Running Newton Coupling, Scale Identification and Black Hole Thermodynamics

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Based on collaboration with
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"Running Newton coupling, scale identification, and black hole thermodynamics,"

Phys. Rev. D 105 (2022) no.10, 106026 [arXiv:2204.09892 [hep-th]].

1 Introduction – quantum effects –

Black holes ⋯ have singularites at the center:

Curvature diverges, and the Einstein equation does not make sense

Can the quantum effects escue the theory?

Here we use the approach to quantum gravity using the functional renormalization group (FRG) – Asymptotic safety

A central object in the FRG is the effective average action Γ_k, defined in the path integral by integrating over UV modes beyond a cutoff energy scale k.

The quantum effects ⇒ Newton coupling depends on the energy scale k.

What consequence this may have to black holes?

We make renormalization group (RG) improvement at the level of the solutions, where we replace the couplings appearing in the classical solutions with the running couplings.

To get physical effects ⇒ Suitable choice of the identification of the energy scale with some length scale in the solutions

2 Running Newton Couplings

The action of the Einstein theory with a cosmological constant:

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda)\,,$$

Running Newton coupling

Renormalization group improvement shows that the Newton coupling at energy scale k has a dependence (under certain approximation)

$$G(k) = \frac{G(k_0)}{1+\omega G(k_0)k^2}. \quad \left(\tilde{G}(k) = \frac{G(k_0)k^2}{1+\omega G(k_0)k^2} \textbf{ has NGFP.}\right)$$

k_0 is a fixed energy scale, which may be taken $k_0 = 0$.

This means that the Newton constant tend to vanish at high energy!

The scale identification – on dimensional grounds

How the energy scale is related to the distance scale?

$$k = \frac{\xi}{d(P)} \cdots \ d \textbf{ is a distance, } \xi \textbf{ is a number of order 1.}$$

3 Scale identification

How to identify $d(P)$?

1. Geodesic distance **(A. Bonnano and M. Reuter, PRD 62 (2000) 043008.)**

2. Kretschmann invariant **(J. M. Pawlowski and D. Stock, PRD 98 (2018) 106008)**

1. Geodesic distance:

The Schwarzschild black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

with

$$f(r) = 1 - \frac{2G_0 M}{r}$$

The behaviors of the geodesic distance are

$$\begin{aligned} d(r) &\to r, \qquad r \to \infty. \\ d(r) &\to r^{3/2}, \qquad r \to 0 \end{aligned}$$

If we replace G_0 by $G(r)$, then the singularity at the origin $r = 0$ disappears!

$$G(r) \to r^{3/2}, \qquad r \to 0$$

This identification depends on coordinate system.

2. Kretschmann invariant, which does not depend on the coordinates:

$$k = (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})^{1/4}$$

Short distance behavior is even better

$$G(r) \to r^2 \quad \textbf{for} \quad r \to 0.$$

The singularity at the origin may be resolved, since the gravity becomes weak there or even zero.

The same problem has been studied for Reisnner-Nordström solution: with Kretschmann identification and found similar conclusion (singlarity may be resolved).

A. Ishibashi, N. Ohta and D. Yamaguchi, "Quantum improved charged black holes," Phys. Rev. D 104 (2021) 066016

So far, so good.

4 Kerr Black Holes

For Kerr solution, both identifications chose fixed angle, and the results in general depend on the angle.

This would be strange! ⇒ Then should we keep arbitrary angle?

This leads to several difficulties:

- **The surface gravity is not constant on the horizon ··· black hole may not be thermal equilibrium!**
- **The entropy does not satisfy the 1st law of thermodynamics!**

$$dM = TdS + \Omega dJ$$

  M**: Black hole mass,** $J = Ma$**: angular momentum (**a **is the angular momentum parameter),** Ω**: Angular velocity.**

- **In general, there appears singularity on the horizon.**

cf. Aaron Held's talk on Monday.

None of these may be accepted for physical situation.

Need to specify what is the correct identification of k **with distance** r**.**

The problem

There has not been any physical principle to determine the scale identification.
What is physical principle to specify the identification?

We have studied this problem with Chiang-Mei Chen, Yi Chen (NCU student), A. Ishibashi and D. Yamaguchi (Kindai student)

We propose that we must impose the consistency of the first law of thermodynamics as a physical guiding principle, at least near horizon.

The first law of black hole:

$$dM = TdS + \Omega dJ,$$

Mass is determined by the vanishing condition of lapse at the horizon:

$$M(r_+, a) = \frac{r_+^2 + a^2}{2Gr_+} \qquad \textbf{(independent variable } M \to r_+\textbf{)}$$

Writing everything as (choose the horizon radius r_+ and angular momentum parameter $a(= J/M)$ as independent variable)

$$dS = \partial_+ S dr_+ + \partial_a S da$$

we find

$$\partial_+ S = \frac{\partial_+ M - \Omega \partial_+ J}{T}, \qquad \partial_a S = \frac{\partial_a M - \Omega \partial_a J}{T}$$

The consistency condition

$$\partial_a(\partial_+ S) = \partial_+(\partial_a S)$$

Assuming that the Newton coupling depends on the coordinate r and a, we calculate the temperature

$$T = \frac{\kappa}{2\pi} = \frac{(r_+^2 - a^2)G(r_+, a) - r_+(r_+^2 + a^2)\partial_+ G(r_+, a)}{4\pi r_+(r_+^2 + a^2)G(r_+, a)}$$

κ is the surface gravity.

The the derivatives of S from the first law are

$$\partial_+ S = \frac{2\pi r_+}{G(r_+, a)}, \qquad \partial_a S = \frac{2\pi \left[r_+^2(r_+^2 + a^2)\partial_a G(r_+, a) - a(r_+^2 - a^2)G(r_+, a) \right]}{G(r_+, a)\left[r_+(r_+^2 + a^2)\partial_+ G(r_+, a) - (r_+^2 - a^2)G(r_+, a) \right]}.$$

The consistency condition gives rather complicated equation

$$\partial_+ \left[\frac{r_+^2(r_+ \partial_a G - a\partial_+ G)}{r_+(r_+^2 + a^2)\partial_+ G - (r_+^2 - a^2)G} \right] = 0,$$

(This may look simple, but what you really get is much more messy equation, with the derivative acted on the expression.)

This can be integrated once, but we have to integrate it more (difficult)! However it is easy to find simple solution.

That is the Newton constant should be a function of the horizon area $A = 4\pi(r_+^2 + a^2)$!

Substituting this condition back into the derivatives of S

$$\partial_+ S = \frac{2\pi r_+}{G(A)}, \qquad \partial_a S = \frac{2\pi a}{G(A)}.$$

This gives us the quantum entropy

Universal formula

$$\Rightarrow S = \int \frac{dA}{4G(A)} \quad \cdots \quad \textbf{a universal formula} \quad (A = 4\pi(r_+^2 + a^2))$$

Evidence of the correctness: this gives Bekenstein-Hawking formula $\frac{A}{4G}$ for constant G.

cf. K. Falls and D. F. Litim, "Black hole thermodynamics under the microscope," Phys. Rev. D 89 (2014), 084002 [arXiv:1212.1821 [gr-qc]].

The formula was given starting with the postulate of possible dependence of S on A.

We have thus found that this leads to a solution that the scale identification should be made through surface area at fixed radius.

Very interestingly, we find that this is a universal formula valid not only for Kerr solutions but also for

Schwarzschild black holes
Reissner-Nordström black holes
Five-dimensional Myers-Perry black holes (with 2 angular momenta)
Kaluza-Klein Black Strings.

Simple dimensional analysis suggests the identification

$$k = \frac{\xi}{\sqrt{A}} = \frac{\tilde{\xi}}{\sqrt{r_+^2 + a^2}}, \quad (\xi \text{ is a dimensionless constant})$$

The entropy is then

$$S = \frac{4\pi(r_+^2 + a^2)}{4G_0} + \pi\tilde{\omega}\ln(r_+^2 + a^2).$$

Typical logarithmic corrections to the Bekenstein-Hawking formula (first term)!

But this identification is only on or near the horizon.
⇒ We have to extend it away from horizon.

$\Rightarrow$ **By continuity, it is natural to extend it to the surface area away from the horizon.**

$$A = 4\pi(r_+^2 + a^2) \Rightarrow \mathcal{A} = 4\pi(r^2 + a^2)$$

Then we may consider something like

$$k = \frac{\xi}{\sqrt{\mathcal{A}}}\left(1 + \frac{G_0}{\sqrt{\mathcal{A}}}\right)^b, \quad \textbf{Sufficiently large } b$$

which diverges faster in the limit of $\mathcal{A} \to 0$, and such modification may resolve singularity. This was the case for Schwarzschild and RN black holes.

But this does not work for Kerr black hole because $\mathcal{A}$ never vanishes.

We have determined that the identification should be made by horizon area on the horizon (small progress), but do not have a criterion which form is correct.

In particular, there is some evidence that our identification should be modified away from the horizon (high energy behavior).

Then there still remains the important question of what is the physical principle for the identification away from the horizon.

5 Summary and discussions

Study of various black holes with coordinate (and angular momentum) dependence, we find

- **The consistency of the thermodynamics gives a physical principle to determine the dependence**
- **The Newton coupling should be a function of the horizon area.**
- **We find a universal formula**

$$S = \int \frac{dA}{4G(A)}$$

Problems to be understood

- **Black holes have extremal cases ⇒ stable remnants?**
  Phases of black hole states (under study)
- **Away from the horizon, what is the physical principle to determine the scale identification?**
- **Can the black hole singularity be resolved? $\cdots$ to some extent**
- **What about other situation?**
  Cosmology etc.