Coupling Fermions to [Unimodular] Einstein-Cartan Gravity

or

Does the quark condensate contribute to the cosmological constant?

Reinhard Alkofer

Institute of Physics, University of Graz

11th International Conference on the Exact Renormalization Group

Berlin, July 25 - 29, 2022

in collaboration with Antonio D. Pereira & Arthur F. Vieira (Niterói) and Tristan Feyer & Niko Heinemann (Graz)

Disclaimer

No ERG (yet) . . .

Motivation: The Cosmological Constant Problem

Is there a conflict between the SM's **vacuum condensates** and the phenomenological value of the **cosmological constant**?

See, *e.g.*, S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. **61** (1989) 1

The cosmological constant Λ from observation:

- Tiny positive Λ (only observable for very large distances)
  (NB: Vanishing and non-vanishing case is *qualitatively* different!
  Cf., e.g., A. Ashtekar, 1706.07482.)
- ΛCDM model: Attributed to energy density of Dark Energy

$$\rho^{DE} = \Lambda/8\pi G_N = \Lambda M_{Pl}^2$$

- Parameter fit to ΛCDM model:

$$\rho^{DE} \approx (\mathrm{meV})^4$$

.

Motivation: QCD and Quark Condensate

QCD dynamics:
(Light) quarks undergo Dynamical Chiral Symmetry Breaking (DχSB).

Consequences:

- Generation of dynamical masses (quark constituent masses)
- Pions as would-be Goldstone bosons are light
- Non-pert. quark-gluon interactions of the scalar / tensor type otherwise forbidden by χS / suppressed by current masses
- Lowering of the **vacuum energy density**

Effective quantity to describe magnitude of QCD-related DχSB:
Quark condensate $\langle \bar{q}(x) q(x) \rangle$.

Motivation: QCD and Quark Condensate

On $\langle \bar{q} q \rangle$:

- RG scheme and RG scale dependent (*cf.*, anom. mass dimension)
- Typical size (MOM scheme / RG scale 2 GeV): - $(250\ \mathrm{MeV})^3$ for *u* and *d* quarks
- Lorentz scalar, colour singlet, appr. isosinglet, electrically neutral
- Note: Auxiliary quantity!
- But: Related to observables, *e.g.*, via GMOR relation

$$f_\pi^2 m_\pi^2 = \tfrac{1}{2}(m_u + m_d)\langle \bar{u}u + \bar{d}d \rangle$$

Lowering in vacuum energy density:

$$\rho^{D\chi SB} \approx 0.2\,\mathrm{GeV/fm}^3 = (0.2\,\mathrm{GeV})^4$$

see, *e.g.*, RA, PhD thesis, TUM, 1988.

Quark Condensation vs. Cosmological Constant

If quark condensation were to contribute to Λ:

- Mismatch in scale from meV to GeV!

Somehow quark condensation does not contribute to the cosmological constant?

"Quark condensate is not a source of gravity!?!"

— Effect not to be confused with the zero-point energy density ($\frac{1}{2}\hbar\omega$ per mode) contribution to the cosmological constant.

— Effect of Higgs' v.e.v. significantly larger.

— "Jump" in the cosmological constant during a phase transition / crossover related to spontaneous / dynamical symmetry breaking? *Cf.*, corresponding discussion in Weinberg's review [1989] . . .

Some facts about Unimodular Gravity

Unimodular gravity:
Fix the determinant of the metric to a background volume form

$$|g| = \omega(x)$$

Not to be confused with "unimodular gauge"!

- $\exists$ coordinate system s.t. $|g| = 1$
- Diffeomorphism (Diff) invariance is reduced to invariance under transverse diffeomorphisms (TDiff, resp. SDiff)
- No cosmological constant Λ in action (trivial!), but cosmological constant reappears as constant of integration.
- Einstein tensor does not couple to the trace of the energy-momentum tensor ☺ & thus vacuum condensates do not contribute to the cosmological constant?!? ☹☺☹...

Recent "Status Report": R. Carballo-Rubio *et al.*, 2207.08499

Some facts about Unimodular Gravity

Classical theory:

Difference between General Relativity and Unimodular Gravity is only the treatment of the cosmological constant.
(NB: Applies also when adding higher-order terms.)

Quantum theory:

There exists a quantisation procedure that makes the respective functional integrals equivalent.
[G.P. de Brito, O. Melichev, R. Percacci, A.D. Pereira, JHEP **12** (2021) 090]

For more details: R. Carballo-Rubio *et al.*, 2207.08499

Some facts about Unimodular Gravity

The term “unimodular gravity” is not unique!

Although the same name is used
different modifications of GR are studied
in different papers!

(NB:
Important symmetry aspect is the role of extra Weyl rescaling symmetry of the metric.)

Dirac fermions in Riemann-Cartan gravity theories

- Vielbein formalism
- Einstein-Cartan framework
- (allow) non-vanishing torsion
  - spin connection incl. contorsion
  - Holst term in gravity action (Barbero-Immerzi parameter),
  - 2 add. kinetic terms for fermions in matter action
    (non-minimal couplings, mixed term incl. parity violation)

Resolve for torsion:
Equivalent metric theory with 4-fermion interactions (dim-6 operators).

[N. J. Poplawski, Gen. Rel. Grav. **44** (2012) 491; 1102.5667;
J. Magueijo et al., Phys. Rev. D **87** (2013) 063504; 1212.0585;
M. Shaposhnikov et al., JHEP **10** (2020) 177; 2007.16158 (NB: Erratum).]

Dirac fermions in Riemann-Cartan gravity theories

Consider 4-quark interactions for the two light flavours in Riemann-Cartan spacetimes

sourced by

- QCD and the related DχSB,
- torsion,

and proportional to

- effective coupling $g_{4q} \propto \Lambda_{QCD}^{-2}$,
- Barbero-Immerzi parameter (γ) and non-minimal fermion-gravity couplings (α, β),

respectively.

No unimodularity condition:

Induced cosmological constant $\Lambda^{\bar{q}q} \propto \langle \bar{q}q \rangle^2$ with

- QCD term $\propto M_{Pl}^{-2} g_{4q}$ (NB: $\propto N_D N_c N_f^{eff}$)
- gravity term s.t. $\propto M_{Pl}^{-4} \times \frac{\gamma^2}{1+\gamma^2} \times (1 + \alpha^2 - \beta^2 + 2\beta/\gamma)$

Dirac fermions in Riemann-Cartan gravity theories

No unimodularity condition:

Contribution to the cosmological constant $\Lambda^{\bar{q}q} \propto \langle \bar{q}q \rangle^2$:

- QCD-induced $\propto 1/\Lambda^2_{QCD} M^2_{Pl}$
- torsion-induced $\propto 1/M^4_{Pl}$

As $M^2_{Pl}/\Lambda^2_{QCD} \approx 10^{37}$
cancelation of QCD-induced vs. gravity-induced contribution would require, as expected, unrealistic fine-tuning.

With unimodularity condition:

. . . sorry, work in progress . . .

Conclusions & Outlook

Quarks in Riemann-Cartan gravity theories:

- ☺ Torsion sources 4-fermion interactions.
- ☺ Induced cosmological constant contains gravity-induced and QCD-induced terms of relative magnitude $\propto \Lambda_{QCD}^2/M_{Pl}^2$.

Outlook:

- Is there a QCD-induced contribution to the cosmological constant in unimodular setting? If so, magnitude?
- Quantum theory via ERG?