Cosmological constant problem and Hubble tension in scale-dependent cosmology

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with
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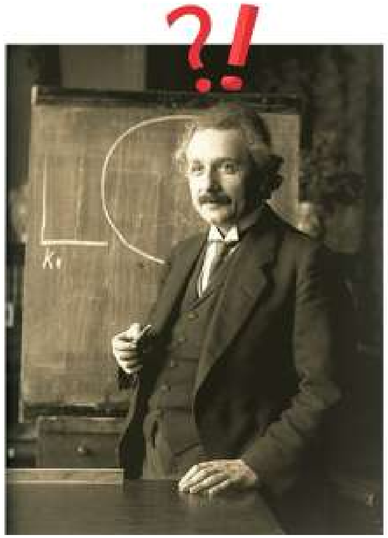
JCAP no 1, 21, 2020, arXiv:1812.10526
JCAP no 6, 019, 2021, arXiv: 2009.02311
arXiv:2205.05592

Content

A (very brief) History of the Cosmological Constant



1917

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

$\Rightarrow \dot{a} \equiv 0$ static

1929

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

$\Rightarrow \dot{a} > 0$ not static

1998

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

$$\Lambda > 0 \Rightarrow \ddot{a} > 0$$

A (very brief) History of the Cosmological Constant

Yakov Zeldovich

Quantum fluctuation predict value of Λ

1967

A (very brief) History of the Cosmological Constant

Steven Weinberg

Quantum fluctuation
predict value of Λ

Problem since '98
ref[3]: S. Wienberg '98

A (very brief) History of the Cosmological Constant

Quantum fluctuation
predict value of Λ

Different methods

$$\left(\frac{\Lambda}{G}\right)_{\text{eff}} \sim M^4(1+...)$$

Effective quantum value

cut-off
mass scale of QFT

Use

$$G \sim \frac{1}{M_{pl}^2}$$

A (very brief) History of the Cosmological Constant

Quantum fluctuation predict value of Λ

Different methods

$$\Lambda_{eff} \sim \frac{M^4}{M_{pl}^2}(1 + ...)$$

Effective quantum value

cut-off mass scale of QFT

Planck mass

A (very brief) History of the Cosmological Constant

Quantum fluctuation predict value of Λ

Highest physical mass scale

$$\Lambda_{eff} \sim \frac{M^4}{M_{pl}^2}(1 + ...)$$

Observed value

$$\Lambda_0 = \frac{\rho_c}{M_{pl}^2} \approx \frac{10^{-47}\mathrm{GeV}^4}{M_{pl}^2}$$

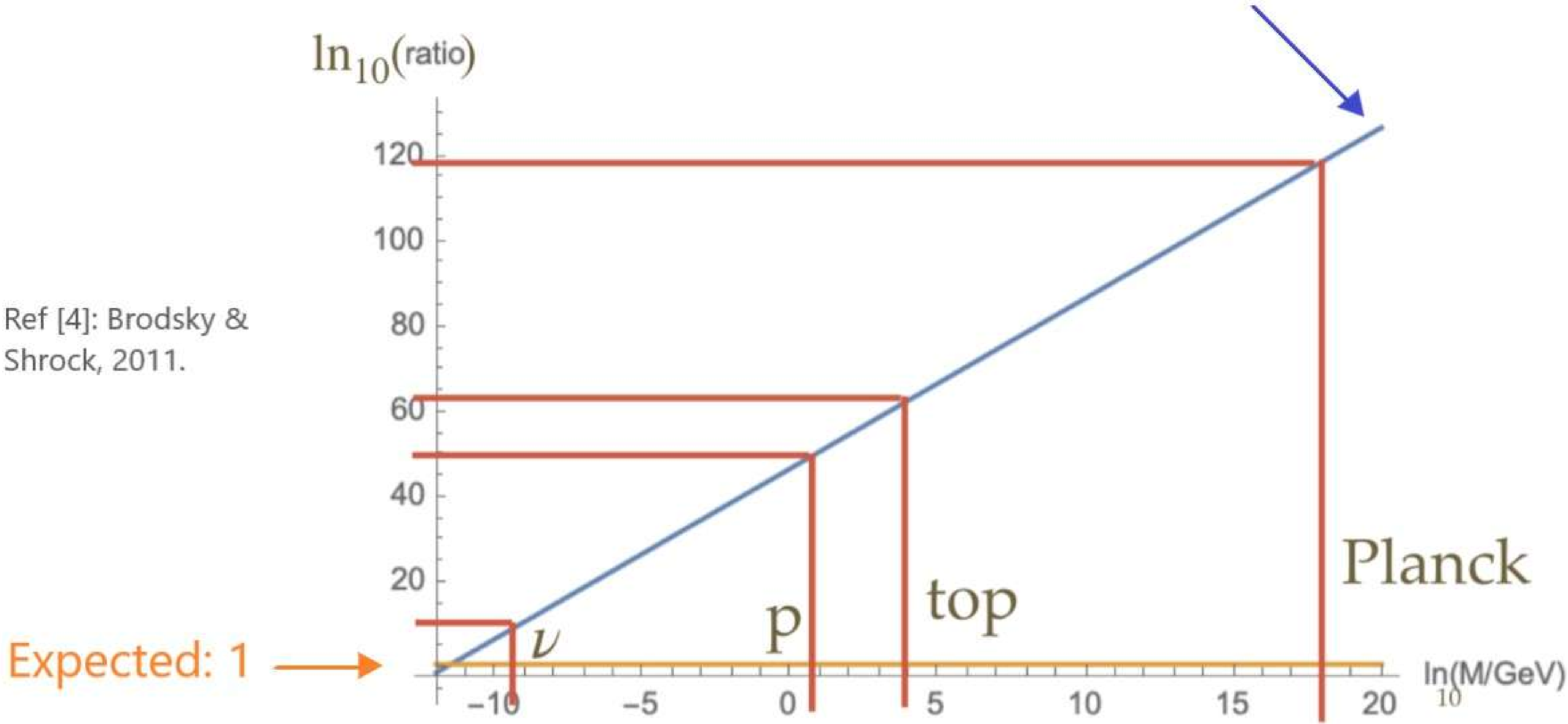
ρ_c = observed critical energy density

A (very brief) History of the Cosmological Constant

The problem view as a ratio

$$\ln\left(\frac{\Lambda_{eff}}{\Lambda_0}\right) \sim \ln\left(\frac{M^4}{\rho_c}\right)$$

Ref [4]: Brodsky & Shrock, 2011.



A (very brief) History of the Cosmological Constant

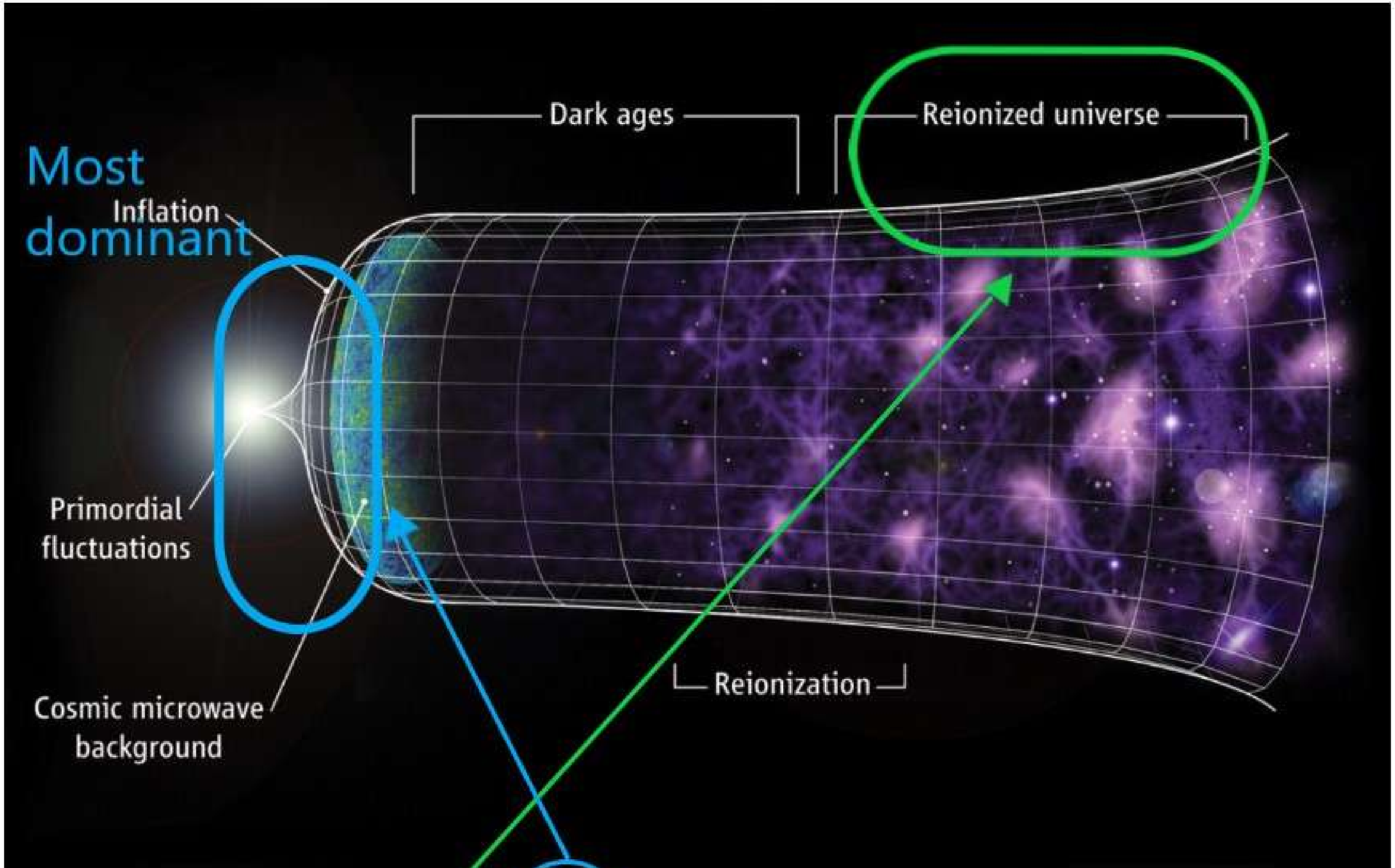
The problem view as a ratio

$$\frac{\Lambda_{eff}}{\Lambda_0} \sim \frac{1}{G_N \Lambda_0} \sim \frac{M_{pl}^4}{\rho_c} \approx 10^{120}$$

We try to address
this problem

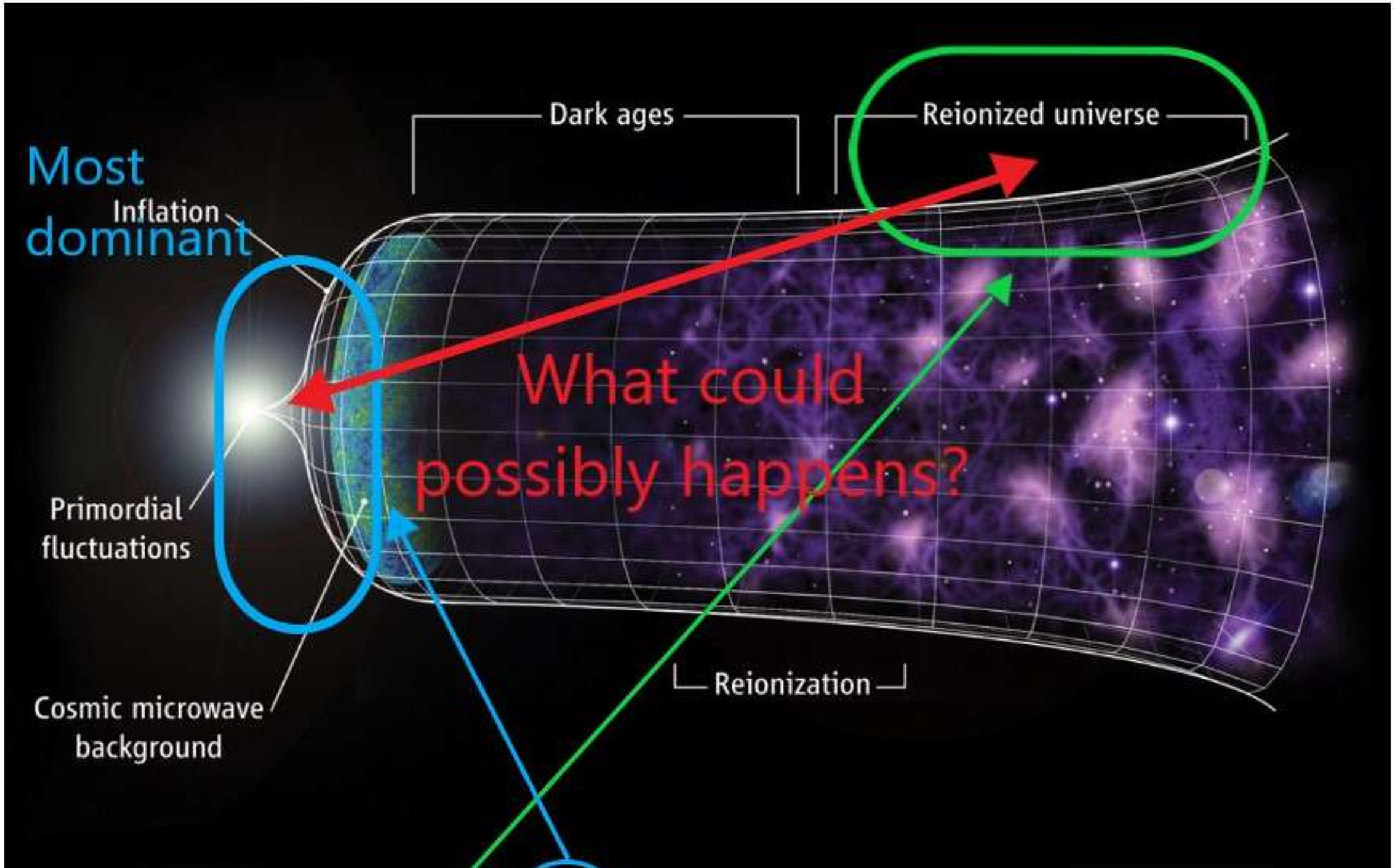
Assuming there are quantum fluctuations
of gravity associated to the Planck scale

Evolving Universe Issue



$$\frac{\Lambda_{eff}}{\Lambda_0} \sim \frac{M_{pl}^4}{\rho_c}(1 + \dots) \approx 10^{120}$$

Evolving Universe Issue



$$\frac{\Lambda_{eff}}{\Lambda_0} \sim \frac{M_{pl}^4}{\rho_c}(1 + ...) \approx 10^{120}$$

Scale-dependent Framework

Gravity as classical theory

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\frac{R}{G_N} - 2\frac{\Lambda_0}{G_N}\right)$$

Gravity as an effective QFT

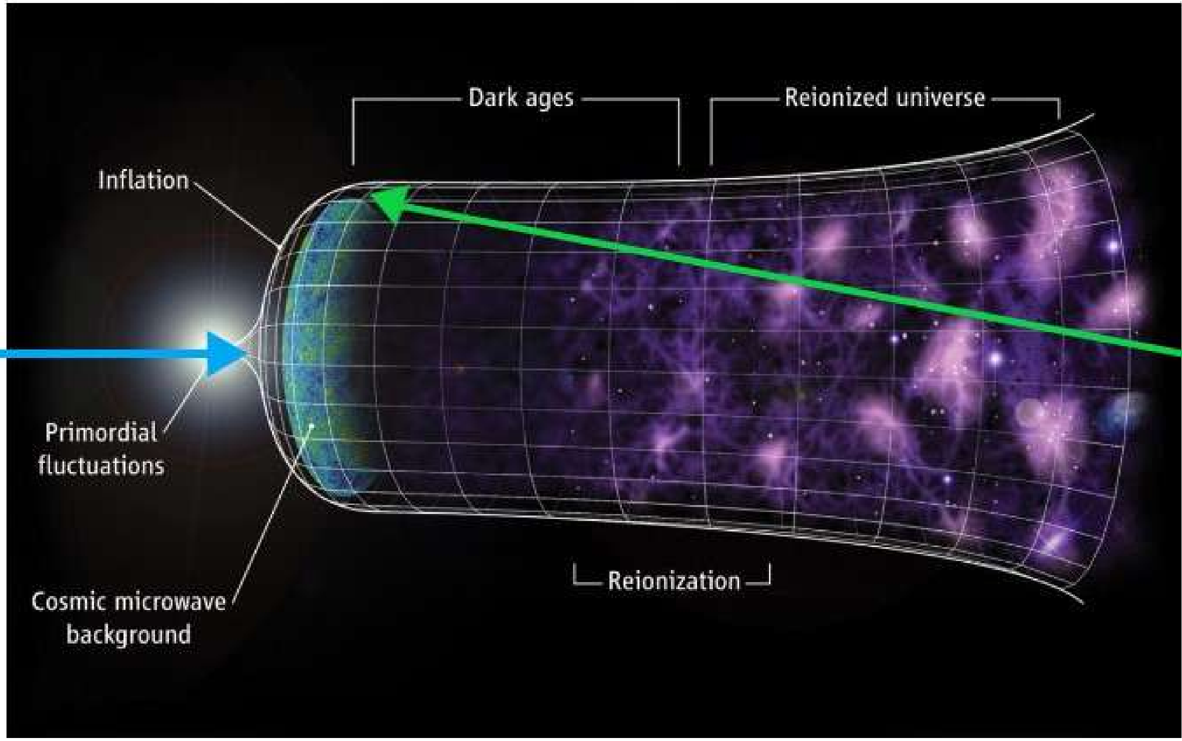
$$\Gamma_k = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k}\right) + \ldots$$

Scale Dependent Framework

$$\Gamma_k = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k}\right) + \dots$$

Implications for CCP?

$$\frac{1}{G_k\Lambda_k} \approx 1$$



$$\frac{1}{G_{k'}\Lambda'_k} \neq 1$$

$$k \gg k'$$

Deflation During Inflation

$$\Gamma_k = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k}\right)$$

We need to solve the gap equations

$$G_{\mu\nu} = -\Lambda_k g_{\mu\nu} - \Delta t_{\mu\nu}$$

with

$$\Delta t_{\mu\nu} = G_k \left(g_\nu \nabla^\alpha \nabla_\alpha - \nabla_\mu \nabla_\nu\right) G_k^{-1}$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda_k}{3} = \frac{\rho_{SD}}{3}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda_k = -p_{SD}$$

Scale-dependent quantities

Since $k = k(t) \Rightarrow G_k = G(t) \quad \& \quad \Lambda_k = \Lambda(t)$

$$\frac{1}{3}\rho_{SD} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right) \quad \& \quad -p_{SD} = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

Problem: 2 Equations
3 Unknown functions $a(t)\, G(t)\, \&\, \Lambda(t)$

Solution: Impose an energy condition

Deflation During Inflation

Null Energy Condition (NEC)

$$\Delta t_{\mu\nu}\ell^{\mu}\ell^{\nu} = 0$$

where

$$\frac{\mathrm{d}\ell^{\mu}}{\mathrm{d}t} + \Gamma^{\mu}_{\alpha\beta} = 0 \Rightarrow \ell^{\mu} = \frac{c_0}{a}\left(1, \frac{(1-\kappa r^2)^{-\frac{1}{2}}}{a}, 0, 0\right)$$

thus

$$-2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\dot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 0$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

NEC

$$-2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\dot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 0$$

3 equations, 3 unknowns ✔

Deflation During Inflation

Solution

$$a(t) = a_i \exp\left(\frac{t}{\sqrt{\Lambda_0/3}}\right)$$

$$G(t) = \frac{G_0}{1+\xi a(t)}$$

$$\Lambda(t) = \Lambda_0 \left(\frac{1+2\xi a(t)}{1+\xi a(t)}\right)$$

Three integration constants: G_0, Λ_0, ξ

What does it mean for the CCP?

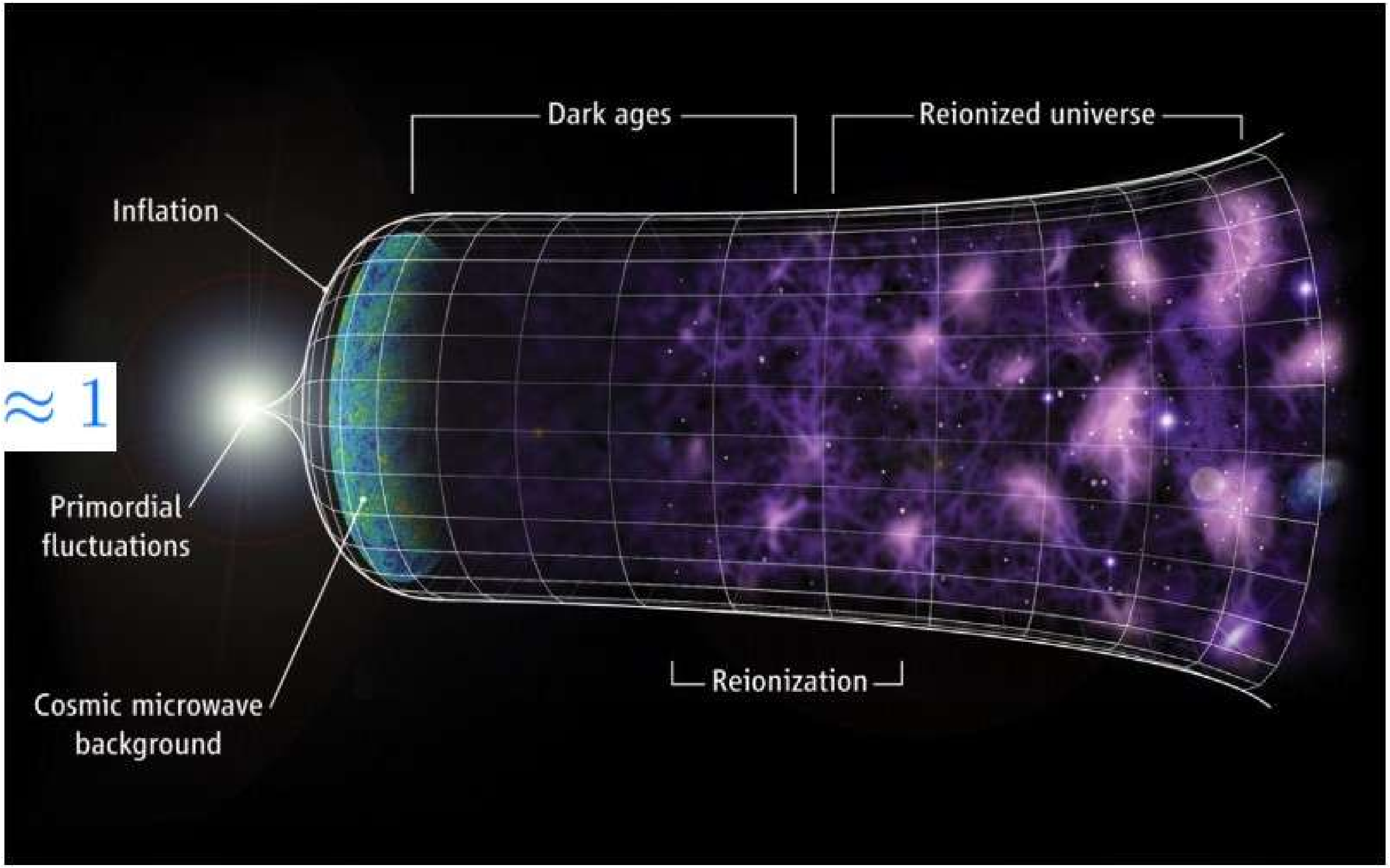
Deflation During Inflation

What does this means for the CCP?

$$G_k \cdot \Lambda_k = G(t) \cdot \Lambda(t) \longrightarrow G(t_f) \cdot \Lambda(t_f) \ll 1$$

$$\downarrow$$

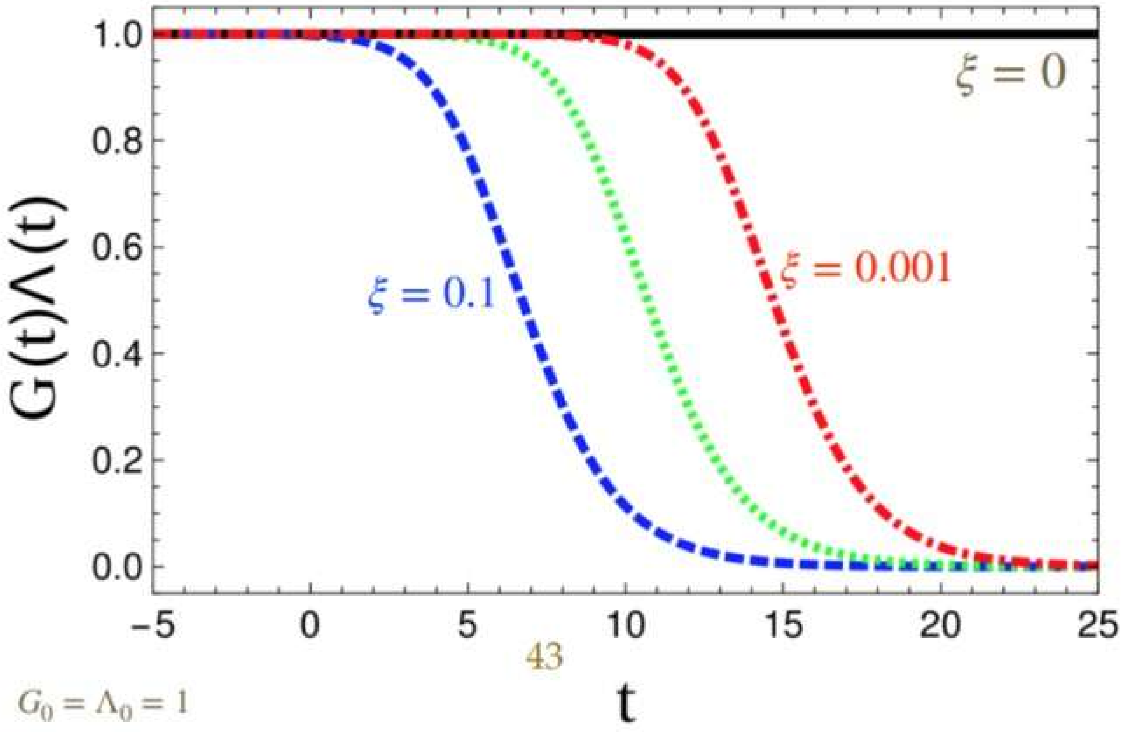
$$G(t_i) \cdot \Lambda(t_i) \approx 1$$



Deflation During Inflation

What does this means for the CCP?

$$G(t) \cdot \Lambda(t) = \frac{G_0}{1+\xi a(t)} \cdot \Lambda_0 \left(\frac{1+2\xi a(t)}{1+\xi a(t)} \right)$$



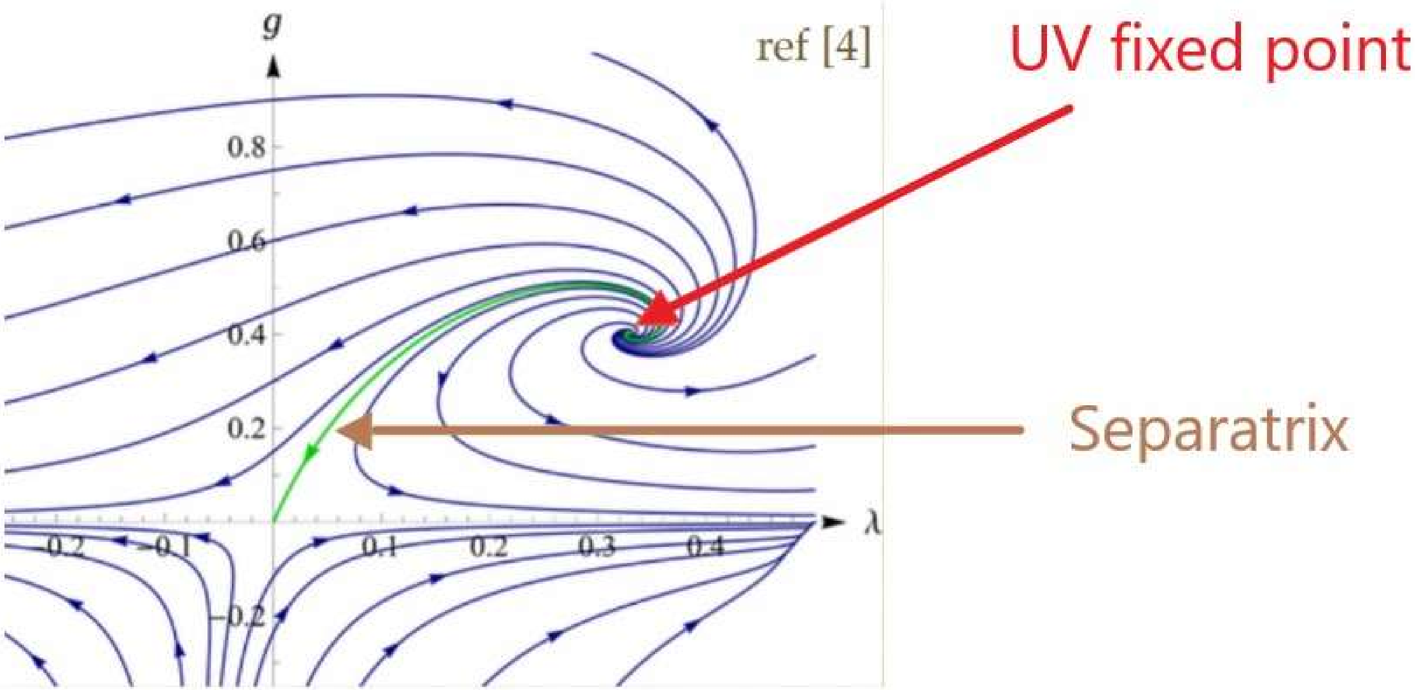
Nice, link to AS?

Link to AS?

Remember:

$$G_k = \frac{\hat{g}_k}{k^2}$$

$$\Lambda_k = \hat{\lambda}_k k^2$$



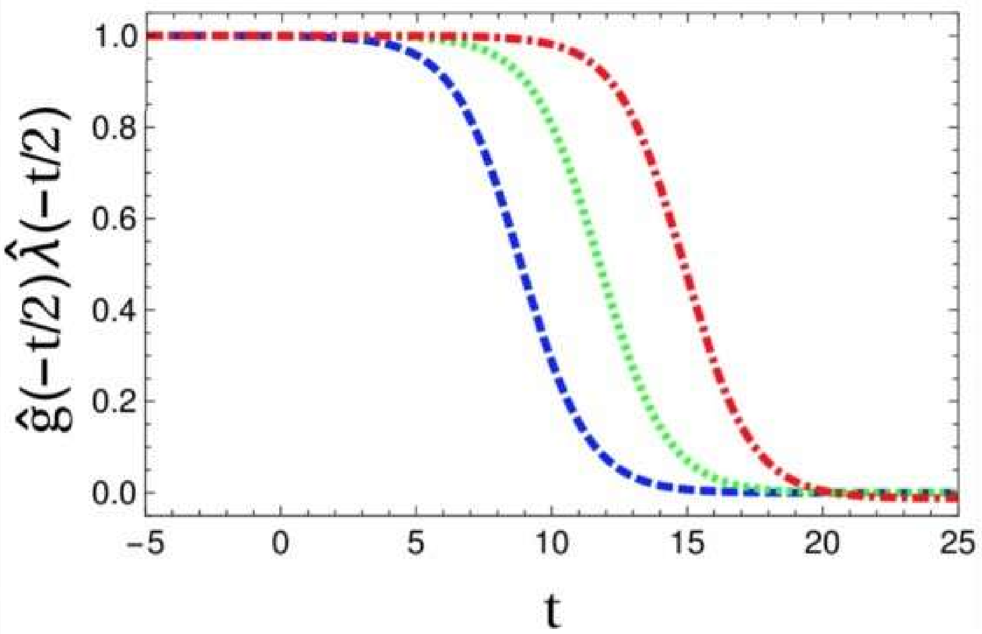
$$\hat{g}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0(e^{2\hat{t}})/g^*} \quad \hat{\lambda}(\hat{t}) = \frac{g^* \lambda_0 + e^{-2\hat{t}} \left(e^{4\hat{t}} - 1\right) g_0 \lambda^*}{1 + g_0 \left(e^{2\hat{t}} - 1\right) / g^*}$$

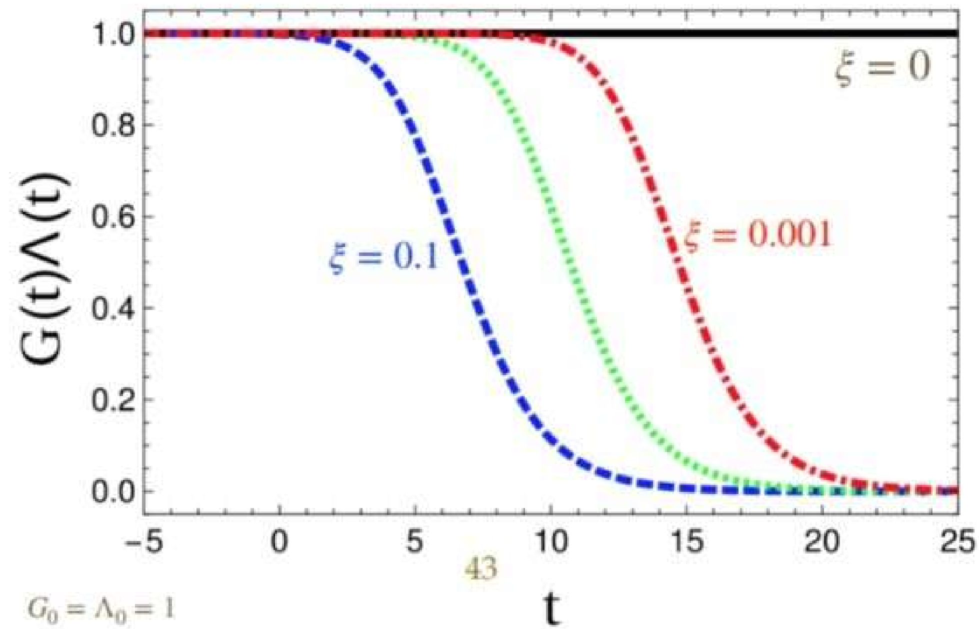
Link to AS?

AS renormalization flow | VS | SD & NEC

$$G_k \cdot \Lambda_k = \frac{\hat{g}_k}{k^2} k^2 \hat{\lambda}_k = \hat{g}_k \cdot \hat{\lambda}_k$$

$$G(t) \cdot \Lambda(t)$$

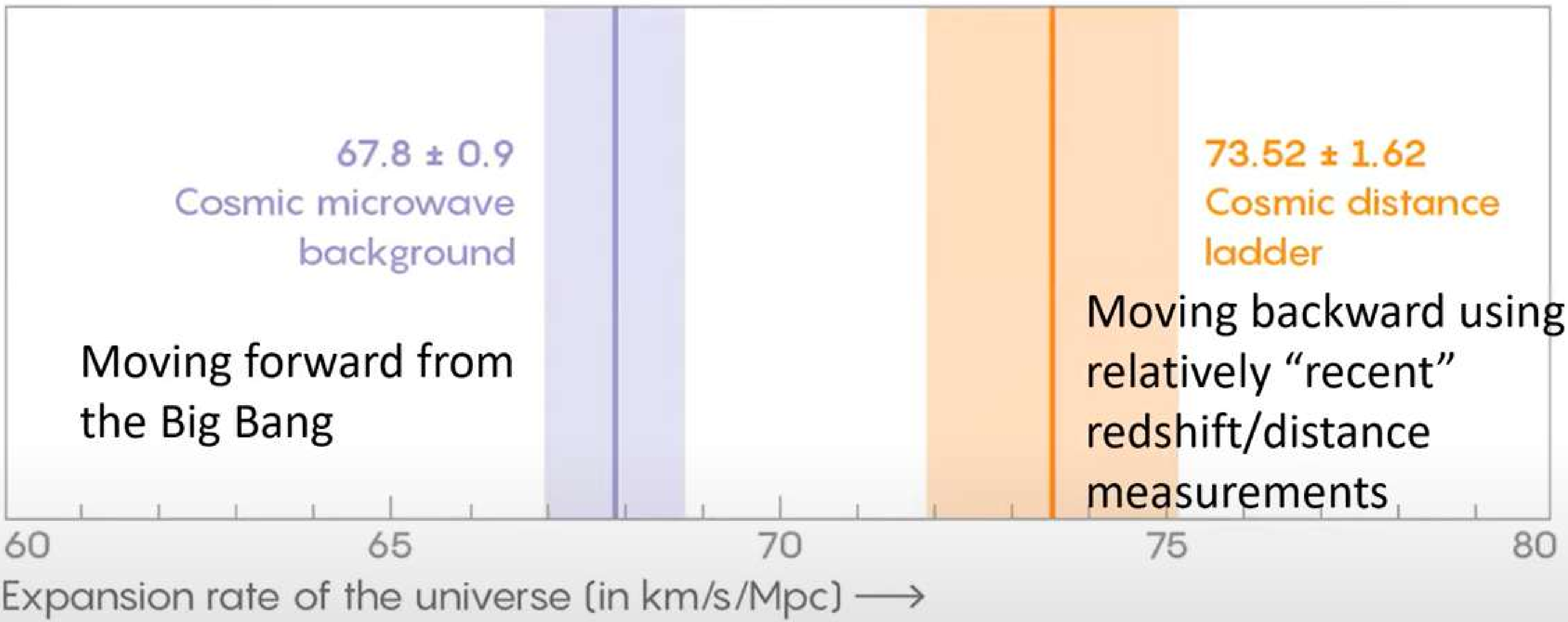




Looks familiar!

The Hubble constant problem

A Discrepancy in the Hubble Constant **Using two different Methods**



Moving forward from the Big Bang

Moving backward using relatively "recent" redshift/distance measurements

Can SD cosmology alleviate the H0 tension?

Time $\Longleftrightarrow$ Redshift

Tension:

$$\Delta(z) \equiv \frac{H(z)}{H^{(\Lambda\mathrm{CDM})}(z)} - 1\,,$$

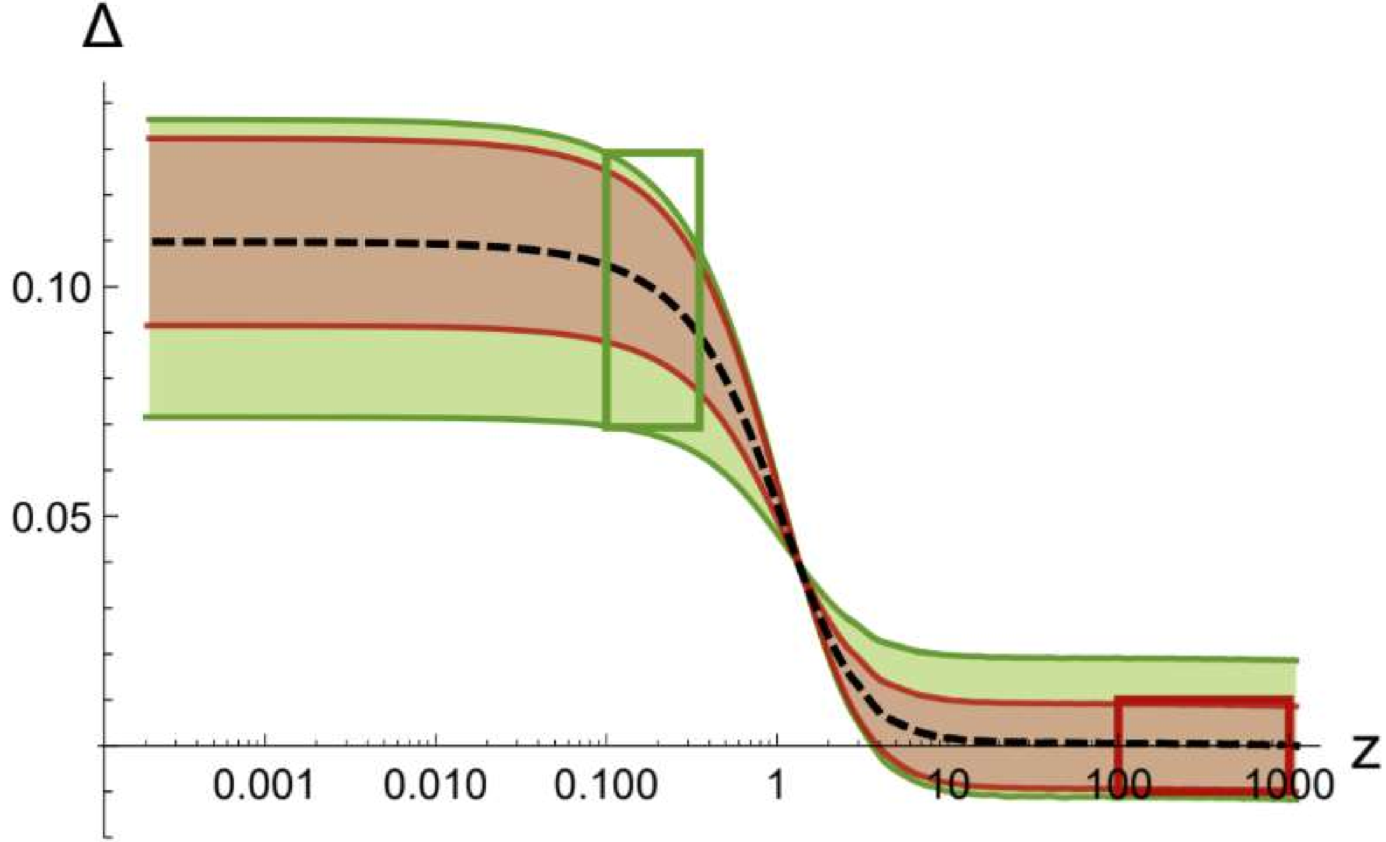
Conflict between CMB and SN Ia

$$\Delta = \frac{H_0^{(\mathrm{late})}}{H_0^{(\mathrm{early})}} - 1 \approx 0.09 \pm 0.02\,.$$

Can SD cosmology alleviate the H0 tension?

Conflict between CMB and SN Ia

$$\Delta = \frac{H_0^{(\mathrm{late})}}{H_0^{(\mathrm{early})}} - 1 \approx 0.09 \pm 0.02\,.$$



Concluding comments

- CCP: watch out SD during inflation
- Showed: with SD CCP is alleviated
- Beautiful matching between AS & SD
- SD scenario of gravity could offer an alleviation to the conflict between early and late-time measurements

Backup

$$\hat{g}(\hat{t}) \cdot \hat{\lambda}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0(e^{2\hat{t}})/g^*} \cdot \frac{g^* \lambda_0 + e^{-2\hat{t}} \left(e^{4\hat{t}} - 1\right) g_0 \lambda^*}{1 + g_0 \left(e^{2\hat{t}} - 1\right) / g^*}$$

Approximate to UV FP and separatrix

$$\hat{g}(\hat{t}) \cdot \hat{\lambda}(\hat{t}) = g^* \lambda^* \left(\frac{g^* \lambda_0}{g_0 \lambda^*} + e^{2\hat{t}}\right) \left(e^{2\hat{t}} + \frac{g^*}{g_0}\right)^{-2} \equiv G(t) \cdot \Lambda(t)$$

For

$$g^* \lambda^* \to G_0 \Lambda_0$$
$$g_0 \to \frac{G_0}{a_i \xi}$$
$$\hat{t} \to -\frac{t}{2\tau}$$

Backup

Modified Friedmann equations

$$\frac{1}{H_0^2}\left(H^2 - H\frac{\dot{g}}{g}\right) = \Omega_\Lambda \lambda(t) + \frac{\Omega_r}{a^4}g + \frac{\Omega_m}{a^3}g$$

$$\frac{1}{H_0^2}\left(2\dot{H} + 3H^2 - 2H\frac{\dot{g}}{g} + 2\frac{\dot{g}^2}{g^2} - \frac{\ddot{g}}{g}\right) = 3\Omega_\Lambda \lambda(t) - \frac{\Omega_r}{a^4}g$$

$$\frac{\ddot{g}}{g} - H\frac{\dot{g}}{g} - 2\frac{\dot{g}^2}{g^2} = 0\,.$$

Initial conditions

$$a(t_0) = 1\,, \quad \dot{a}(t_0) = H_0\,,$$
$$g(t_0) = 1\,, \quad \dot{g}(t_0) \sim 0.1\,,$$

Numerical Survey

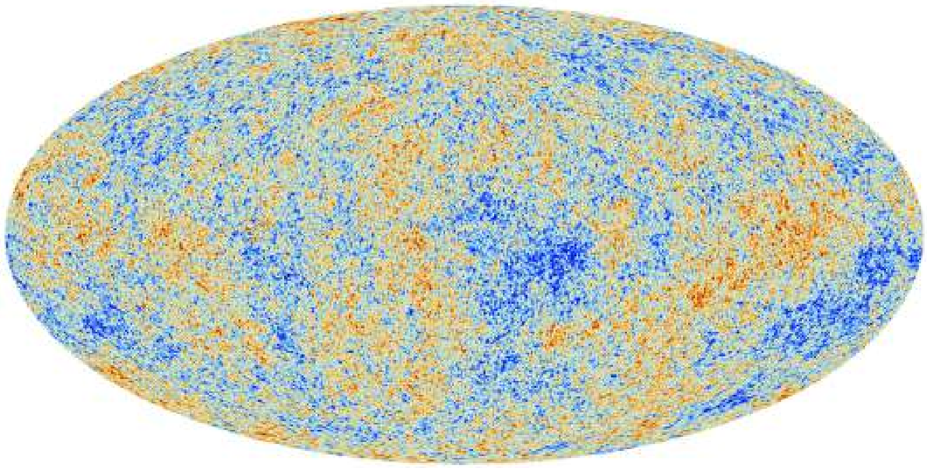
Numerical constraints

- Expansion rate of the Universe at present as implied by late time observations

Numerical Survey

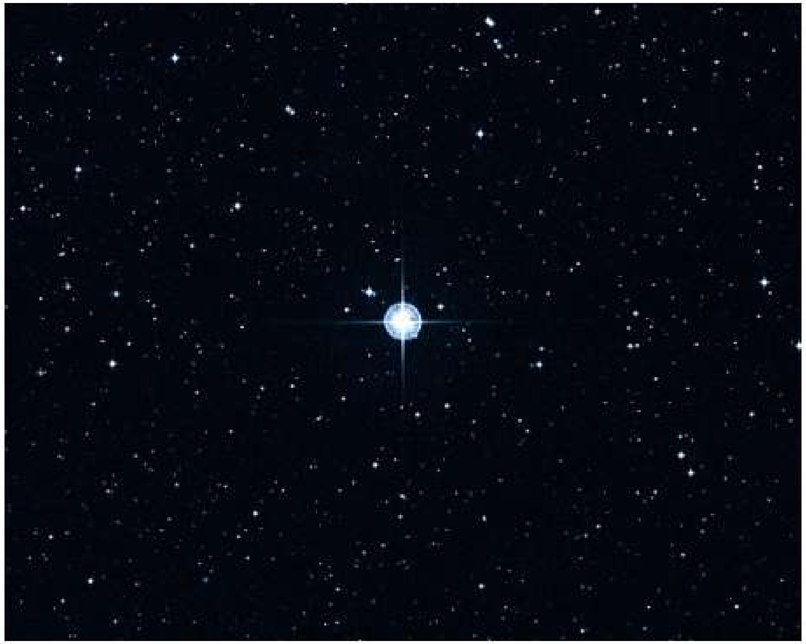
Numerical constraints

- Expansion rate at $z \sim 10^3$ as it is inferred from CMB anisotropies



Numerical Survey

Numerical constraints



- Age of the Universe coming from stellar evolution

Bond et all, ApJ letters '13

Numerical Survey

- Dimensionless function

$$x = \frac{a(t)}{a_0}\,,$$

- System of 2 FE + NEC

$$\frac{x'(\tau)^2}{x(\tau)^2} - \frac{g'(\tau)x'(\tau)}{g(\tau)x(\tau)} = \Omega_m g(\tau)x(\tau)^{-3} + \Omega_r g(\tau)x(\tau)^{-4} + \Omega_\Lambda \lambda(\tau)\,,$$

$$\frac{x'(\tau)^2}{x(\tau)^2} - 2\frac{g'(\tau)x'(\tau)}{g(\tau)x(\tau)} + 2\frac{g'(\tau)^2}{g(\tau)^2} + 2\frac{x''(\tau)}{x(\tau)} - \frac{g''(\tau)}{g(\tau)} = -\Omega_r g(\tau)x(\tau)^{-4} + 3\Omega_\Lambda \lambda(\tau)\,,$$

$$\frac{g''(\tau)}{g(\tau)} - \frac{g'(\tau)x'(\tau)}{g(\tau)x(\tau)} - 2\frac{g'(\tau)^2}{g(\tau)^2} = 0\,.$$

Numerical Survey

- Expansion rate of the Universe at present as implied by late time observations

$$\Sigma_{(i)} = \frac{\text{abs}\left[\left. \frac{\dot{x}/x}{\dot{x}^{(\Lambda\text{CDM})}/x^{(\Lambda\text{CDM})}} \right|_{z\sim 0} - \frac{H_0^{(\text{late})}}{H_0^{(\text{early})}} \right]}{\frac{H_0^{(\text{late})}}{H_0^{(\text{early})}} \left(\left(\frac{\Delta H_0^{(\text{late})}}{H_0^{(\text{late})}} \right)^2 + \left(\frac{\Delta H_0^{(\text{early})}}{H_0^{(\text{early})}} \right)^2 \right)^{1/2}}$$

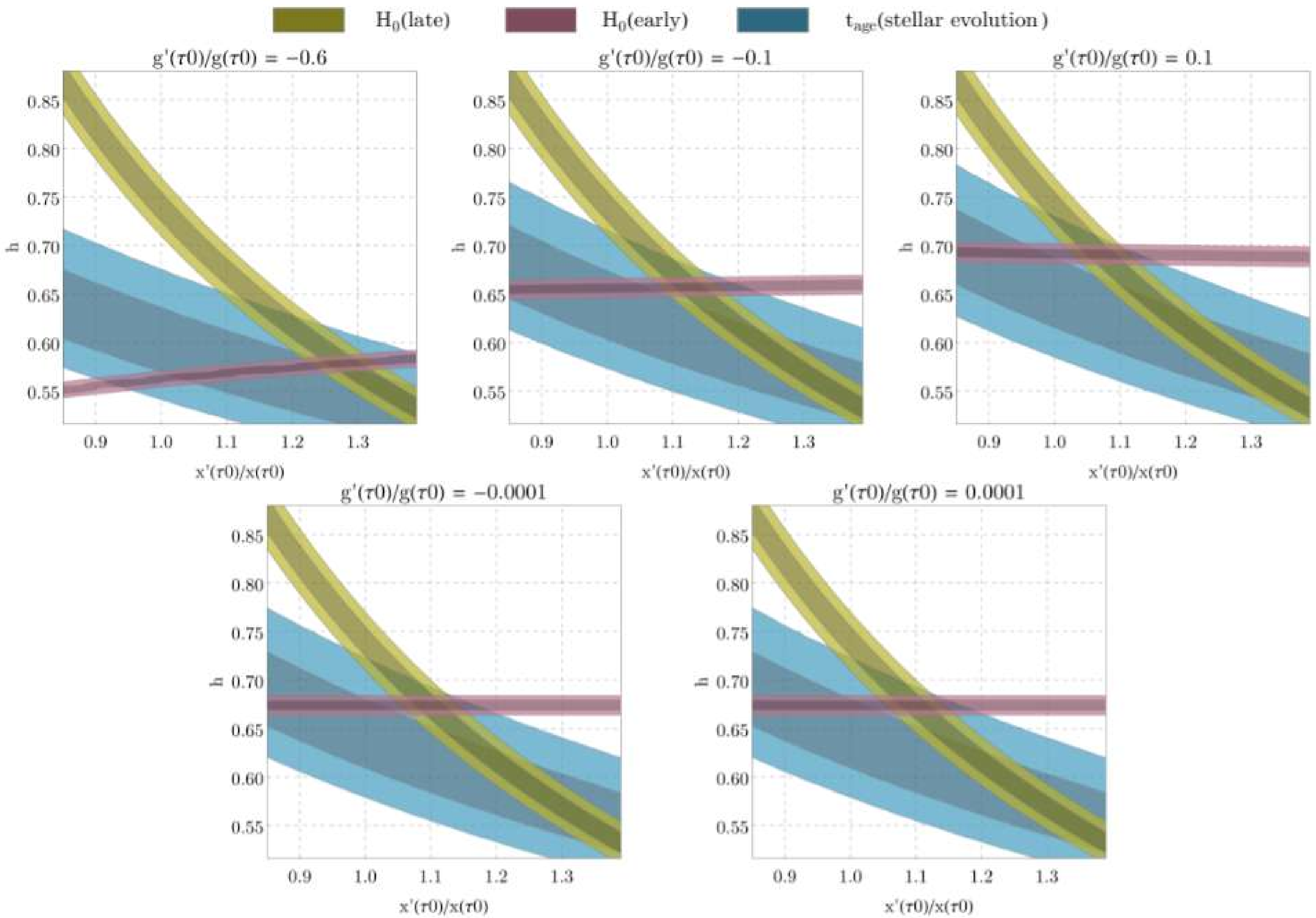
- Expansion rate at $z \sim 10^3$ as it is inferred from CMB anisotropies

$$\Sigma_{(ii)} = \frac{\text{abs}\left[\left. \frac{\dot{x}/x}{\dot{x}^{(\Lambda\text{CDM})}/x^{(\Lambda\text{CDM})}} \right|_{z\sim 10^3} - 1 \right]}{\Delta H_0^{(\text{early})}/H_0^{(\text{early})}}$$

- Age of the Universe coming from stellar evolution

$$\Sigma_{(iii)} = \frac{\text{abs}\left[t_{\text{age}} - t^*_{\text{age}} \right]}{\Delta t^*_{\text{age}}} .$$

Numerical Survey



Numerical Survey