The weak-gravity bound in asymptotically safe gauge-gravity systems

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Marc Schiffer, Perimeter Institute

based on

A. Eichhorn, J. Kwapisz, MS: **Phys.Rev.D 105 (2022) 106022**

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AS in gravity with matter

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  - Does asymptotically safe quantum gravity support a UV-complete matter sector?
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  - Is there a viable phenomenology?
    [Shaposhnikov, Wetterich; 2009], [Harst, Reuter; 2011], [Eichhorn, Held; 2017, 2018], [Eichhorn, Versteegen; 2017], · · ·
    [Draper, Knorr, Ripken, Saueressig; 2020], [Knorr, Pirlo, Ripken, Saueressig; 2022]
    [Reichert, Smirnov; 2019], [Kowalska, Sessolo; 2020], [Eichhorn, Pauly; 2020], · · ·
  - . . .

The "weak gravity bound"

- Specifically in asymptotically safe gravity:
  - There exist indications that metric fluctuations must not be too strong.
  - Interacting nature of gravity induces novel interactions in the matter sector.
    [Eichhorn and Gies, 2011], [Eichhorn, 2012], [Meibohm and Pawlowski, 2016], [Eichhorn, Held and Pawlowski, 2016], [Christiansen and Eichhorn, 2017], [Eichhorn and Held, 2017], [Eichhorn, Lippoldt and Skinjar, 2017] [Eichhorn, Lippoldt and MS, 2018]
  - Beyond the weak-gravity regime, metric fluctuations can induce novel divergences in these interactions.

Induced interactions

- Example: Abelian gauge field A_μ
  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
  - From kinetic term:

$$S_{\rm kin} = \frac{Z_A}{4} \int \mathrm{d}^d x \sqrt{g} F_{\mu\nu} F^{\mu\nu}$$

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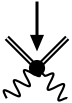

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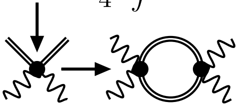

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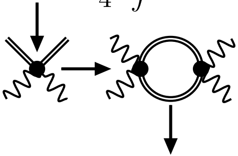

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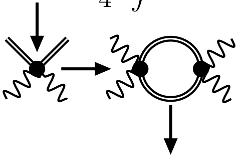

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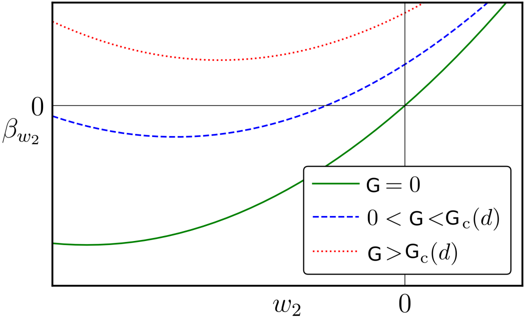

- Schematically:

$$\beta_{w_2} = B_0(G) + w_2\, B_1(G) + w_2^2 B_2$$

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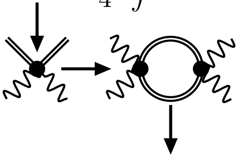

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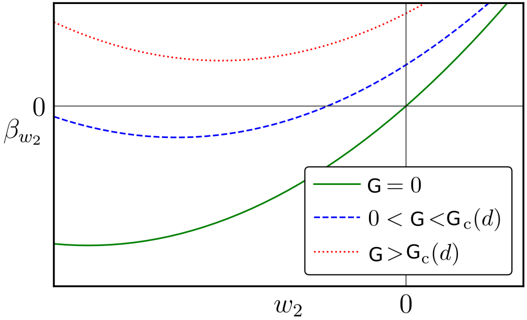

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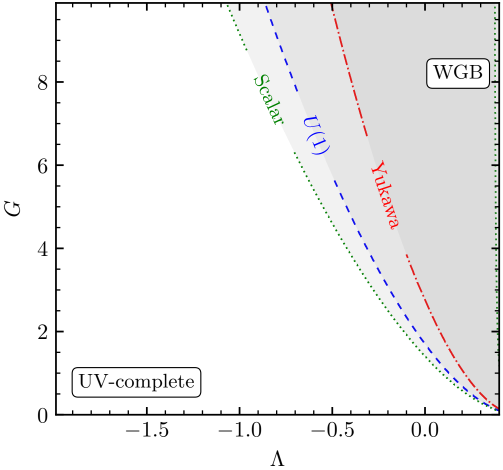
$$\beta_{w_2} = B_0(G) + w_2\, B_1(G) + w_2^2 B_2$$

- $\exists$ real FP only for $B_0 \leq \frac{B_1^2(G)}{4B_2}$

Excluded strong gravity regime: The weak-gravity bound



[MS; 2021]

- WGB explored for:
  - Scalar interactions
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  - Fermionic interactions
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  - Gauge interactions
    [Christiansen, Eichhorn; 2017], [Eichhorn, MS; 2019] [Eichhorn, Kwapisz, MS; 2021]
  - Scalar-Fermion interactions
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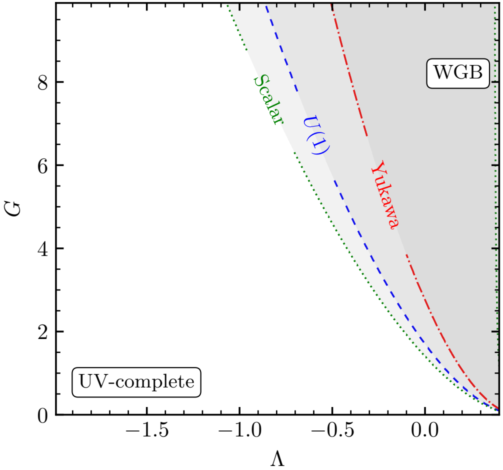

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- further "pheno" consequences
  [Eichhorn, MS; 2019], [Hamada, Pawlowski, Yamada; 2020] [de Brito, Eichhorn, MS; 2020], · · ·

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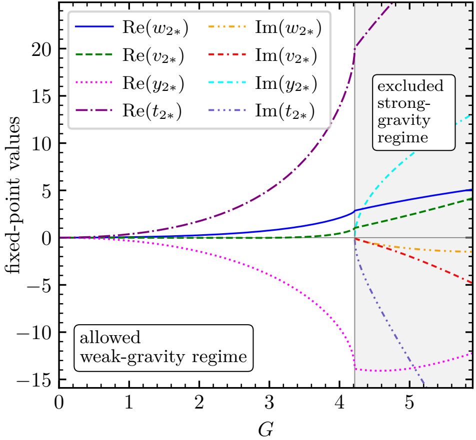
  $x_2,\ z_2,\ u_2,\ s_2$ break it $\Rightarrow$ not induced see also [de Brito, Eichhorn, Robson Linos dos Santos; 2021]

Two gauge fields: Excluded strong-gravity regime



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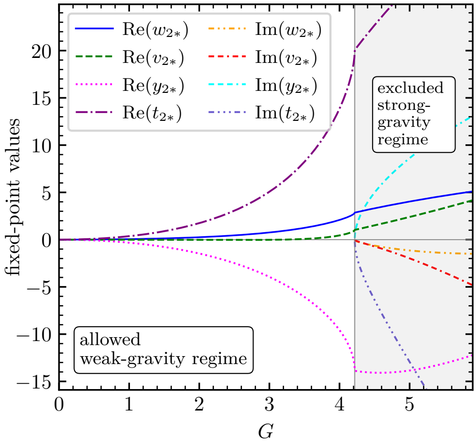
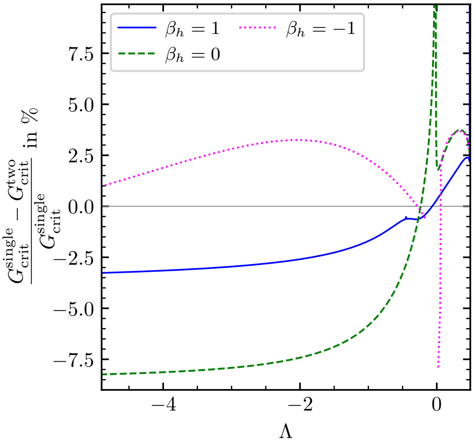
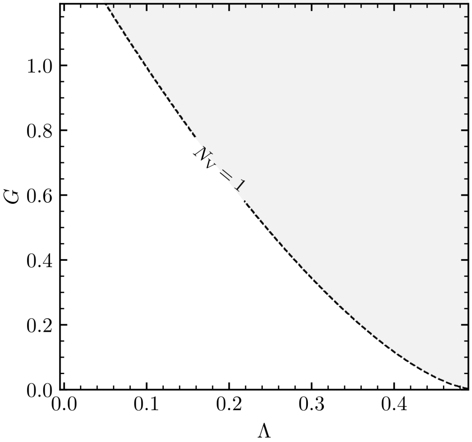




From one to many gauge fields



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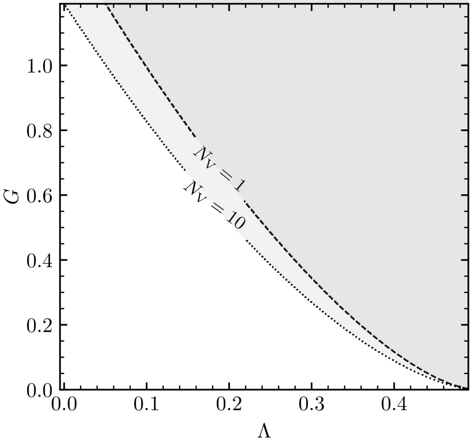

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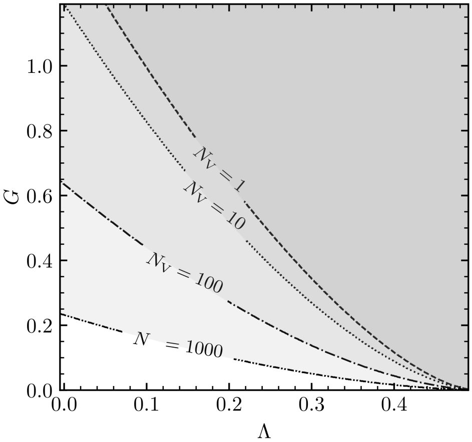

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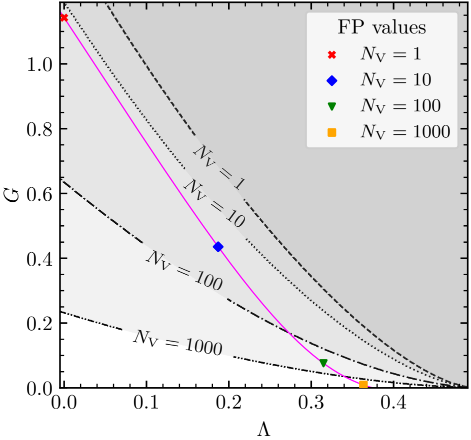

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- Compare to gravitational FP-values (background approximation):
  FP avoids WGB for all N_V (including $N_V \to \infty$)

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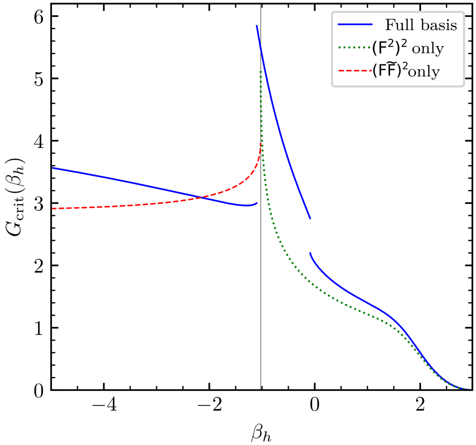
- Residual gauge dependence: estimate for robustness of results

Gauge dependence of WGB

Single gauge field:



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Gauge dependence of WGB

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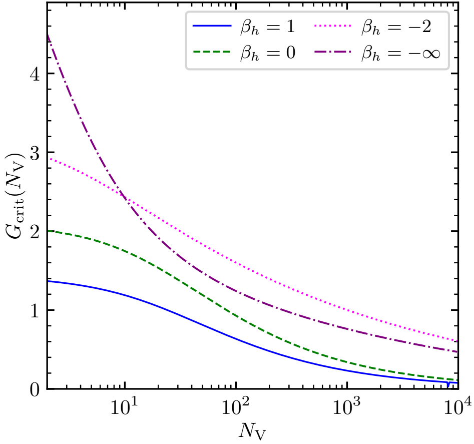

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    see also [Laporte, Locht, Pereira, Saueressig; 2022]

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Thank you for your attention!

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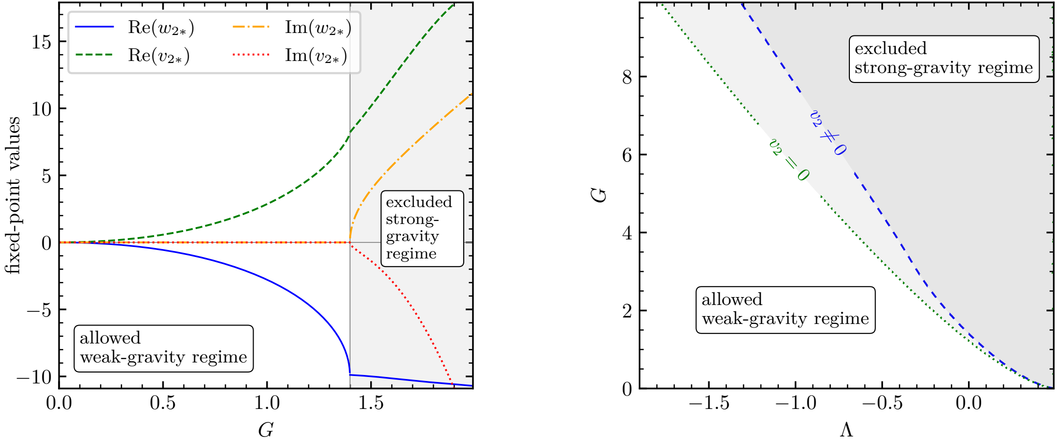
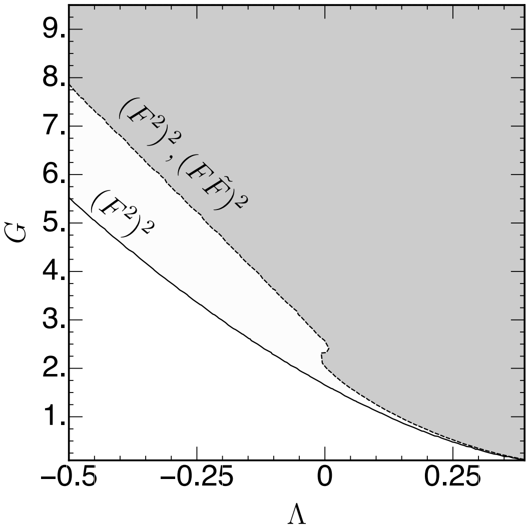
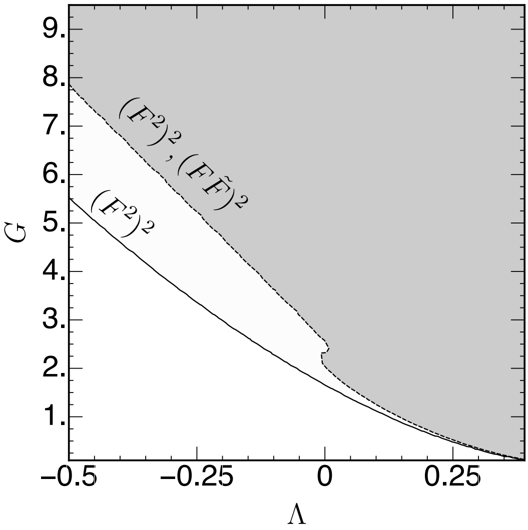


Jump in G_{crit}



Critical Dimension



Solution to the triviality problem in $d > 4$

- $[\bar{g}_Y] = \frac{4-d}{2}$

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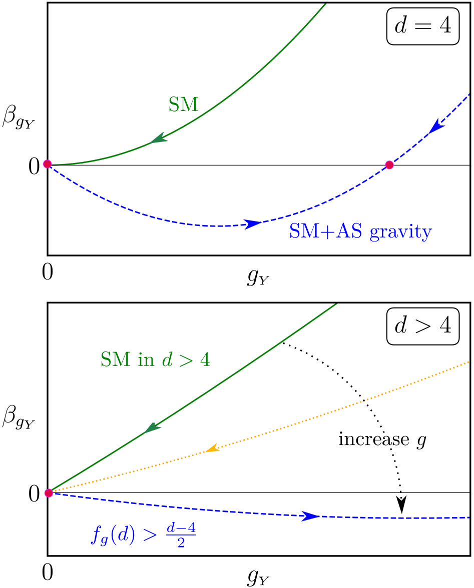
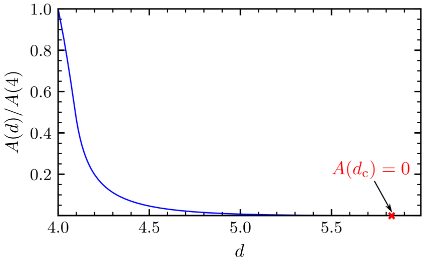
- Competition of $f_g(d)$ with canonical mass term.
- Necessary condition for UV completion:
  Effective dimensionality below four,

$$f_g(d) > \frac{d-4}{2}$$



UV complete matter sector beyond $d = 4$?

- Area of allowed region for $G \in (0, 1000)$ and $\Lambda \in (-1500, 0.5)$.
- Area shrinks to zero at $d_c \approx 5.8$.



[Eichhorn, MS; 2019]

UV complete matter sector beyond $d = 4$?

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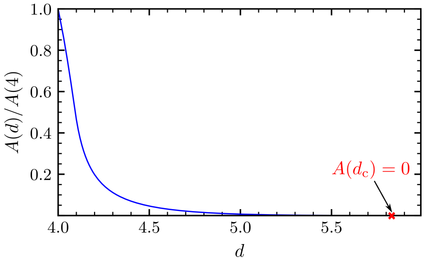

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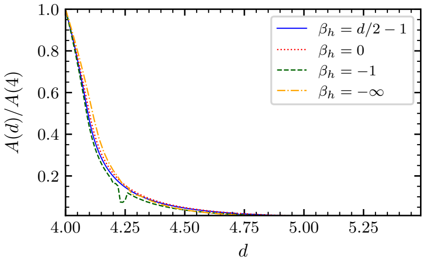

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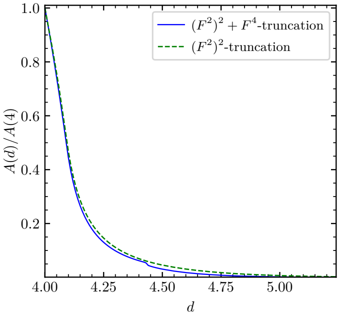

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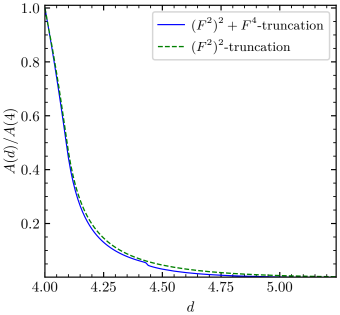

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The predictive power of the asymptotic-safety paradigm could extend to fundamental parameters of the geometry.