Sextic tensor field theories

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with D.Benedetti, N.Delporte and R.Sinha
and on arXiv:2109.08034

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Tensor models: from random geometry to melonic CFTs

- First introduced in zero dimension: random geometry and quantum gravity [Gurau, Bonzom, Rivasseau, . . .]
- Strongly coupled QFTs and holography : SYK model without disorder [Witten, Klebanov, Tarnopolsky, . . .]
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- Long-range $O(N)^3$ bosonic tensor model with quartic interactions Razvan's talk

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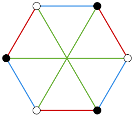
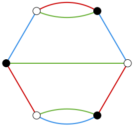
$\Rightarrow$ What about sextic interactions?

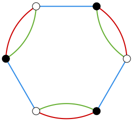
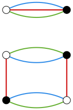
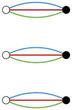
Outline

Sextic $U(N)^3$ model

- Complex field ϕ_{abc}, $\bar{\phi}_{abc}$, $U(N)^3$ symmetry

$$S[\phi,\bar{\phi}] = \int d^d x \, \bar{\phi}_{abc} (-\partial_\mu \partial^\mu)^\zeta \phi_{abc} + S_{\text{int}}[\phi,\bar{\phi}],$$

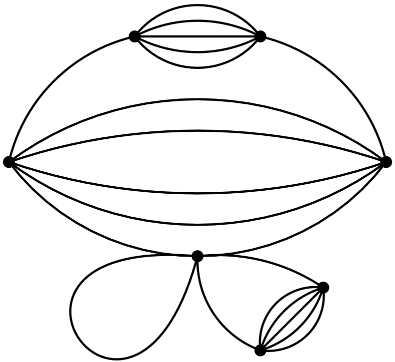
$S_{\text{int}}[\phi,\bar{\phi}] = \int d^d x \Bigg(\frac{\lambda_1}{6N^3}$  $+ \frac{\lambda_2}{6N^4}$ 

$+ \frac{\lambda_3}{6N^4}$  $+ \frac{\lambda_4}{6N^5}$  $+ \frac{\lambda_5}{6N^6}$  $\Bigg) .$

- $\zeta = 1$: short-range
- $0 < \zeta < 1$: long-range

Large-N expansion

- Factor N per face of color $0i$
- Large-N expansion:
  - Melons with wheel vertices
  - Double-tadpoles with any vertex



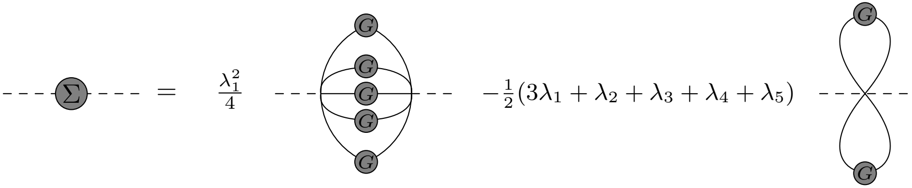
Power counting and renormalization scheme

- Marginality:
  - Short-range: $d = 3$
  - Long-range: $\zeta = d/3$, $d < 3$
- Two and four-point functions: power divergent
- Six-point functions: log divergent
- Choice of scheme
  - Short-range: $d = 3 - \epsilon$
  - Long-range: $\zeta = \frac{d+\epsilon}{3}$
  - Subtraction at non-zero external momentum

Schwinger-Dyson equation: short-range

- Closed equation for G: characteristic of tensor models at large N

$$G(p)^{-1} = C(p)^{-1} - \Sigma(p)\,,$$



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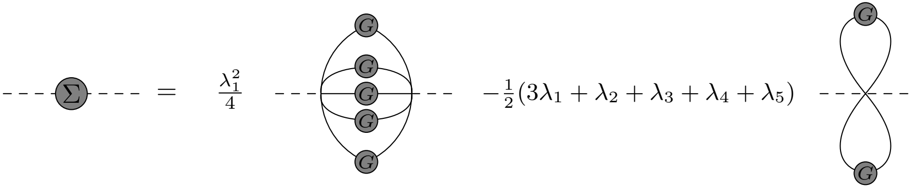


- UV: free scaling regime
- IR: anomalous scaling $G(p)^{-1} \sim p^{2\Delta}$ with $\Delta = d/3$
- Wave function renormalization: $\phi = \phi_R \sqrt{Z}$
- Z such that $\lim_{\epsilon \to 0} \frac{d\Gamma_R^{(2)}(p)}{dp^2}\,|_{p^2=\mu^2} = 1$

$$Z = 1 - \mu^{-4\epsilon} \frac{\lambda_1^2 \pi^2}{6\,(4\pi)^6\,\epsilon}$$

Schwinger-Dyson equation: long-range

- Free propagator reproduces IR scaling
- Only one scaling regime $\Rightarrow$ no wave function renormalization
- SDE solved by ansatz $G(p) = \frac{\mathcal{Z}}{p^{2d/3}}$
- $\mathcal{Z}$ coefficient of the full two-point function

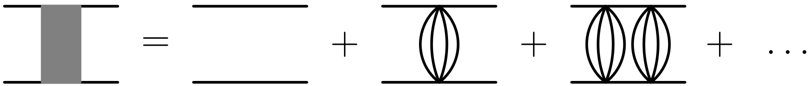
$$\mathcal{Z} = 1 - \frac{\lambda_1^2}{4(4\pi)^{2d}} \frac{3\Gamma(1-\frac{d}{3})\Gamma(\frac{d}{5})^5}{d\Gamma(\frac{d}{3})^5\Gamma(5\frac{d}{6})}$$

- Finite for $d < 3$

$\Rightarrow$ Characteristic feature of long-range models

Four-point function

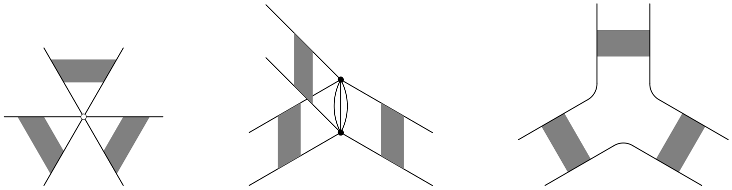
- Computation of four-point kernel
- No contribution from tadpoles in dimensional regularization
- Four-point function as a series of ladders



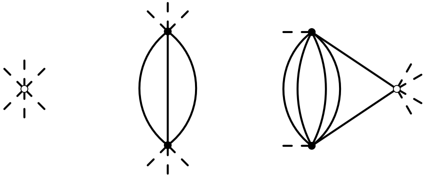
- No divergences in the four-point kernel
- Take four-point couplings to zero from the beginning

Beta functions: bare expansion

- Structure known thanks to large-N limit



- Here only up to order 3 in the coupling constants



Fixed points: short-range

- Two interacting fixed points:

$$\bar{g}_1^* = \pm\frac{\sqrt{\epsilon}}{\pi}\,; \qquad \bar{g}_2^* = \frac{9}{2\pi}\left(-1 \mp 2\sqrt{\epsilon}\right)\,; \qquad \bar{g}_3^* = 0\,;$$
$$\bar{g}_4^* = \frac{9}{7\pi}\left(5 \pm 7\sqrt{\epsilon}\right)\,; \qquad \bar{g}_5^* = \frac{-109 \mp 126\sqrt{\epsilon}}{42\pi}\,.$$

- Critical exponents:

$$(4\epsilon;\ 4\epsilon;\ 6\epsilon;\ 14\epsilon;\ 30\epsilon)$$

- Infrared stable
- Eigenvalue of multiplicity two: non-diagonalizable stability matrix
- Logarithmic CFT $\Rightarrow$ non-unitary

Fixed points: long-range

- Line of fixed points parametrized by g_1:

$$\bar{g}_2^* = -9\bar{g}_1 + \frac{9\Gamma(d/3)\Gamma(2d/3)}{\Gamma(-d/6)\Gamma(d/6)}\,; \qquad \bar{g}_3^* = 0\,;$$
$$\bar{g}_4^* = 9\bar{g}_1 - \frac{90}{7}\frac{2\Gamma(d/3)\Gamma(2d/3)}{\Gamma(-d/6)\Gamma(d/6)}\,;$$
$$\bar{g}_5^* = -3\bar{g}_1 + \frac{109\Gamma(d/3)\Gamma(2d/3)}{21\Gamma(-d/6)\Gamma(d/6)}\,.$$

- Critical exponents:

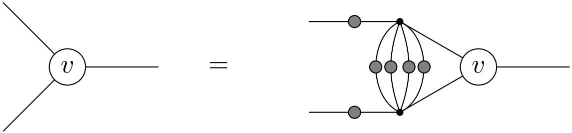
$$\left(\frac{15\bar{g}_1^2\alpha}{2};\ \frac{7\bar{g}_1^2\alpha}{2};\ \frac{3\bar{g}_1^2\alpha}{2};\ \bar{g}_1^2\alpha\right)\,,\ \alpha > 0$$

- Infrared stable
- Compatible with unitarity

Spectrum of bilinears

Three-point function of two fields and a primary:

$$\langle \mathcal{O}_h(x_0)\phi_{abc}(x_1)\bar{\phi}_{abc}(x_2)\rangle \equiv v(x_0,x_1,x_2)$$



- Bilinears $\leftrightarrow$ eigenfunctions of eigenvalue 1 of the kernel

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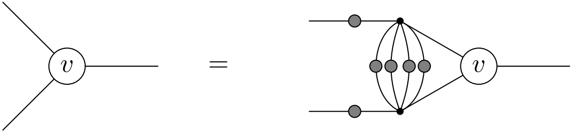


- Bilinears $\leftrightarrow$ eigenfunctions of eigenvalue 1 of the kernel
- Short-range and spin $J=0$

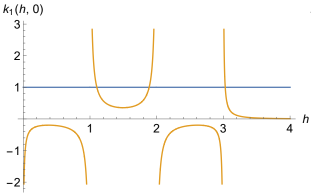
$$h_0 = 1 + \frac{29}{3}\epsilon + \mathcal{O}(\epsilon^2),\ h_1 = 3 + 3\epsilon + \mathcal{O}(\epsilon^2)$$

$$h_q = 2 - \frac{32}{3}\epsilon + \mathcal{O}(\epsilon^2)$$

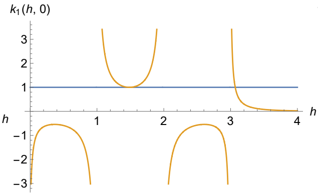
$$h_n = 1 + 2n - \frac{\epsilon}{3} + \frac{20}{3n(n-1)(4n^2-1)}\epsilon^2 + \mathcal{O}(\epsilon^3) \quad , \quad n > 1.$$

Appearance of complex dimensions

- Small $\epsilon > 0$: real dimensions
- For $\epsilon > 0.02819$, h_0 and h_q merge and become complex



$\epsilon = 0.01$



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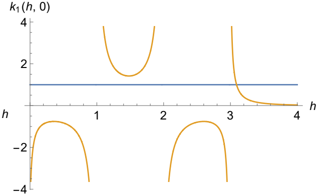

$\epsilon = 0.04$

- Similar situation for long-range case
- Instabilities when complex dimensions of the form $d/2 + \mathrm{i}\, r$ [Kim et al., Benedetti]

$1/N$ corrections

- Start with a generic multi-scalar model with sextic interactions

$$S[\phi] = \int d^d x \left[\frac{1}{2} \bar{\phi}_{\mathrm{a}}(x) \left(-\partial^2\right)^{\zeta} \phi_{\mathrm{a}}(x) \right.$$
$$\left. + \frac{1}{(3!)^2} \lambda_{\mathrm{abc;def}} \phi_{\mathrm{a}}(x) \phi_{\mathrm{b}}(x) \phi_{\mathrm{c}}(x) \bar{\phi}_{\mathrm{d}}(x) \bar{\phi}_{\mathrm{e}}(x) \bar{\phi}_{\mathrm{f}}(x) \right],$$

- Compute beta functions
- Specify to desired symmetry group

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  - Two infrared stable fixed points
  - Diagonalizable stability matrix
  - Non-unitary

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- Short-range
  - Two infrared stable fixed points
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- Long-range
  - Non-perturbative in $\epsilon \Rightarrow$ No precursor of the large-N fixed point

Conclusion and perspectives

- Rigorous RG study of tensor models with sextic interactions
- Short-range: IR **stable** fixed point but **non-unitary**
- Long-range: IR **stable** line of fixed points and hints of unitarity
- Model with $O(N)^5$ symmetry: **trivial** fixed points
- Role of order of interactions and order of the tensors

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- SR: properties of logarithmic CFT
- LR: Study of CFT data $\rightarrow$ further hints of unitarity?
- What about higher order interactions?