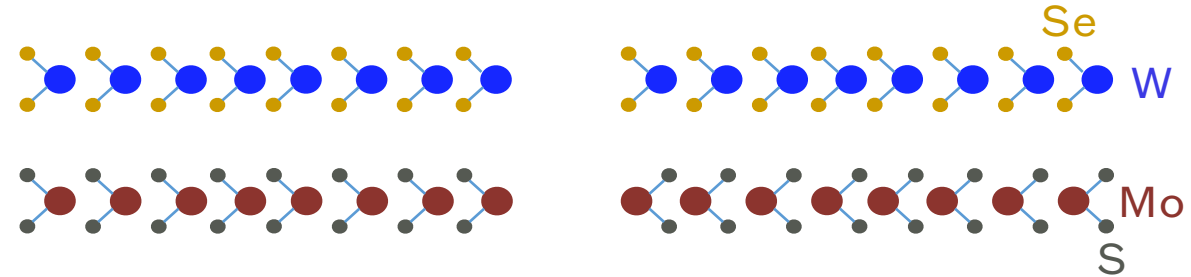




Moiré transition metal dichalcogenides

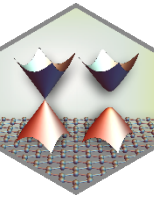


- Homo- vs hetero-bilayers
- AA vs AB stacking (0° vs 180°)



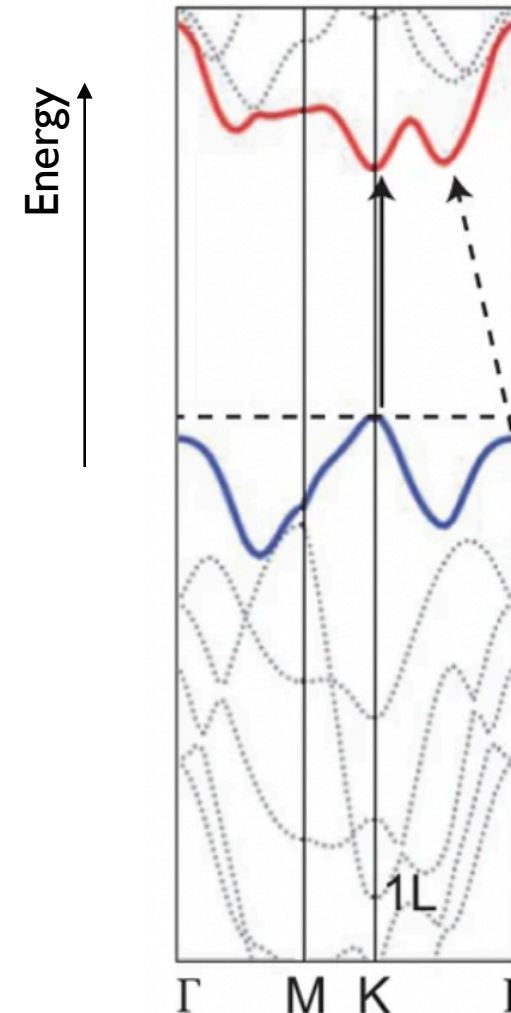


Moiré transition metal dichalcogenides



- Valence-band maxima near K, K'
- Spin-valley locking: (K, \uparrow) and (K', \downarrow)
- Band gap depends on material

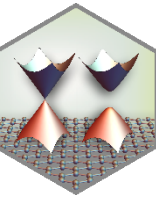
Band structure of WSe₂



Chhowalla et al
Nature Chem. (2013)

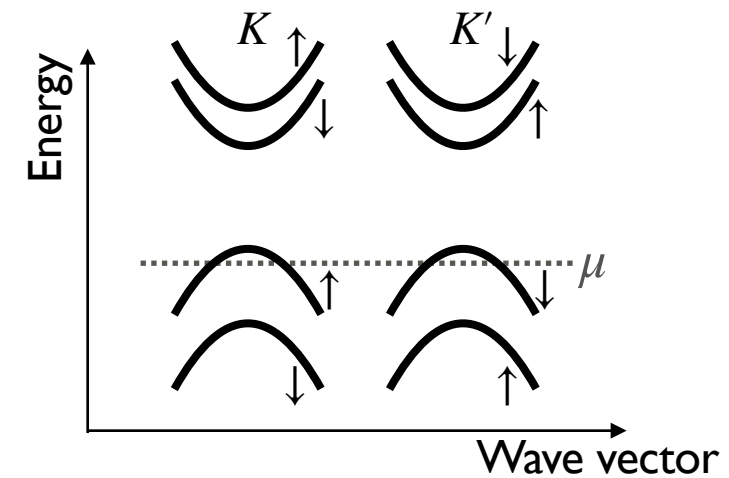


Moiré transition metal dichalcogenides



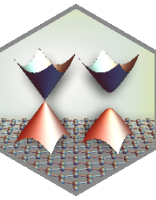
- Valence-band maxima near K, K'
- Spin-valley locking: (K, \uparrow) and (K', \downarrow)
- Band gap depends on material

Schematic band structure WSe₂





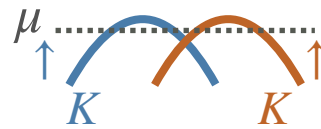
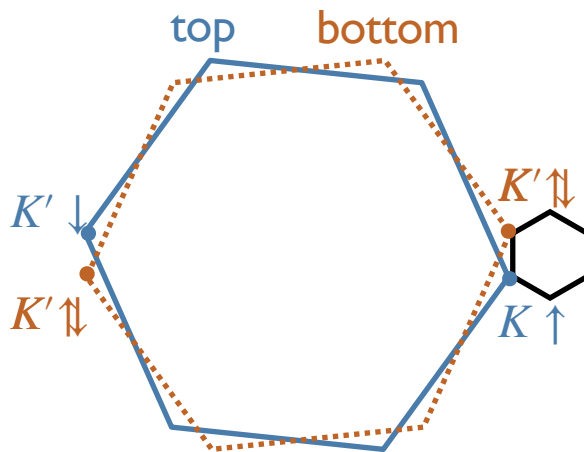
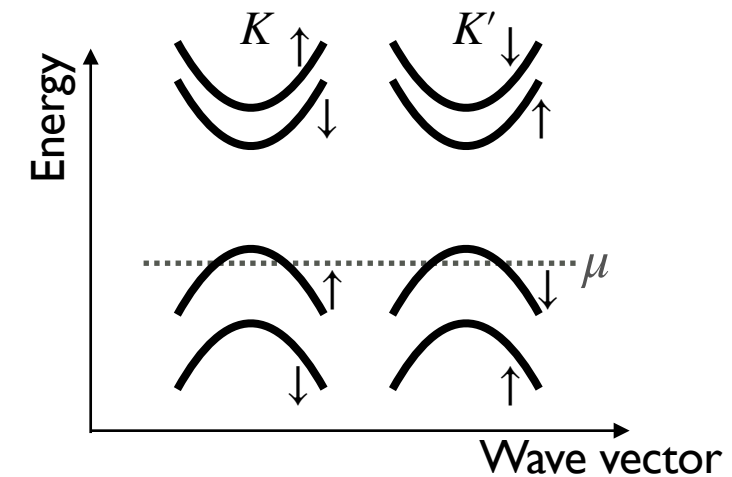
Moiré transition metal dichalcogenides



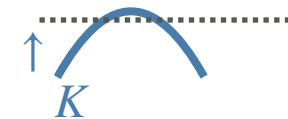
- Valence-band maxima near K, K'
- Spin-valley locking: (K, \uparrow) and (K', \downarrow)
- Band gap depends on material

- Differences in set-up for moiré bands:

Schematic band structure WSe₂



homo-bilayer
AA stacking



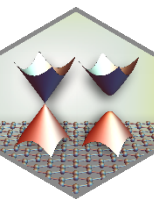
hetero-bilayer
AA stacking



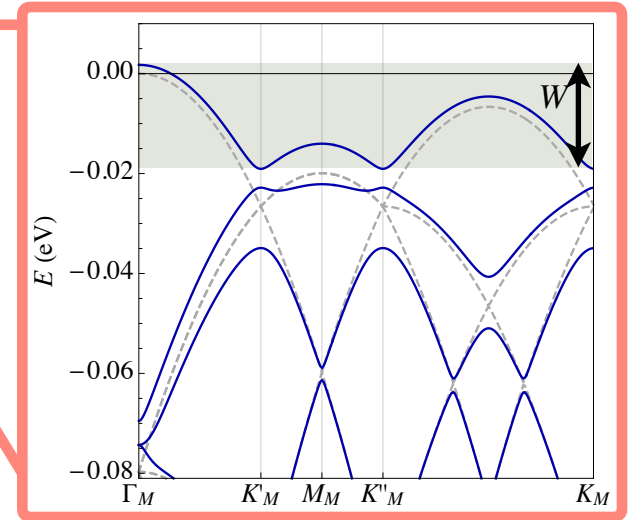
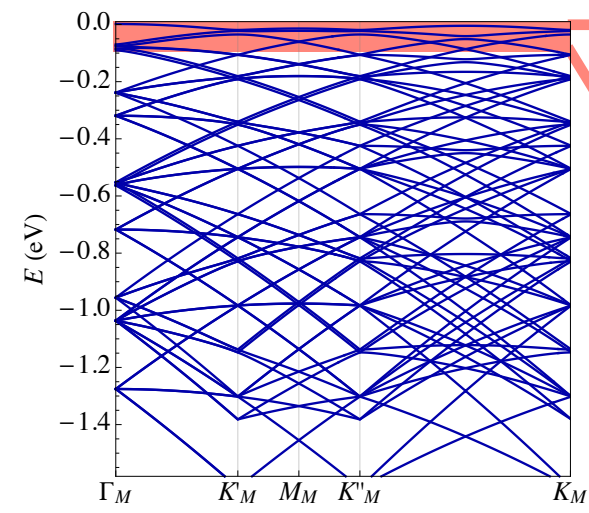
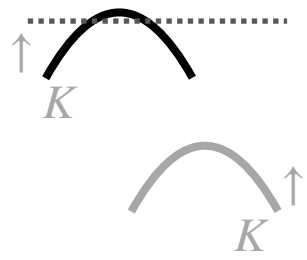
homo-bilayer
AB stacking



Simulate triangular-lattice Hubbard models

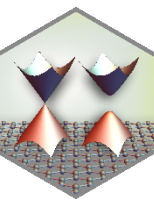


- Add moiré potential: $\Delta(\mathbf{r}) = \sum_{\mathbf{G}} V(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}$
- Huge increase of unit cell \rightarrow bands folded back into reduced zone

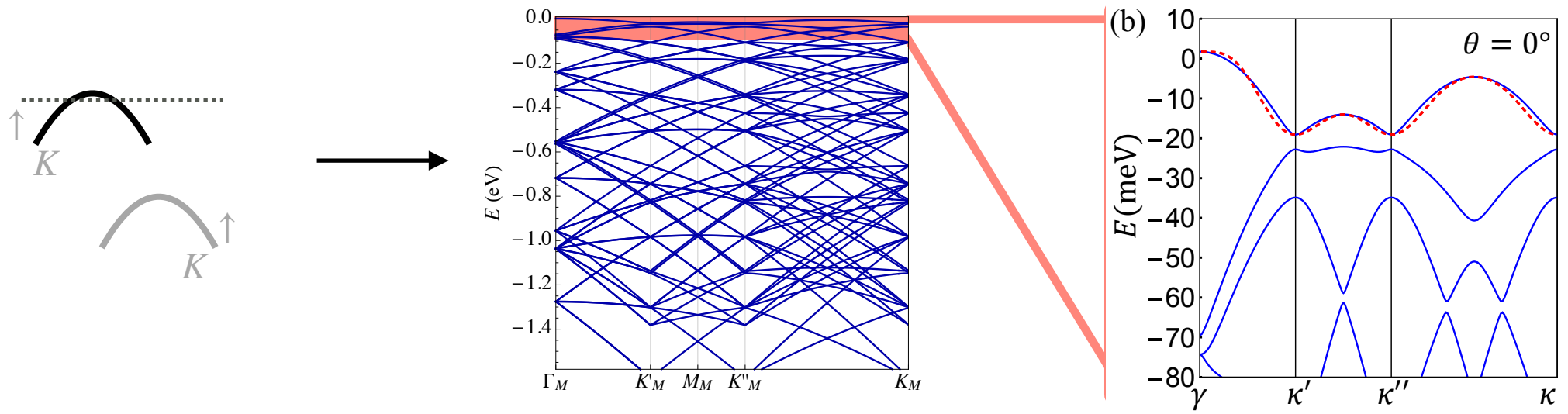




Simulate triangular-lattice Hubbard models



- Add moiré potential: $\Delta(\mathbf{r}) = \sum_{\mathbf{G}} V(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}}$
- Huge increase of unit cell \rightarrow bands folded back into reduced zone

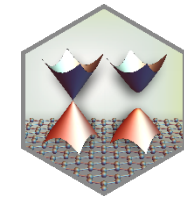


- Highest valence band: well described by triangular lattice Hubbard model

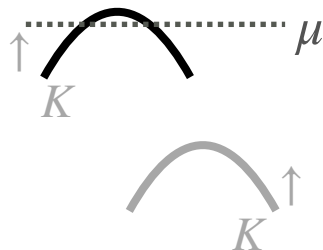
Wu, Lovorn, Tutuc, MacDonald, PRL (2018)



Simulate triangular-lattice Hubbard models



hetero-bilayer
AA stacking



Triangular-lattice Hubbard model with SU(2) between $(K \uparrow)$ & $(K' \downarrow)$

Wu, Lovorn, Tutuc, MacDonald, PRL (2018)

homo-bilayer
AB stacking



- Inter-layer hopping suppressed due to spin conservation
- SU(4) triangular-lattice Hubbard model $(K \uparrow \text{ top}), (K' \downarrow \text{ top}), (K \uparrow \text{ bot.}), (K' \downarrow \text{ bot.})$

Wu, Lovorn, Tutuc, Martin, MacDonald, PRL (2019)

Zhang Sheng, Vishwanath, PRL (2021)

homo-bilayer
AA stacking



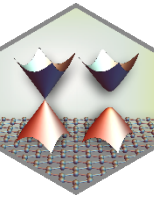
- Displacement field breaks SU(2) between $(K \uparrow)$ & $(K' \downarrow)$ and tunes energy states

Pan, Wu Das Sarma, PRR (2020)

Zang, Wang, Cano, Millis, PRB (2021)

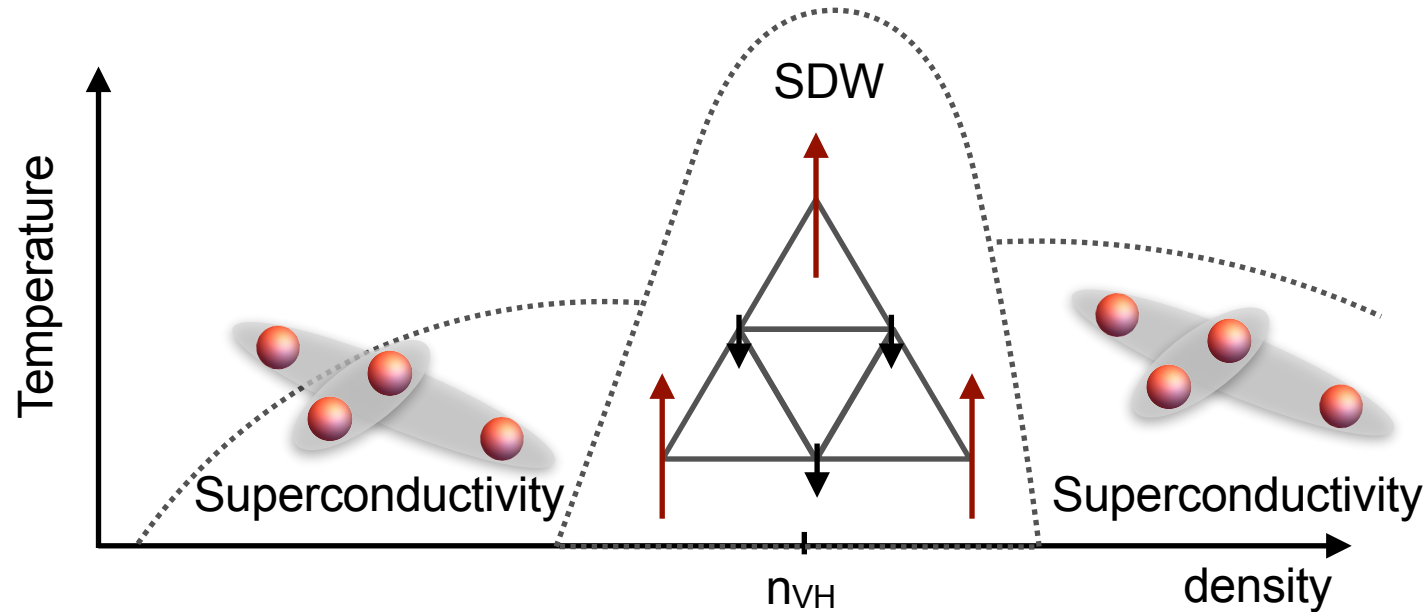


Interplay of orders for triangular lattice



- Two ordering tendencies at special filling:
 - Van Hove singularity
 - Nested Fermi surface
- Magnetism: spin-density-wave
- Unconventional superconductivity

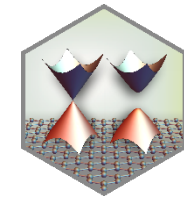
- 📄 Honerkamp, PRB (2003)
- 📄 Raghu et al, PRB (2010)
- 📄 Nandkishore et al, Nat. Phys. (2012)
- 📄 Chern, Batista PRL (2012)
- 📄 Nandkishore et al, PRB (2014)



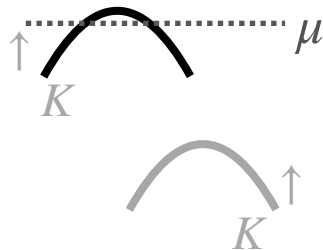
→ Change for moiré models?





Effects on interplay of orders



hetero-bilayer
AA stacking



- Effect of longer-ranged Coulomb repulsion?

 M. Scherer, D. Kennes, LC, arXiv:2108.11406
 N. Gneist, LC, M. Scherer, arXiv:2203.01226

homo-bilayer
AB stacking



- Effect of more flavours in SU(4)?

 Classen, Honerkamp Scherer, PRB 99, 195120 (2019)

homo-bilayer
AA stacking

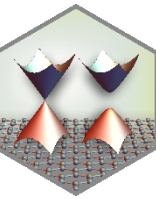


- Effect of Fermi surface geometry and Van Hove location?

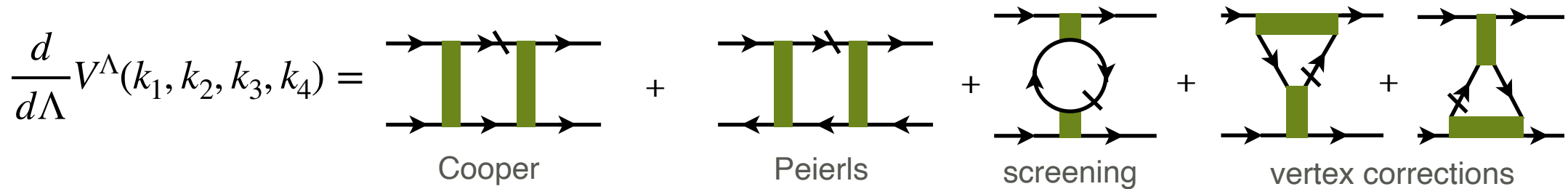
 L. Klebl, A. Fischer, LC, M. Scherer, D. Kennes arXiv:2204.00648



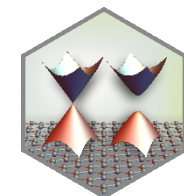
Competing orders from FRG



- Functional RG: discovery tool for ordering tendencies
- Calculate static 2-particle correlation function $V(p_1, p_2, p_3)$ dressed by interactions
neglect ≥ 6 -point vertex, self-energy feedback, frequency-dependence
- Unbiased, momentum-resolved!



Metzner, Salmhofer, Honerkamp, Meden, Schönhammer, Rev. Mod. Phys. (2012)

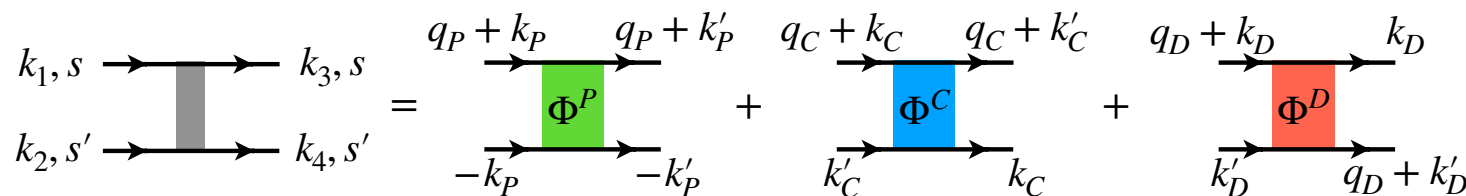


Momentum resolution

- 3 schemes: Fermi-surface patching, TUFGRG, momentum-mesh (+ non-SU(2) symmetry)

- Channel decomposition:

$$V = \Phi^P + \Phi^C + \Phi^D$$

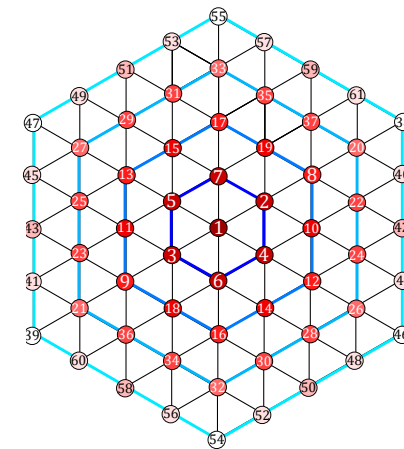
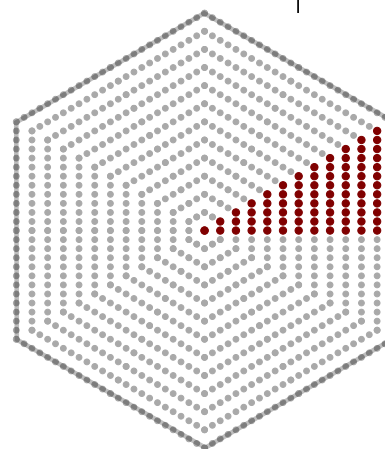





- TUFGRG:

$$\Phi^X(q, k, k') = \sum_{l, l'} X^{l, l'}(q) f_l(k) f_{l'}^*(k')$$

Channel X	P	C	D
Interaction type	Pairing	Magnetic	Density
Transfer momentum q_X	$\mathbf{k}_1 + \mathbf{k}_2$	$\mathbf{k}_1 - \mathbf{k}_4$	$\mathbf{k}_1 - \mathbf{k}_3$
Momentum k_X	$-\mathbf{k}_2$	\mathbf{k}_4	\mathbf{k}_3
Momentum k'_X	$-\mathbf{k}_4$	\mathbf{k}_2	\mathbf{k}_2

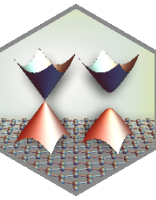
- High-resolution of transfer momenta q
- Form-factor expansion for k, k' $f_l(k) = e^{i\mathbf{k} \cdot \mathbf{R}_l}$



 Karrasch et al, J. Phys. Cond. Mat. (2008)
 Husemann, Salmhofer, PRB (2009)
 Lichtenstein et al, Comp. Phys. Com. (2017)



AA hetero-bilayer



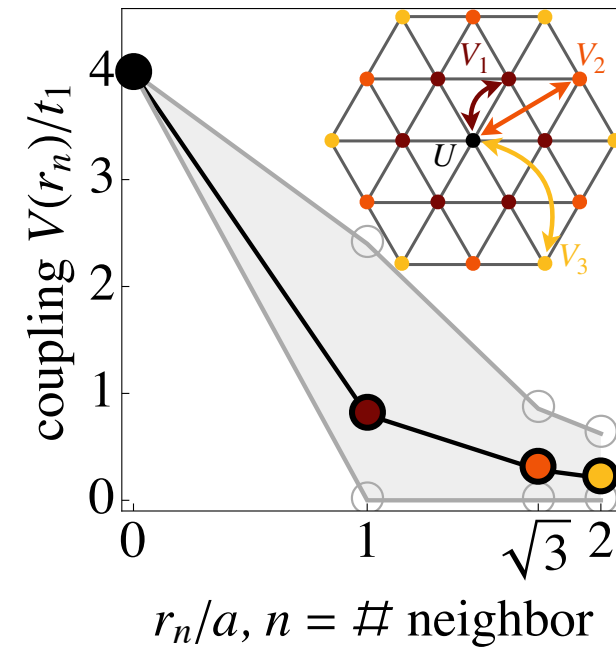
- SU(2) Hubbard model

$$H = \sum_{i,j} \sum_{\sigma} t_{i-j} c_{i,\sigma}^{\dagger} c_{j,\sigma} - \mu \sum_{i\sigma} c_{i,\sigma}^{\dagger} c_{i\sigma} + \frac{U}{2} \sum_{i,\sigma,\sigma'} n_{i\sigma} n_{i\sigma'} + \sum_{\sigma,\sigma'} \sum_{i,j} V_{i-j} n_{i\sigma} n_{j\sigma'} \quad n_{i\sigma} = c_{i\sigma}^{\dagger} c_{i\sigma}$$

- **Longer-ranged** interactions important! Up to V_3 .
- Overall strength tunable, e.g., substrate engineering
- Specifically: WSe2/MoS2

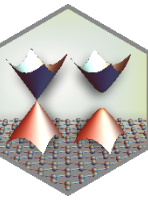
$\theta = 0 : t_1 \approx 2.5 \text{ meV}, t_2 \approx 0.5 \text{ meV}, t_3 \approx 0.25 \text{ meV}$

 Wu, Lovorn, Tutuc, MacDonald, PRL (2018)

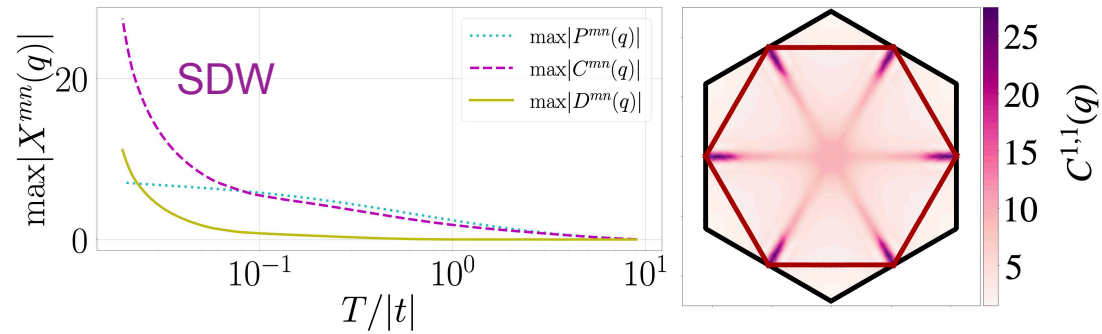




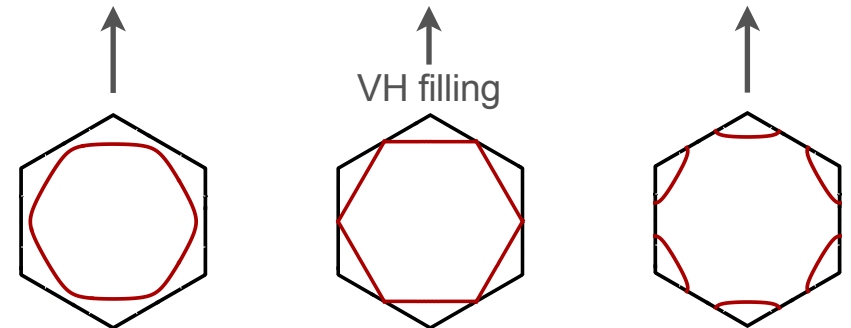
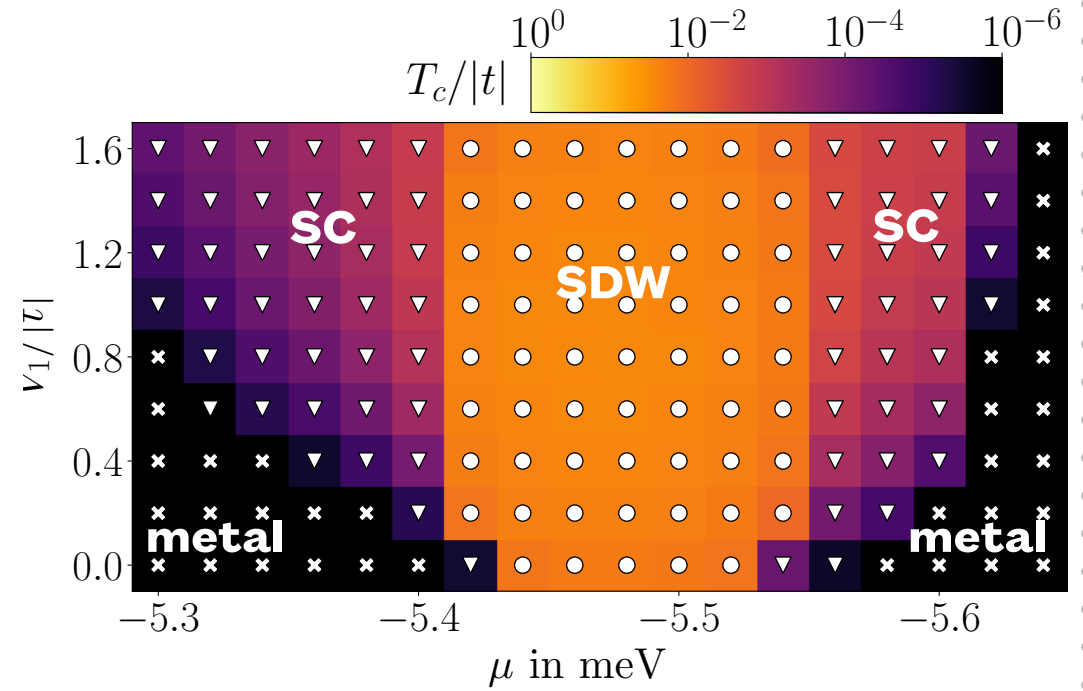
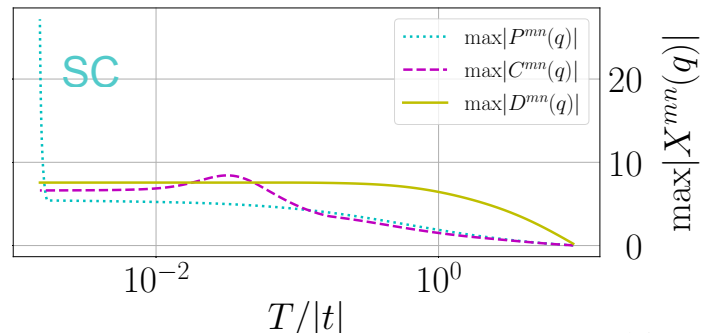
AA hetero-bilayer



- Spin density wave instability:

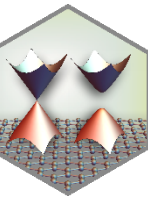


- Longer-ranged interactions stabilise unconventional pairing state

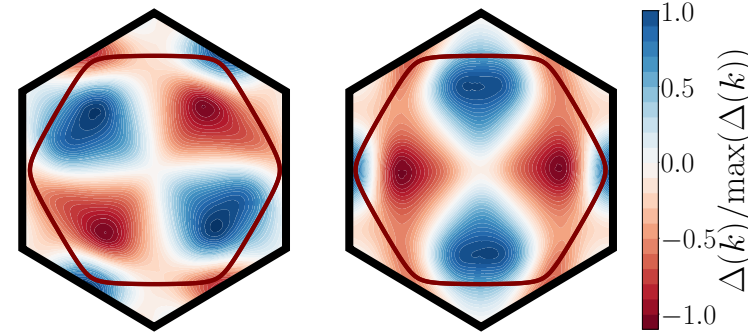
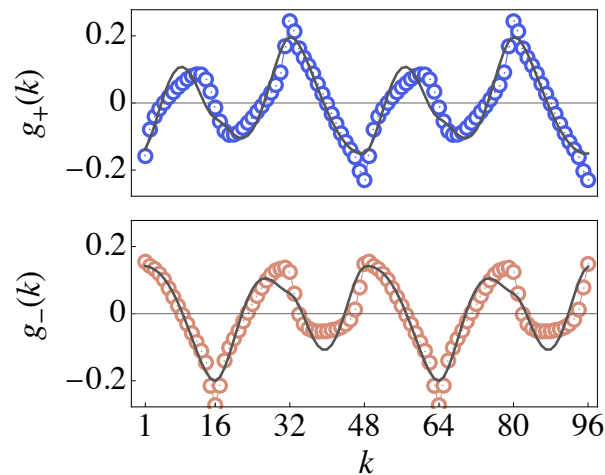




Pairing symmetry

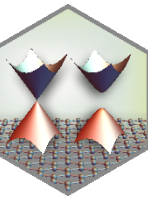


- SC gap: $\Delta(\mathbf{k}) = - \sum_{\mathbf{k}'} V_{pair}(\mathbf{k}, \mathbf{k}') \frac{\Delta(\mathbf{k}')}{2\xi_{\mathbf{k}'}} \tanh \frac{\xi_{\mathbf{k}'}}{2T_c}$
- Fitted well by 2nd nearest-neighbour harmonics of irrep E_2 of lattice symmetry C_{6v}
→ “g-wave”
- 2 degenerate solutions (symmetry!)





Superconducting state

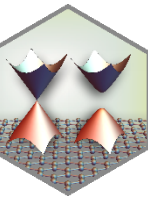


- 2 degenerate pairing solutions \rightarrow gap $\Delta(\mathbf{k}) = \Delta_1 g_1(\mathbf{k}) + \Delta_2 g_2(\mathbf{k})$
- Which linear combination realised in ground state?
- Minimize Landau free energy

$$\mathcal{L} = \alpha(|\Delta_1|^2 + |\Delta_2|^2) + \beta(|\Delta_1|^2 + |\Delta_2|^2)^2 + \gamma|\Delta_1^2 + \Delta_2^2|^2$$



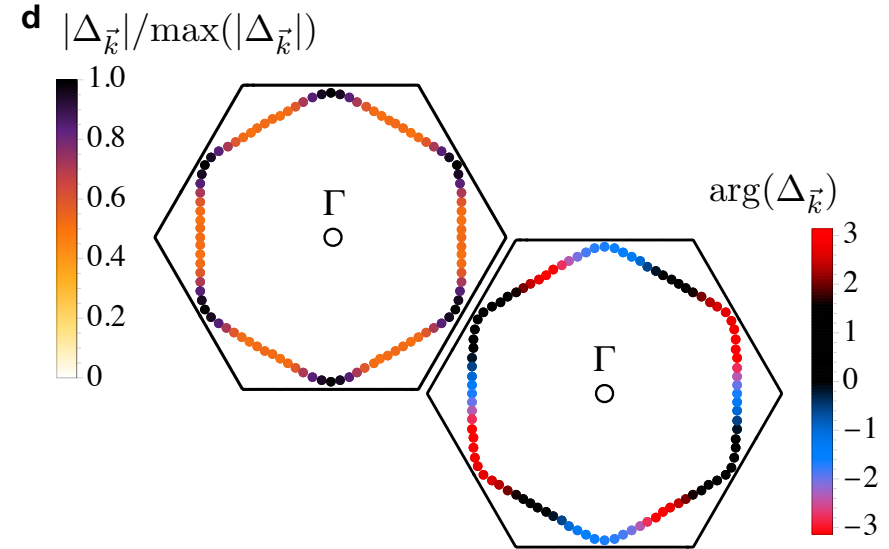
Superconducting state



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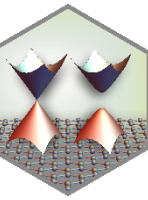
$$\mathcal{L} = \alpha(|\Delta_1|^2 + |\Delta_2|^2) + \beta(|\Delta_1|^2 + |\Delta_2|^2)^2 + \gamma|\Delta_1^2 + \Delta_2^2|^2$$

- FRG data as input to calculate α, β, γ
 $\Rightarrow \Delta(\mathbf{k}) = \Delta_1[g_1(\mathbf{k}) \pm ig_2(\mathbf{k})]$
- $|\Delta(\mathbf{k})|$ has no nodes, $\arg \Delta(\mathbf{k})$ winds 4 times





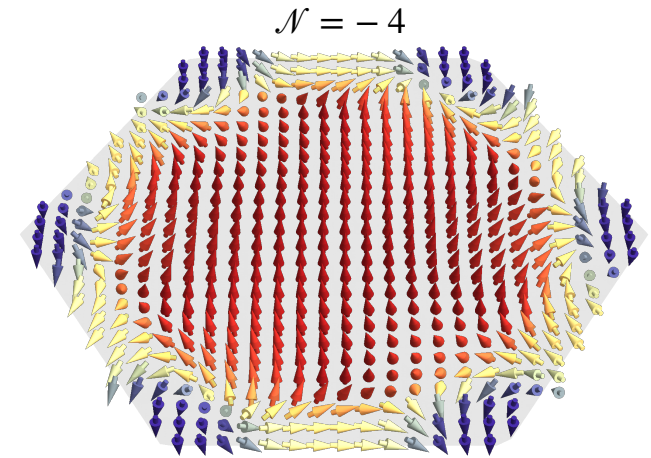
Topological superconductivity



- Spontaneous breaking of time-reversal: $g_1 + ig_2$ vs $g_1 - ig_2$

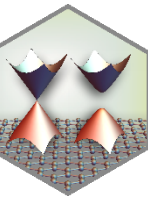
- “Skyrmion” number $\mathcal{N} = \frac{1}{4\pi} \int_{\text{BZ}} d^2k \mathbf{m} \cdot \left(\frac{\partial \mathbf{m}}{\partial k_x} \times \frac{\partial \mathbf{m}}{\partial k_y} \right)$

based on “pseudo-spin” $\mathbf{m} = \frac{(\text{Re}\Delta_{\mathbf{k}}, \text{Im}\Delta_{\mathbf{k}}, \xi_{\mathbf{k}})}{\sqrt{\xi^2 + \Delta_{\mathbf{k}}^2}}$





Topological superconductivity

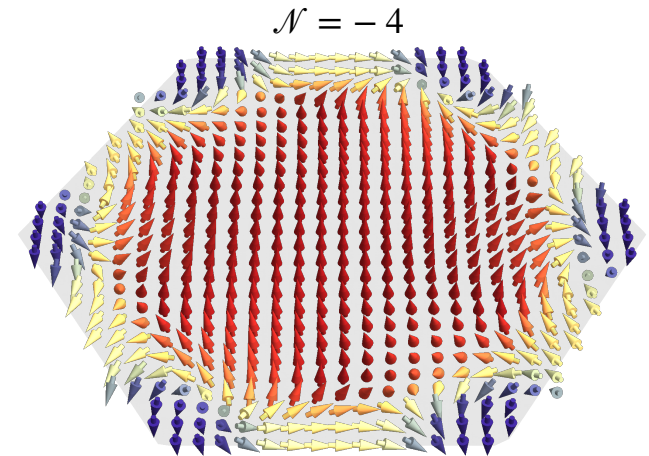


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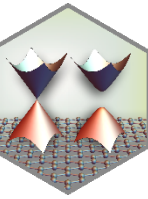
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- $\left. \begin{array}{l} g + ig : \mathcal{N} = 4 \\ d + id : \mathcal{N} = 2 \end{array} \right\}$ same symmetry under C_6 , different topology





Topological superconductivity



- Spontaneous breaking of time-reversal: $g_1 + ig_2$ vs $g_1 - ig_2$

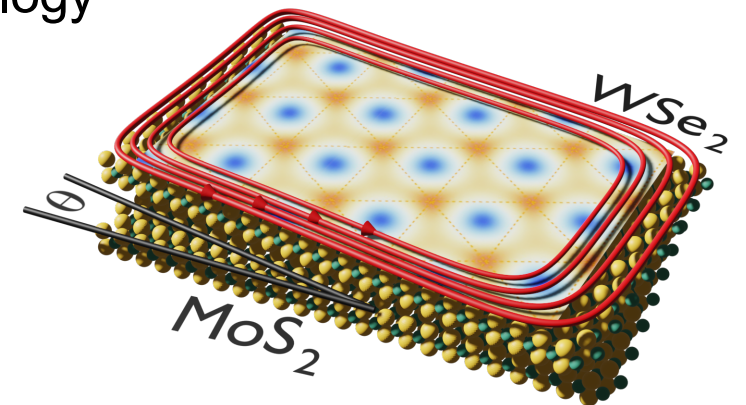
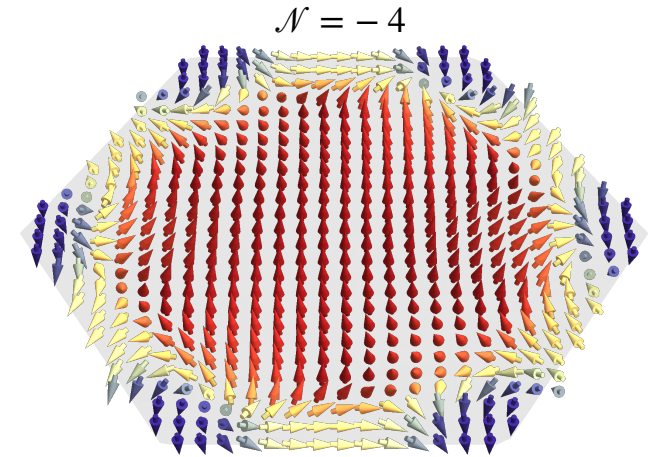
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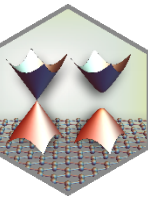
- \mathcal{N} chiral edge modes \rightarrow enhanced, quantized response

- Spin Hall conductance $\sigma_{xy}^s = \mathcal{N}\hbar/(8\pi)$
- Thermal Hall conductance $\kappa = \mathcal{N}\pi k_B^2/(6\hbar)$





AB homobilayer



- SU(4) Hubbard model

$$H = -t \sum_{\langle i,j \rangle} \sum_{\alpha} (c_{i\alpha}^{\dagger} c_{j\alpha} + h.c.) + U \sum_i \left(\sum_{\alpha} n_{i,\alpha} \right)^2 + J \sum_{\langle i,j \rangle} \sum_x (c_i^{\dagger} T^x c_i) (c_j^{\dagger} T^x c_j)$$

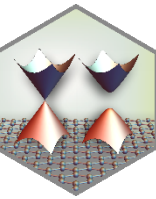
$\alpha \in \{(K \uparrow t), (K' \downarrow t), (K \uparrow b), (K' \downarrow b)\}$

SU(4) generators

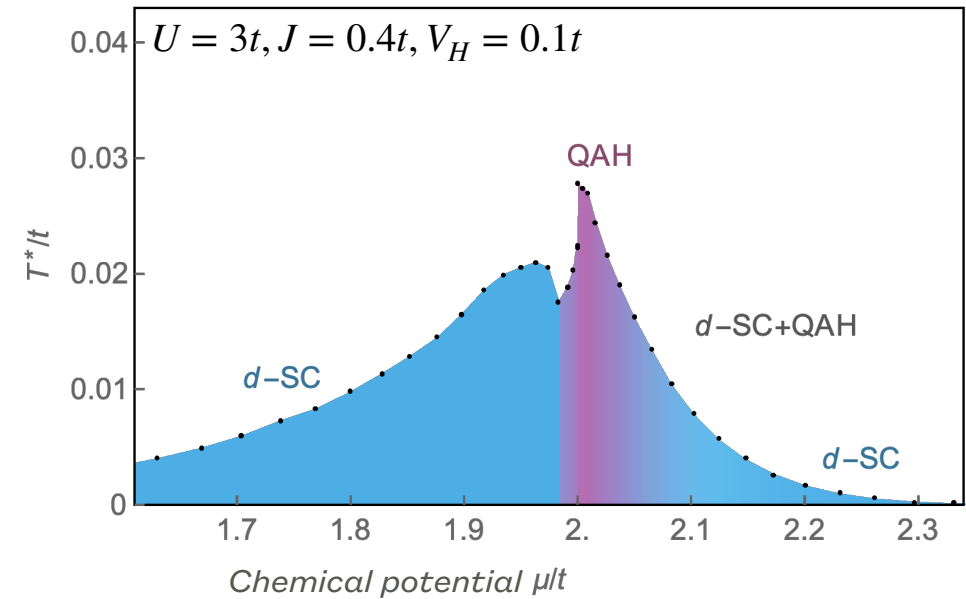
- Include exchange coupling J
- Also tested robustness regarding Hund's coupling SU(4) \rightarrow SU(2) x SU(2)



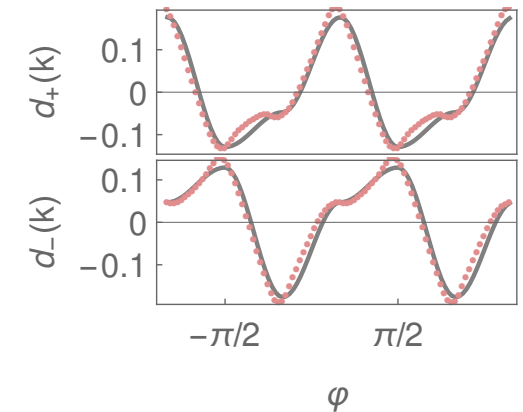
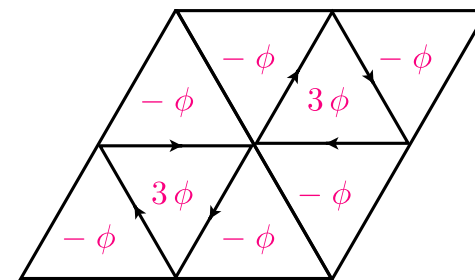
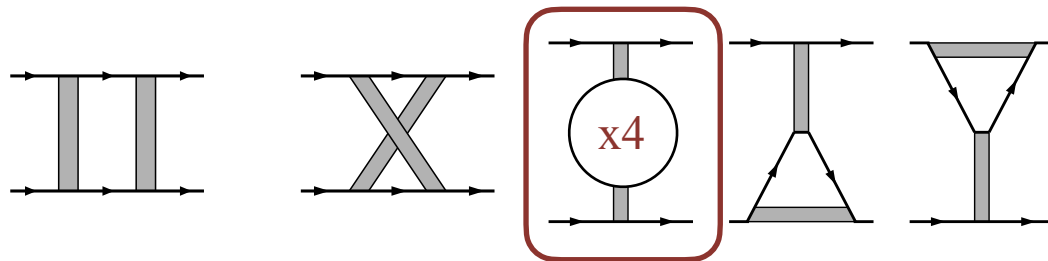
AB homobilayer



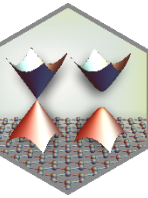
- Main ordering tendencies:
 - QAH: interaction-induced quantum anomalous Hall state (same as iCDW: loop currents)
 - d-wave superconductivity



- QAH instead of SDW due to more flavours



See also Lin, Nandkishore PRB(2019)

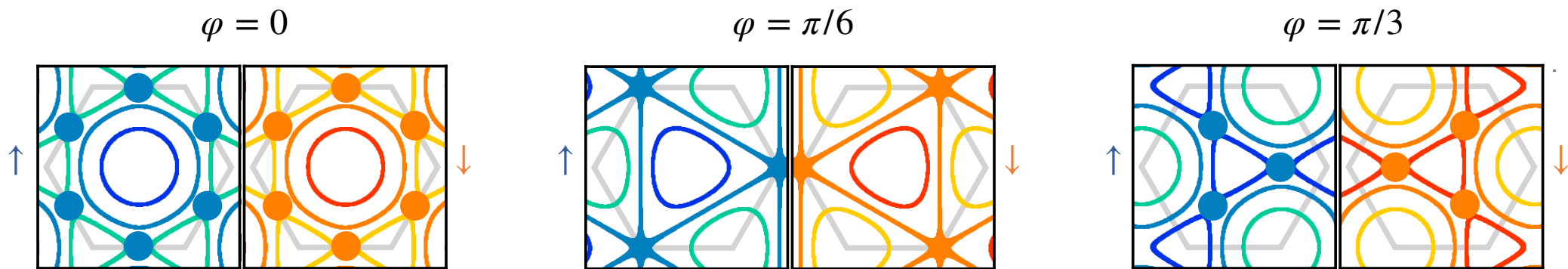


AA homobilayer

- Hubbard model with SU(2) breaking from displacement field

$$H = - \sum_{\langle i,j \rangle} \sum_{\sigma} |t| e^{i\sigma\varphi} c_{i\sigma}^{\dagger} c_{j\sigma} + \frac{U}{2} \sum_{i,\sigma,\sigma'} n_{i\sigma} n_{i\sigma'}$$

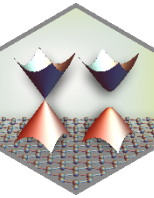
- Displacement field yields Peierls phase $\varphi \in [0, \pi/3]$
- Tunable Van Hove singularities \rightarrow rich phase diagram of symmetry-broken phases



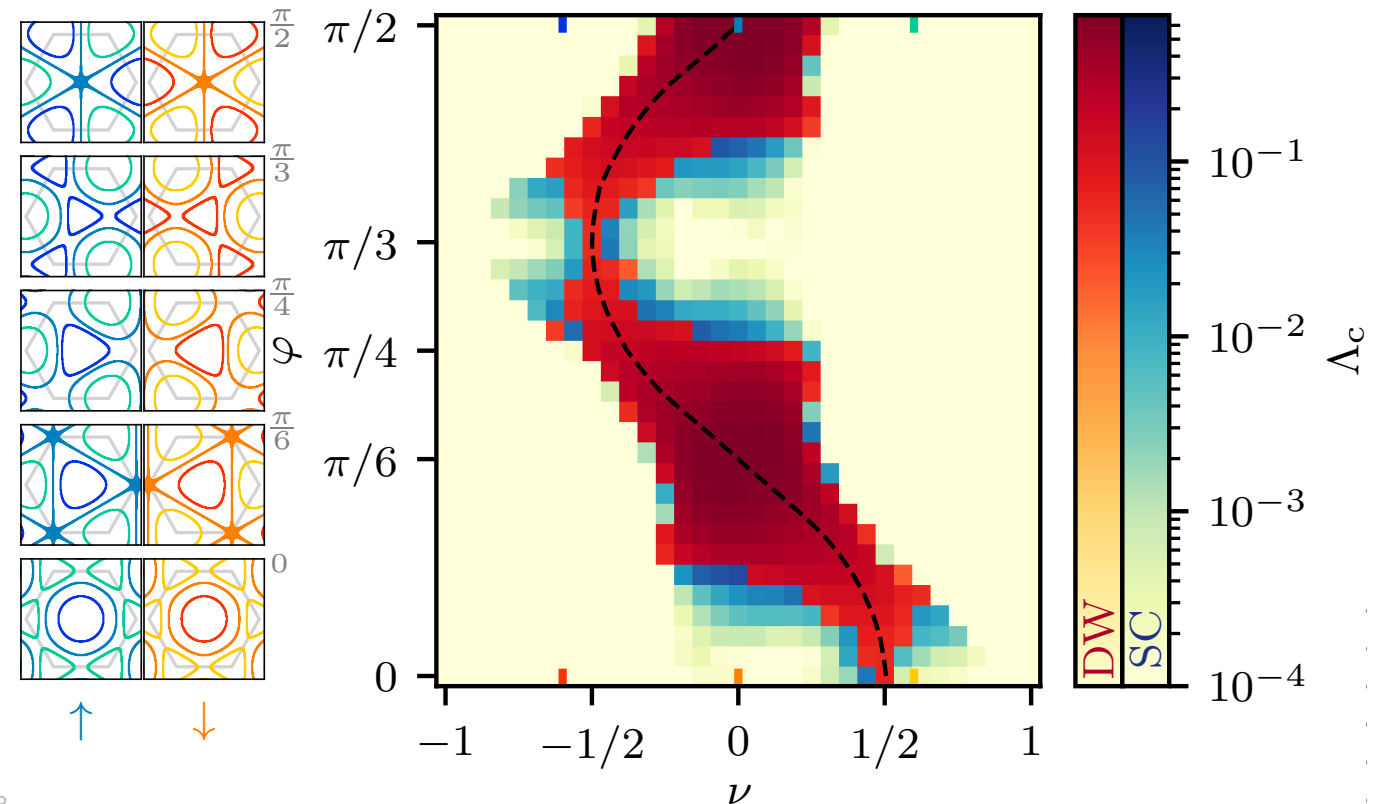
 Pan, Wu Das Sarma, PRR (2020); Zang, Wang, Cano, Millis, PRB (2021)



AA homobilayer

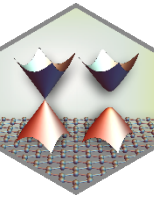


- Density-wave instabilities follow Van Hove singularity
- Fluctuations mediate pairing in vicinity

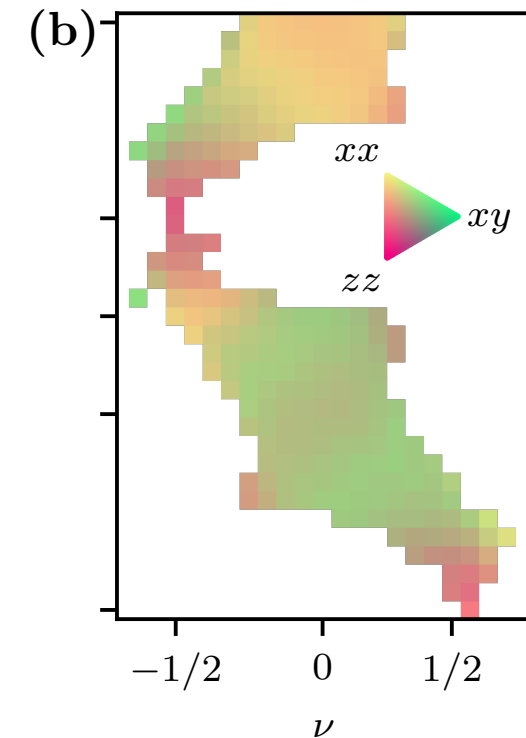
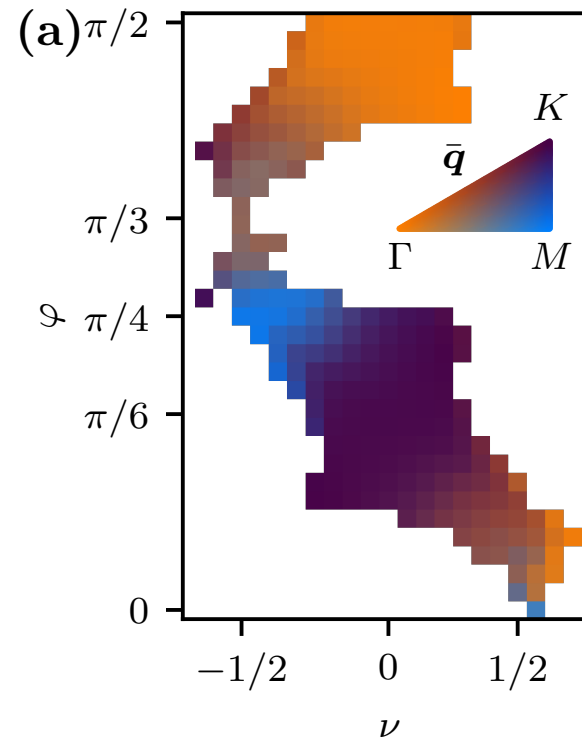




AA homobilayer

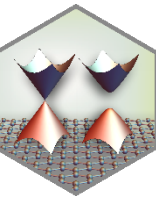


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- (Incommensurate) wave vectors of DWs follow Fermi surface nesting:
3 stripes \rightarrow 120° spiral \rightarrow FM

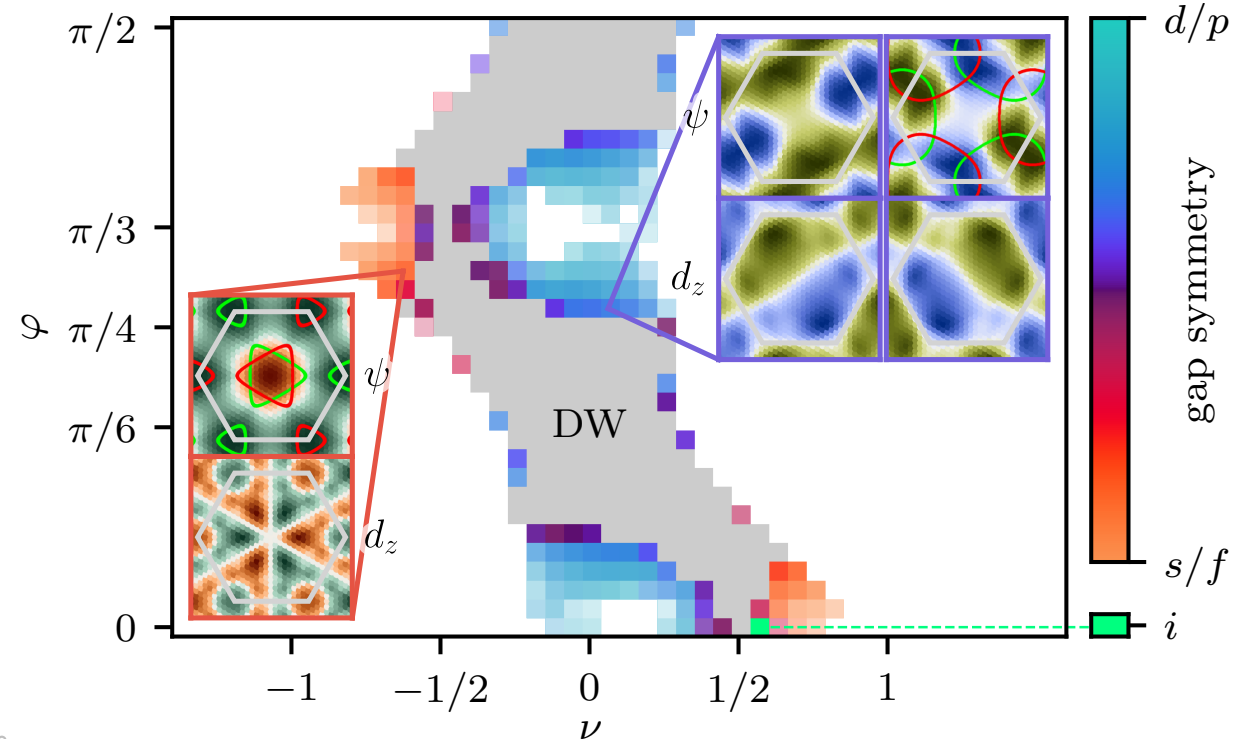




AA homobilayer

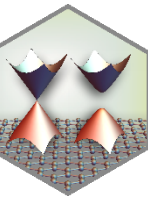


- Density-wave instabilities follow Van Hove singularity
- Fluctuations mediate pairing in vicinity
- (Incommensurate) wave vectors of DWs follow Fermi surface nesting:
3 stripes \rightarrow 120° spiral \rightarrow FM
- Pairing symmetry: mixed s/f or p/d (p+ip/d+id)
- Mainly determined by filling (i.e. Fermi surface): maximise gap

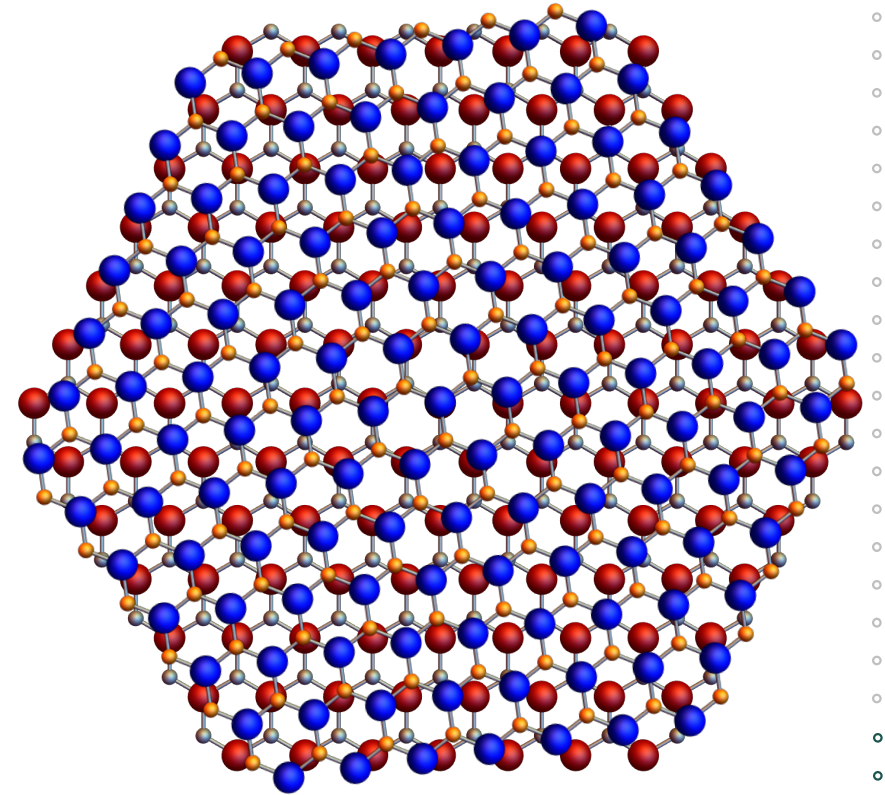




Summary



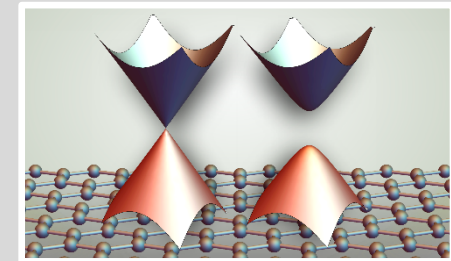
- Study correlated phases and mechanism for superconductivity
- Moiré materials: stacked and twisted 2D materials
- Moiré transition metal dichalcogenides as “simulators” for triangular-lattice Hubbard model
- FRG to investigate interplay of orders → unbiased method with high enough momentum resolution





Thank you!

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Lennart Klebl, Ammon Fischer, Dante Kennes (RWTH Aachen)



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