Gross–Neveu SO(3) criticality in spin-orbital liquids: FRG vs higher-order perturbation theory

TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

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ct.qmat Complexity and Topology in Quantum Matter

Würzburg-Dresden Cluster of Excellence

Criticality in spin-orbital liquids

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma$$

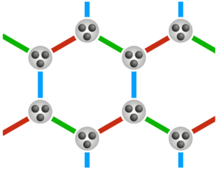
$K > 0$

σ^α (τ^γ) — Pauli matrices in spin (orbital) space

- Spin-orbital generalization (still soluble!) of Kitaev model on honeycomb lattice

  $\Rightarrow$ fermionic excitations from fractionalization

  Chulliparambil *et al.*, Phys. Rev. B(R) '20



Criticality in spin-orbital liquids

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

$K > 0, J > 0$

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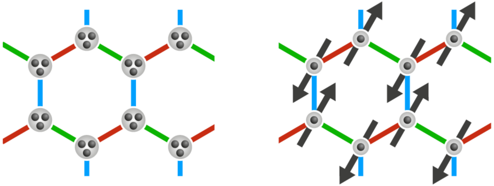

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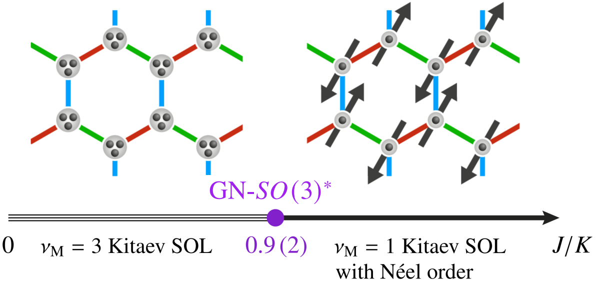

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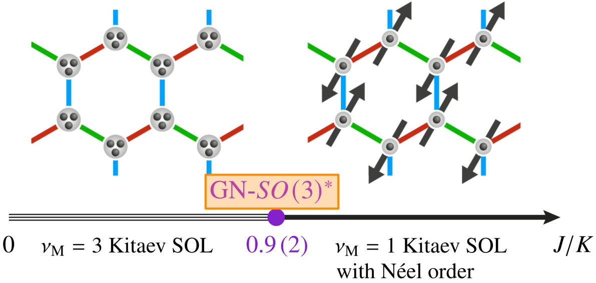


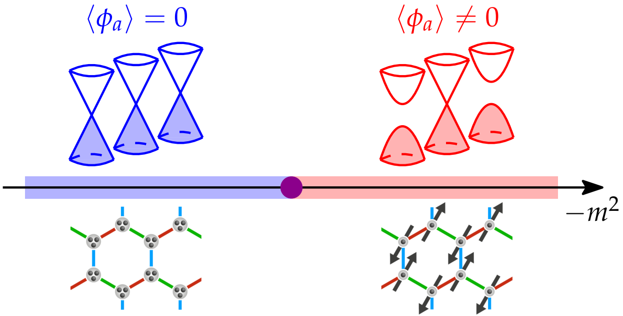
Gross–Neveu SO(3) universality class

Effective field theory Lagrangian [Seifert *et al.*, Phys. Rev. Lett. '20]

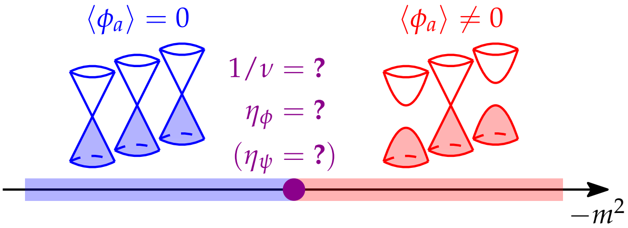
$$\mathcal{L} = \overline{\psi}_i \partial\!\!\!/ \psi_i + \tfrac{1}{2}(\partial_\mu \phi_a)^2 - g\phi_a \overline{\psi}_i T^a_{ij} \psi_j + \tfrac{1}{2} m^2 \phi^2 + \tfrac{1}{4!}\lambda \phi^4$$

$\mu = 0,1,2$: $(2+1)$D Euclidean spacetime; T^a: $SO(3)$ generators ($a = 1,2,3$)

- 3 fermion flavours, $SO(3)$ symmetry
- one flavour remains gapless after SSB
  contrast with chiral Ising or chiral Heisenberg!
- Novel universality class, state of the art: first-order ϵ and $1/N$ expansion
  ibid.



Our work / this talk . . .



. . . series expansion parameters ϵ, $1/N$ in reality ~ 1

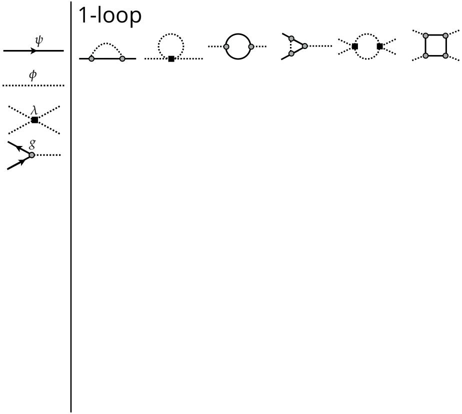
This talk:

- Series expansions to high(-er) orders
- Complementary non-perturbative calculation (FRG)
- Combined estimates with high-confidence error bars

Perturbative series I

ϵ expansion

- Basic idea: loop expansion; need small interaction strengths
- Couplings at criticality of order ϵ in $D = 4 - \epsilon$ spacetime dimensions



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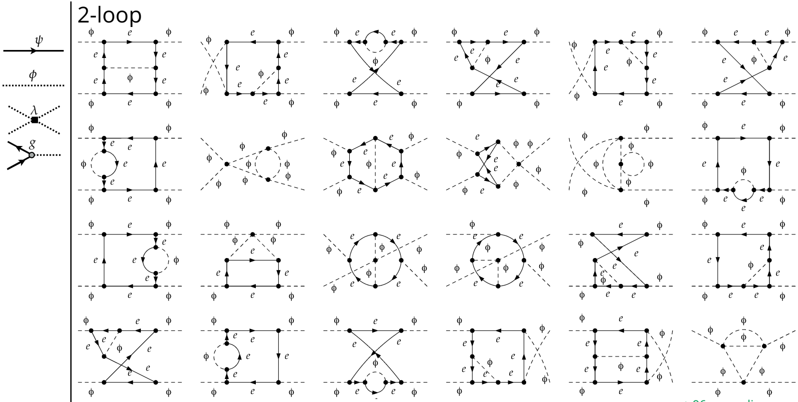

…+ 96 more diagrams

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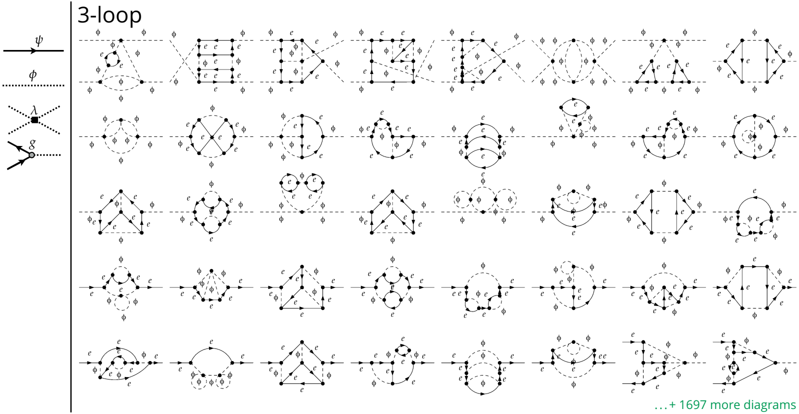

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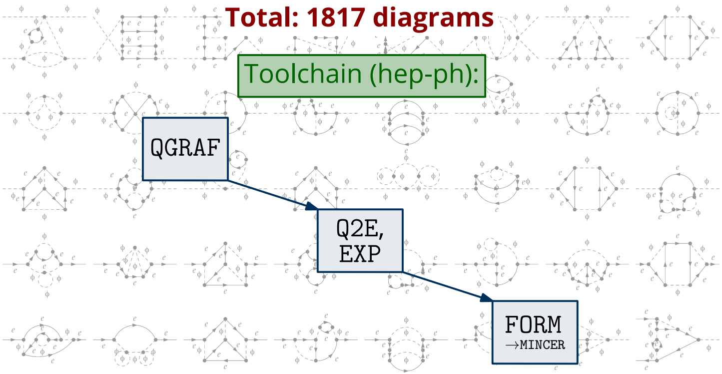


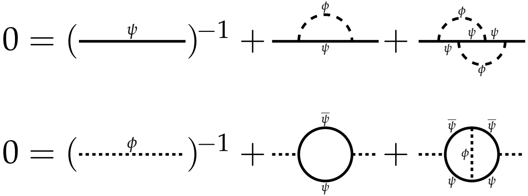
cf.: Nogueira J Comput. Phys. '93; Nogueira Nucl. Instrum. Methods Phys. Res. '06; Harlander *et al.* Phys. Lett. B '98; Seidensticker et al. arXiv:hep-ph/9905298; Vermaseren arXiv:math-ph/0010025; Kuipers et al. Comput. Phys. Commun. '13; Ruijl et al. arXiv:1707.06453; Czakon Nucl. Phys. B '05; Gorishnii et al. Comput. Phys. Commun. '89; Larin et al. NIKHEF-H-91-18 '91

Perturbative series II

$1/N$ expansion

- Basic idea: flavour number $N \to \infty$ with gN fixed $\simeq$ mean-field theory
- Systematic corrections in $1/N$: resummation of (infinitely many!) loop diagrams
  …perturbation theory reorganized according to 'closed fermion loops per Yukawa vertex'

  $\Rightarrow$ self-consistent Dyson-Schwinger equations



(+ 6 more diagrams at NNLO)

cf.: Vasil'ev *et al*. Theor. Math. Phys. '81a,b; '93;
Gracey Int. J. Mod. Phys. A '94; '18 and refs. therein

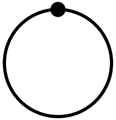
Non-perturbative

Functional Renormalization Group (FRG)

- Basic idea: solve Wetterich–Morris–Ellwanger equation self-consistently

  Wetterich, Phys. Lett. B '93; Morris, Int. J. Mod. Phys. A '94; Ellwanger, Z. Phys. C '94

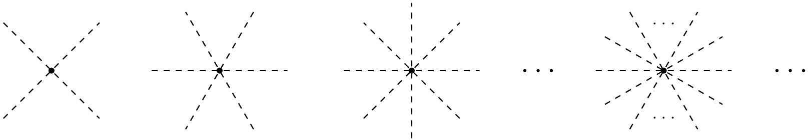
$$\frac{\partial \Gamma_k[\Phi]}{\partial \ln k} = \mathrm{STr}\left[\left(\frac{\delta^2 \Gamma_k[\Phi]}{\delta\Phi\delta\Phi^\top} + R_k[\Phi]\right)^{-1} \frac{\partial R_k[\Phi]}{\partial \ln k}\right] =$$



cf., e.g.: Berges *et al*. Phys. Rep. '02; Metzner *et al*. Rev. Mod. Phys. '12; Dupuis *et al*. Phys. Rept. '21; and refs. therein

$\Phi = (\phi_a, \psi, \overline{\psi})^\top$; k — RG scale; R — regulator

- truncation: improved local potential approximation (LPA')
  - ∗ standard wavefct. and Yukawa vertex renorm.: $\psi, \phi, g \longrightarrow \sqrt{Z_\psi}\psi, \sqrt{Z_\phi}\phi, Z_g g$
  - ∗ arbitrary renormalized boson effective potential: $m^2\phi^2/2 + \lambda\phi^4/4! \longrightarrow V(\phi)$



Post processing

- Series only asymptotic in $\epsilon \to 0$ (i.e., convergence radius typically 0)
- Physical case: $\epsilon = 1 \longrightarrow$ extrapolation; here: **Padé approximants**

$$[m/n] := \frac{a_0 + a_1\epsilon + \ldots + a_m\epsilon^m}{1 + b_1\epsilon + \ldots + b_n\epsilon^n} = c_0 + c_1\epsilon + \ldots + c_{m+n}\epsilon^{m+n} + \mathcal{O}(\epsilon^{m+n+1})$$

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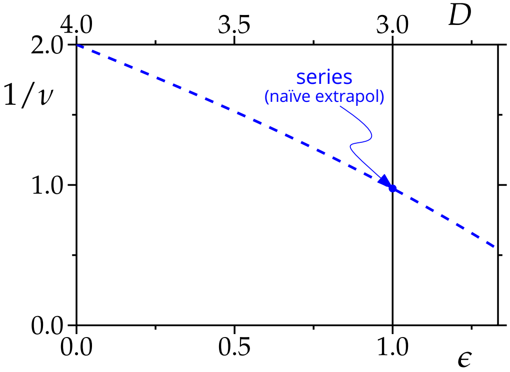

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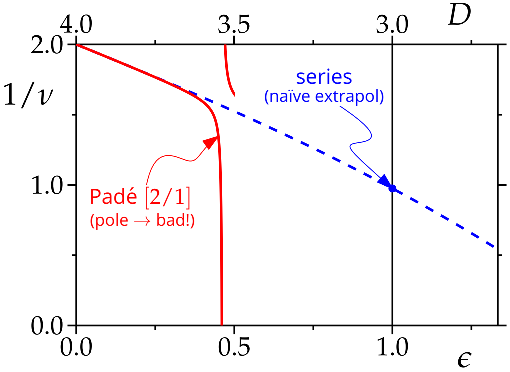

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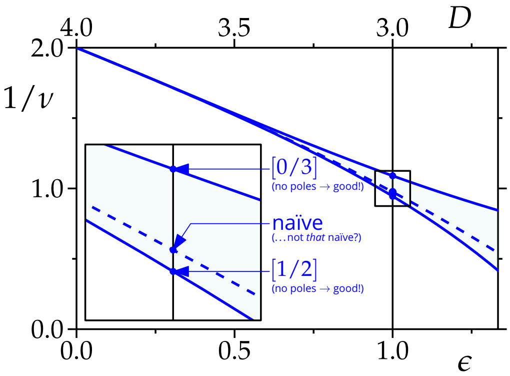

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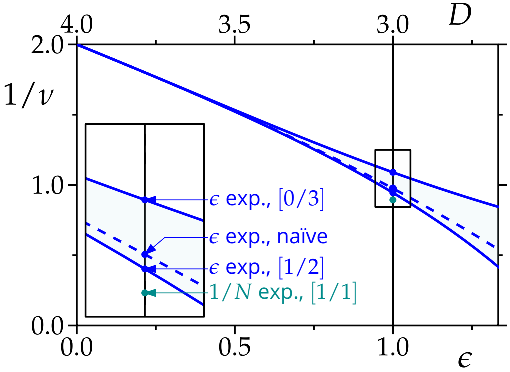

$1/N$ expansion
Padés (*mutatis mutandis*)
N.B.: only one 'good' extrapolation (a bit extreme . . .)

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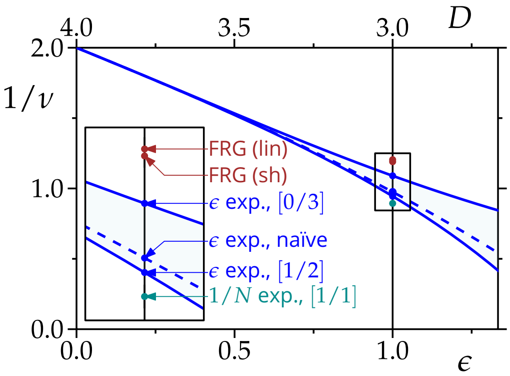

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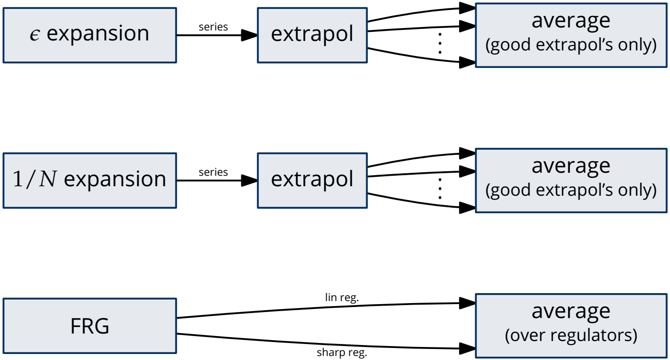
FRG

regulator-dependence as proxy for truncation error

Recall: LPA' $\sim 0^{\text{th}}$ order derivative expansion + wavefct. renorm.; higher orders too much work . . .

Combined estimates — Workflow



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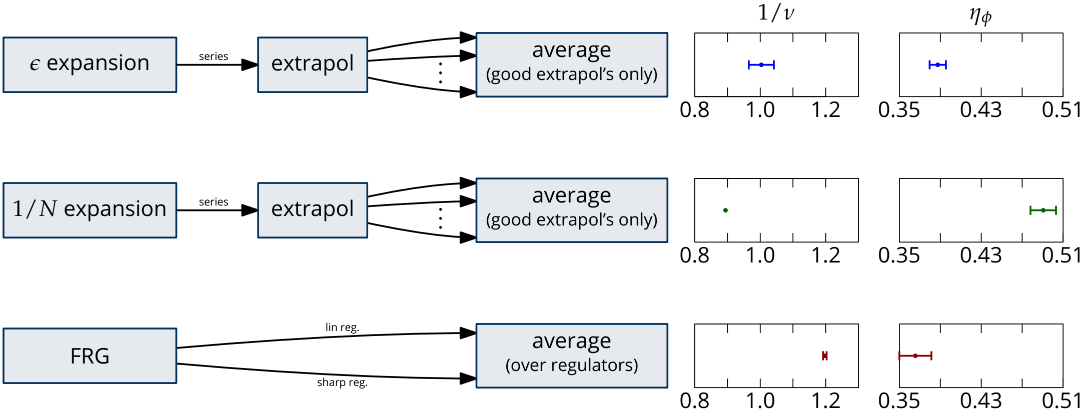

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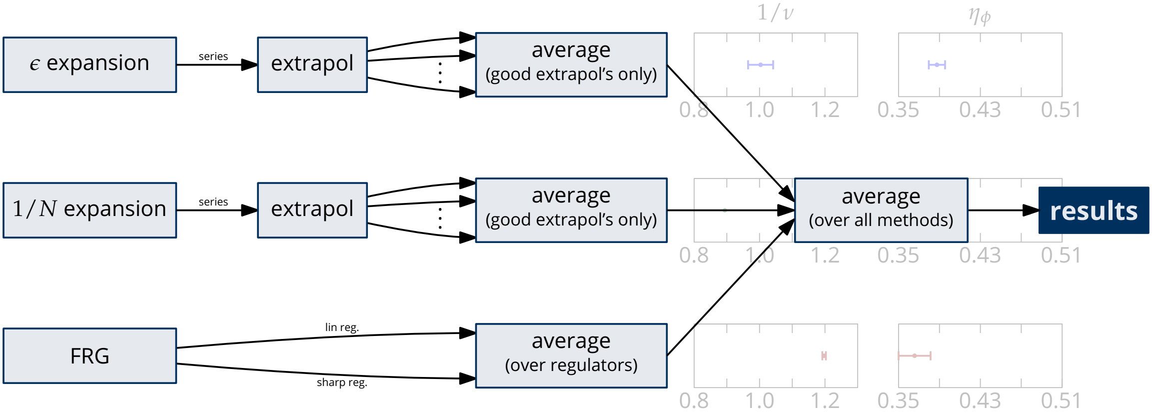
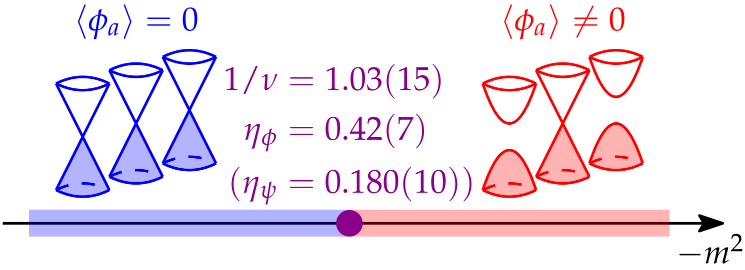


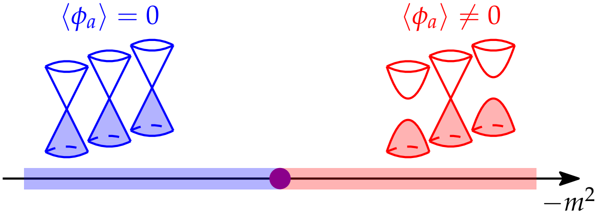
Numbers

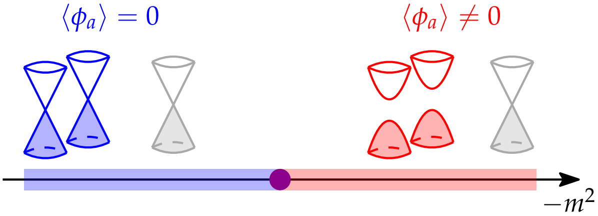


$\langle \phi_a \rangle = 0$
$\langle \phi_a \rangle \neq 0$
$1/\nu = 1.03(15)$
$\eta_\phi = 0.42(7)$
$(\eta_\psi = 0.180(10))$
$-m^2$

Quantum criticality as diagnostic?

- Two possible scenarios compatible with gapless DOF count





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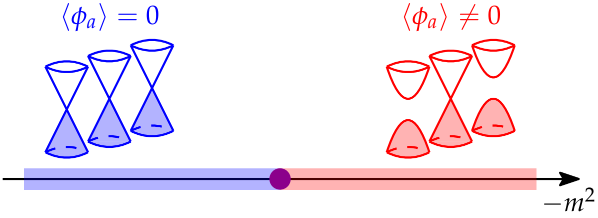

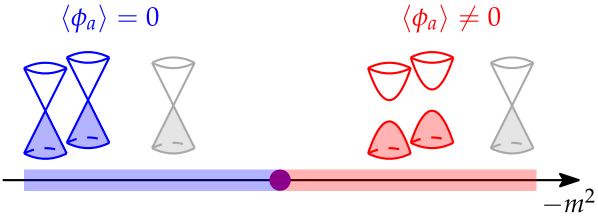

$$\eta_\phi = 0.53(1)$$

Lang & Läuchli, Phys. Rev. Lett. '19

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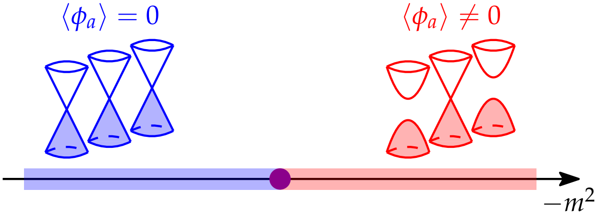
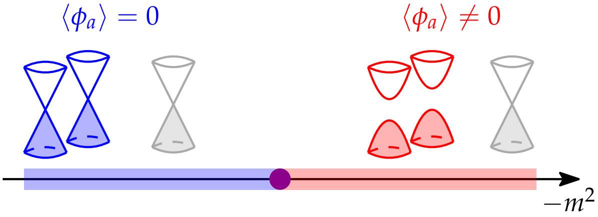


$$\eta_\phi = 0.42(7)$$

this work



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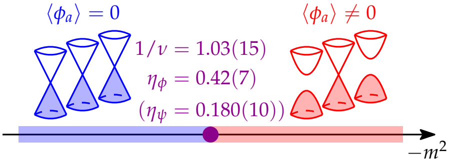
Closing remarks



- Error bars not 'small', but high-confidence
  ...three methods with distinct regimes of validity

  $\Rightarrow$ point of reference for future complementary efforts
  e.g., conf. bootstrap, QMC, DMRG, ...

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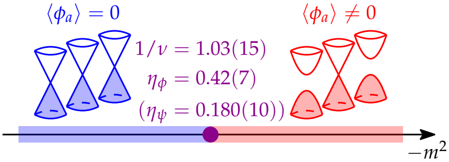

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  ...extrapolation more reliable if more partial sums are available ($4-\epsilon$, large-N) or at all possible (FRG in derivative expansion)

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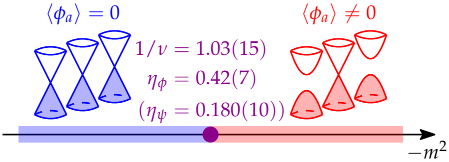

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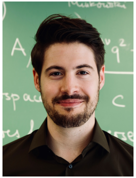
Further details and results:

S.R., B. Ihrig, D. Kruti, J.A. Gracey, M.M. Scherer, and L. Janssen (2021):
Phys. Rev. B **103**, 155160

Acknowledgement



B. Ihrig

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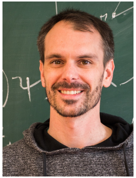

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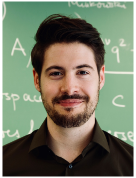

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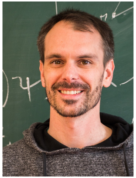

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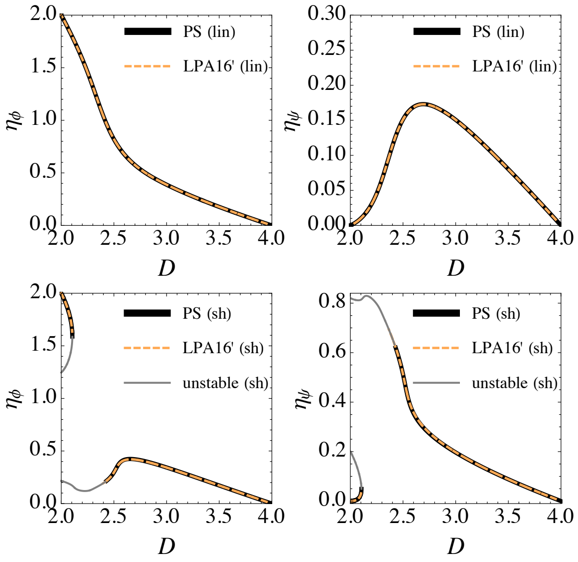
Thank you!

Non-perturbative fixed-point collisions



Quantum Monte Carlo ($N = 12$ flavours)

Liu *et al.*, Phys. Rev. Lett. '22