Novel critical phenomena in compressible polar active fluids

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Overview

1. Compressible polar active matter has a rich phase diagram
   - Interplay between **ordered** and **disordered motion** with **phase-separation**
2. There is a **multicritical point** where all phases merge
3. We discover **3 new nonequilibrium universality** classes using the FRG

Hydrodynamic Theory

Under assumptions of:

- Mass conservation
- Translation sym.
- Rotation sym.
- Chiral sym.

Continuity equation for **density**

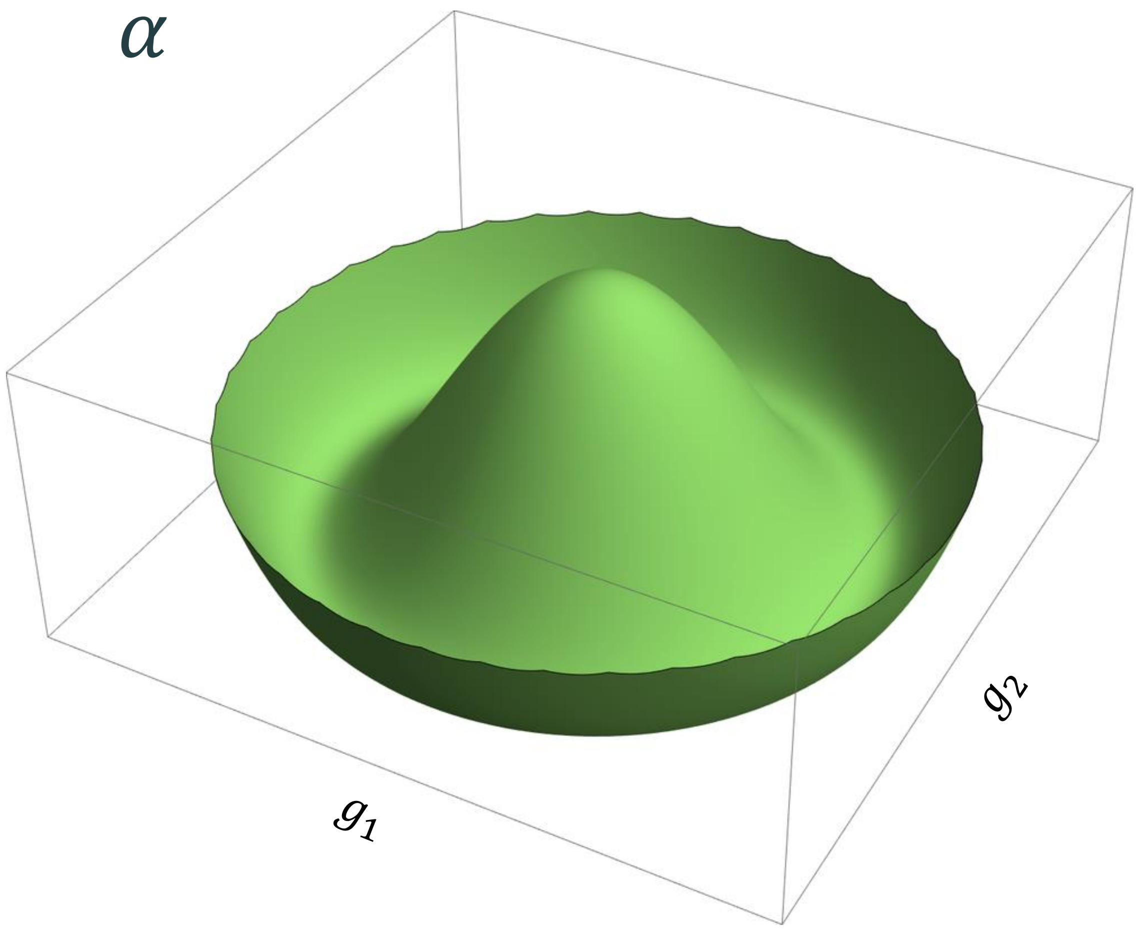
$$\partial_t \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{g}$$

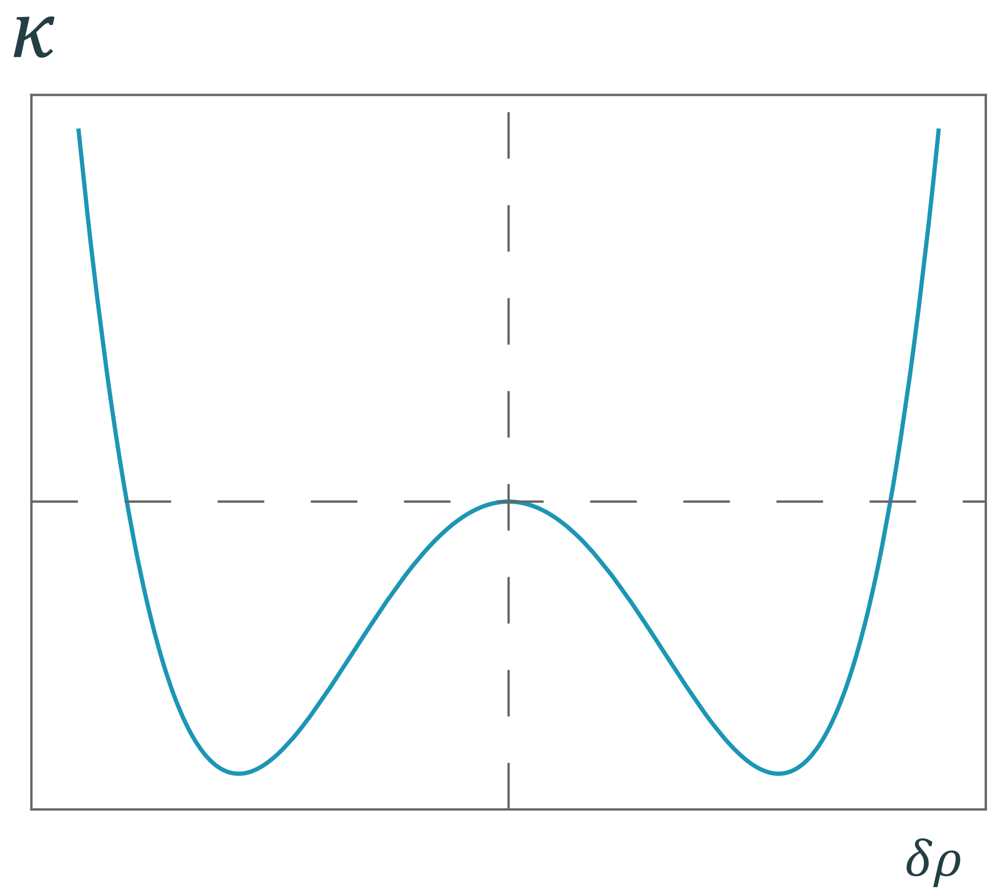
Toner-Tu equation for **momentum** density

$$\gamma \partial_t \boldsymbol{g} + \lambda_1 \boldsymbol{g} \cdot \boldsymbol{\nabla} \boldsymbol{g} + \lambda_2 \boldsymbol{g} \cdot (\boldsymbol{\nabla} \boldsymbol{g})^T + \lambda_3 \boldsymbol{g} \boldsymbol{\nabla} \cdot \boldsymbol{g}$$
$$= -\alpha(\rho, g^2)\boldsymbol{g} - \kappa(\rho, g^2)\boldsymbol{\nabla}\rho + (\zeta - \mu)\nabla^2 \boldsymbol{g} + \mu \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{g}) + \eta \nabla^2 \boldsymbol{\nabla} \rho$$
$$+\kappa_2 \boldsymbol{g}(\boldsymbol{g} \cdot \boldsymbol{\nabla} \rho) + \boldsymbol{f}$$

Toner and Tu, Physical Review Letters (1995)

Spontaneous Symmetry Breaking





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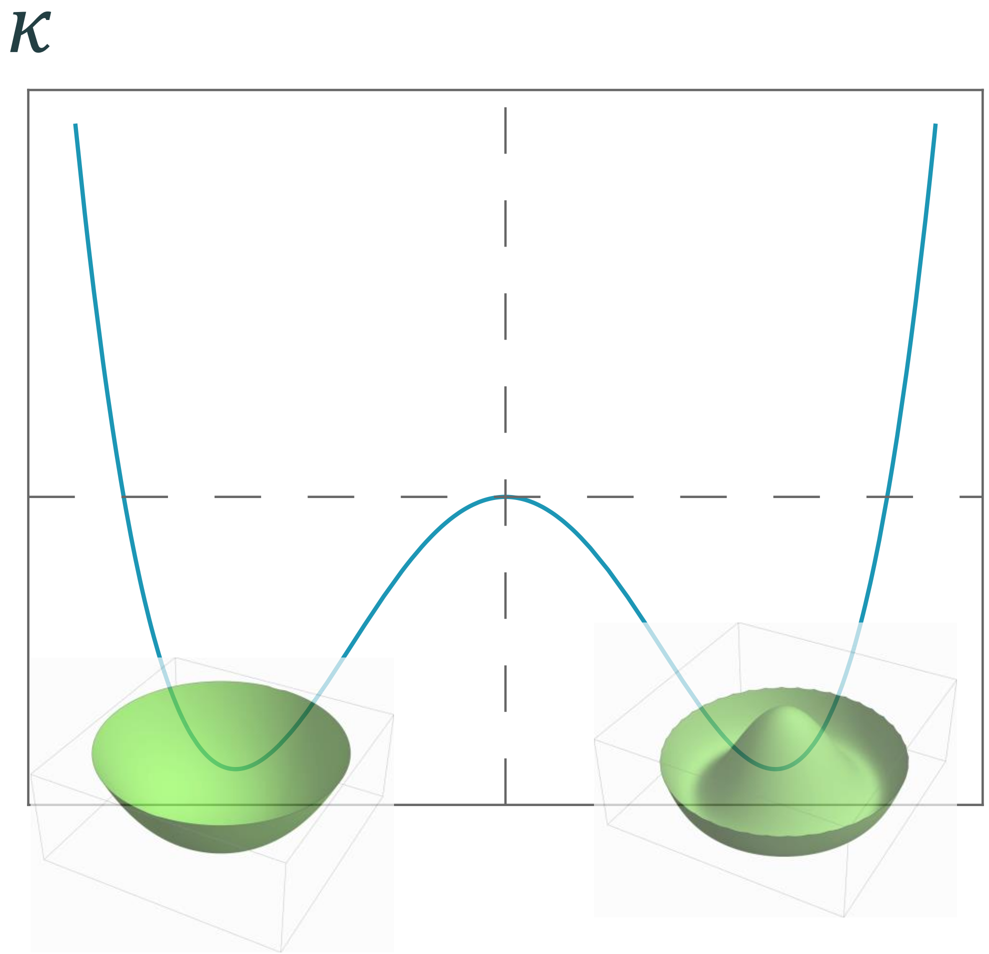
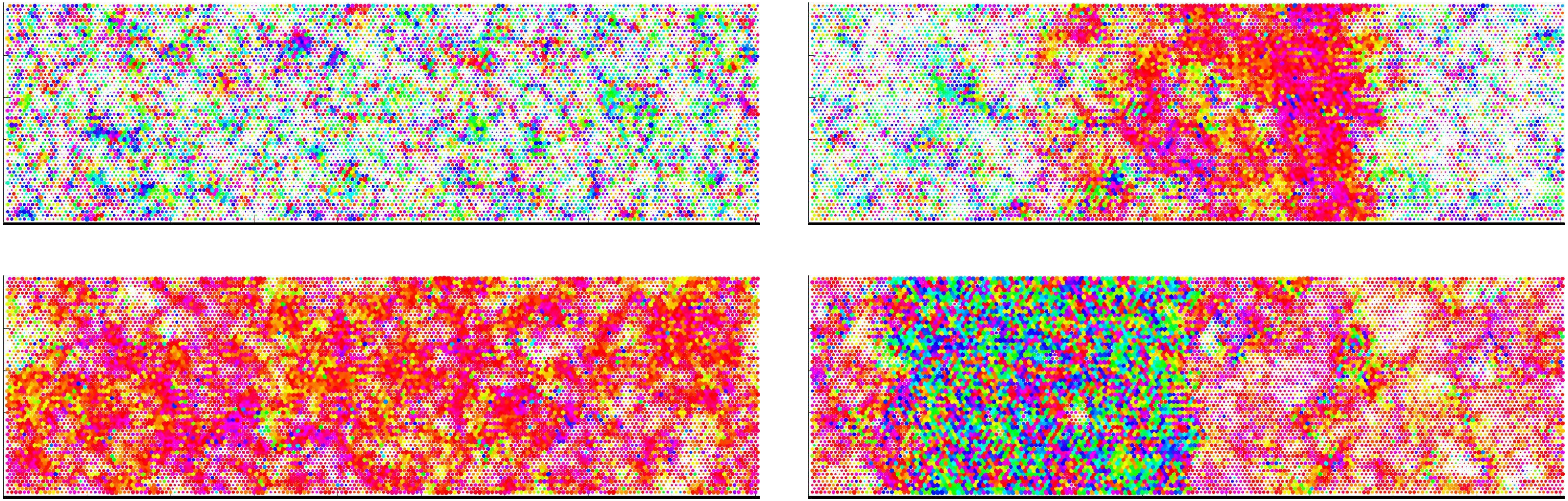
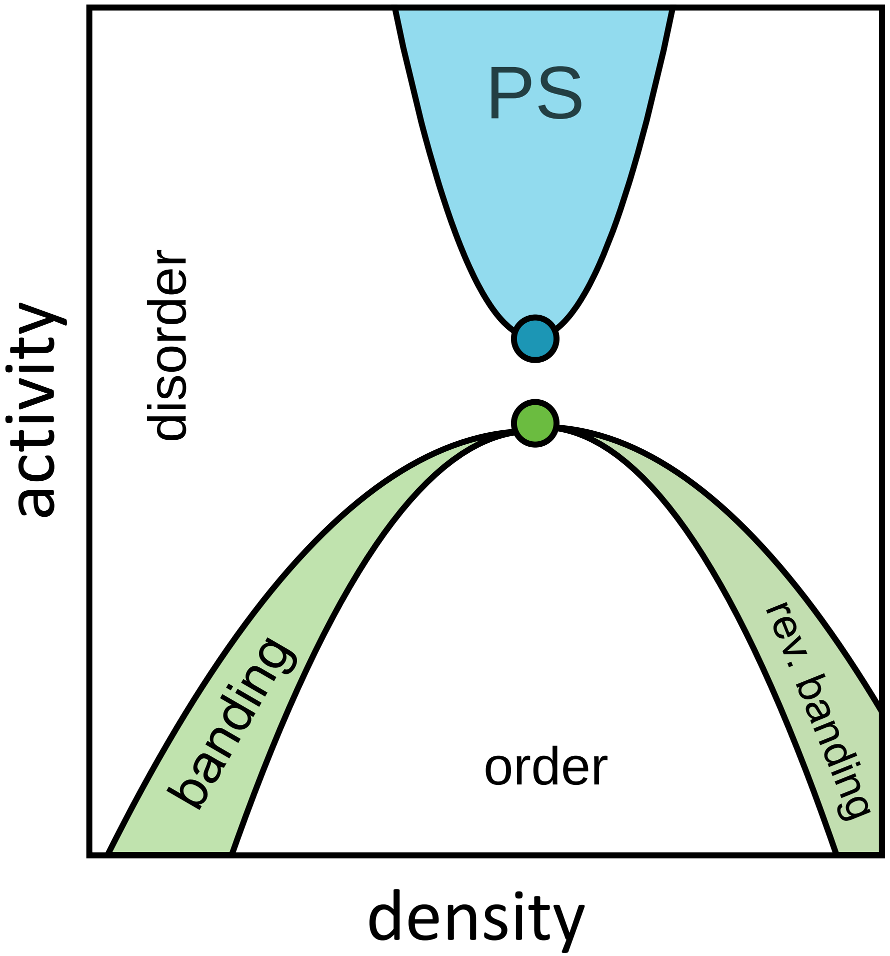


Simulations



Nesbitt, Pruessner, Lee, New Journal of Physics (2021)

Phase Diagram



- Critical order-disorder

  Nesbitt, Pruessner, Lee, New Journal of Physics **23**, 43047 (2021)

- Critical phase separation

  Partridge, Lee, PRL (2019)

  Maggi *et al.*, Soft Matter (2021)

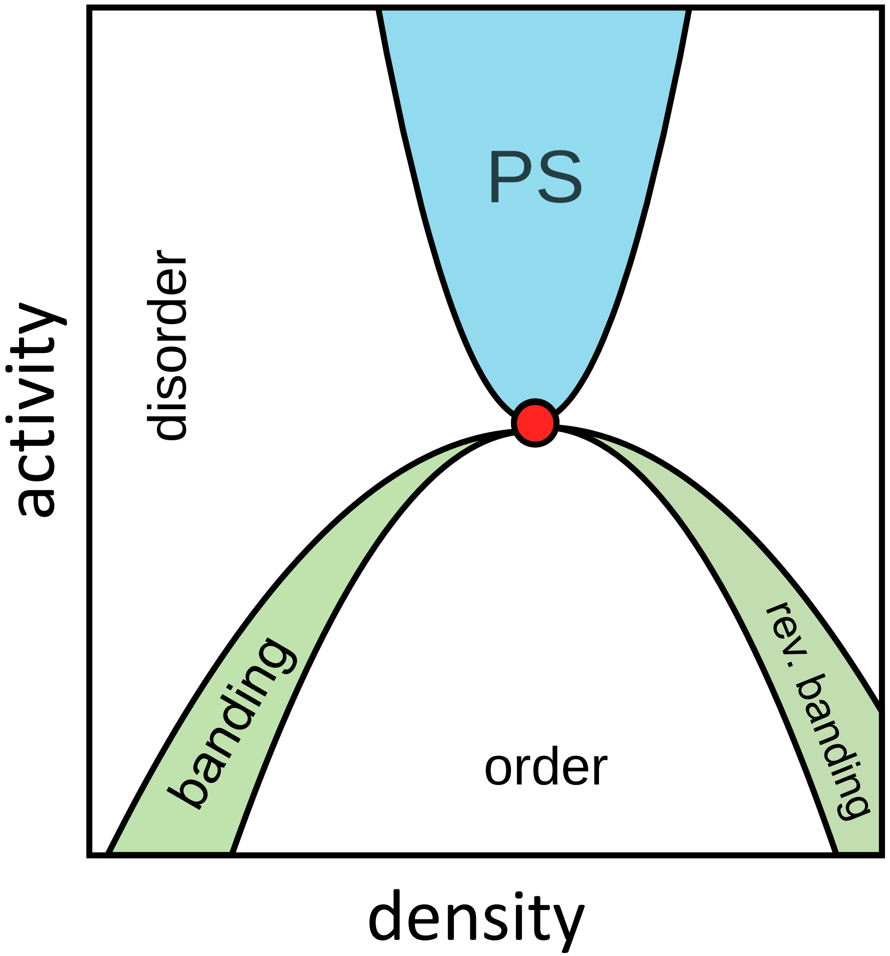
  Siebert *et al.*, Phys. Rev. E (2018)

  Caballero, Nardini, Cates, J. Stat. Mech. Theory Exp., (2018)

- Multicritical Point

  Bertrand, Lee, arXiv:2012.05866 (2020)

Phase Diagram



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Multicritical Point

Bertrand, Lee, arXiv:2012.05866 (2020)

EOM near the multicritical point in $d = 6 - \epsilon$

$$\partial_t \boldsymbol{g} = -\alpha(\rho)\boldsymbol{g} + (\mu_\parallel - \mu_\perp)\nabla^2 \boldsymbol{g} + \mu_\perp \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{g}) - \kappa(\rho)\boldsymbol{\nabla}\rho + \zeta \nabla^2 \boldsymbol{\nabla}\rho + \boldsymbol{f}$$

Momentum density

"pressure"

Noise

$$\partial_t \rho = -\boldsymbol{\nabla} \cdot \boldsymbol{g}$$

Continuity eq.

$$< \boldsymbol{f}(\boldsymbol{r}, t)\boldsymbol{f}(\boldsymbol{r}', t') > = 2D\, \delta^d(\boldsymbol{r} - \boldsymbol{r}')\delta(t - t')$$

Noise

Critical Exponents

Anomalous dimensions η_g and η_ρ

Dynamic exponent z

$$\langle \boldsymbol{g}(\boldsymbol{q},\omega)\boldsymbol{g}(-\boldsymbol{q},-\omega)\rangle = q^{-2+\eta_g} S_g\left(\frac{\omega}{q^z}\right)$$

$$\langle \rho(\boldsymbol{q},\omega)\rho(-\boldsymbol{q},-\omega)\rangle = q^{-4+\eta_\rho} S_g\left(\frac{\omega}{q^z}\right)$$

Nonequilibrium exponent

$$\frac{\langle \boldsymbol{g}(\boldsymbol{q},\omega)\boldsymbol{g}(-\boldsymbol{q},-\omega)\rangle}{\frac{1}{\omega}\operatorname{Im}\chi(\boldsymbol{q},\omega)} = q^{\eta_{NE}}\ S_{NE}\left(\frac{\omega}{q^z}\right)$$

Functional Renormalization Group

Exact Wetterich equation

$$\partial_k \Gamma_k = \mathrm{Tr}\left\{\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right\}$$

Wetterich, Phys. Lett. B (1993)
Morris, Int. J. Mod. Phys. A (1994)
Ellwanger, Zeitschr. f. Phys. C (1994)

Ansatz for effective Action

$\partial_t \boldsymbol{g} = -\alpha(\rho)\boldsymbol{g} + (\mu_\parallel - \mu_\perp)\nabla^2 \boldsymbol{g} + \mu_\perp \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{g})$ (Momentum density) $-\kappa(\rho)\boldsymbol{\nabla}\rho + \zeta\nabla^2\boldsymbol{\nabla}\rho$ ("pressure") $+\boldsymbol{f}$ (Noise)

$\partial_t \rho = -\boldsymbol{\nabla}\cdot\boldsymbol{g}$ (Continuity eq.)

$< \boldsymbol{f}(\boldsymbol{r},t)\boldsymbol{f}(\boldsymbol{r}',t') > = 2D\,\delta^d(\boldsymbol{r}-\boldsymbol{r}')\delta(t-t')$ (Noise)

Martin-Siggia-Rose-de Dominicis-Janssen formalism

Canet, Chaté, Delamotte, *J. Phys. A: Math. Theor. (2011)*

$$\Gamma_k\,[\bar{\boldsymbol{g}}, \boldsymbol{g}, \bar{\rho}, \rho] = \int_{\boldsymbol{r},t} \{ \underbrace{-2D_k \bar{\boldsymbol{g}}^2}_{\text{Noise}} \underbrace{+\,\bar{\rho}(\partial_t \rho + \boldsymbol{\nabla}\cdot\boldsymbol{g})}_{\text{Continuity eq.}} \underbrace{+\,\bar{\boldsymbol{g}}(\kappa_k(\rho)\boldsymbol{\nabla}\rho - \zeta_k \nabla^2 \boldsymbol{\nabla}\rho)}_{\text{"pressure"}}$$

$$\underbrace{+\,\bar{\boldsymbol{g}}(\gamma_k \partial_t \boldsymbol{g} + \alpha_k(\rho)\boldsymbol{g} - (\mu_{\parallel,k} - \mu_{\perp,k})\nabla^2 \boldsymbol{g} - \mu_{\perp,k}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{g}))\}}_{\text{Momentum density}}$$

Ansatz for effective Action

Martin-Siggia-Rose-de Dominicis-Janssen formalism   Canet, Chaté, Delamotte, *J. Phys. A: Math. Theor. (2011)*

$$\Gamma_k\,[\bar{\boldsymbol{g}},\boldsymbol{g},\bar{\rho},\rho] = \int_{\boldsymbol{r},t}\{-2D_k\bar{\boldsymbol{g}}^2 \quad + \bar{\rho}(\partial_t\rho + \boldsymbol{\nabla}\cdot\boldsymbol{g}) \quad +\bar{\boldsymbol{g}}(\kappa_k(\rho)\boldsymbol{\nabla}\rho - \zeta_k\nabla^2\boldsymbol{\nabla}\rho)$$

$$+\bar{\boldsymbol{g}}(\gamma_k\partial_t\boldsymbol{g} + \alpha_k(\rho)\boldsymbol{g} - (\mu_{\parallel,k} - \mu_{\perp,k})\nabla^2\boldsymbol{g} - \mu_{\perp,k}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{g}))\}$$

Momentum dependent terms

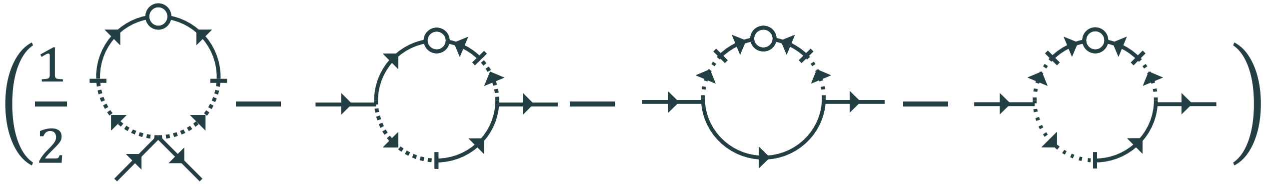
Projections at non-zero density fluctuations

$$\partial_k \mu_\parallel = \frac{1}{2VT}\frac{1}{d-1}\frac{\mathrm{d}^2}{\mathrm{d}q^2}\mathrm{Tr}\frac{\boldsymbol{q}\otimes\boldsymbol{q}}{q^2}\frac{\delta^2\partial_k\Gamma_k}{\delta\,\overline{\boldsymbol{g}}(\boldsymbol{q},0)\delta\boldsymbol{g}(-\boldsymbol{q},0)}\bigg|_{\rho=\rho_{\min}}$$

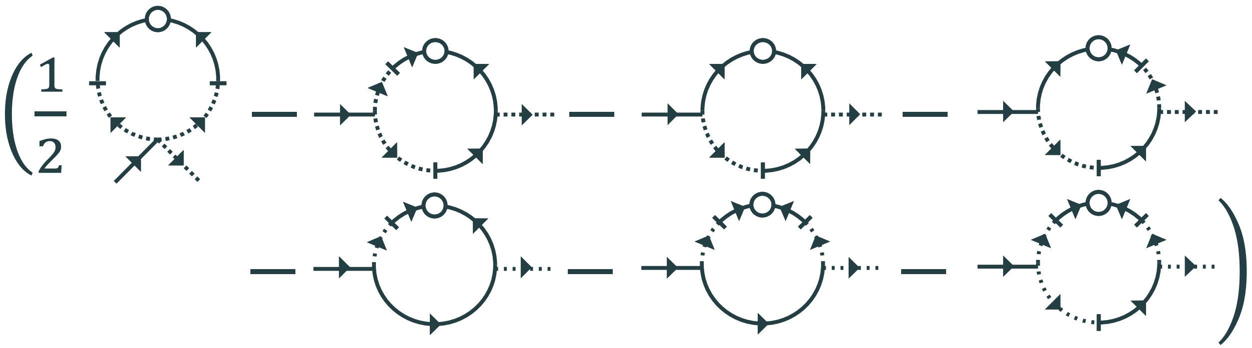
Nonperturbative in $\mu_\perp/\mu_\parallel$ and $\zeta\gamma/{\mu_\parallel}^2$

Flow equations

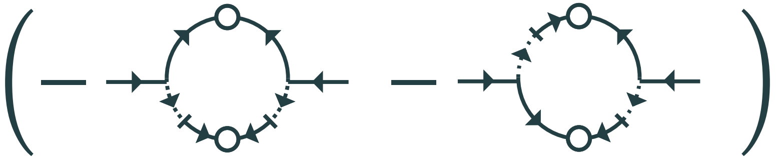
$$\partial_k \Gamma_k^{(1,1,0,0)} = \tilde{\partial}_k$$



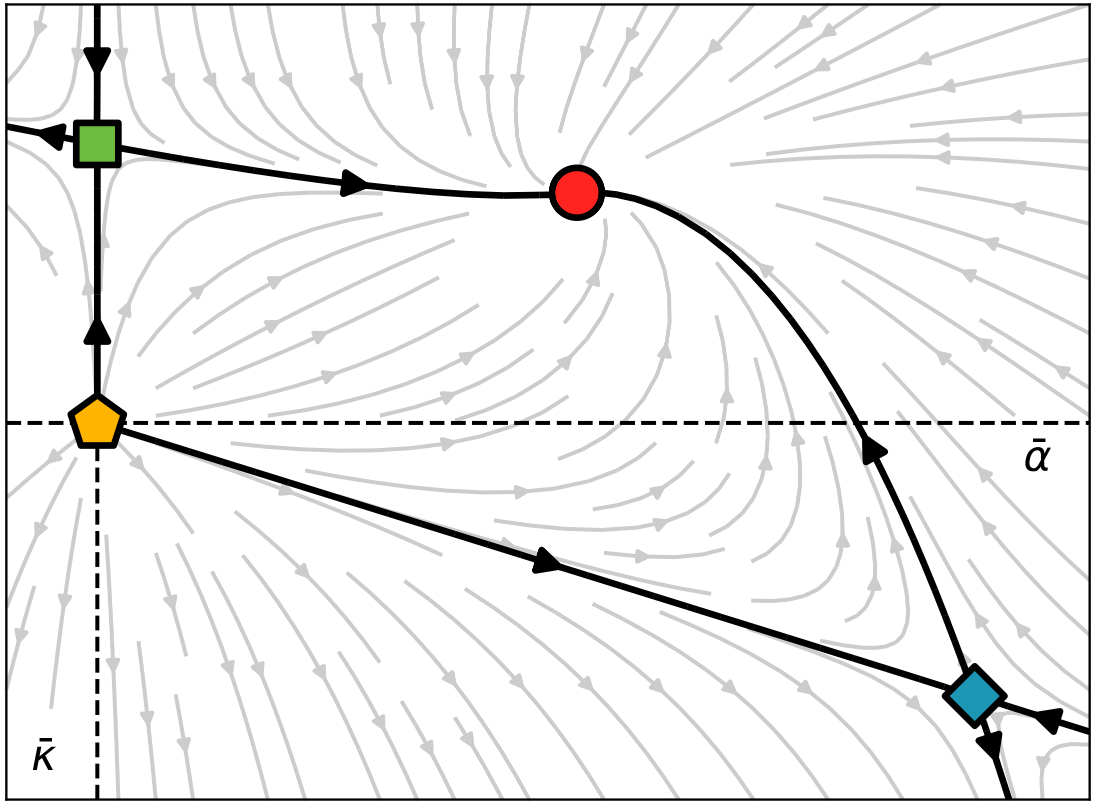
$$\partial_k \Gamma_k^{(1,0,0,1)} = \tilde{\partial}_k$$



$$\partial_k \Gamma_k^{(1,1,0,0)} = \tilde{\partial}_k$$



Flow diagram (2D projection)



- Gaussian
- Unstable FP
- Multicritical Point
- Unstable FP

Critical Exponents and Universal Ratios

	η_ρ	η_g	η_{NE}	$z-2$	$\mu_\perp/\mu_\parallel$	$\zeta\gamma/\mu_\parallel{}^2$
⬟	0	0	0	0		
■	0	0	0	0		1.43
●	$0.066\epsilon^2$	$0.043\epsilon^2$	$0.076\epsilon^2$	$0.011\epsilon^2$	1.45	1.37
◆	$0.223\epsilon^2$	$-0.026\epsilon^2$	$0.039\epsilon^2$	$0.084\epsilon^2$	0.31	0

Conclusion

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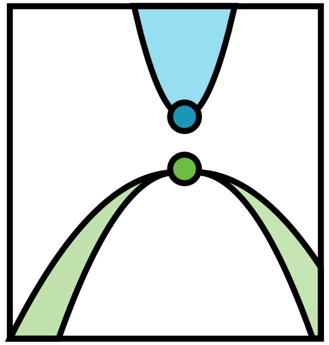
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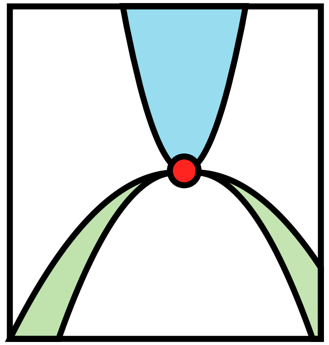
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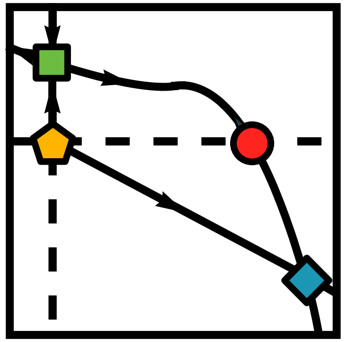

Thank you!

Preprint available:
arXiv:2205.01610

Universality in Biology Group

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