Asymptotic safety of Yang-Mills/gauge theory in 5 dimensional spacetime

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Contents

1. Motivations
   - Why gauge theory in 5d?
2. SU(N) Yang-Mills theory in $R^4 \times S^1$.
   - Aharonov-Bohm phases and Hosotani mechanism
3. Flow of gauge coupling
   - Fixed point and critical exponent

Motivations
Why 5d gauge theory?

- Phenomenological side
  - A model of Gauge-Higgs unification $A_M^a = (A_\mu^a, H^a)$
- Theoretical side
  - Perturbatively non-renormalizable
  - Non-perturavitve Renormalizability: Asymptotic safety
  - Towards asymptotically safe gauge theory

YM theory in compactified spacetime

- Phase structure (due to the Hosotani mechanism)
  - Spontaneous gauge symmetry breaking
  - Order of phase transitions
- Dynamical change of dimension of the system.
  - Running gauge coupling from 4D to 5D.

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Model

- Consider a Yang-Mills theory:

$$S_{\mathrm{YM}} = \frac{1}{2g^2}\int \mathrm{d}^5x\,\mathrm{tr}\left[F_{MN}F^{MN}\right]$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + f^{abc}A^b_M A^c_N$$

- Compactification: $R^4 \times S^1$

$$x_M = (x_\mu, x_5)$$

$$|x_5| \leq R$$

Compactification

- Kaluza-Klein (KK) expansion (p_μ, Ω_n)

$$A_M(x, x_5) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} A_M^{(n)}(p) e^{ip\cdot x + i\Omega_n x_5}$$

- KK modes $\Omega_n = \dfrac{n}{R}$

Aharonov-Bohm phases

- A background configuration of $A_M = A_M^a T^a$

$$\bar{A}_\mu = 0 \qquad (M = \mu = 0, \ldots, 3)$$

$$\bar{A}_5 = \frac{1}{2\pi R} \begin{pmatrix} \theta_1 & & \\ & \ddots & \\ & & \theta_N \end{pmatrix}, \quad \sum_{i=1}^{N} \theta_i = 0 \quad (M = 5)$$

- θ_i : Aharonov-Bohm (AB) phases

For finite AB phases

- Covariant derivative

$$\bar{D}_M^2 A_M = (\partial_\mu^2 + \bar{D}_5^2) A_M = \left[\partial_\mu^2 + (\partial_5 + i\bar{A}_5)^2\right] A_M$$

$$= \left[\partial_\mu^2 + \underbrace{\frac{1}{R^2}\left(n - \frac{\theta_i - \theta_j}{2\pi}\right)^2}_{\text{mass}}\right] A_M$$

- The gauge field (zero mode) obtain finite masses.

$$m_{A(0)}^2 = \frac{1}{R^2}\left(\frac{\theta_i - \theta_j}{2\pi}\right)^2 \qquad n = 0$$

Hosotani mechanism

Y. Hosotani, Phys. Lett. B 126, 309 (1983)
Phys. Lett. B 129, 193 (1983)
Annals Phys. 190, 233 (1989)

- Due to the non-trivial configuration of AB phases, the gauge symmetry is spontaneously broken.

- e.g. SU(3)

$\mathcal{H}_{\text{sym}}$/Phase	config.	θ_1	θ_2	θ_3
$SU(3)$/confined	X	—	—	—
	A_1	0	0	0
$SU(3)$/deconfined	A_2	$\frac{2}{3}\pi$	$\frac{2}{3}\pi$	$\frac{2}{3}\pi$
	A_3	$-\frac{2}{3}\pi$	$-\frac{2}{3}\pi$	$-\frac{2}{3}\pi$
	B_1	α	α	-2α
$SU(2)\times U(1)$/split	B_2	α	-2α	α
	B_3	-2α	α	α
	C_1	β_1	β_2	$-\beta_1-\beta_2$
$U(1)\times U(1)$/reconfined	C_2	β_1	$-\beta_1-\beta_2$	β_2
	C_3	$-\beta_1-\beta_2$	β_1	β_2

Physical quantities

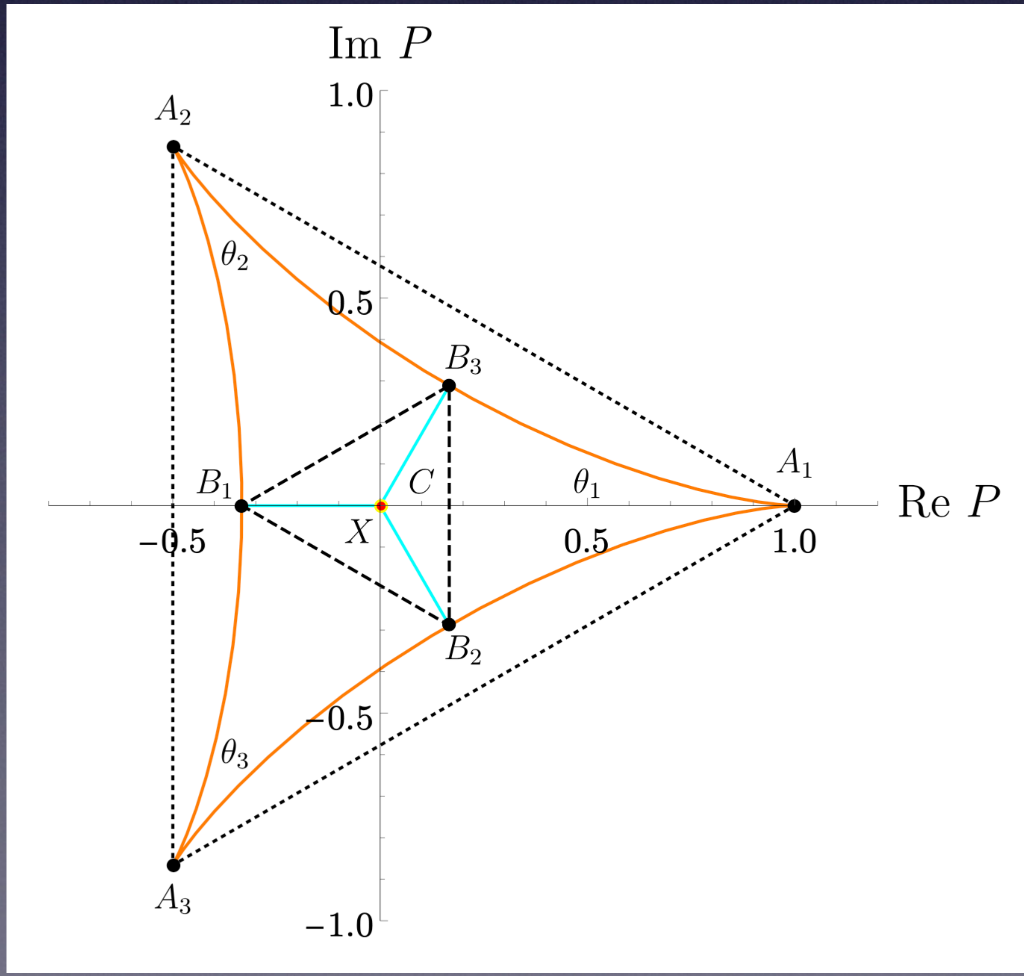
- Wilson line $W(x) = \mathcal{P} \exp\left\{ -i \int_0^{2\pi R} \mathrm{d}x_5\, \bar{A}_5(x, x_5) \right\} \sim \begin{pmatrix} e^{i\theta_1} & & \\ & e^{i\theta_2} & \\ & & e^{i\theta_3} \end{pmatrix}$
  - Broken generators: $[W, T^a] \neq 0$

SU(3)

- Polyakov loop $P = \frac{1}{N} \mathrm{tr} W$
  - e.g. SU(3)

$$P = \frac{1}{3}(e^{i\theta_1} + e^{i\theta_2} + e^{i\theta_3})$$

  - Confinement $P = 0$



Flow equation of AB phases

- Potential of AB phases

$$\mathcal{I}_N(\bar{R};\theta_H) = \pi\bar{R}\sum_{i,j=1}^{N}\frac{\sinh(2\pi\bar{R})}{\cosh(2\pi\bar{R})-\cos(\theta_H)}$$

  - Pure YM

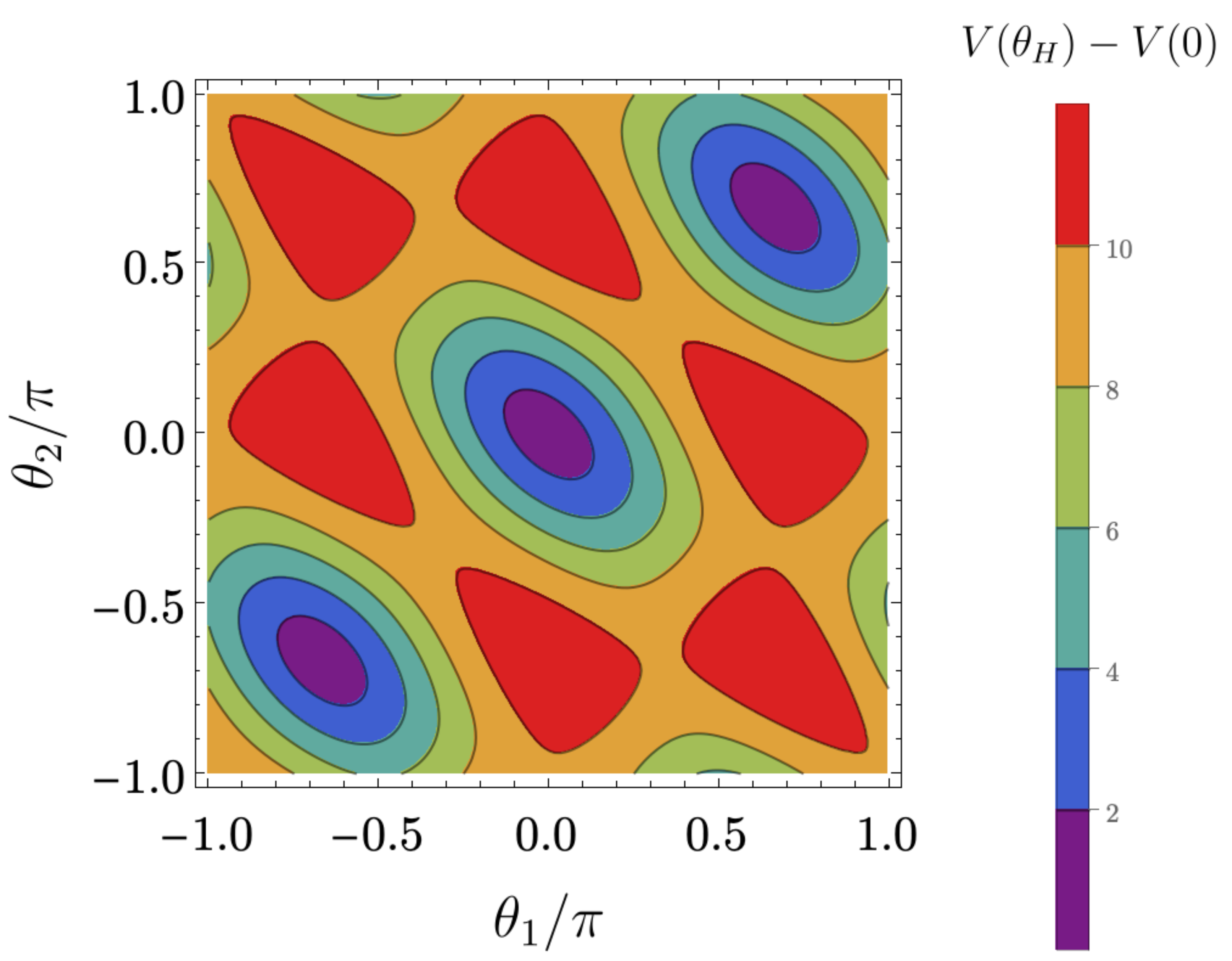
$$\partial_t V(\theta_H) = \underbrace{\frac{5k^4}{2(4\pi)^2}\left(1-\frac{\eta_g}{6}\right)\mathcal{I}_N(\bar{R};\theta_i-\theta_j)}_{\text{gauge fields}} - \underbrace{\frac{k^4}{(4\pi)^2}\left(1-\frac{\eta_{\mathrm{gh}}}{6}\right)\mathcal{I}_N(\bar{R};\theta_i-\theta_j)}_{\text{ghost fields}}$$

  - Inclusion of fundamental and adjoint fermions
    - fundamental $-\frac{4N_{\mathrm{f}}k^4}{(4\pi)^2}\left(1-\frac{\eta_{\mathrm{f}}}{6}\right)\mathcal{I}_N(\bar{R};\theta_i)$
    - adjoint $-\frac{4N_{\mathrm{ad}}k^4}{(4\pi)^2}\left(1-\frac{\eta_{\mathrm{ad}}}{6}\right)\mathcal{I}_N(\bar{R};\theta_i-\theta_j)$

Solving the flow equation

- SU(3) case
  - two independent AB phases $\theta_1,\ \theta_2$ $\theta_3 = -\theta_1 - \theta_2$
- Set $V(\theta_H) = 0$ at $k = \Lambda$ and neglect η
  - No background at classical level $\bar{A}_M = 0$
- Solve until $k = 0$ to obtain $V(\theta_H)$ in IR.

Pure gauge theory (N_f=0, N_{ad}=0)



Minima at

$$(\theta_1, \theta_2, \theta_3) = \begin{cases} (0,0,0) \\ (\pm\frac{2}{3}\pi, \pm\frac{2}{3}\pi, \pm\frac{2}{3}\pi) \end{cases}$$

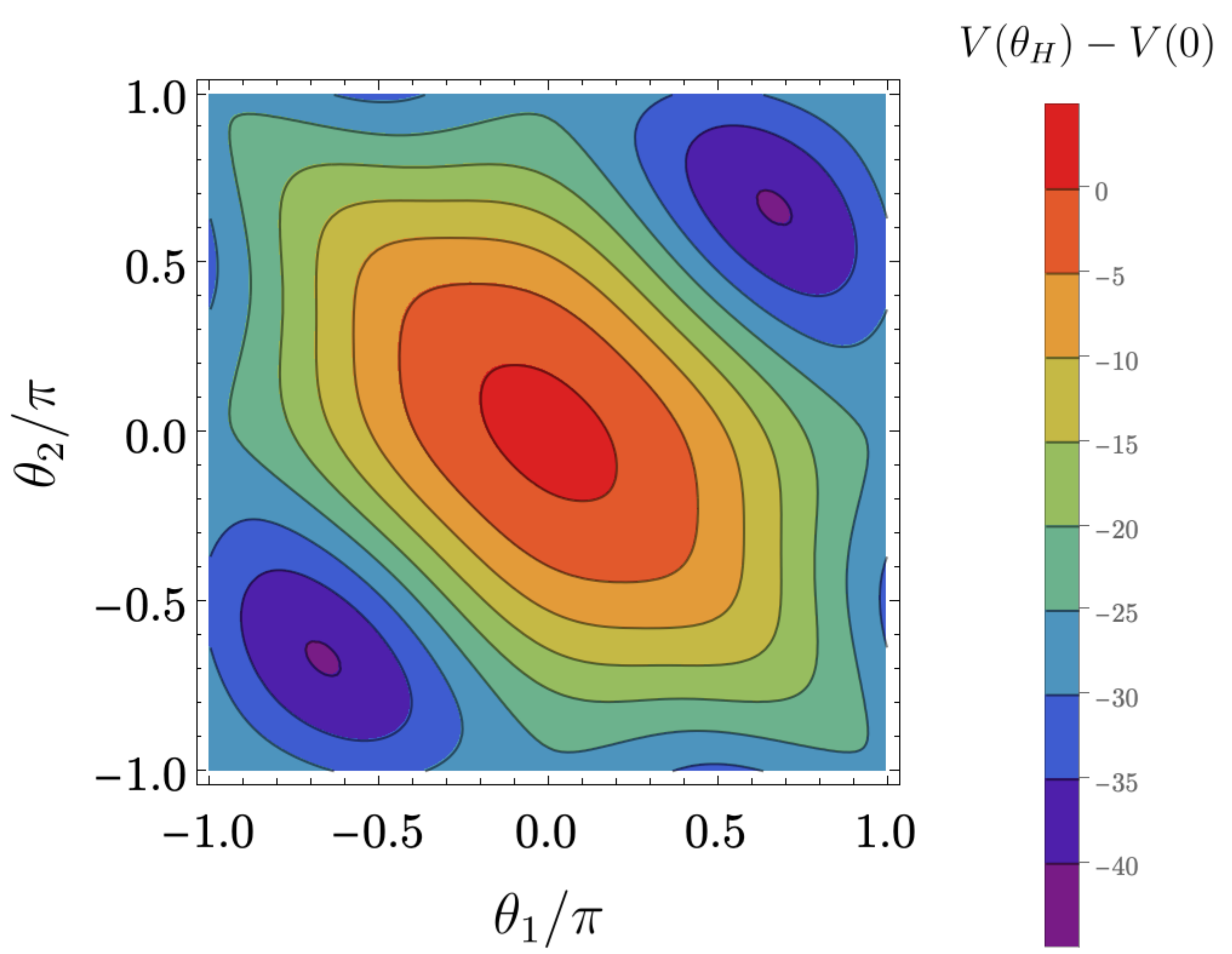
→ $P \neq 0$

deconfinement

$SU(3)$

is unbroken.

Including fundamental fermion ($N_f=1$, $N_{ad}=0$)



Minima at

$$(\theta_1, \theta_2, \theta_3) = (\pm\tfrac{2}{3}\pi, \pm\tfrac{2}{3}\pi, \pm\tfrac{2}{3}\pi)$$

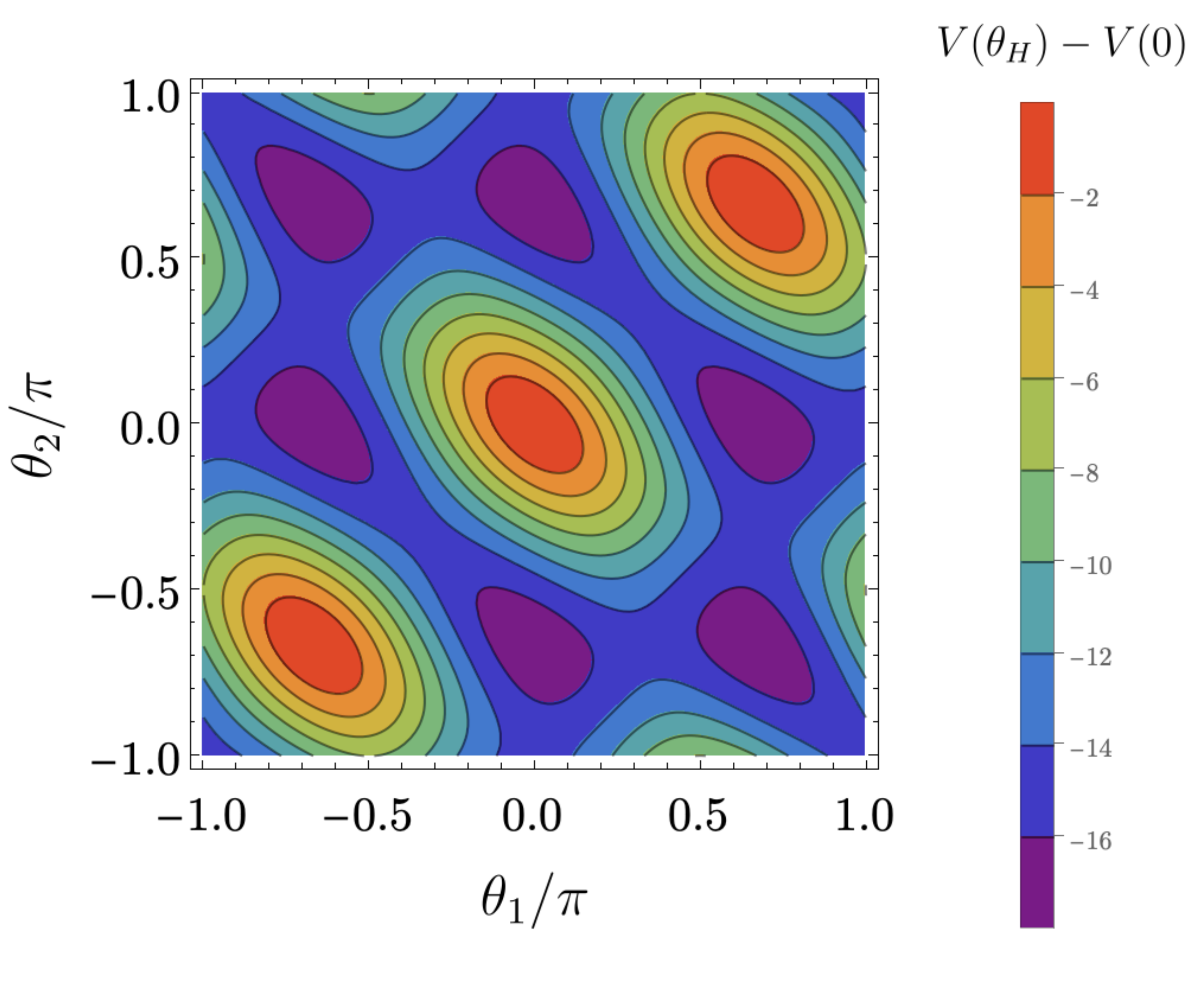
→ $P \neq 0$

deconfinement

$SU(3)$

is unbroken.

Including adjoint **mass**less fermion (N_f=0, N_{ad}=1)



Minima at

$$(\theta_1, \theta_2, \theta_3) = (0, \tfrac{2}{3}\pi, -\tfrac{2}{3}\pi)$$

and perm.

→ $P = 0$

confinement

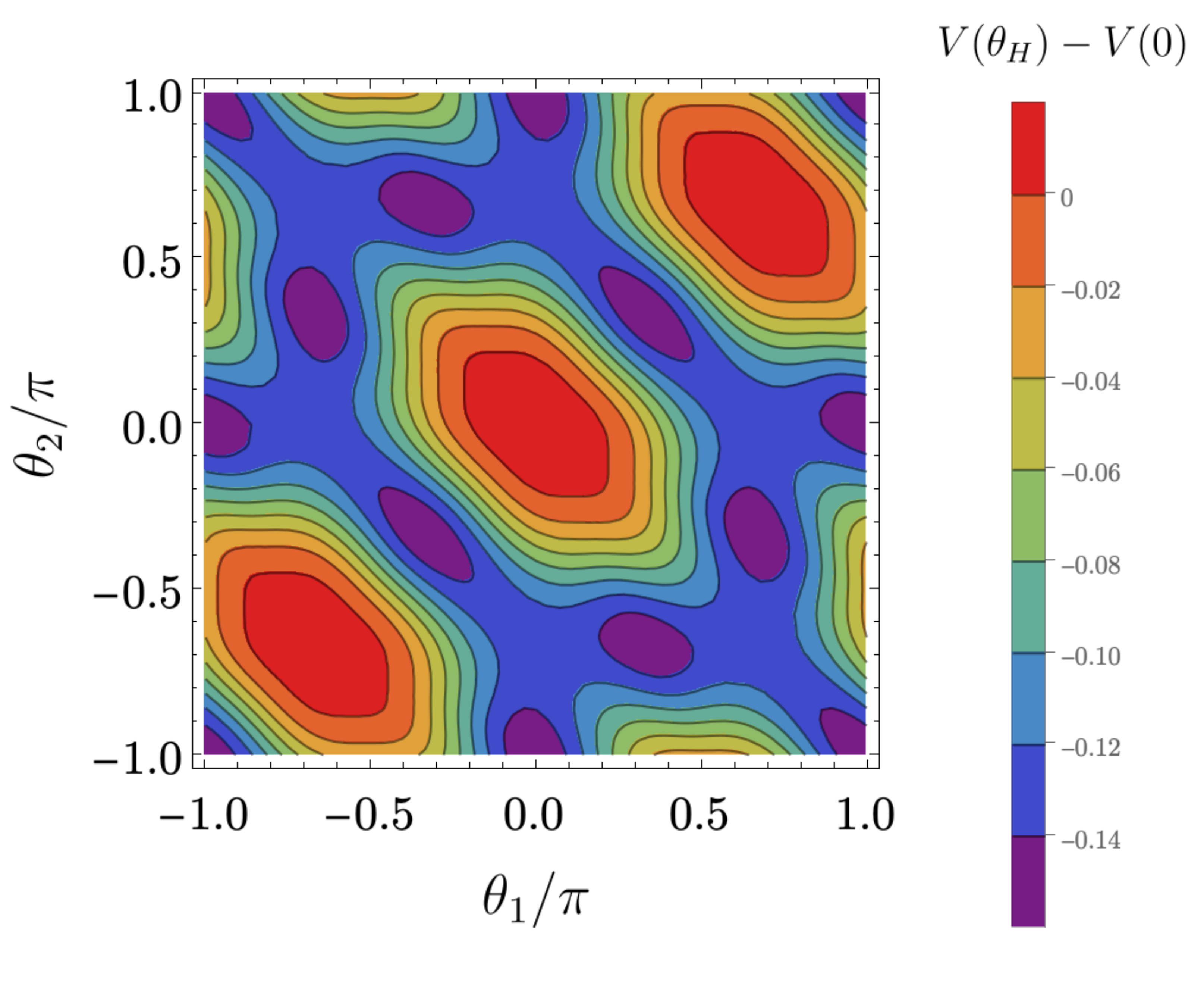
$SU(3)$

↓

$U(1) \times U(1)$

Including adjoint **massive** fermion (N_f=0, N_{ad}=1)

$m_{\rm ad}/k = 0.67$



Minima at

$$(\theta_1, \theta_2, \theta_3) = (\pi, \pi, 0)$$

$$(\pm\tfrac{\pi}{3}, \pm\tfrac{\pi}{3}, \mp\tfrac{2}{3}\pi)$$

and perm.

→ $P \neq 0$

deconfinement

$$SU(3)$$

↓

$$SU(2) \times U(1)$$

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Flow of gauge coupling

- Consider pure YM theory
  - Symmetric vacuum is realized: $(\theta_1, \theta_2, \theta_3) = (0, 0, 0)$
  - Consider $\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} \neq 0$ $\quad \bar{A}_5 = 0$
- Using the heat kernel technique

$$\frac{\partial_t Z_k}{2}\frac{1}{g^2}\int \mathrm{d}^5x\, \bar{F}_{MN}\bar{F}^{MN} = \frac{\partial_t Z_k}{2}\underbrace{\frac{2\pi R}{g^2}}_{\frac{1}{g_{\mathrm{4D}}^2}}\int \mathrm{d}^4x\, \bar{F}_{\mu\nu}\bar{F}^{\mu\nu}$$

Refs. on
gauge coupling running in higher dim.

- 4+ε expansion (one-loop): M. E. Peskin, *Phys.Lett.B* 94 (1980) 161-165

  see D.I.Kazakov, *JHEP* 03 (2003) 020

- higher dim. YM theory by fRG: H. Gies, *Phys.Rev.D* 68 (2003) 085015

- 4+ε expansion (four-loop): T.R. Morris , *JHEP 01 (2005) 002*

Flow of gauge coupling

- Contributions to beta function

$$\frac{\partial_t Z_k}{2}\frac{2\pi R}{g^2} = \frac{N}{2(4\pi)^2}\left[\underbrace{\frac{20}{3}\left(1-\frac{\eta_g}{2}\right)}_{A_\mu} - \underbrace{\frac{1}{3}\left(1-\frac{\eta_g}{2}\right)}_{A_5} + \underbrace{\frac{2}{3}\left(1-\frac{\eta_{\rm gh}}{2}\right)}_{\text{ghosts}}\right]\frac{1}{N^2}\mathcal{I}_N(\bar{R};\theta_H=0)$$

- Running gauge coupling

$$\tilde{g}^2 = \frac{Z_k^{-1}g^2}{2\pi R}\frac{\mathcal{I}_N(\bar{R};0)}{N^2} = Z_k^{-1}g_{\rm 4D}^2\frac{\mathcal{I}_N(\bar{R};0)}{N^2}$$

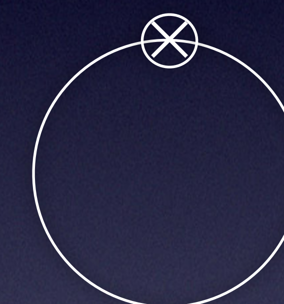
  - The scale derivative acts on Z_k^{-1} and $\mathcal{I}_N(\bar{R};0)$.

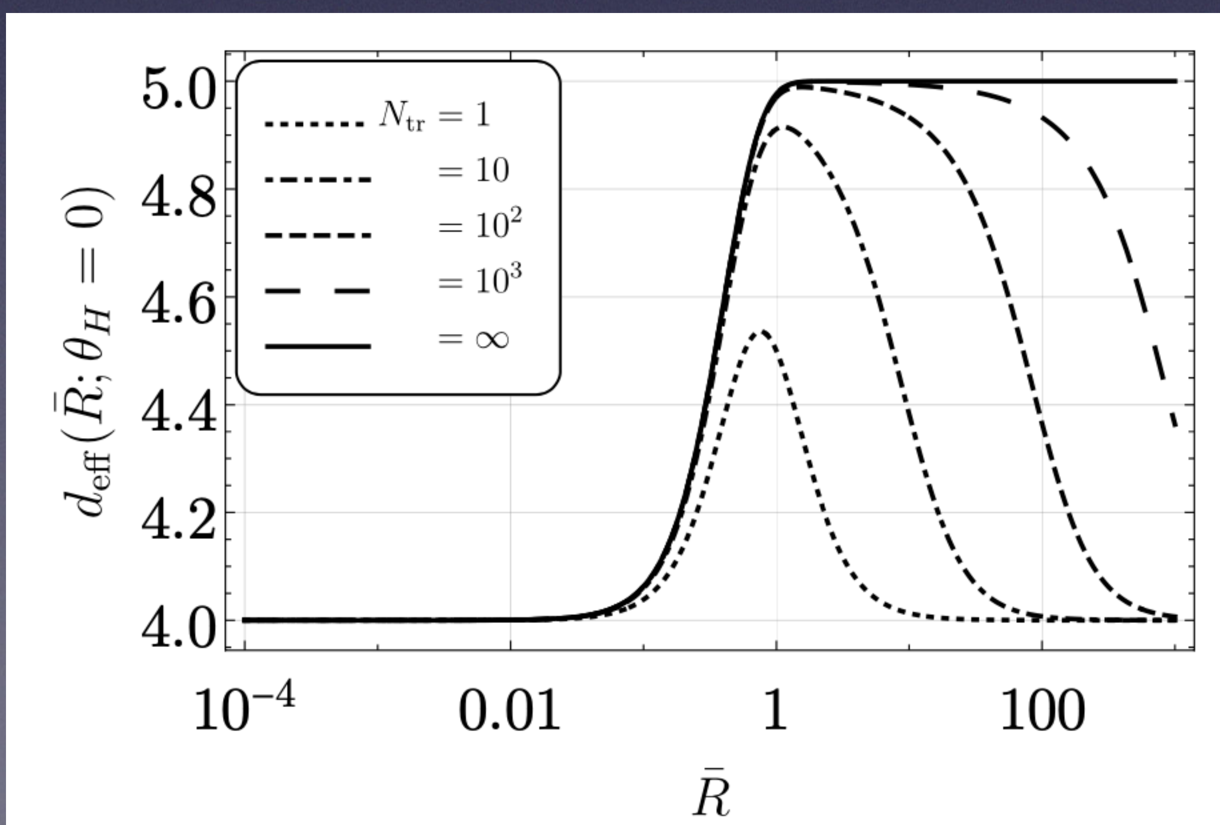
$$\partial_t\tilde{g}^2 = \left(d_{\rm eff}(\bar{R};0) - 4 + \eta_g\right)\tilde{g}^2$$

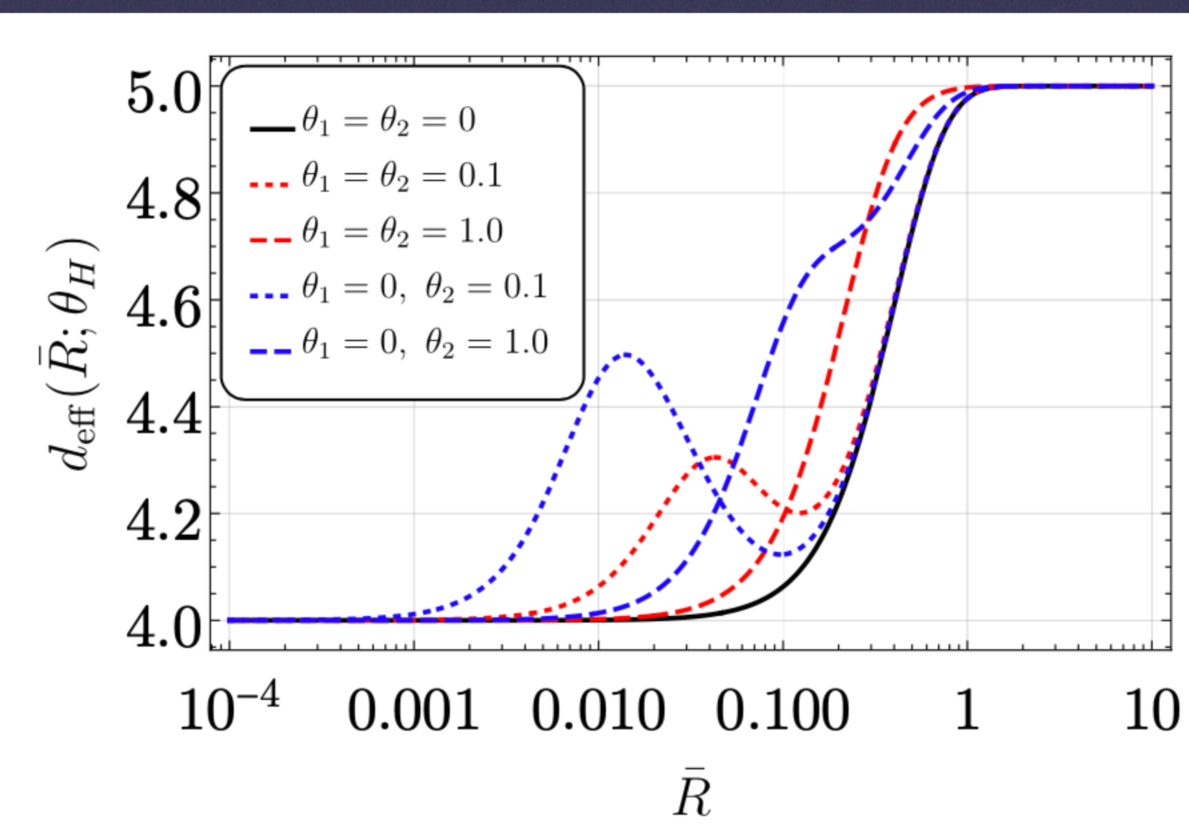
Effective dimension

- The threshold function contains the dynamical dimension in the system.

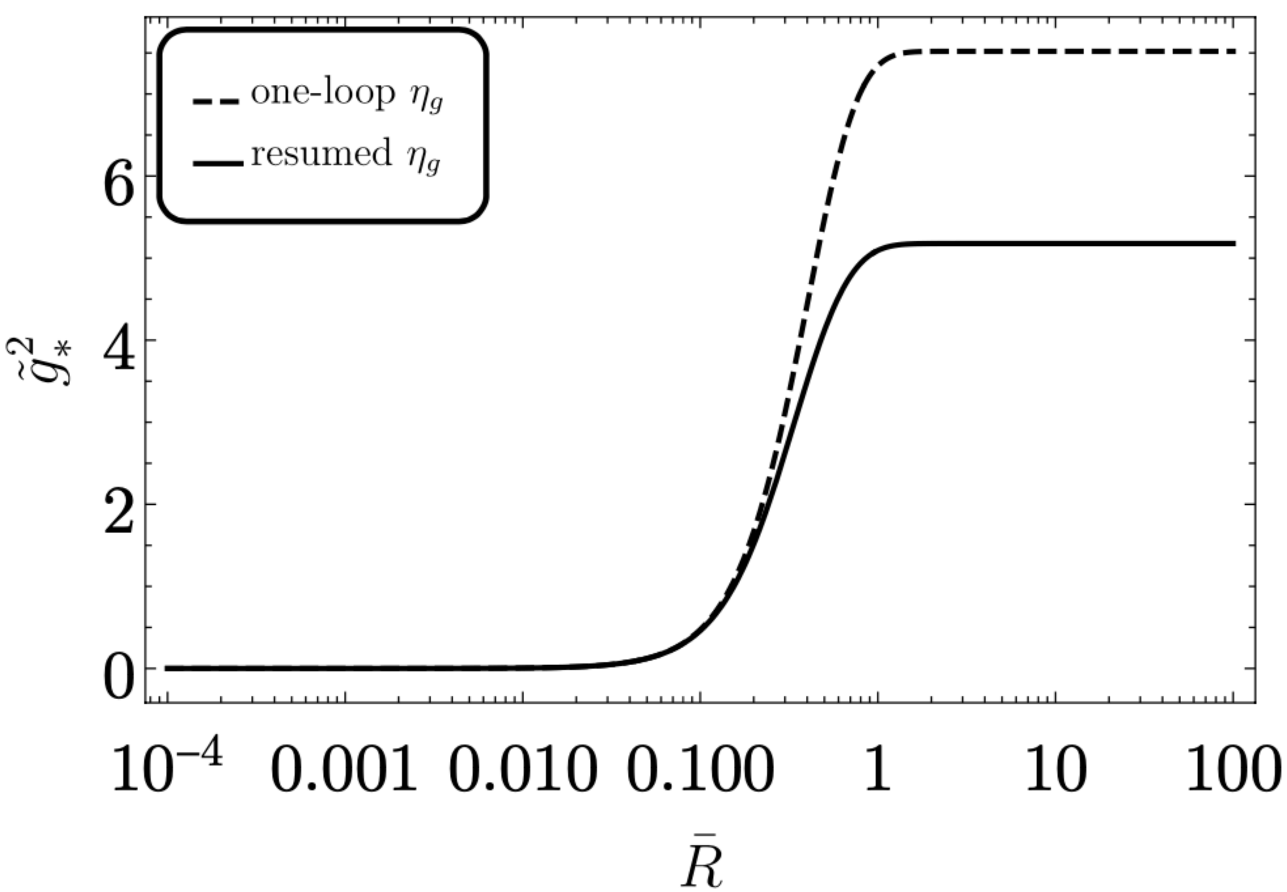
$$d_{\mathrm{eff}}(\bar{R};\theta_H) = 4 + \frac{\mathrm{d}\log \mathcal{I}_N(\bar{R};\theta_H)}{\mathrm{d}\log \bar{R}} \qquad \mathcal{I}_N(\bar{R};\theta_H) =$$

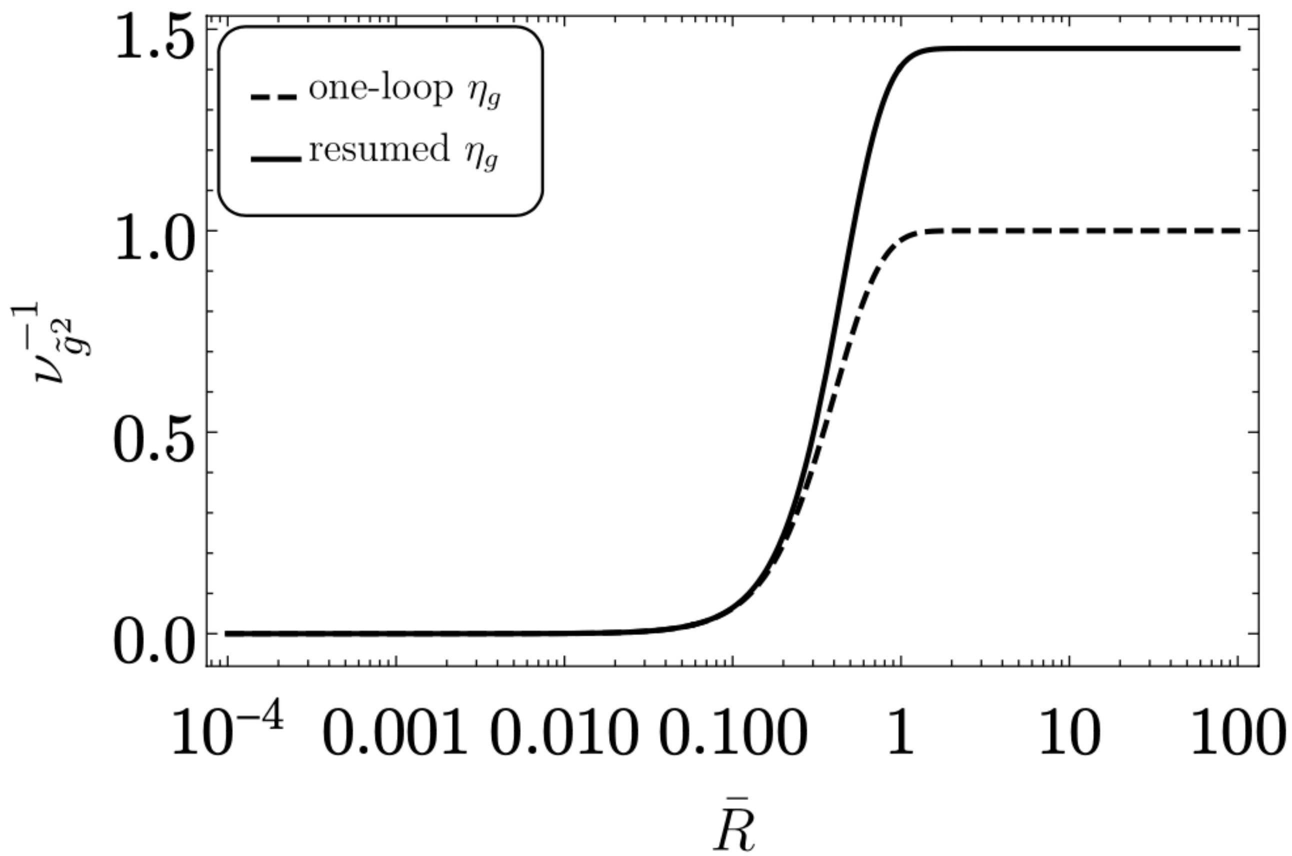






There exits a UV Fixed point at which gauge coupling is relevant.



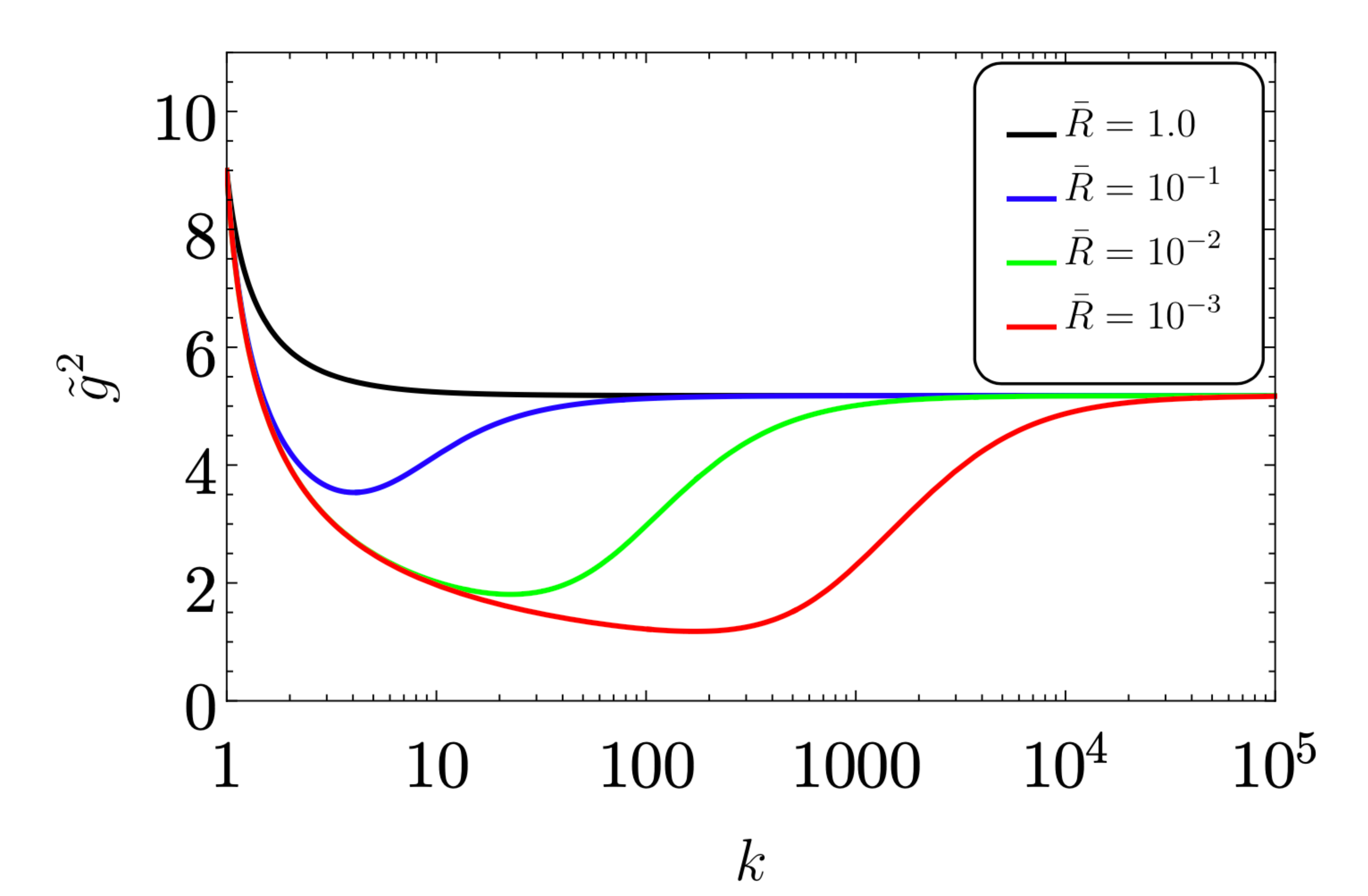


$$\partial_t \tilde{g}^2 = \left(d_{\rm eff}(\bar{R};0) - 4 + \eta_g\right)\tilde{g}^2$$

$$\eta_g = \begin{cases} -\dfrac{\frac{21}{(4\pi)^2}\tilde{g}^2}{1-\frac{57}{6(4\pi)^2}\,\tilde{g}^2} & \text{(resumed)} \\[2em] -\dfrac{21}{(4\pi)^2}\tilde{g}^2 & \text{(one-loop)} \end{cases}$$

Flow of gauge coupling in pure YM theory

$$\partial_t \tilde{g}^2 = \left(d_{\text{eff}}(\bar{R};0) - 4 + \eta_g\right)\tilde{g}^2$$



Summary

- 5 dimensional gauge theory
  - Hosotani mechanism
    - nontrivial background field is induced.
  - Phase structure
  - Non-trivial fixed point
- fRG captures well the dynamics of 5d YM theory.
- Phase diagram (future work)