Formation and evaporation of quantum black holes from the decoupling mechanism

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Motivation

Einstein's General Relativity (GR)

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_0} \int d^4x \sqrt{-g}(R - 2\Lambda)$$

Problems:

classical spacetime singularities [Penrose & Hawking, 1969]

quantum UV divergences ['t Hoft & Veltmann, 1974; Gorroff & Sagnotti, 1985]

→ **loss of predictivity** beyond regime of **effective theory**

Asymptotically Safe Gravity (AS)

Non-perturbative approach to quantum gravity postulating **quantum scale invariance** [Weinberg, 1976]

↓

Question: Potential **modifications to classical spacetimes** due to **quantum-gravitational fluctuations**?

Asymptotic Safety in Gravity via the Functional Renormalization Group

Γ_k scale-dependent effective average action (EAA) $\Gamma_k = \sum_i c_i(k)\mathcal{O}_k$

IR UV

$\Gamma_{k=0} \equiv \Gamma$

$\Gamma_{k=\Lambda_{\mathrm{UV}}\to\infty} \equiv S$

Flow equation:
[Wetterich, 1993; Reuter, 1996]

$$k\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{STr}\Big[\big(\Gamma_k^{(2)} + \mathcal{R}_k\big)^{-1} k\partial_k\mathcal{R}_k\Big]$$

<u>Asymptotic Safety:</u> [Weinberg, 1976]

$\exists$ **UV-attractive non-Gaussian fixed point** (UV completeness) of the RG flow with a **finite number of relevant directions** (predictivity)

<u>Einstein-Hilbert Truncation:</u>

$$\Gamma_k = -\frac{1}{16\pi G_k}\int d^4x\sqrt{g}(R - 2\Lambda_k)$$

[Bonanno & Reuter, 2000]

$$G_k = \frac{G_0}{1 + {g_*}^{-1}G_0 k^2}$$

$\lim_{k\to 0} G_k = G_0$ (observed Newton constant)

$G_k = g_* k^{-2},\ k \gg m_{pl}$

anti-screening at high energies

Renormalization Group Improvement

Motivation:

qualitative understanding of the **effect of quantum fluctuations** at high energies on a **classical theory**

Idea:

$c_i \to c_i(k)$ in classical **actions/equations/solutions**

Challenge:

$k = k(x)$ **scale-identification**

$\Leftrightarrow$

"short-cut" from UV to IR

based on simple local truncation instead of integrating more general ansatz for EAA including higher-derivative & non-local terms

Typical scale-identifications in gravity:

symmetry & dimensional analysis: e.g. $k \sim 1/d(r),\ (R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})^{1/4},\ \rho^{1/4},\ \dots$

Bianchi identities & minimal scale-dependence: $2\Lambda_k' + (R - 2\Lambda_k)G_k' {G_k}^{-1} = 0$

[Babić et al, 2000; Domazet & Stefancić, 2011/12; Koch et al, 2015; Koch & Ramirez, 2016]

New proposal:

use **decoupling mechanism** (close to original spirit of RG-improvement)

Scale-Identification from the Decoupling Mechanism

k_{dec}

IR UV

$\Gamma_{k=0} \equiv \Gamma$ $\Gamma_{k=\Lambda_{\text{UV}}\to\infty} \equiv S$

<u>**decoupling scale:**</u> [Reuter & Weyer, 2000]

effect of artificial IR-cutoff $\mathcal{R}_k$ becomes negligible compared to physical IR-scales, Γ_k flows very slowly for $k < k_{\text{dec}}$

$$\Gamma_{k=k_{\text{dec}}} \approx \Gamma$$

use decoupling condition for scale-identification in RG-improvement:

$$k(x) \equiv k_{\text{dec}}$$

<u>Expectation:</u> mimic effect of higher-derivative & non-local terms not taken into account in the original truncation

Setup: Iterative RG-Improvement + Scale-Identification $k(x) \equiv k_{\rm dec}$

Iterative RG-improvement for self-consistent spacetimes [Platania, 2019]

Aim: dynamical adjustment of cutoff function & taking into account backreaction effects

0. start: $ds^2_{(0)} \supset G_{(0)} \equiv G_0$ classical solution incl. classical Newton constant
1. RG-step: $ds^2_{(1)} \supset G_{(1)}$ new spacetime after 1 x RG-improvement

$G_0 \to G_1(k_1)$

...

n. RG-step: $ds^2_{(n)} \supset G_{(n)}$ new spacetime after n x RG-improvements

$$G_n(k_n) = \frac{G_0}{\alpha + {g_*}^{-1} G_0 k_n^2}$$

$$k_n = k_n(\text{spacetime}_{n-1})$$

... assume sequence $(G_{(n)})$ converges to an **effective Newton coupling** $G_{(\infty)}$ ...

$$G_\infty = \frac{G_0}{\alpha + {g_*}^{-1} G_0 k_\infty^2}$$

$$k_\infty = k_\infty(G_\infty, G'_\infty, \dots)$$

$G_{(\infty)}$ determines a **self-consistent spacetime** $ds^2_{(\infty)}$ invariant under repeated RG-improvements

$\alpha \equiv 0:$
fixed-point scaling

$\alpha \equiv 1:$
approximation interpolating between UV & IR
[Bonanno & Reuter, 2000]

+ combine with scale-identification from decoupling mechanism

Difficulty: determining $k_n(\text{spacetime}_{n-1}) \equiv k_{\rm dec}(\text{spacetime}_{n-1})$ from the decoupling condition would require spacetime_{n-1} to arise from a variational principle for a gravitational action

Application: Quantum-Corrected Vaidya Spacetimes

0. start:

Classical (imploding) Vaidya spacetimes [Vaidya, 1951; Vaidya, 1966]

$$ds^2_{(0)} = -f_{(0)}(r,v)dv^2 + 2dvdr + r^2 d\Omega^2 \,, \quad f_{(0)}(r,v) = 1 - \frac{2G_{(0)}m(v)}{r}$$

$$G^{(0)}_{\mu\nu} = 8\pi \; G_{(0)} T^{(0)}_{\mu\nu} \,, \quad T^{(0)}_{\mu\nu} = \mu_{(0)} u_\mu u_\nu \,, \quad \mu_{(0)} = \frac{\dot{m}(v)}{4\pi G_{(0)} r^2}$$

exact solution to GR sourced by energy-momentum tensor of perfect fluid

n. RG-step:

RG-improved Vaidya spacetimes

$$f_{(n)}(r,v) = 1 - \frac{2M_{(n)}(r,v)}{r} \,, \quad M_{(n)}(r,v) \equiv G_{(n)}(r,v)m(v)$$

$$G^{(n)}_{\mu\nu} = 8\pi \; G_{(n)} T^{(n)}_{\mu\nu} \,, \quad T^{(n)}_{\mu\nu} = \mu_{(n)} l_\mu l_\nu + (\rho_{(n)} + p_{(n)})(l_\mu n_\nu + l_\nu n_\mu) + p_{(n)} g^{(n)}_{\mu\nu}$$

$$\mu_{(n)} \equiv \frac{\dot{M}_{(n)}}{4\pi G_{(n)} r^2} \,, \quad \rho_{(n)} \equiv \frac{M'_{(n)}}{4\pi G_{(n)} r^2} \,, \quad p_{(n)} \equiv -\frac{M''_{(n)}}{8\pi G_{(n)} r}$$

exact solution to GR – "Generalized Vaidya spacetimes" [Wang & Wu, 1999]

Application: Quantum-Corrected Vaidya Spacetimes

Scale-identification

Fact no. 1: $ds^2_{(0)}$ follows from variational principle for a gravitational action [Ray, 1972]

– associated Euclidean EAA:

$$\Gamma_k[g] = -\int d^4x\sqrt{g}\Big(\frac{1}{16\pi G_k}R-\mu\Big) \qquad \text{(I)}$$

Fact no. 2: energy density and pressure determine Ricci scalar of spacetime_n

$$R_{(n)} = 16\pi G_{(n)}\big(\rho_{(n)}-p_{(n)}\big) \qquad \text{(II)}$$

Although a derivation of spacetime_{n-1} from a gravitational action is not available for $n>1$, we can combine the **decoupling condition for (I) with the expression for the Ricci scalar (II)**, to identify an RG-scale parameter at each step of the iteration which includes all physical quantities characterizing the spacetime at the previous step:

$$k_{\text{dec}}{}^2_{(n)} \equiv G_{(n-1)}\Big(\mu_{(n-1)}+\frac{32}{3}\pi\big(\rho_{(n-1)}-p_{(n-1)}\big)\Big) \qquad \lim_{n\to\infty}$$

Master equation for the effective Newton coupling of the self-consistent RG-improved spacetime

$$\Big(G_0\omega m(v)\big(16\pi r G''_\infty(r,v)+32\pi G'_\infty(r,v)+3\dot{G}_\infty(r,v)\big)+3G_0\omega m(v)G_\infty(r,v)+12\pi\alpha r^2\Big)G_\infty(r,v)-12\pi G_0 r^2=0$$

$\omega \equiv g_*^{-1}$ 2nd-order nonlinear PDE

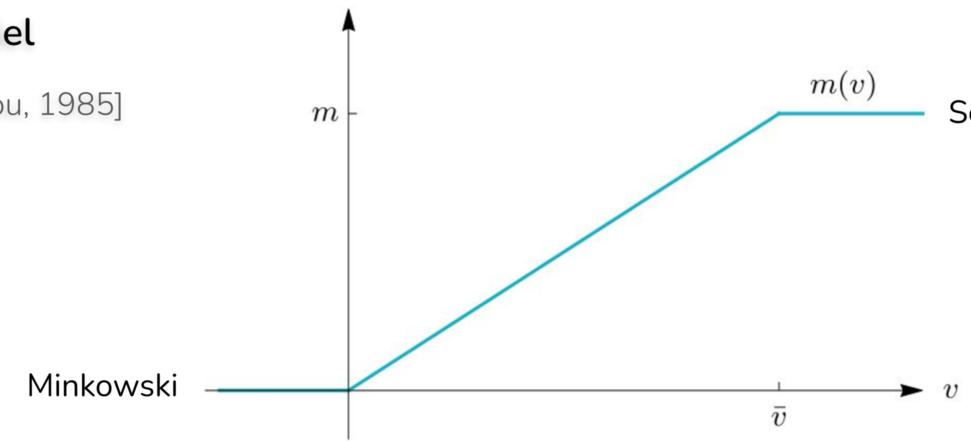
– specify classical mass function & study different regimes…

RG-Improved VKP Spacetime: Numerical Solution for the Dynamics

Vaidya-Kuroda-Papapetrou model

[Vaidya, 1966; Kuroda, 1984; Papapetrou, 1985]

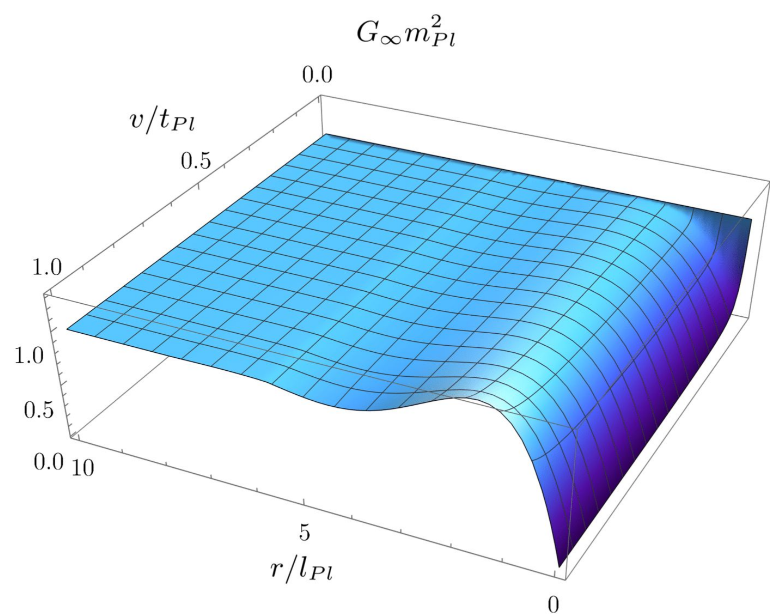
classical gravitational collapse



Numerical solution

effective Newton coupling
of the RG-improved spacetime

$\omega \equiv m \equiv \bar{v} \equiv 1$



Analytical Approximations in the Static Phase

Static limit of PDE: $m(v) \equiv m = \text{const}\ , \quad G_\infty = G_\infty(r)$

UV fixed-point regime

$$G(k) = g_* k^{-2} \ \ (\leftrightarrow \alpha = 0)$$

PDE reduces to ordinary differential equation which admits exact solution...

$$G_\infty(r) = \frac{1}{\sqrt{5\omega m}} r^{3/2}$$

for small r (same exponent found in [Pawlowski & Stock, 2018])

In particular:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \propto \frac{1}{r^3} \quad \text{vs.} \quad \propto \frac{1}{r^6}$$

(for classical Schwarzschild spacetime)

➡ **anti-screening Newton coupling at high energies results in weakening of classical singularity**

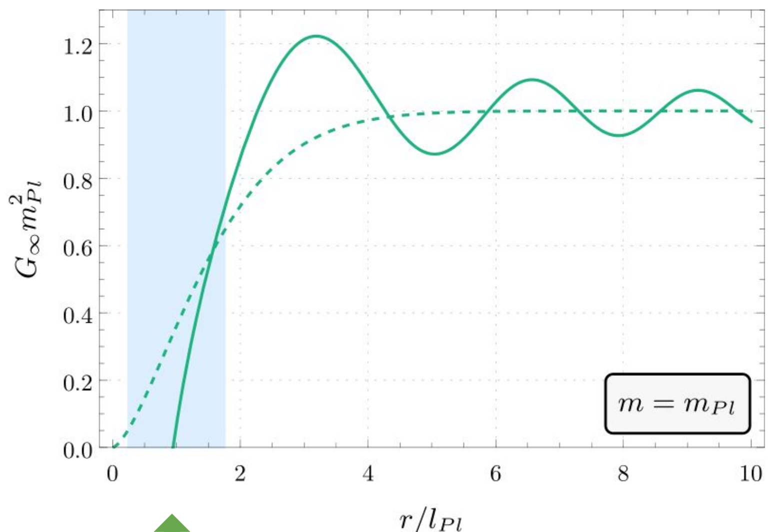
Analytical Approximations in the Static Phase

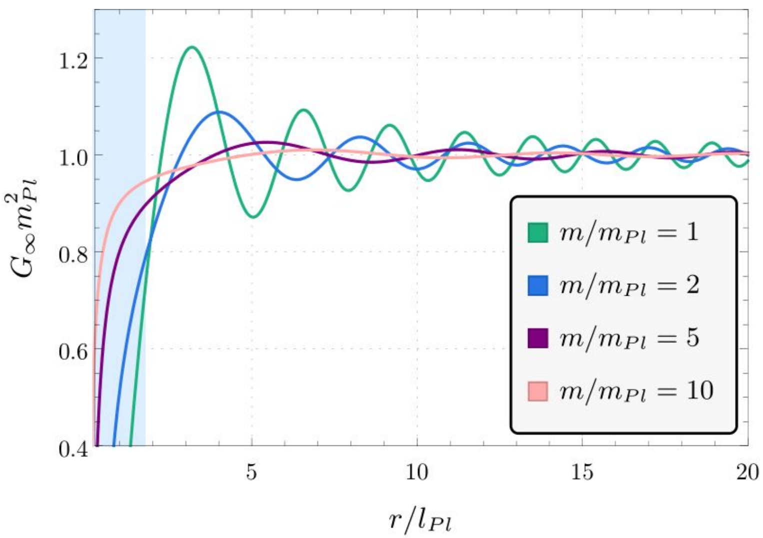
IR-regime

$$G(k) = \frac{G_0}{1+\omega G_0 k^2} \quad (\leftrightarrow \alpha = 1,\; k \ll m_{Pl})$$

oscillatory solutions analogous to results in higher-derivative gravity – consistent with aim of RG-improvement [Bonanno & Reuter, 2013; Zhang et al, 2015]

PDE reduces to ordinary Stokes-type differential equation with solutions given by damped Airy functions...





Planck-scale transition region

$$\lim_{k\to 0} G_k = G_0$$

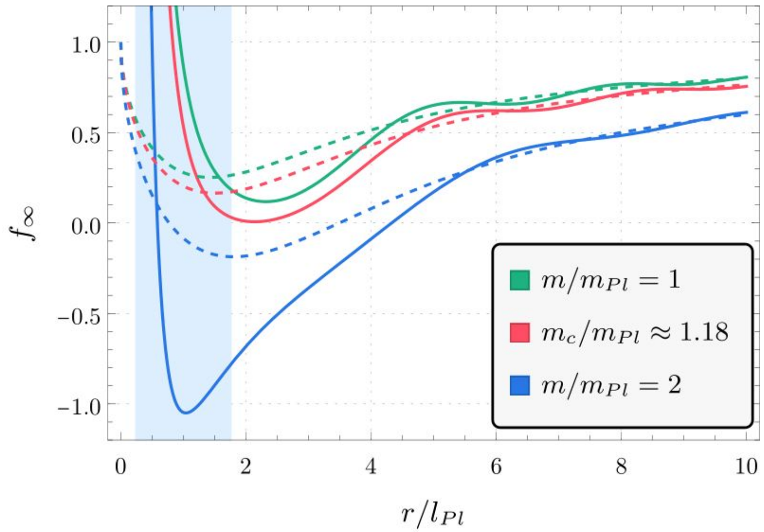
$$\lim_{k\gg m_{Pl}} G_k \approx \frac{1}{\sqrt{5\omega m}} r^{3/2}$$

...combine to define an effective spacetime...

Effective Static Spacetime after Collapse

$$G_\infty(r) = G_0\left(1 - e^{-\frac{r^{3/2}}{\sqrt{5\omega m}}}\right)\,, \quad f_\infty(r) = 1 - \frac{2mG_\infty(r)}{r}$$

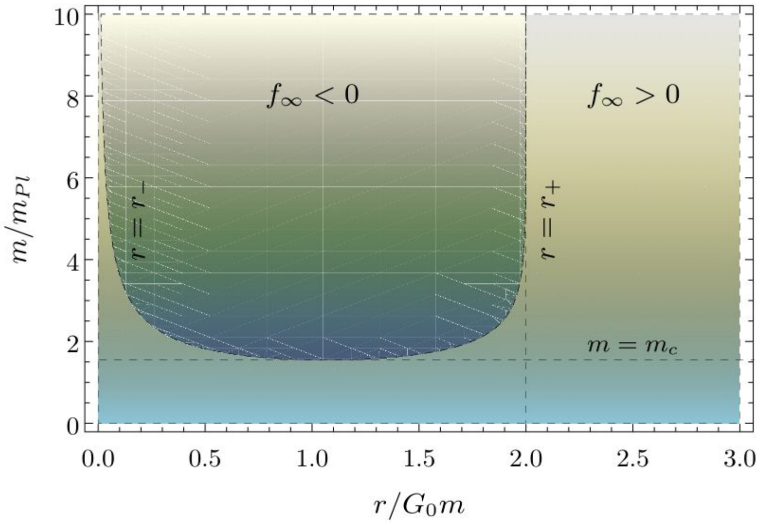
...reminiscent of Dymnikova spacetime resulting from iterative RG-improvement in [Platania; 2019]



large-distance analytical solution (solid)

&

effective metric smoothly interpolating between UV & IR (dashed)



horizon structure of the effective spacetime for different masses

Evaporation of the Quantum Black Hole

Black-hole temperature [Hawking, 1975; Gibbons & Perry, 1978; Hawking, 1978]

$$T_{\rm BH} = \frac{f'(r_+)}{4\pi}$$

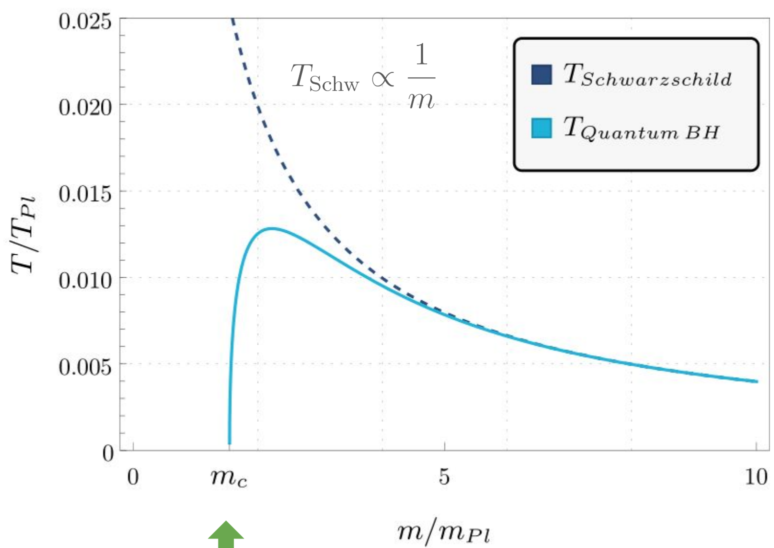
r_+ (outer event horizon) simple zero of f, in particular $m > m_{\rm crit}$

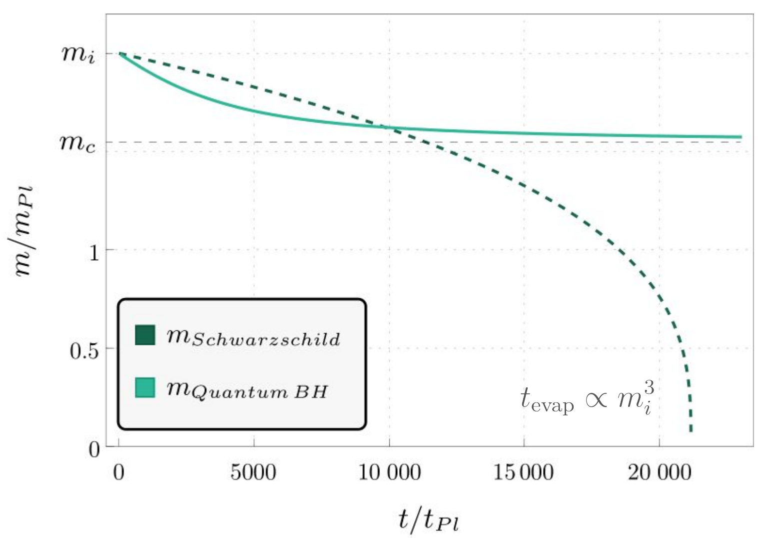
Stefan-Boltzmann law

$$\dot{m}(t) = -\sigma A(m) T_{\rm BH}(m)^4$$

$A(m)$ horizon area

mass-loss per unit proper time





cold black-hole remnant...

...reached after **infinitely long period of time** for evaporation

results qualitatively consistent with [Bonanno & Reuter, 2000] where RG-improvement was not iterated & different scale-identification was used

Summary & Conclusion

New way to study the **effect of quantum fluctuations of gravity on classical spacetimes** which...

...is based on **iterative RG-improvement** combined with **scale-identification from the decoupling mechanism.**

...provides a **complete dynamical model** for **quantum-corrected gravitational collapse**, followed by **static phase** and subsequent **evaporation.**

...is in **qualitative agreement with previous RG-improvements** using different implementation schemes and scale-identifications

...consistently reproduces **features of corresponding solutions in higher-derivative gravity**

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