Interacting fixed points of fermionic field theories

Charlie Cresswell-Hogg

based on 2207.10115 with Daniel Litim

c.cresswell-hogg@sussex.ac.uk

Exact renormalisation group 2022, Berlin

UNIVERSITY OF SUSSEX

Gross-Neveu

N ***Dirac fermions*** *in d dimensions*

$$S = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 \right\}$$

Gross-Neveu

*N **Dirac fermions** in d dimensions*

$$S = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 \right\}$$

GN universality class

Gross-Neveu

*N **Dirac fermions** in d dimensions*

$$S = \int_x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 \right\}$$

GN universality class

toy model for nonpertubative physics

Gross-Neveu

N ***Dirac fermions*** *in d dimensions*

$$S = \int_x \left\{ \bar{\psi}_a \partial\!\!\!/ \psi_a + \frac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 \right\}$$

asymptotically...

- ***free*** *in d = 2*
- ***safe*** *in d = 3*

[*Gross, Neveu, PRD ’74*]
[*Gawędzki, Kupiainen, Nucl.Phys.B ’85*]
[*Rosenstein, Warr, Park, PRL ‘88*]
[*de Calan et al., PRL ’91*]

Gross-Neveu

N ***Dirac fermions*** *in d dimensions*

$$S=\int_x \left\{\bar{\psi}_a \not{\partial}\psi_a+\frac{1}{2}G\left(\bar{\psi}_a\psi_a\right)^2\right\}$$

$$\psi\to\gamma^5\psi,\quad \bar{\psi}\to-\bar{\psi}\gamma^5,\quad \bar{\psi}\psi\to-\bar{\psi}\psi$$

asymptotically…

- ***free*** *in d = 2*
- ***safe*** *in d = 3*

Gross-Neveu

N ***Dirac fermions*** *in d dimensions*

$$S = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \tfrac{1}{2} G \left(\bar{\psi}_a \psi_a \right)^2 + \underline{\tfrac{1}{3!} H \left(\bar{\psi}_a \psi_a \right)^3} \right\}$$

$$\cancel{\psi \to \gamma^5 \psi, \quad \bar{\psi} \to -\bar{\psi}\gamma^5, \quad \bar{\psi}\psi \to -\bar{\psi}\psi}$$

asymptotically...

- ***free*** *in d = 2*
- ***safe*** *in d = 3*

[CCH, Litim, 2207.10115]

RG flow

LPA for fermions:
[*Jakovác, Patkós, 1306.2660*]
[*Jakovác, Patkós, Pósfay, 1406.3195*]

At large N, exact RG flow for

$$\Gamma_k[\psi, \bar{\psi}] = \int_x \{\bar{\psi}_a \partial\!\!\!/ \psi_a + V_k(\bar{\psi}_a \psi_a)\}$$

reduces to

$$\partial_t v = -d\, v + (d-1) z \partial_z v - \frac{1}{1 + (\partial_z v)^2}$$

where

$$t = \log k/\Lambda, \quad z = k^{1-d} \bar{\psi}_a \psi_a, \quad v(z,t) = k^{-d} V_k(\bar{\psi}_a \psi_a)$$

Fixed points

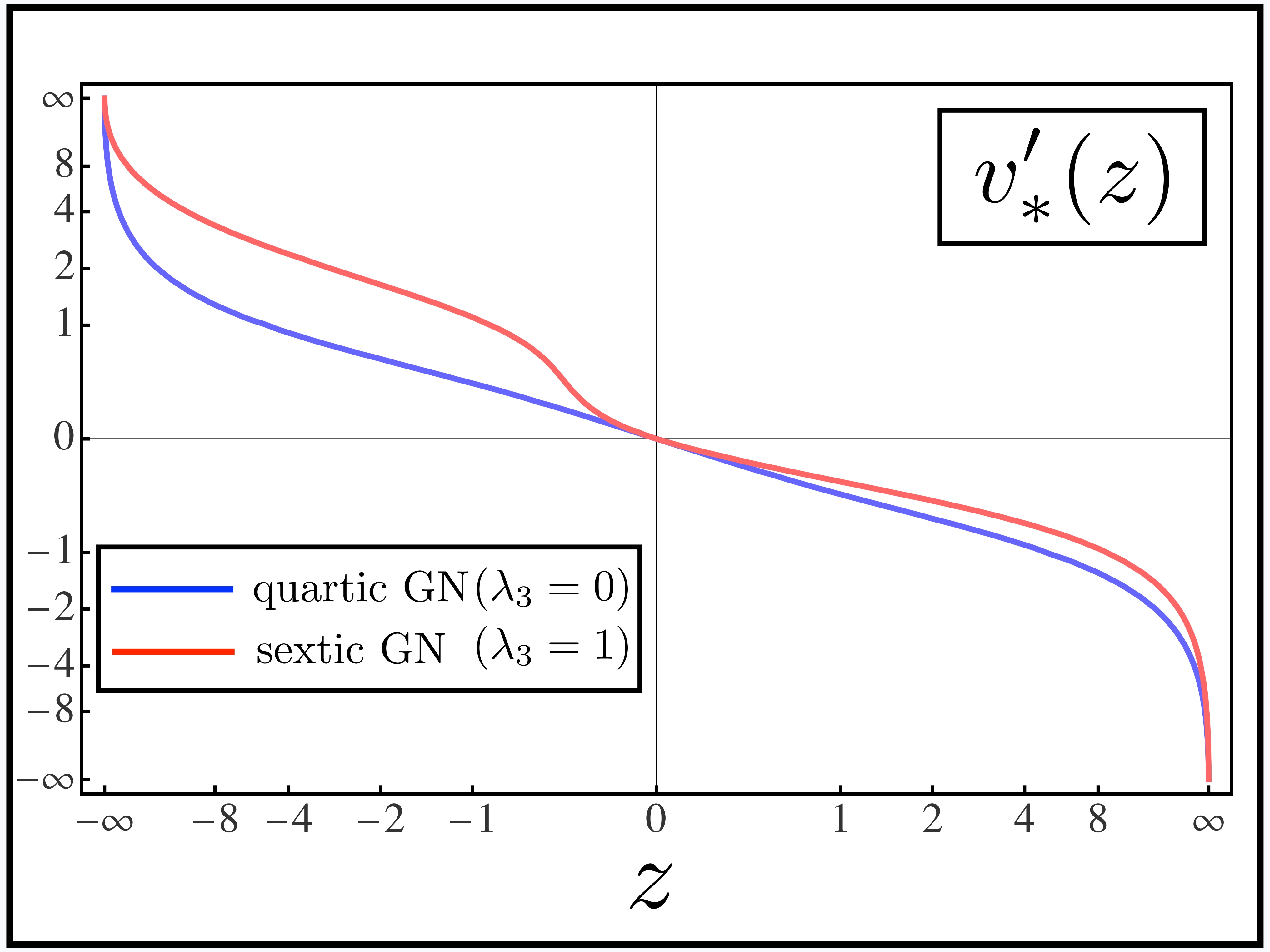
Exact solution

$$z = 4\lambda_3^*(v')^2 - v'\left[\frac{2+3(v')^2}{1+(v')^2} + 3v' \arctan v'\right]$$

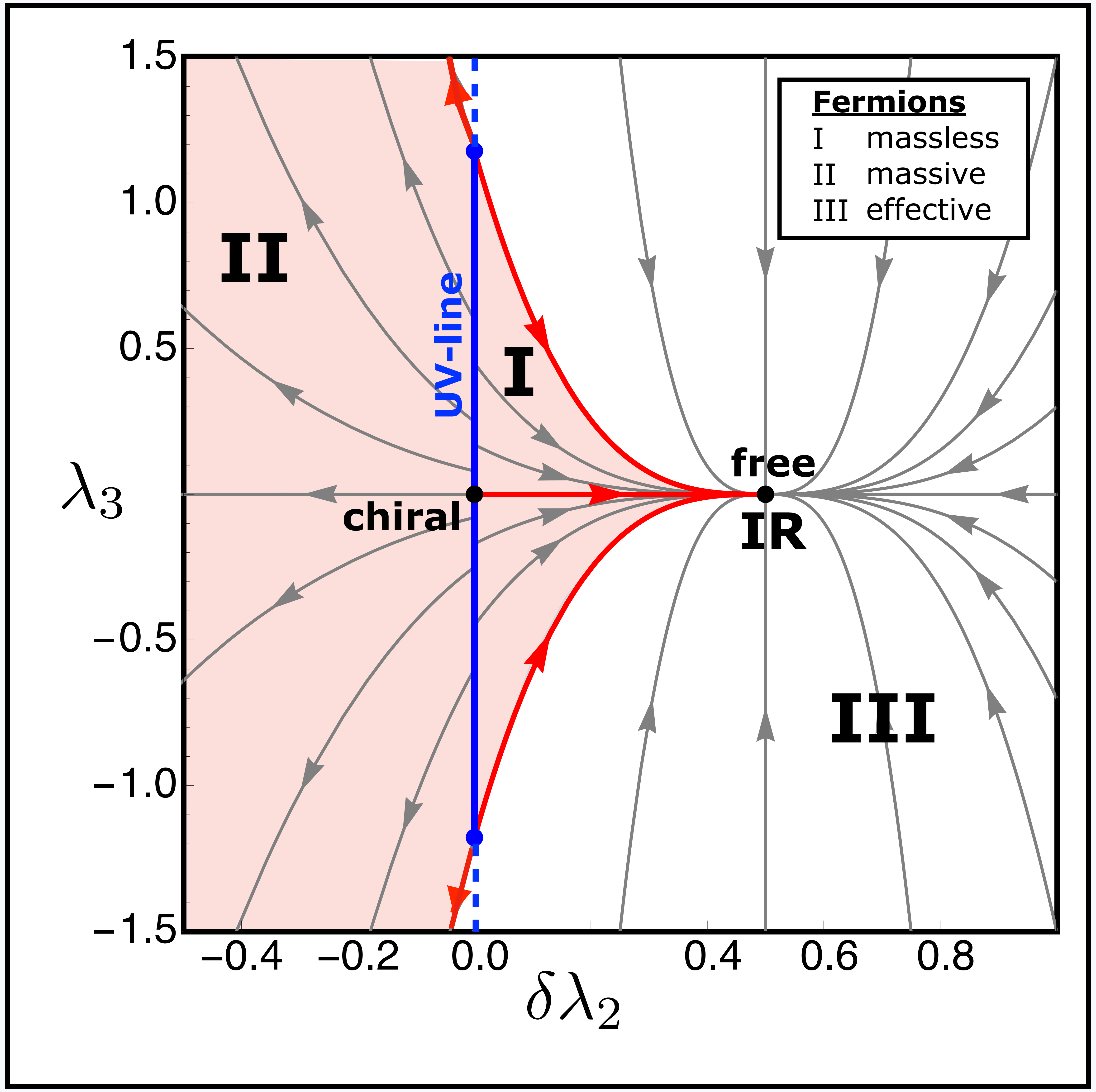
$$0 \le |\lambda_3^*| < \lambda_3^{\mathrm{crit}}$$

For small z

$$v_*(z) = -\frac{1}{4}z^2 + \frac{1}{3!}\lambda_3^* z^3 + \mathcal{O}(z^4)$$



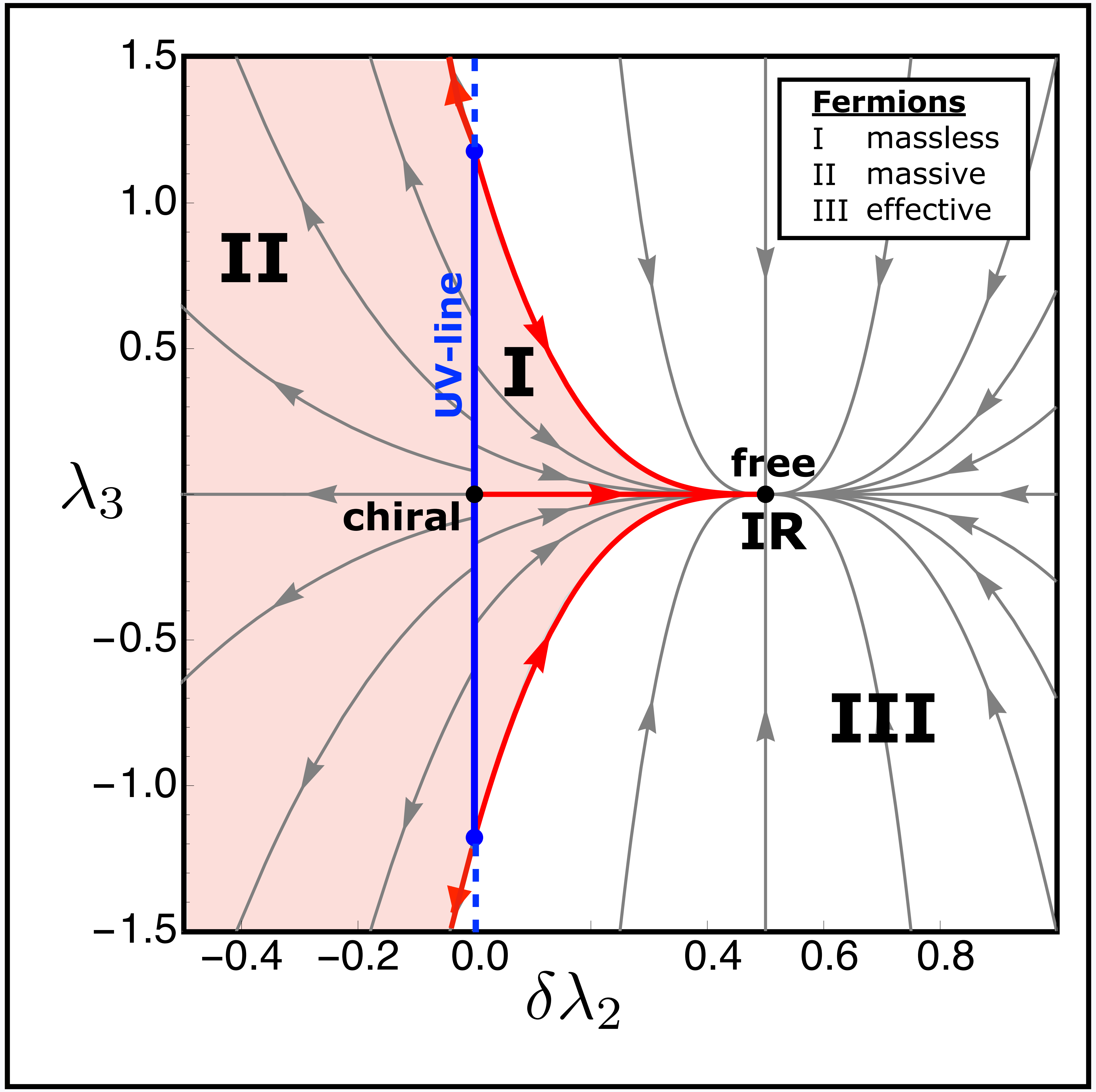
Phase diagram



Phase diagram

IR: 1 relevant

$$\theta = 1,\ -1,\ -3,\ -5, \ldots$$



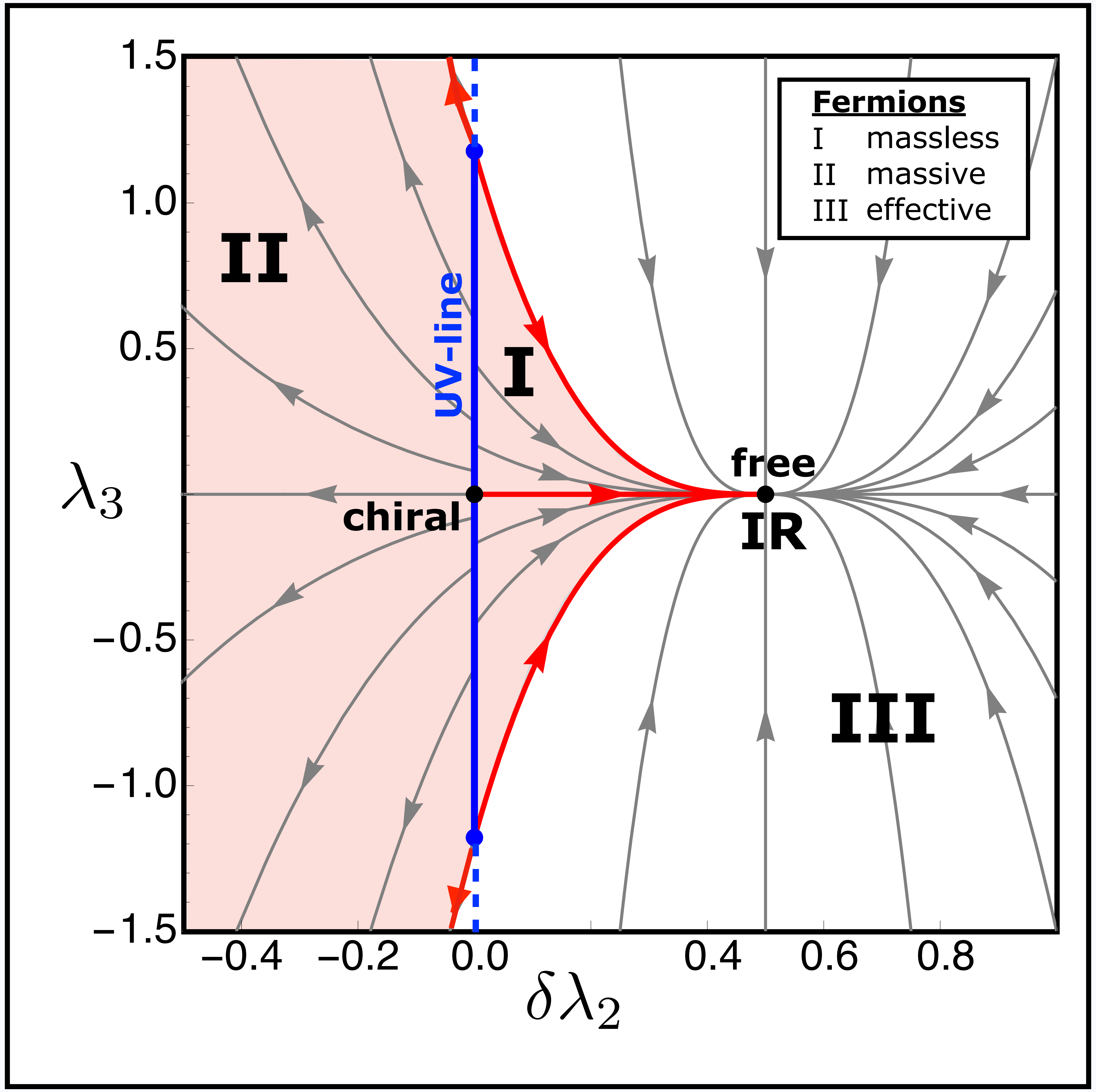
Phase diagram

IR: 1 relevant

$$\theta = 1,\ -1,\ -3,\ -5, \ldots$$

UV: 2 relevant, 1 marginal

$$\theta = 2,\ 1,\ 0,\ -1, \ldots$$

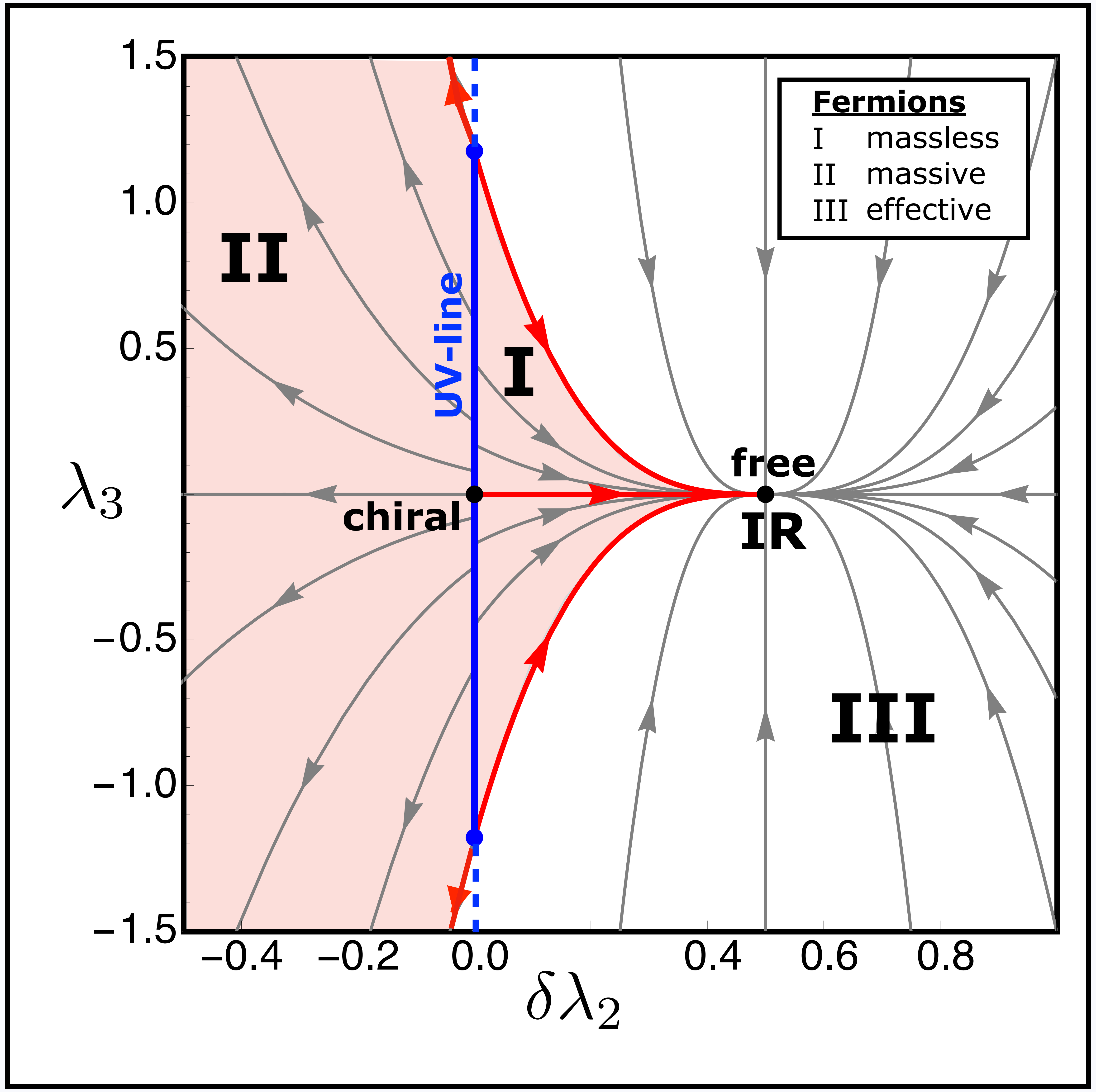


Phase diagram

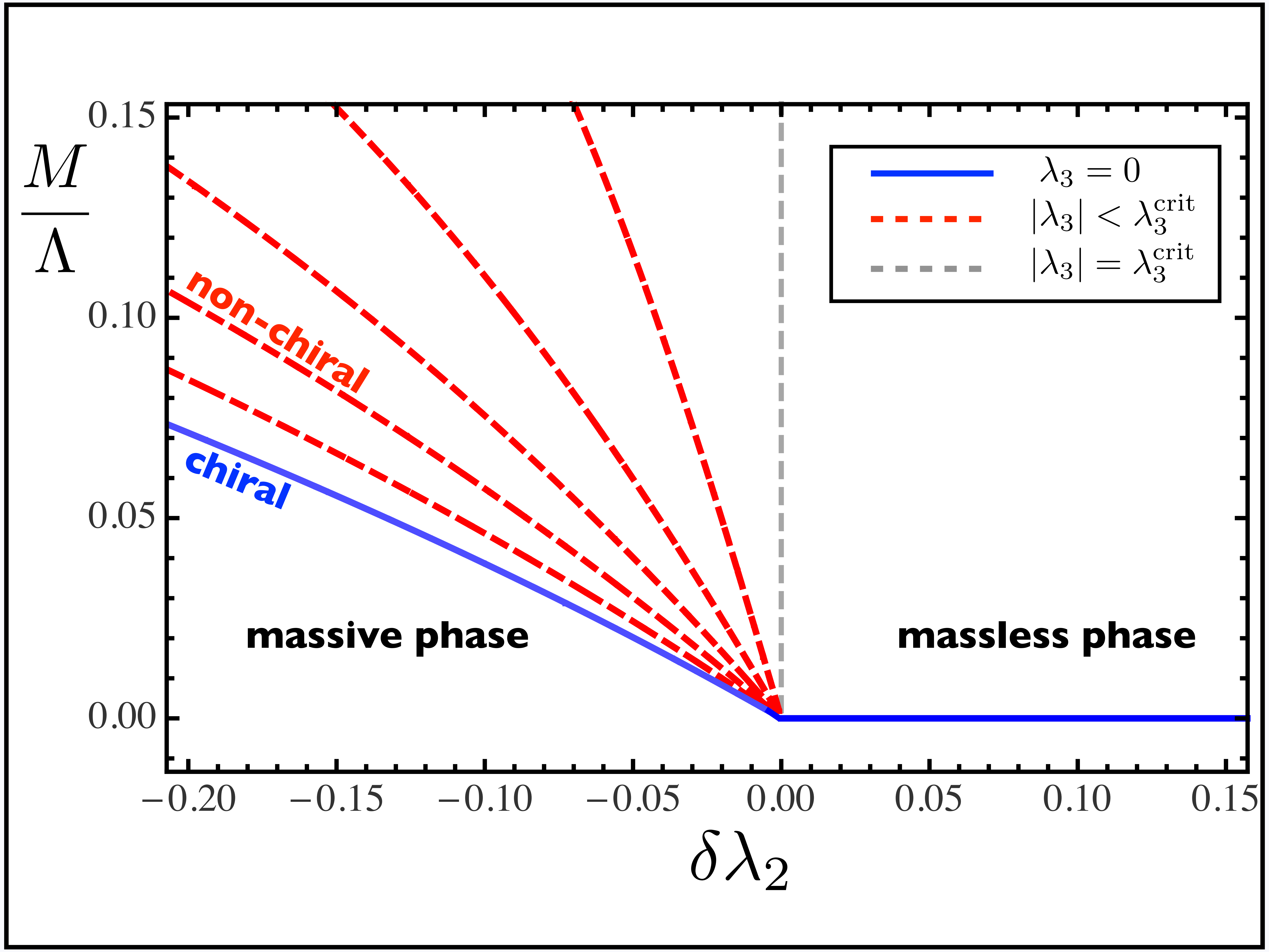
Region I: massless theories

Region II: massive theories

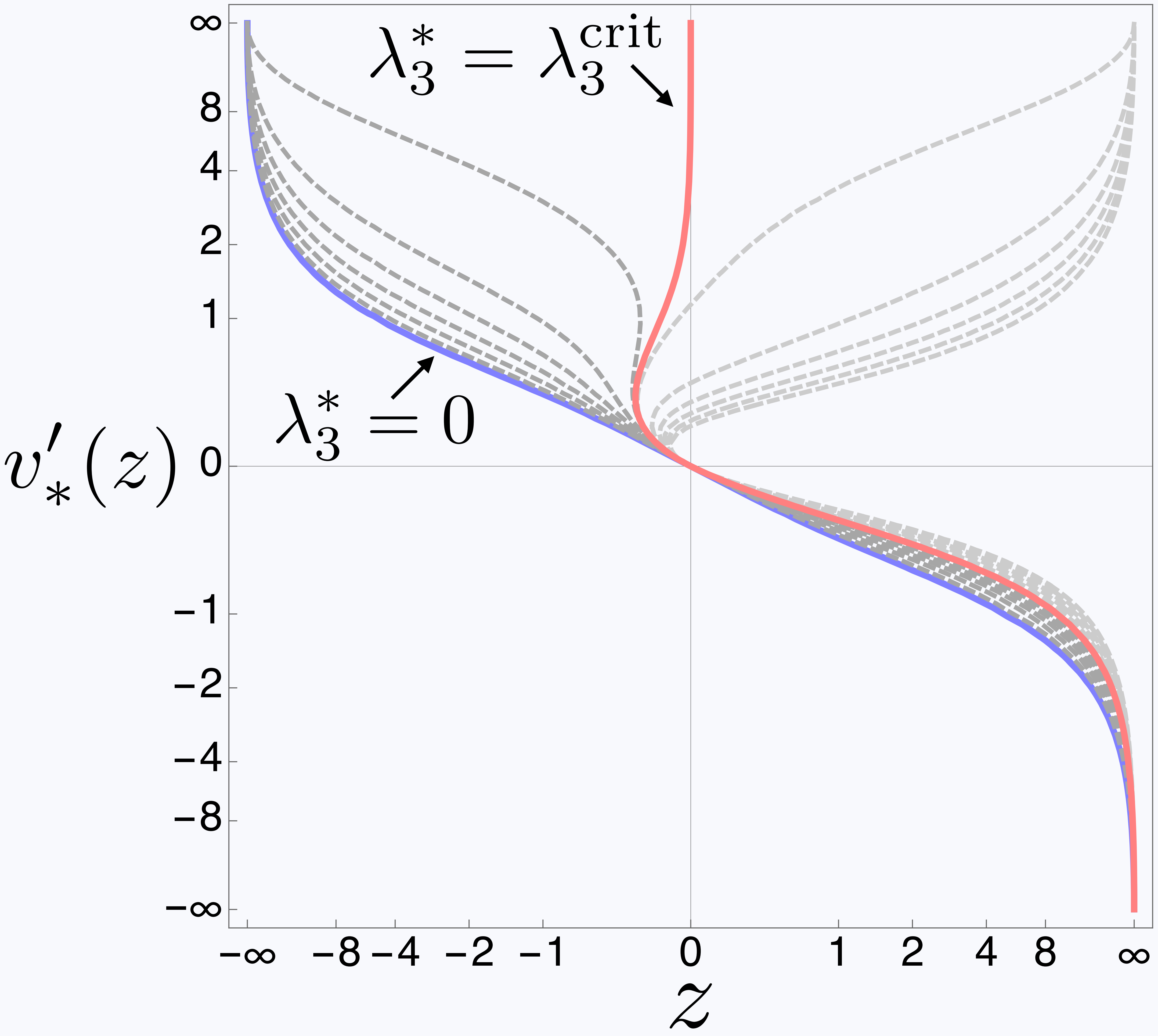
Region III: effective theories



Generation of mass



Endpoints



Endpoints

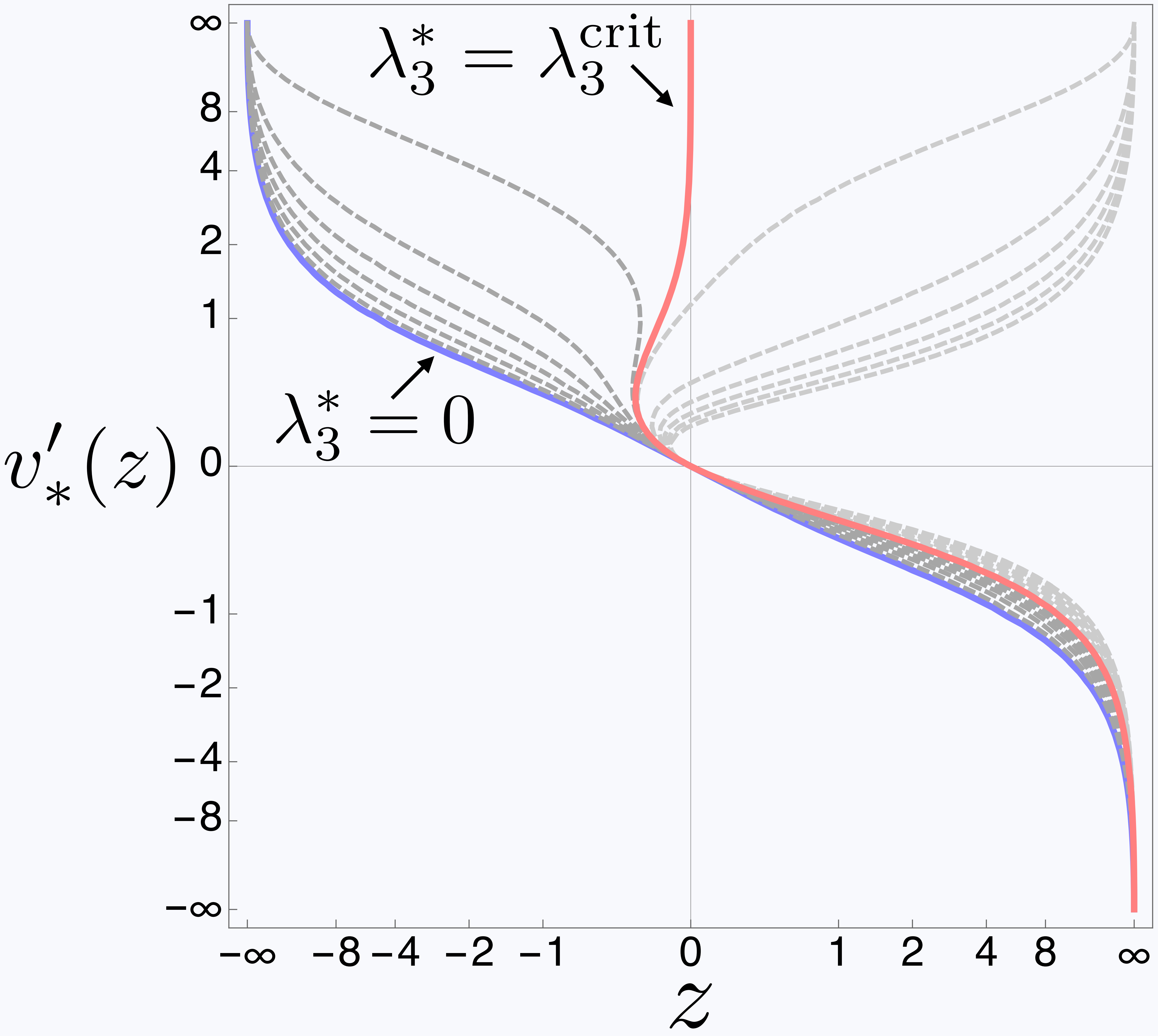
Physical fermion mass

$$M = \lim_{k \to 0} k v'_*(0)$$

satisfies gap equation

$$\left(\lambda_3^* - \lambda_3^{\text{crit}}\right) M = 0$$

[*CCH, Litim, in preparation*]



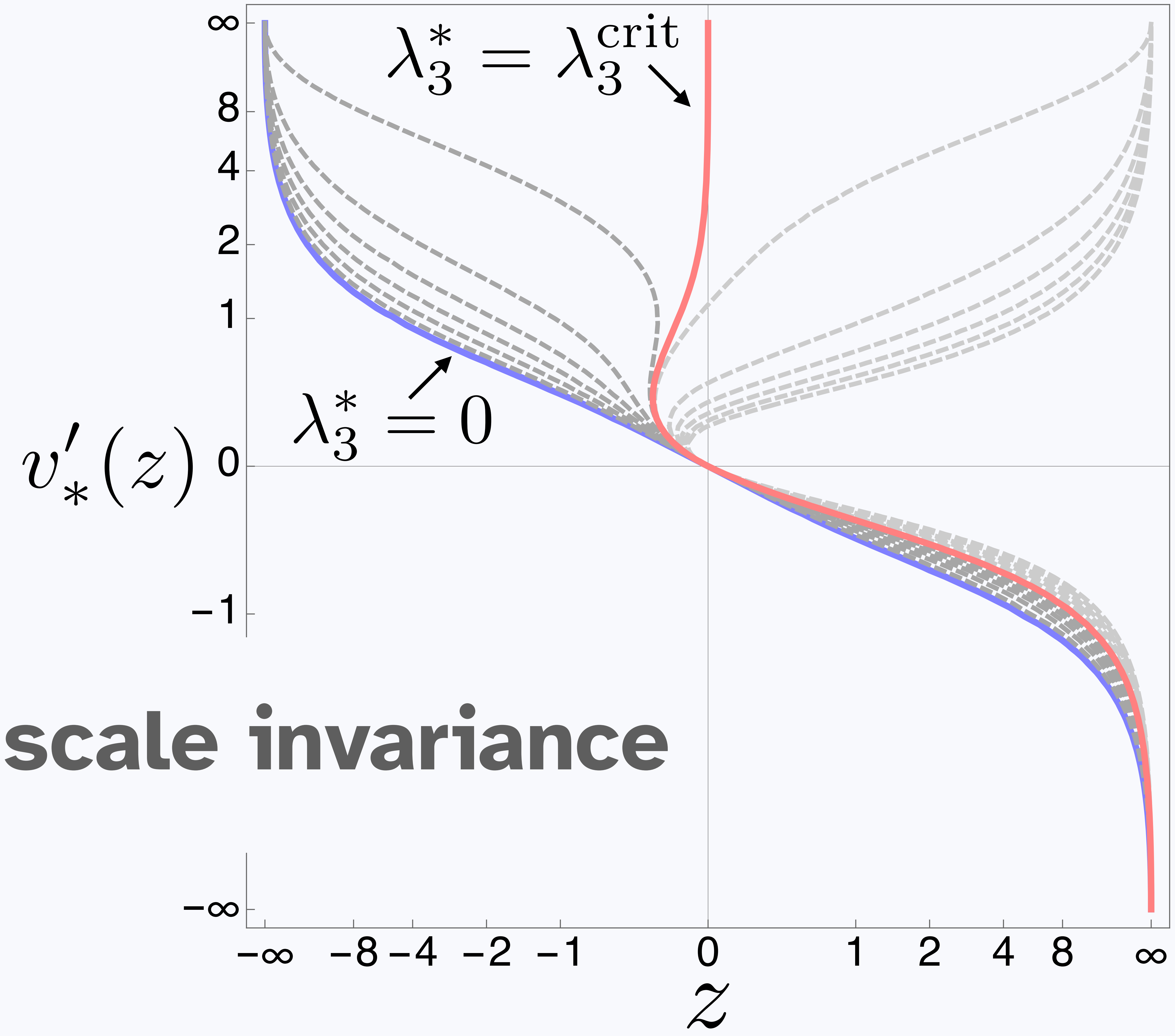
Endpoints

Physical fermion mass

$$M = \lim_{k \to 0} k v'_*(0)$$

satisfies gap equation

$$\left(\lambda_3^* - \lambda_3^{\text{crit}}\right) M = 0$$



spontaneous breaking of scale invariance

[CCH, Litim, in preparation]
cf. O(N) model:
[Bander, Bardeen, Moshe, PRL ’83]
[David, Kessler, Neuberger, PRL ’85]

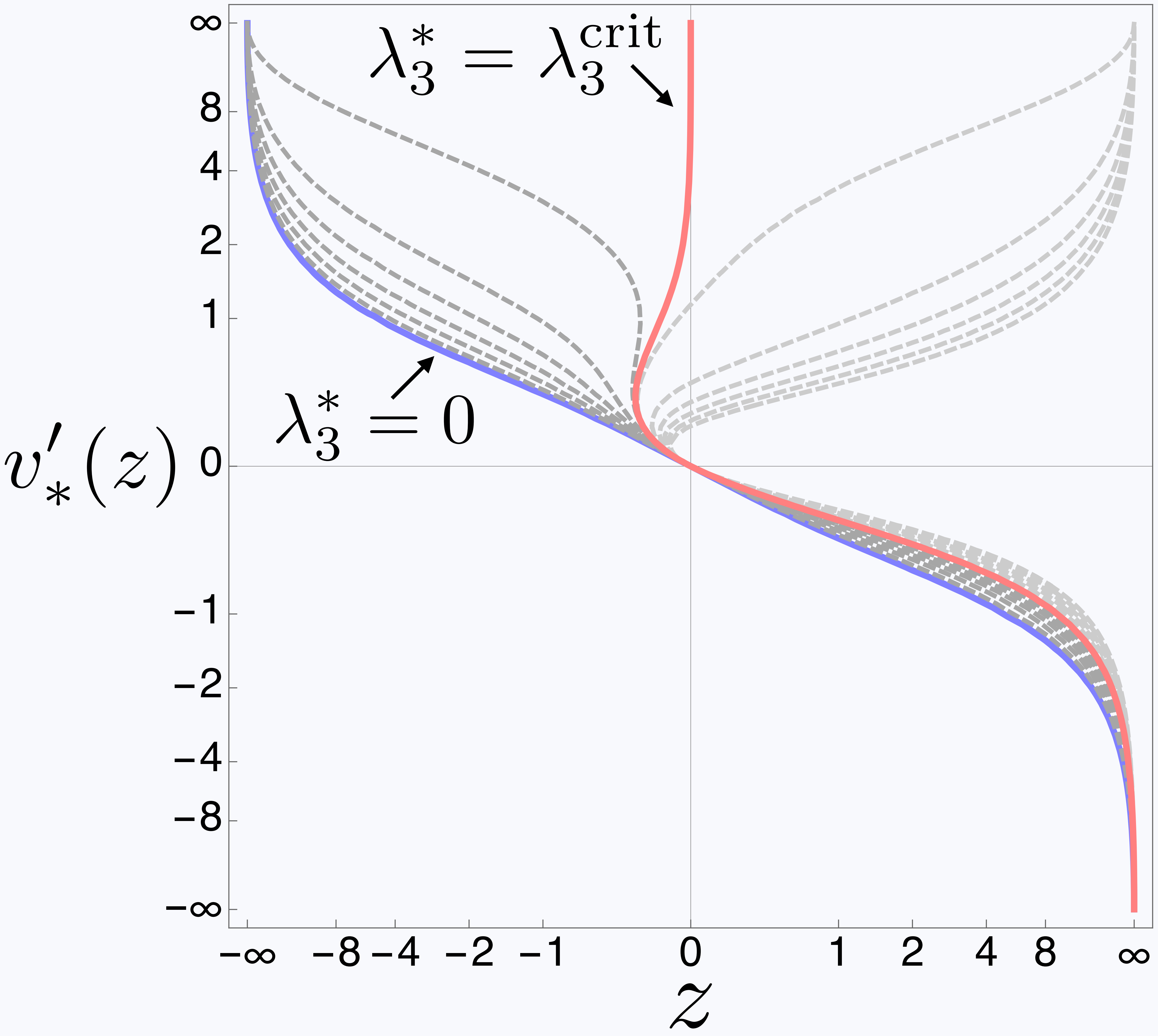
Endpoints

Critical exponents

$$\nu = \alpha = \frac{1}{2}$$

modified at endpoints

$$\nu = \frac{1}{3} \qquad \alpha = \frac{2}{3}$$



[*CCH, Litim, in preparation*]

cf. O(N) model:

[*David, Kessler, Neuberger, PRL ’85*]

Endpoints

Critical exponents

$$\nu = \alpha = \frac{1}{2}$$

modified at endpoints

$$\nu = \frac{1}{3} \qquad \alpha = \frac{2}{3}$$

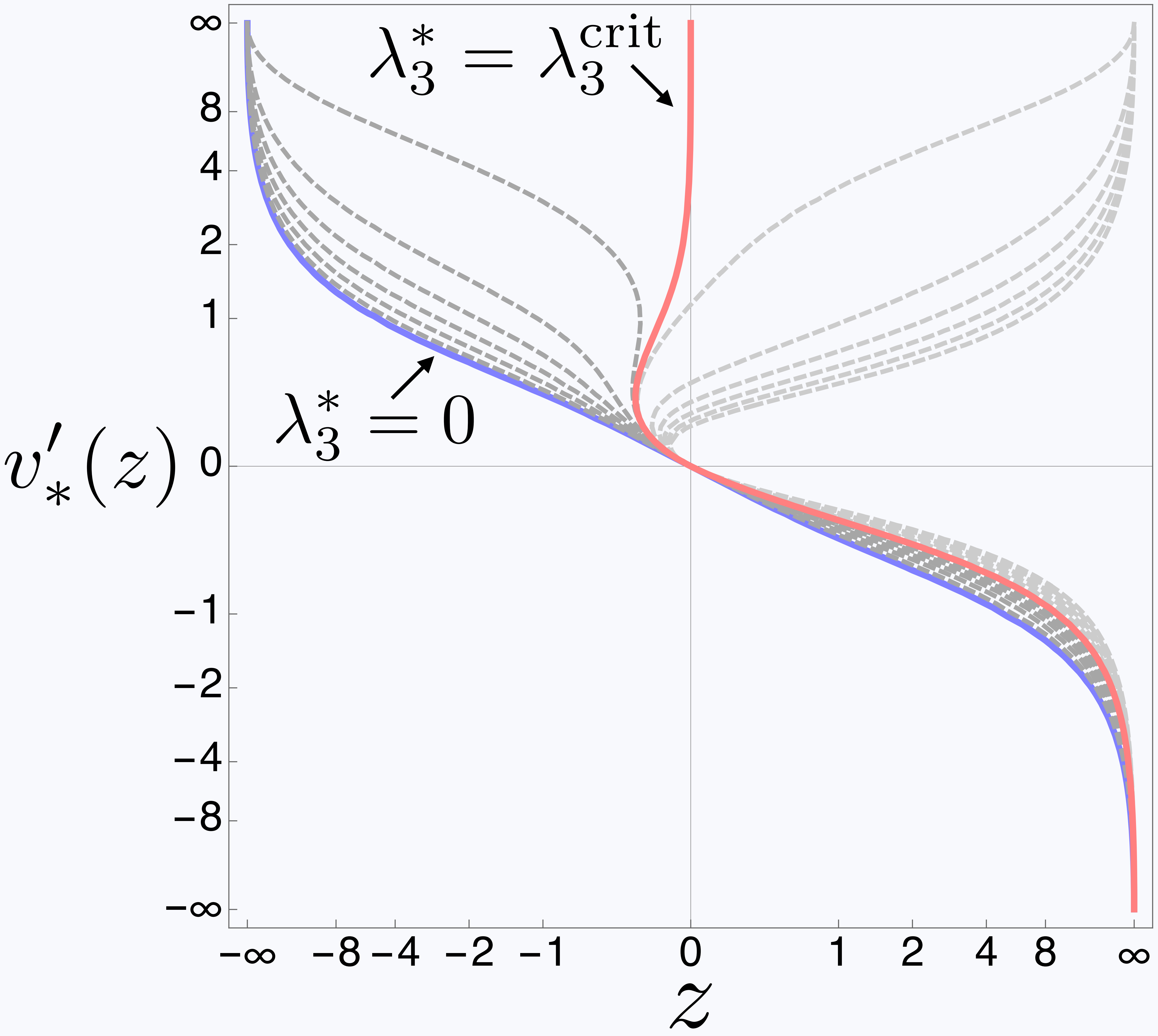
violation of hyperscaling

$$d\,\nu = 2 - \alpha$$

[CCH, Litim, in preparation]

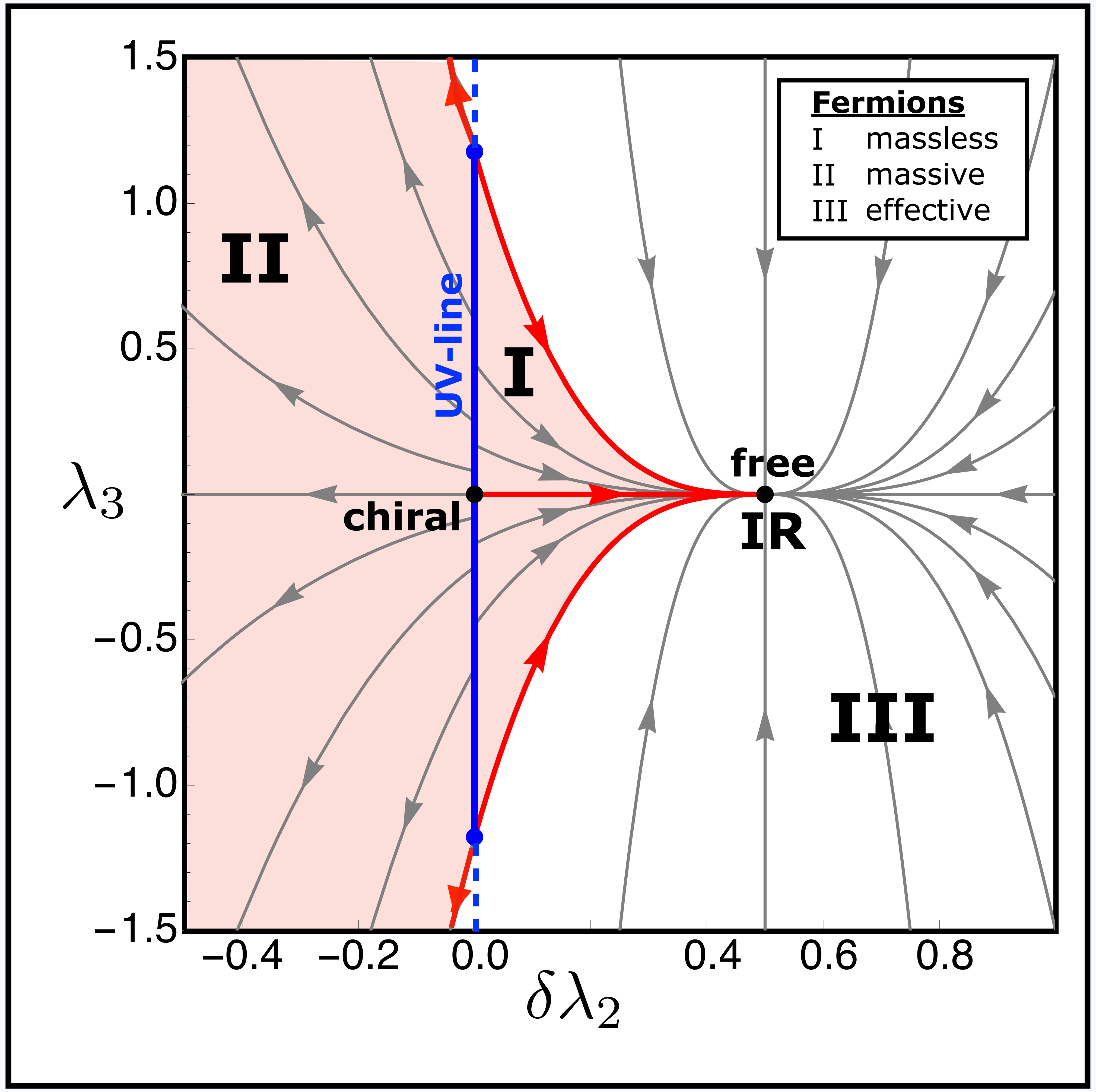
cf. O(N) model:

[David, Kessler, Neuberger, PRL ’85]

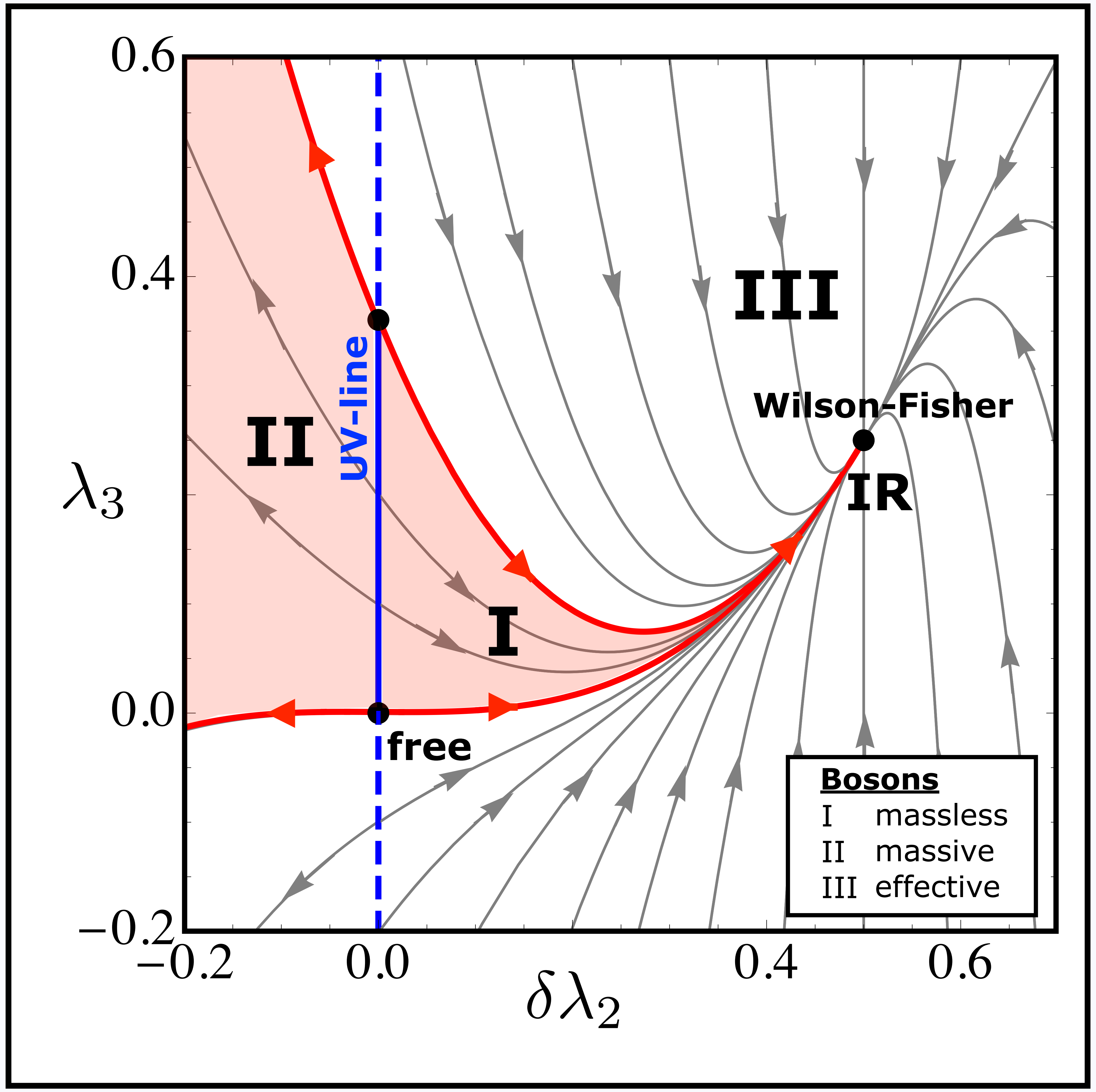


Fermions & bosons

$$V(\bar{\psi}\psi) \sim (\lambda_2^* + \delta\lambda_2)\,(\bar{\psi}\psi)^2 + \lambda_3(\bar{\psi}\psi)^3$$

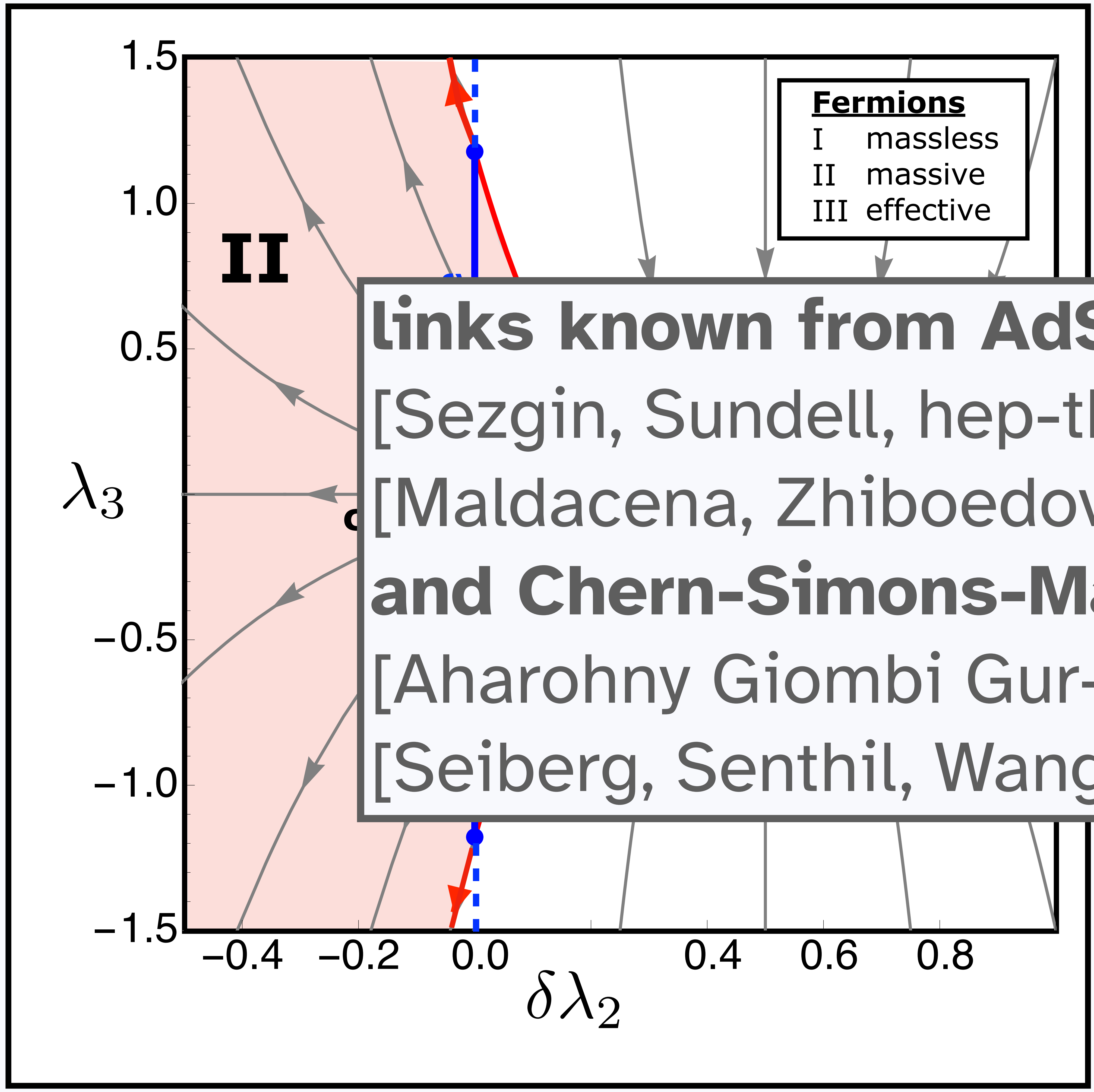


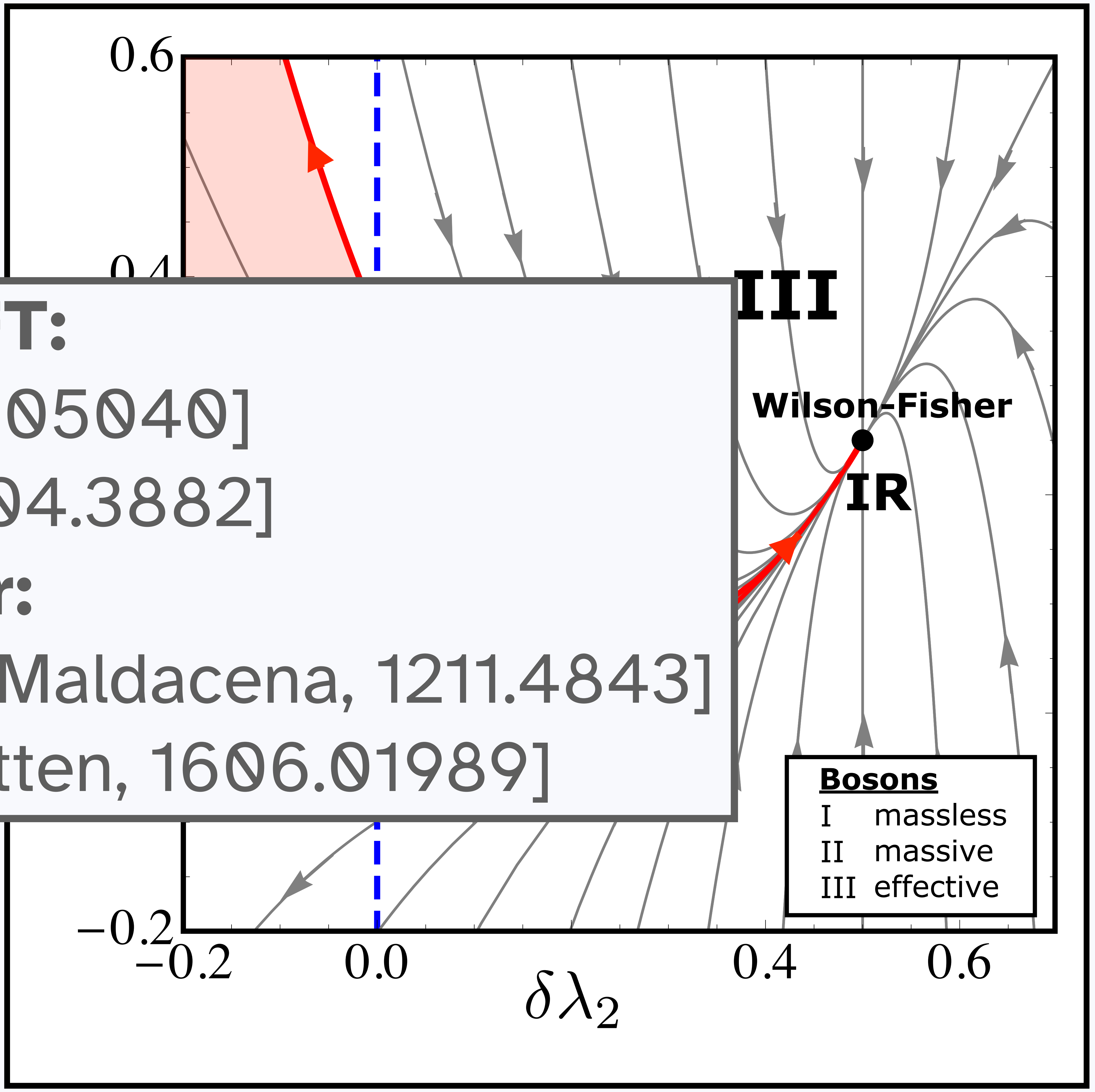
$$U(\phi^2) \sim \delta\lambda_2(\phi^2)^2 + \lambda_3(\phi^2)^3$$



Fermions & bosons

$$V(\bar{\psi}\psi) \sim (\lambda_2^* + \delta\lambda_2)\,(\bar{\psi}\psi)^2 + \lambda_3(\bar{\psi}\psi)^3 \qquad U(\phi^2) \sim \delta\lambda_2(\phi^2)^2 + \lambda_3(\phi^2)^3$$





links known from AdS/CFT:

[Sezgin, Sundell, hep-th/0305040]

[Maldacena, Zhiboedov, 1204.3882]

and Chern-Simons-Matter:

[Aharohny Giombi Gur-Ari, Maldacena, 1211.4843]

[Seiberg, Senthil, Wang, Witten, 1606.01989]

Outlook

Summary:

- Explicit breaking of parity opens up line of 6F fixed points at large-N
- Some theories exactly massless despite explicitly broken symmetry
- Qualitative and quantitative parallels with O(N) models, including fermionic BMB

Ongoing work:

- Counterpart fixed points in Yukawa models
- Boson-fermion correspondence from RG perspective