Gauge Field Theory Vacuum
and
Cosmological Inflation

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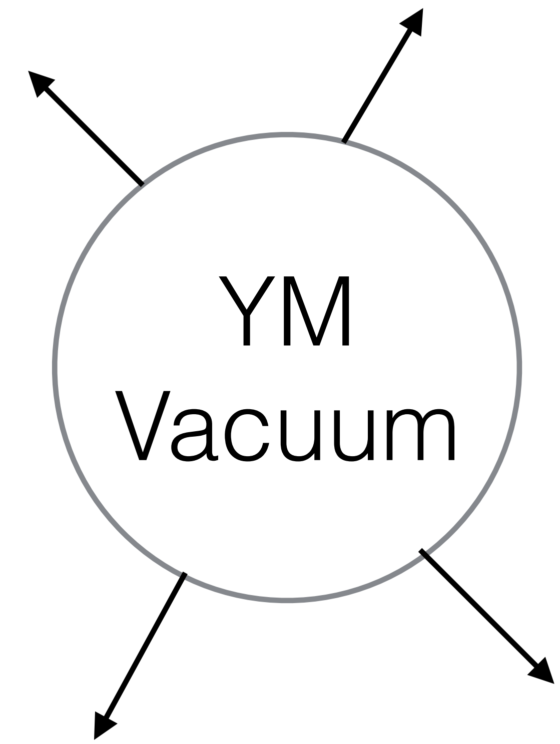
What is the Influence of the Gauge Field Theory Vacuum on the Cosmological Evolution?



Y. B. Zel'dovich, *The Cosmological constant and the theory of elementary particles,* Sov. Phys. Usp. **11** (1968) 381

S. Weinberg, *The Cosmological constant problem,* Rev. Mod. Phys. **61** (1989) 1-23

V. Mukhanov, Physical Foundations of Cosmology, Cambridge University Press, New York, 2005.

G.Savvidy, 1. *Gauge field theory vacuum and cosmological inflation without scalar field,* Annals of Phys. **436** (2022) 168681; PoS Corfu Meeting (2022), arXiv:2204.08933

2. *From Heisenberg–Euler Lagrangian to the discovery of Chromomagnetic Gluon Condensation*, Eur. Phys. J. **C 80** (2020) 165; PoS CORFU2019 (2020) 162

3. *Stability of Yang Mills Vacuum State,* e-Print: 2203.14656

The vacuum energy density

$$E_0 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \omega_p \sim \frac{1}{16\pi^2} \Lambda^4 \qquad \approx 1.44 \times 10^{110} \frac{g}{s^2 cm}$$

The contribution of zero-point energy exceed by many orders of magnitude the observational cosmological upper bound on the energy density of the universe

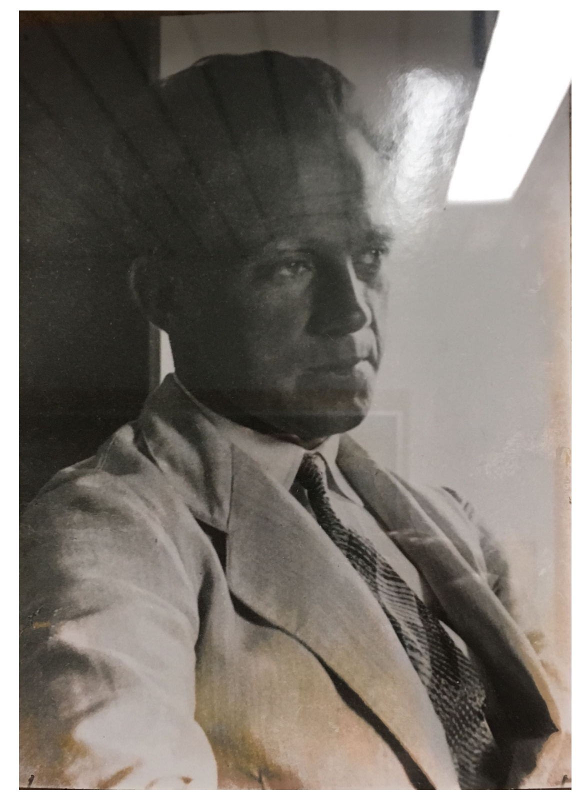
$$\epsilon_{crit} = 3 \frac{c^4}{8\pi G} \left(\frac{H_0}{c} \right)^2 \approx 7.67 \times 10^{-9} \frac{g}{s^2 cm}$$

$$\epsilon_\Lambda = 3 \frac{c^4}{8\pi G} \left(\frac{H_0}{c} \right)^2 \Omega_\Lambda \approx 5.28 \times 10^{-9} \frac{g}{s^2 cm} \qquad 69\%$$

The Effective Lagrangians

Sauter 1931 LMU
Euler and Kockel 1935
Heisenberg and Euler 1936

Hans Euler



Werner Heisenberg

Contribution of Vacuum Fluctuations to the Cosmological Constant

Only the difference between vacuum energy in the presence and in the absence of the external sources has a well defined physical meaning

Heisenberg and Euler - 1936

Heisenberg-Euler, 1936; Schwinger 1951;Coleman-Weinberg 1973; Vanyashin-Terentev 1965; Skalozub:1975; Brown-Duff,1975; Duff — Ramon-Medrano,1975; Nielsen and Olesen 1978; Skalozub 1978; Nielsen 1978; Ambjorn-Nielsen-Olesen1979; Nielsen and Ninomiya,1979; Nielsen and Olesen 1979; Nielsen-Ninomiya 1980; Nielsen-Olesen 1979; Ambjorn-Olesen 1980; Ambjorn-Olesen 1980; Skalozub1980; Leutwyler 1980; Leutwyler 1981; Duff 1977 ; Savvidy 1976, 1977, 2018, 2020, 2022

$$U^{\infty}_{\gamma} = \sum \frac{1}{2}\hbar\omega_k e^{-\gamma\omega_k}$$

$$\lim_{\gamma\to 0}[\ U^{\infty}_{\gamma}(J) - U^{\infty}_{\gamma}(0)\ ] = U_{phys}$$

Lamb shift - 1947 *Casimir effect 1948*

1. Effective Lagrangians in QED and YM theory

2. Quantum Energy Momentum Tensor

3. Vacuum Condensate in YM theories

4. Solution of Friedmann Equations in Gauge Field Theory Vacuum, Inflation

1. Annals of Phys. **436** (2022) 168681
2. PoS Corfu Meeting (2022), arXiv:2204.08933
3. Eur. Phys. J. **C 80** (2020) 165

Heisenberg-Euler Effective Lagrangian in QED

$$\mathcal{L}_{eff} = \frac{\mathcal{E}^2 - \mathcal{H}^2}{2} - \pi mc^2 (\frac{mc}{h})^3 \int_0^\infty \frac{ds}{s^3} e^{-s} \{ \frac{as\cos(as)}{\sin(as)} \ \frac{bs\cosh(bs)}{\sinh(bs)} - 1 + \frac{a^2 - b^2}{3} s^2 \}$$

where dimensionless fields are

$$a = \frac{e\hbar\mathcal{E}}{m^2c^3}, \qquad b = \frac{e\hbar\mathcal{H}}{m^2c^3}$$

$$mc^2 = 8.2 \cdot 10^{-7} \ \frac{g \ cm^2}{s^2} \qquad \lambda_c = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11} cm \qquad \frac{mc^2}{(\frac{\hbar}{mc})^3} = 1.43 \cdot 10^{25} \frac{g}{cm \ s^2}$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \sim 10^{16} \ Volt/cm \qquad \mathcal{H}_c = \frac{m^2c^3}{e\hbar} \sim 4.4 \cdot 10^{13} \ Gauss$$

Contribution of Vacuum Fluctuations

Renormalisation of massless Heisenberg-Euler
and
Yang-Mills Effective Lagrangians

G.S. 1976

$$\frac{\partial \mathcal{L}}{\partial \mathcal{F}}\Big|_{t=\frac{1}{2}\ln(\frac{2e^2|\mathcal{F}|}{\mu^4})=\mathcal{G}=0} = -1, \quad ($$

where $\mathcal{F} = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$ is the Lorentz and gauge invariant form of the YM field strength tensor

Heisenberg-Euler Effective Lagrangian

Massless limit of fermions G.S. 2020

$$\mathcal{L}_e = -\mathcal{F} + \frac{e^2 \mathcal{F}}{24\pi^2}\Big[\ln(\frac{2e^2\mathcal{F}}{\mu^4}) - 1\Big], \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}^2 - \vec{\mathcal{E}}^2}{2}, \quad \mathcal{G} = \vec{\mathcal{E}}\vec{\mathcal{H}} = 0,$$

the energy momentum tensor by using the formula derived by Schwinger in [5]:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial \mathcal{L}}{\partial \mathcal{F}} - \mathcal{G}\frac{\partial \mathcal{L}}{\partial \mathcal{G}}).$$

In massless QED using the one-loop expression (1.2) for $T_{\mu\nu}$ one can get

$$T_{\mu\nu} = T^M_{\mu\nu}\Big[1 - \frac{e^2}{24\pi^2}\ln\frac{2e^2\mathcal{F}}{\mu^4}\Big] + g_{\mu\nu}\frac{e^2}{24\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0.$$

Effective Lagrangian in Yang-Mills theory

The YM effective Lagrangian take the following form

$$\mathcal{L}^{(1)} = -\frac{1}{8\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} \frac{(gF_1 s)\ (gF_2 s)}{\sinh(gF_1 s)\ \sinh(gF_2 s)} - \\ -\frac{1}{4\pi^2}\int \frac{ds}{s^3} e^{-i\mu^2 s} (gF_1 s)\ (gF_2 s) [\frac{\sinh(gF_1 s)}{\sinh(gF_2 s)} + \frac{\sinh(gF_2 s)}{\sinh(gF_1 s)}]$$

$$F_1^2 = -\mathcal{F} - (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}, \qquad F_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$$

Vanyashin and Terentev 1965
Duff and Ramon-Medrano 1975
Skalozub 1976

Bartalin, Matinyan and Savvidy 1976
Savvidy 1977
Matinyan and Savvidy 1978

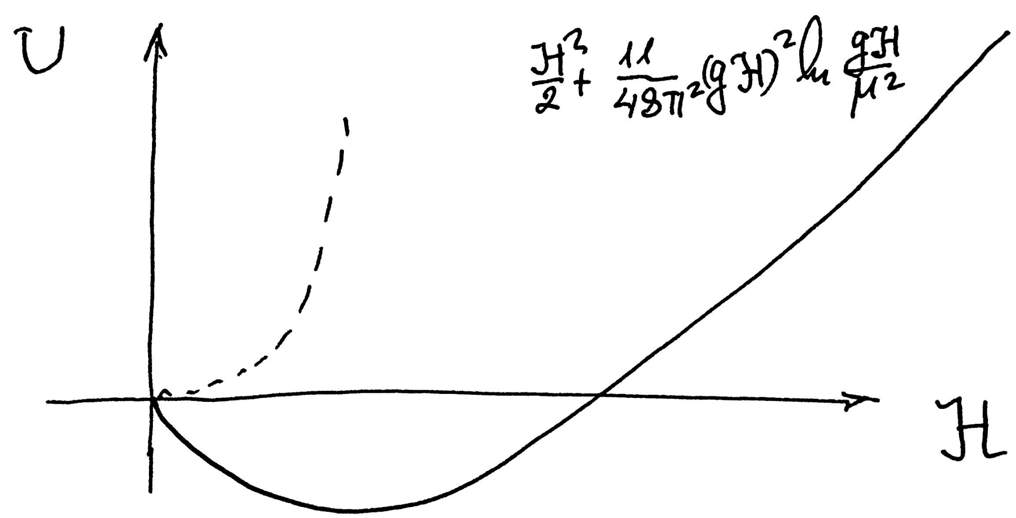
N.Nielsen and Olesen 1978
Ambjorn, N.Nielsen and Olesen 1979
H.Nielsen and Ninomia 1979
H.Nielsen and Olesen 1979
Ambjorn and Olesen1980

Dimensional Transmutation and Condensation

G.S. 1977, 2020

$$\mathcal{L}_g = -\mathcal{F} - \frac{11N}{96\pi^2} g^2 \mathcal{F} \Big(\ln \frac{2g^2 \mathcal{F}}{\mu^4} - 1 \Big) \ , \qquad \mathcal{F} = \frac{\vec{\mathcal{H}}_a^2 - \vec{\mathcal{E}}_a^2}{2} > 0, \quad \mathcal{G} = \vec{\mathcal{E}}_a \vec{\mathcal{H}}_a = 0 \ .$$

$$\mathcal{L}_q = -\mathcal{F} + \frac{N_f}{48\pi^2} g^2 \mathcal{F} \Big[\ln (\frac{2g^2 \mathcal{F}}{\mu^4}) - 1 \Big]$$



$$2g^2 \mathcal{F}_{vac} = \mu^4 \exp\big(-\frac{96\pi^2}{b \ g^2(\mu)}\big) = \Lambda_{YM}^4,$$

where $b = 11N - 2N_f$.

$$T_{\mu\nu} = T_{\mu\nu}^{YM} \Big[1 + \frac{b \ g^2}{96\pi^2} \ln \frac{2g^2 \mathcal{F}}{\mu^4} \Big] - g_{\mu\nu} \frac{b \ g^2}{96\pi^2} \mathcal{F}, \qquad \mathcal{G} = 0.$$

The YM effective Lagrangian and of the Energy momentum tensor and the Renormalisation group

The YM effective Energy Momentum Tensor has the following form:

$$T_{\mu\nu} = (F_{\mu\lambda}F_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}F^2_{\lambda\rho})\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - g_{\mu\nu}(\mathcal{L} - \mathcal{F}\frac{\partial\mathcal{L}}{\partial\mathcal{F}} - \mathcal{G}\frac{\partial\mathcal{L}}{\partial\mathcal{G}}).$$

When $\mathcal{G} = \mathcal{E}^{'}_a\mathcal{H}^{'}_a = 0$, we have

$$\frac{\partial\mathcal{L}}{\partial\mathcal{F}} = -\frac{g^2}{\bar{g}^2(t)}, \qquad \frac{d\bar{g}}{dt} = \beta(\bar{g}) \ , \qquad t = \frac{1}{2}\ln(2g^2\mathcal{F}/\mu^4).$$

Quantum Energy Momentum Tensor in RG :

$$T_{\mu\nu} = -\Big(G_{\mu\lambda}G_{\nu\lambda} - g_{\mu\nu}\frac{1}{4}G^2_{\lambda\rho}\Big)\frac{g^2}{\bar{g}^2(t)} + g_{\mu\nu}\Big(\int \frac{e^{2t}}{\bar{g}^2(t)}dt - \frac{1}{2}\frac{e^{2t}}{\bar{g}^2(t)}\Big)\mu^4.$$

Alternative Renormalisation group approach

M. Reuter and C. Wetterich, *Indications for gluon condensation for nonperturbative flow equations,* hep-th/9411227.

M. Reuter and C. Wetterich, *Search for the QCD ground state,* Phys. Lett. B **334** (1994) 412 doi:10.1016/0370-2693(94)90707-2 [hep-ph/9405300].

M. Reuter and C. Wetterich, *Effective average action for gauge theories and exact evolution equations,* Nucl. Phys. B **417** (1994) 181. doi:10.1016/0550-3213(94)90543-6

C. Wetterich, *Exact evolution equation for the effective potential,* Phys. Lett. B **301** (1993) 90 doi:10.1016/0370-2693(93)90726-X [arXiv:1710.05815 [hep-th]].

R. Schutzhold, H. Gies and G. Dunne, *Dynamically assisted Schwinger mechanism,* Phys. Rev. Lett. **101** (2008), 130404 doi:10.1103/PhysRevLett.101.130404 [arXiv:0807.0754].

C. Wetterich, Exact evolution equation for the effective potential, Phys. Lett. B **301**, 90 (1993), arXiv:1710.05815

J. Berges, N. Tetradis, and C. Wetterich, Coarse graining and first order phase transitions, Phys. Lett. B **393**, 387 (1997), arXiv:hep-ph/9610354.

J. Berges, N. Tetradis, and C. Wetterich, Nonperturbative renormalization flow in quantum field theory and statistical physics, Phys. Rept. **363**, 223 (2002), arXiv:hep-ph/0005122.

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \, ,$$

Quantum Energy Momentum Tensor

$$T_{\mu\nu} = T^{YM}_{\mu\nu}\Big[1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big] - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}, \qquad \mathcal{G} = 0,$$

$$T_{00} \equiv \epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big)$$

$$T_{ij} = \delta_{ij}\ \Big[\frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big)\Big] = \delta_{ij}\ p(\mathcal{F}).$$

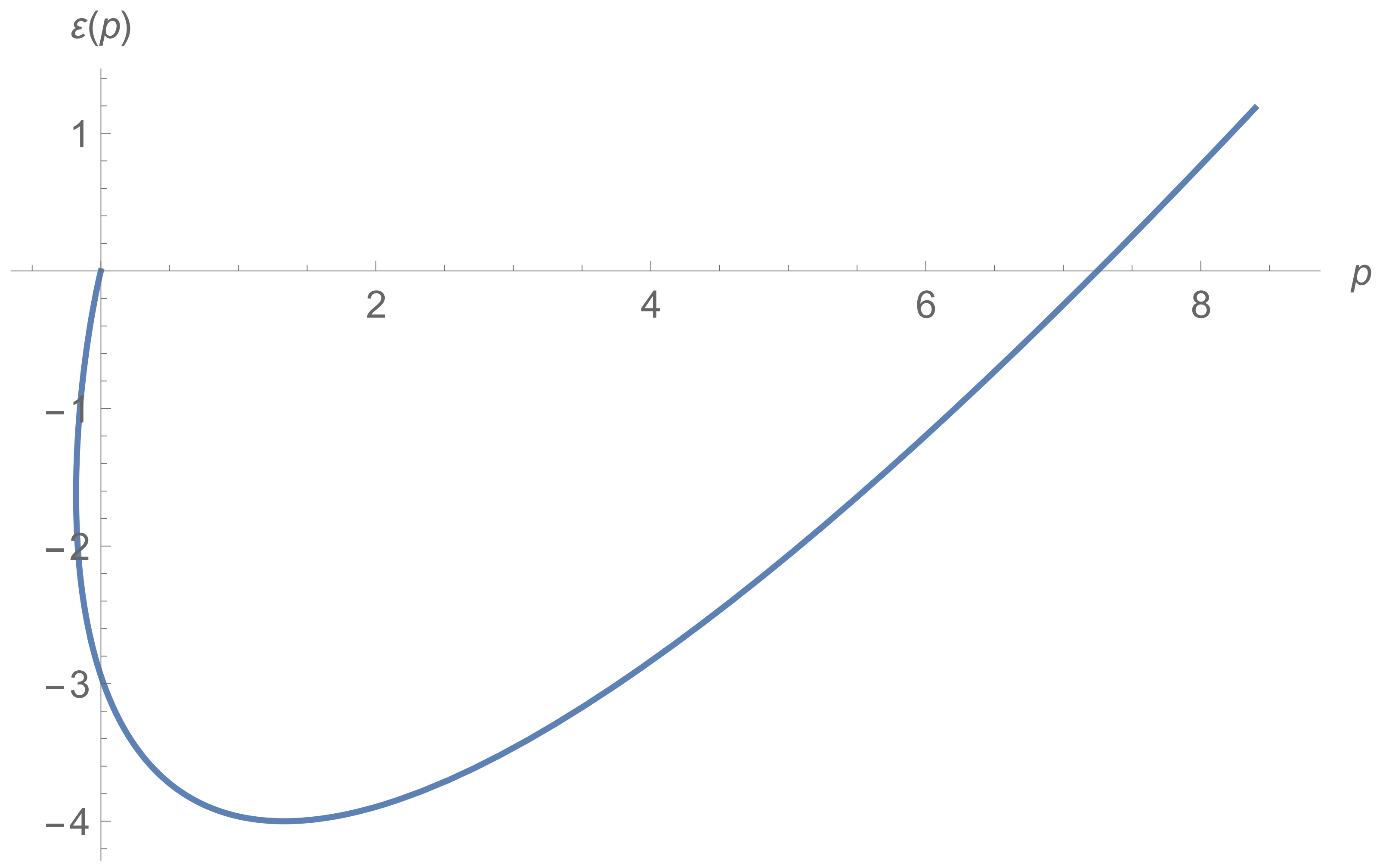
$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big),$$

$$p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

$$\mathcal{F} = \tfrac{1}{4}g^{\alpha\beta}g^{\gamma\delta}G^a_{\alpha\gamma}G_{\beta\delta} \geq 0$$

$$\mathcal{G} = G^*_{\mu\nu}G^{\mu\nu} = 0$$

Yang-Mills Quantum Equation of State



$$\epsilon(\mathcal{F}) = \mathcal{F} + \frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} - 1\Big), \qquad p(\mathcal{F}) = \frac{1}{3}\mathcal{F} + \frac{1}{3}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big(\ln\frac{2g^2\mathcal{F}}{\mu^4} + 3\Big).$$

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p).$$

Equation of State

general parametrisation of the equation of state $p = w\epsilon$

when $w = -1$, $p = -\epsilon < 0$,

the acceleration is positive:

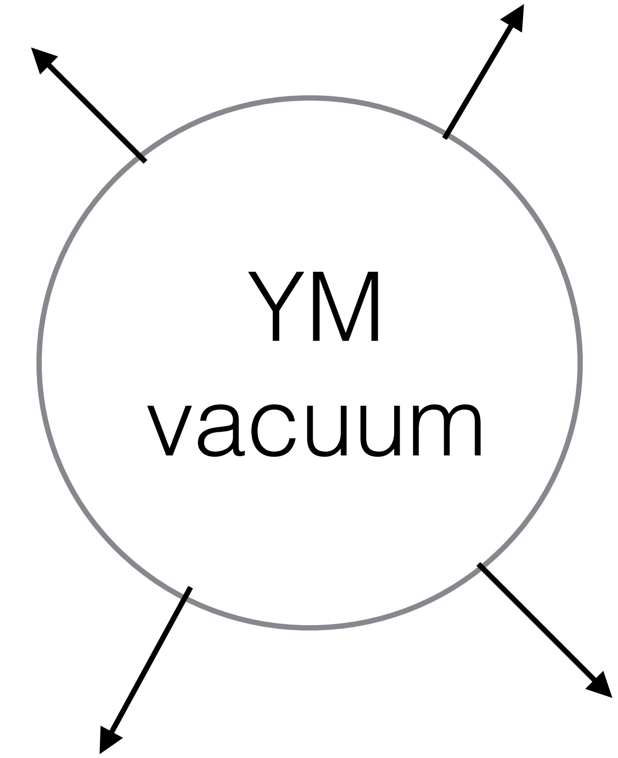
$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3c^4}\epsilon > 0.$$

Yang-Mills Quantum Equation of State

$$p = \frac{1}{3}\epsilon + \frac{4}{3}\frac{b\ g^2\mathcal{F}}{96\pi^2}\Lambda_{YM}^4 \qquad \text{and} \qquad w = \frac{p}{\epsilon} = \frac{\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} + 3}{3\left(\ln\frac{2g^2\mathcal{F}}{\Lambda_{YM}^4} - 1\right)}$$

general parametrisation of the equation of state $p = w\epsilon$

GR Action



$$S = -\frac{c^3}{16\pi G}\int R\sqrt{-g}d^4x + \int \left(\mathcal{L}_q + \mathcal{L}_g\right)\sqrt{-g}d^4x.$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}\Big[T^{YM}_{\mu\nu}\Big(1 + \frac{b\ g^2}{96\pi^2}\ln\frac{2g^2\mathcal{F}}{\mu^4}\Big) - g_{\mu\nu}\frac{b\ g^2}{96\pi^2}\mathcal{F}\Big].$$

$$\Lambda_{eff} = \frac{8\pi G}{c^4}\ \epsilon_{vac} = -\frac{8\pi G}{c^4}\frac{b}{192\pi^2}2g^2\mathcal{F}_{vac} = -\frac{8\pi G}{c^4}\frac{b}{192\pi^2}\Lambda^4_{YM}$$

The YM field strength $\mathcal{F}$ is not a constant function of time but evolve in time in accordance with the Feidmann equations, thus the cosmological term here is time dependent

Friedmann Evolution Equations

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + p) = 0, \quad \longrightarrow \quad \epsilon + p = \frac{4\mathcal{A}}{3}\ (2g^2\mathcal{F})\ \log\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\epsilon + 3p). \quad \longrightarrow \quad \epsilon + 3p = 2\mathcal{A}\ (2g^2\mathcal{F})\ \Big(\log\frac{2g^2\mathcal{F}}{\Lambda^4_{YM}} + 1\Big).$$

the first equation can be solved for the field strength

$$2g^2\dot{\mathcal{F}} + 4(2g^2\mathcal{F})\frac{\dot{a}}{a} = 0 \qquad\qquad 2g^2\mathcal{F}\ a^4 = const \equiv \Lambda^4_{YM}\ a_0^4,$$

Friedmann Evolution Equations

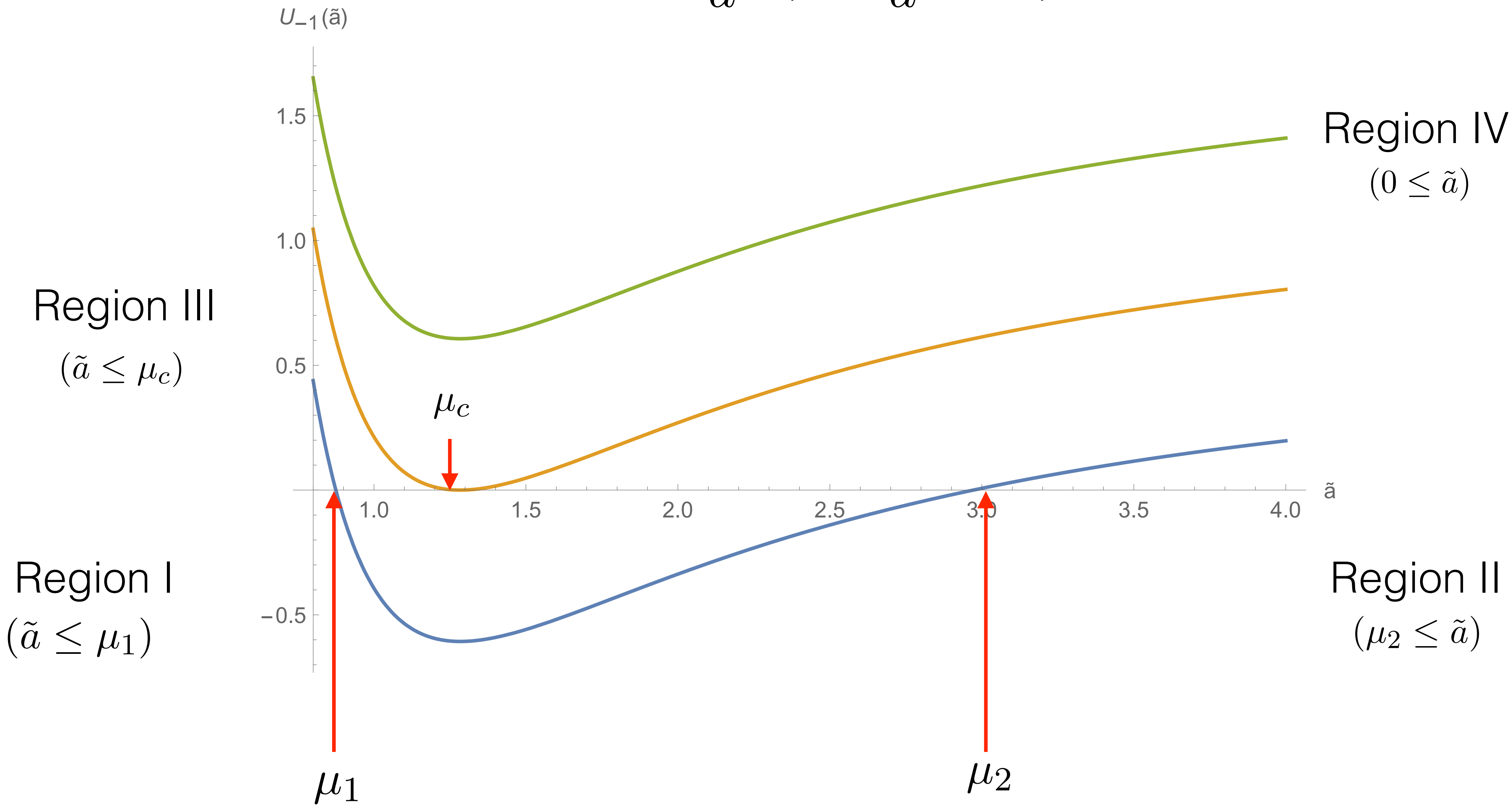
$$a(\tau) = a_0 \; \tilde{a}(\tau), \qquad ct = L \; \tau,$$

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log \frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

$$\frac{1}{L^2} = \frac{8\pi G}{3c^4} \; \mathcal{A} \; \Lambda_{YM}^4 \equiv \Lambda_{eff} \;\; ,$$

$$\mathcal{A} = \frac{b}{192\pi^2} = \frac{11N - 2N_f}{192\pi^2}.$$

$$U_{-1}(\tilde{a}) \equiv \frac{1}{\tilde{a}^2}\left(\log\frac{1}{\tilde{a}^4} - 1\right) + \gamma^2.$$



$$0 \leq \gamma^2 < \gamma_c^2$$

$$\gamma^2 = \gamma_c^2 = \frac{2}{\sqrt{e}}$$

$$\gamma_c^2 < \gamma^2$$

Type II Solution — Initial Acceleration of Finite Duration

$$\frac{d\tilde{a}}{d\tau} = \pm\sqrt{\frac{1}{\tilde{a}^2}\Big(\log\frac{1}{\tilde{a}^4} - 1\Big) - k\gamma^2}, \qquad k = 0, \pm 1, \qquad \gamma^2 = \Big(\frac{L}{a_0}\Big)^2.$$

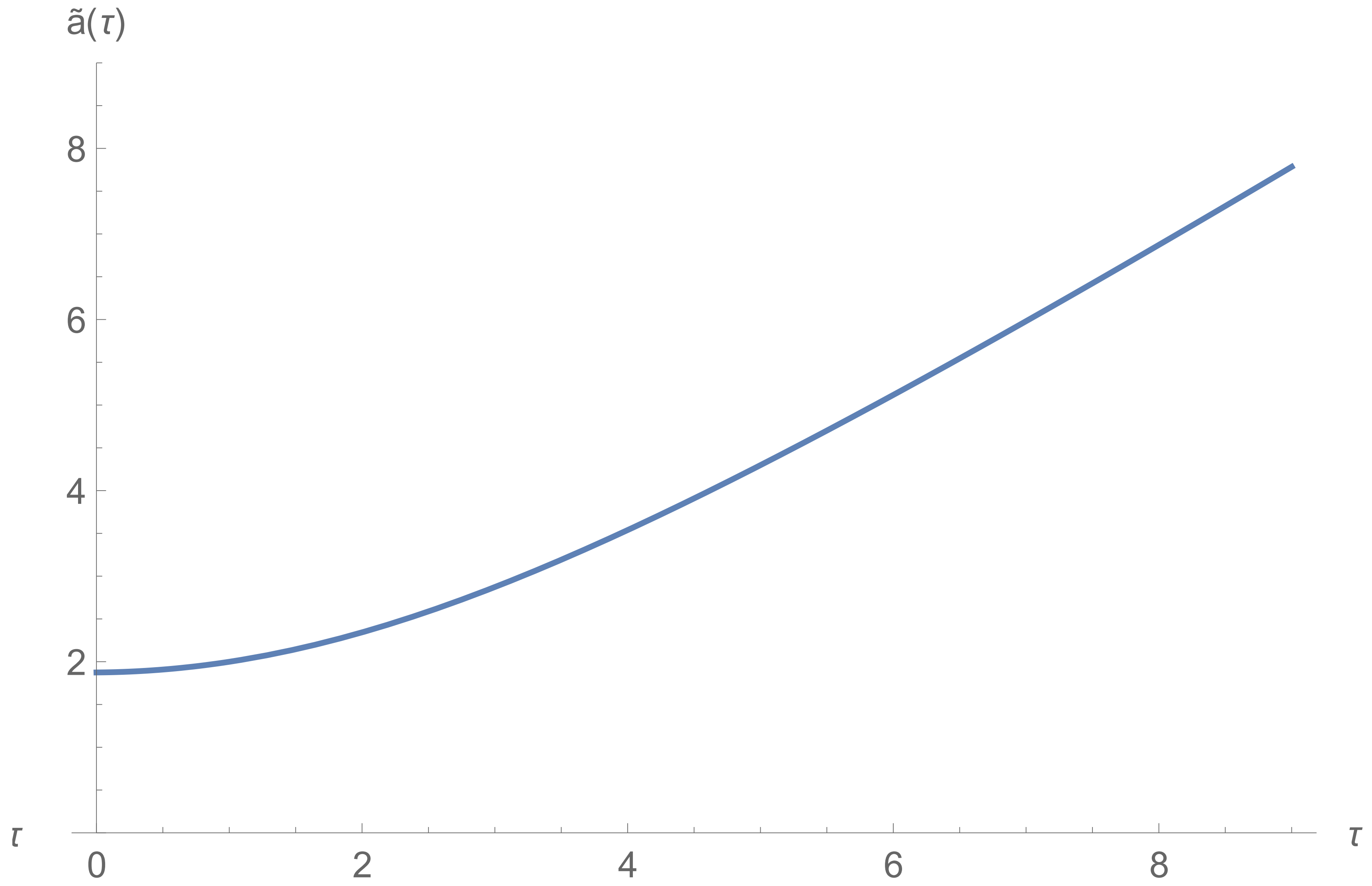
$$\tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$

$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\; e^{-\frac{b^2}{2}}\Big(\frac{\gamma^2\mu_2^2}{b^2}(e^{\frac{b^2}{2}} - 1) - 1\Big)^{1/2}.$$

$$\mu_2^2 = -\frac{2}{\gamma^2}\; W_-\Big(-\frac{\gamma^2}{2\sqrt{e}}\Big), \qquad 0 \le \gamma^2 < \frac{2}{\sqrt{e}} \text{ and } \tilde{a} \ge \mu_2.$$

Type II Solution   Initial Acceleration of Finite Duration

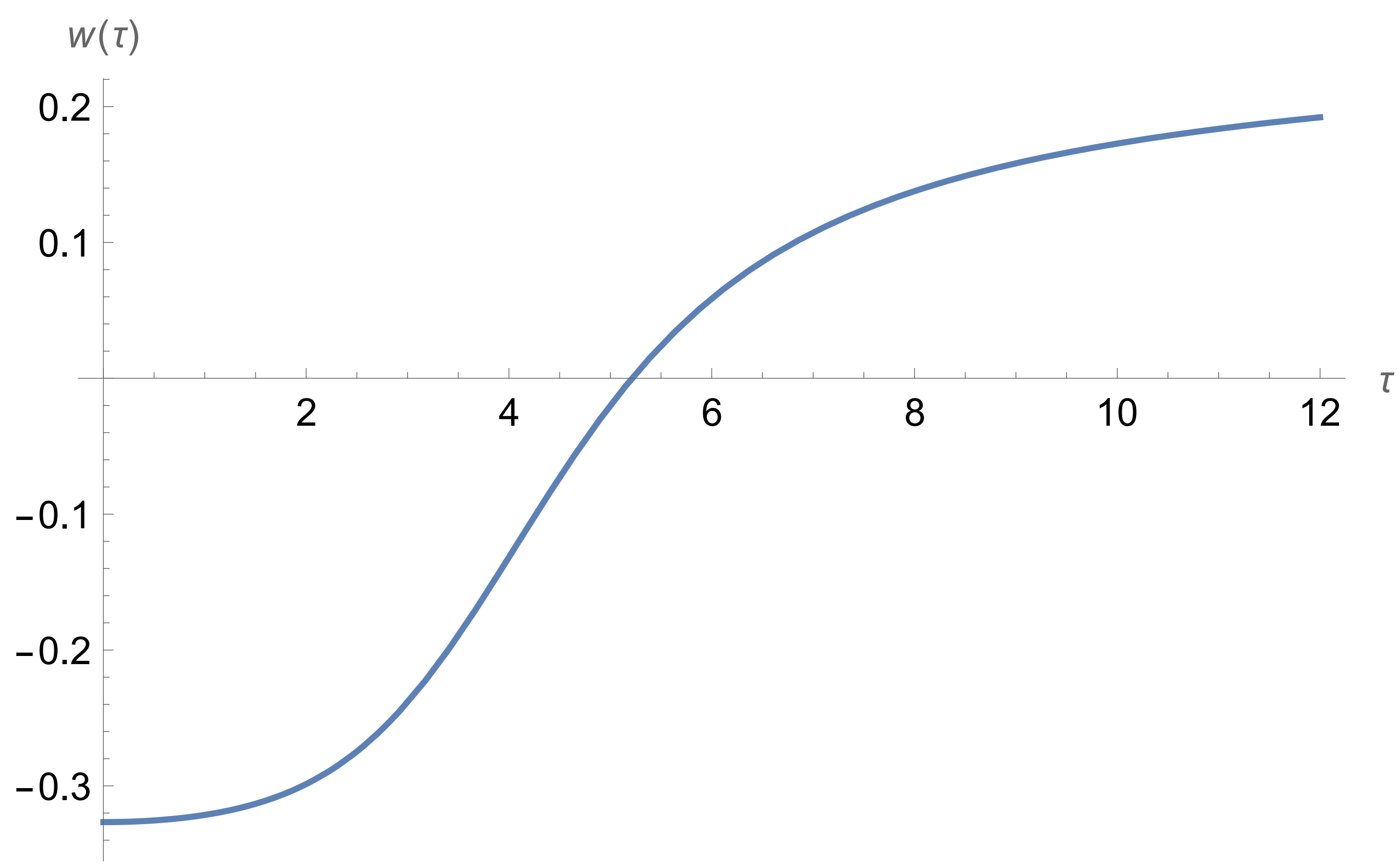
$$\frac{db}{d\tau} = \frac{2}{\mu_2^2}\; e^{-\frac{b^2}{2}} \Big(\frac{\gamma^2 \mu_2^2}{b^2} (e^{\frac{b^2}{2}} - 1) - 1 \Big)^{1/2}. \qquad \tilde{a}^4 = \mu_2^4 e^{b^2}, \qquad b \in [0, \infty],$$



The regime of the exponential growth will continuously transformed into the linear in time growth of the scale factor‡

$$a(t) \simeq \; ct, \qquad a(\eta) \simeq a_0 e^{\eta}. \tag{5.87}$$

Type II Solution — Effective Parameter w



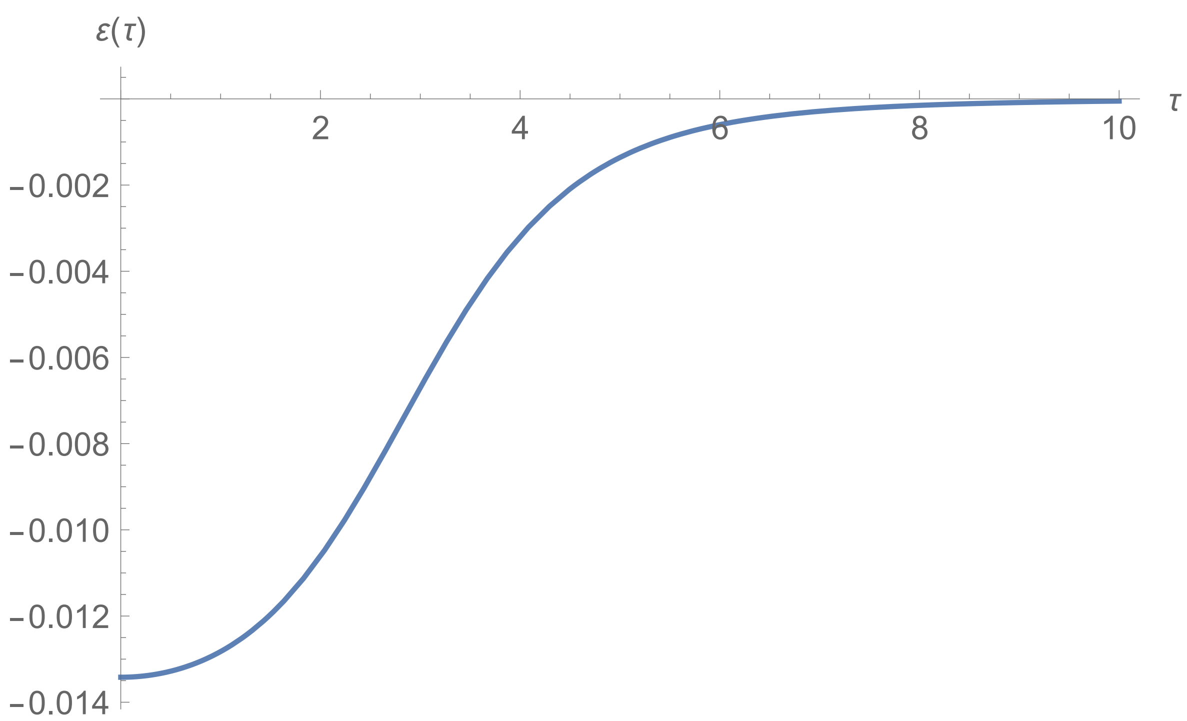
For the equation of state $p = w\epsilon$ one can find the behaviour of the effective parameter w

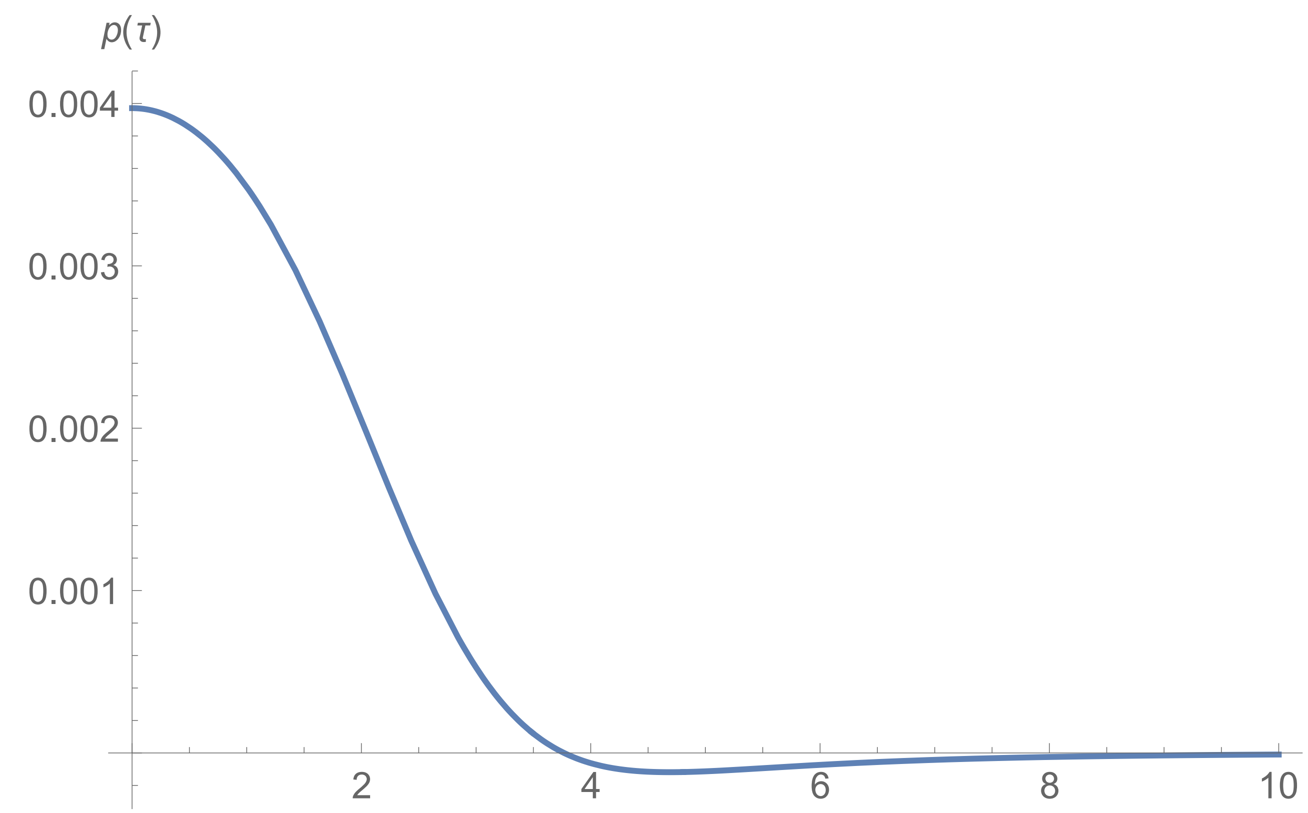
$$w_{II} = \frac{b^2(\tau) + \gamma^2\mu_2^2 - 4}{3\Big(b^2(\tau) + \gamma^2\mu_2^2\Big)}, \qquad -1 \leq w_{II},$$

$$w = \frac{p}{\epsilon} = \frac{\log\frac{1}{\tilde{a}^4(\tau)} + 3}{3\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)}.$$

Evolution of Energy Density and Pressure

$$\epsilon = \frac{\mathcal{A}}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{\mathcal{A}}{3\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} + 3\Big)\Lambda_{YM}^4.$$

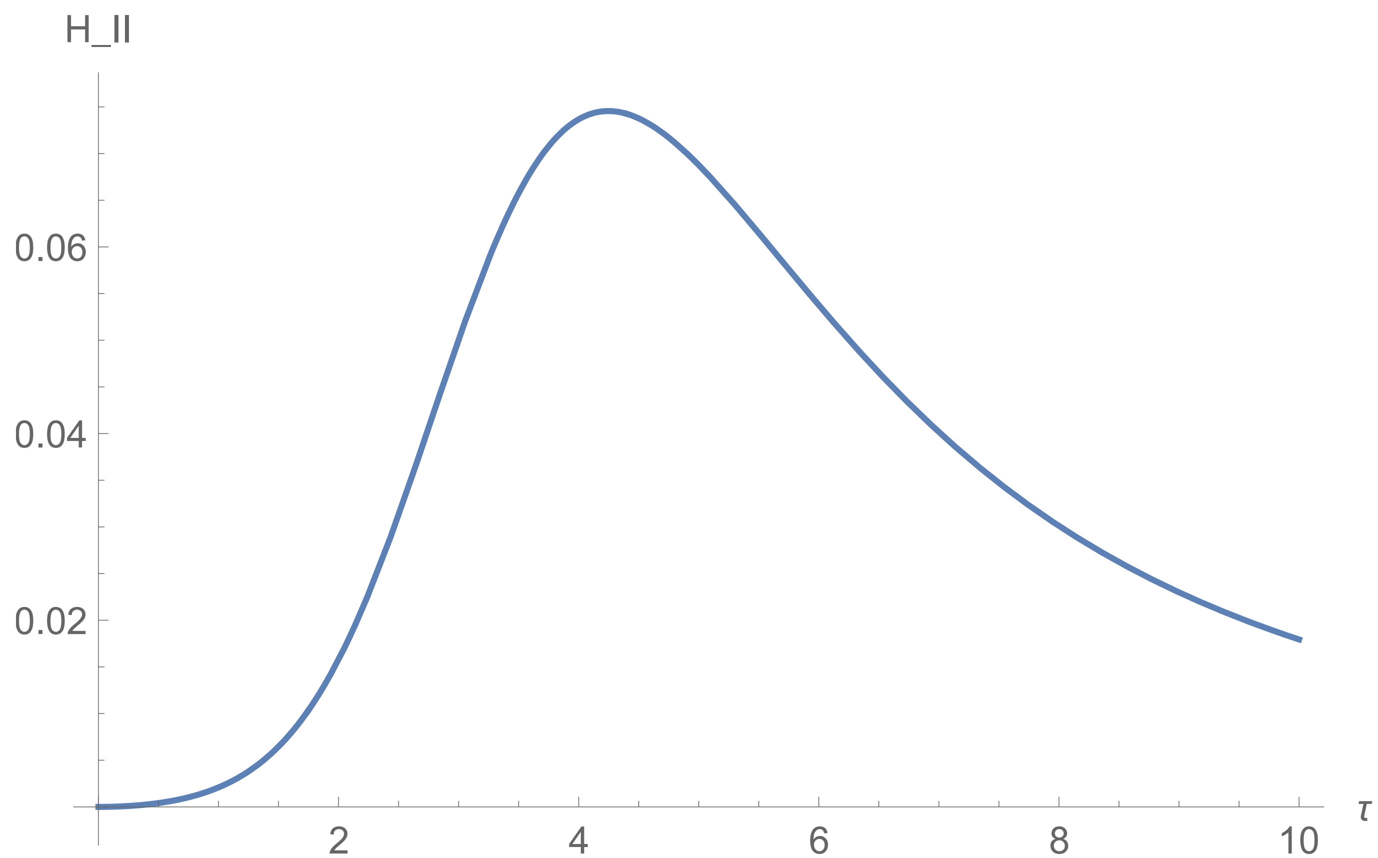




Hubble Parameter

$$L^2H^2 = L^2\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{\tilde{a}^2}\Big(\frac{d\tilde{a}}{d\tau}\Big)^2 = \frac{1}{\tilde{a}^4(\tau)}\Big(\log\frac{1}{\tilde{a}^4(\tau)} - 1\Big) - \frac{k\gamma^2}{\tilde{a}^2(\tau)}$$

$$L^2H^2 = \frac{e^{-b^2}}{\mu_2^4}\Big(\gamma^2\mu_2^2(e^{b^2/2}-1) - b^2\Big).$$



Polarisation of the YM vacuum and the Effective Lagrangians

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2}, \qquad \frac{1}{L^2} = \frac{8\pi G}{3c^4}\ \frac{11N - 2N_f}{196\pi^2}\ \Lambda^4_{YM}$$

Λ^4_{YM} *is the dimensional transmutation scale of YM theory*

$$\epsilon_{YM} = 3\frac{c^4}{8\pi G}\frac{1}{L^2} = \begin{cases} 9.31\times 10^{-3} & eV \\ 9.31\times 10^{29} & QCD \\ 9.31\times 10^{97} & GUT \\ 9.31\times 10^{110} & Planck \end{cases} \qquad \frac{g}{s^2 cm}$$

the YM vacuum energy density is well defined, is finite and is time dependent quantity

Eur.Phys.J. **C 80** (2020) 165
e-Print: 2109.02162

Type IV Solution - Late time Acceleration

The type IV solution is defined in the region $\gamma^2 > \gamma_c^2$ where the equation

$$U_{-1}(\mu) = \frac{1}{\mu^2}\Big(\log\frac{1}{\mu^4} - 1\Big) + \gamma^2 = 0$$

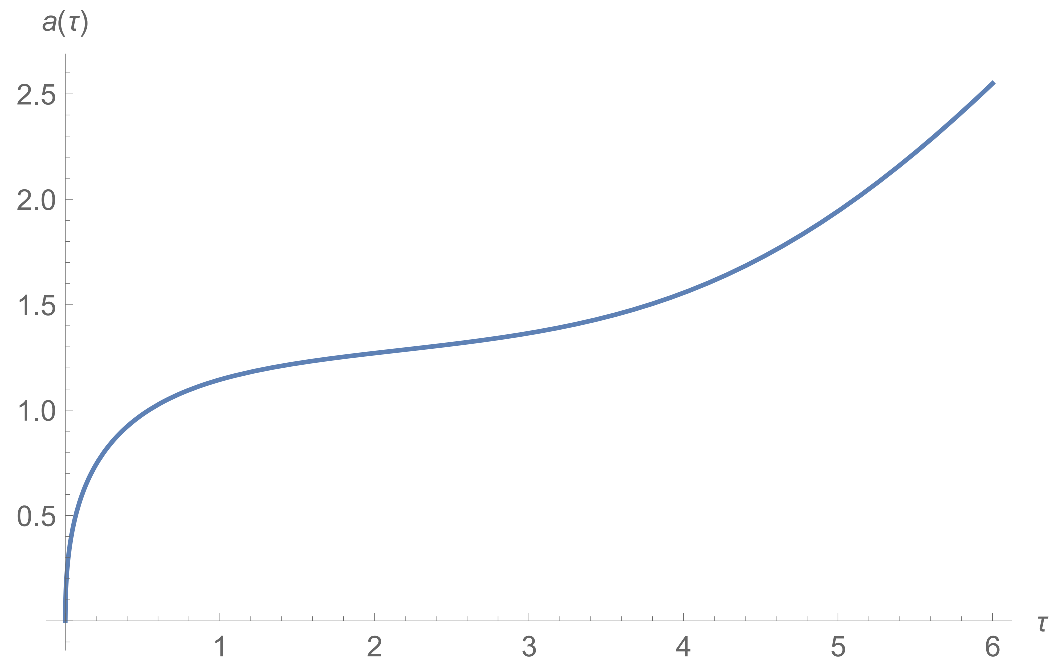
$$\tilde{a} = \mu_c e^b, \qquad b \in [-\infty, \infty], \qquad 2 < \gamma^2 \mu_c^2, \qquad \gamma_c^2 = \frac{2}{\sqrt{e}},$$

$$\frac{db}{d\tau} = \sqrt{\frac{2}{e}}\; e^{-2b}\Big(\frac{\gamma^2}{\gamma_c^2}e^{2b} - 1 - 2b\Big)^{1/2}.$$

$$2g^2\mathcal{F} = e^{-4b(\tau)-1}\Lambda_{YM}^4,$$

$$\epsilon = 2\mathcal{A}e^{-4b(\tau)-1}\Big(-2b(\tau) - 1\Big)\Lambda_{YM}^4, \qquad p = \frac{2\mathcal{A}}{3}e^{-4b(\tau)-1}\Big(-2b(\tau) + 1\Big)\Lambda_{YM}^4.$$

Type IV Solution - Late time Acceleration



$$q_{IV} \simeq -\frac{2}{\gamma^2 \mu_c^2} b e^{-2b} \to 0.$$

$$H = \sqrt{\frac{2}{e}} \frac{e^{-2b}}{L} \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \right)^{1/2} \simeq \frac{1}{ct}.$$

$$\Omega_{vac} = 1 - \frac{\gamma^2}{(\frac{d\tilde{a}}{d\tau})^2} = 1 - \frac{\gamma^2 e^{2b}}{\gamma_c^2 \left(\frac{\gamma^2}{\gamma_c^2} e^{2b} - 1 - 2b \right)} \to 0.$$

Thank You !