Functional renormalisation group for cosmic large-scale structure formation

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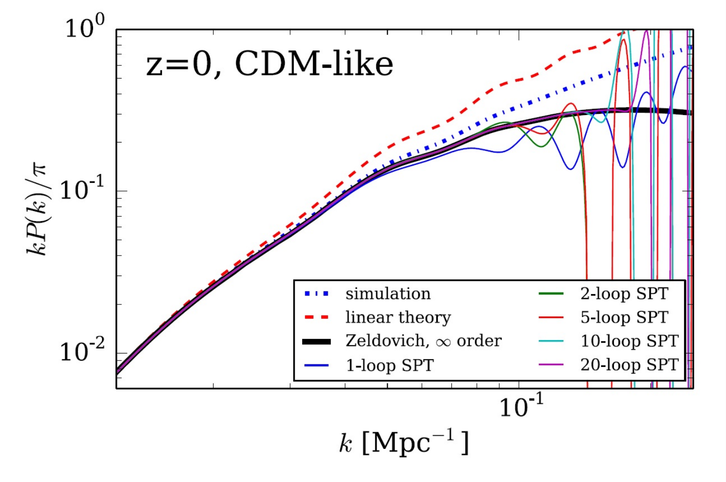
Late-time large-scale cosmic structure formation

- Ensemble of self-gravitating non-relativistic collisionless point particles evolving on an expanding space-time
- Stochastic nature of (Gaussian) initial conditions allow for statistical field theory formulation
- Study statistical properties of dark matter by investigating correlation functions

Standard perturbation theory



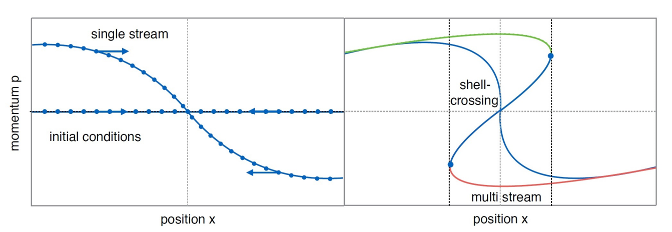
[McQuinn & White '16]

Shell-crossing

- Single-stream approximation

$$f(\tau, \boldsymbol{x}, \boldsymbol{p}) \propto \rho(\tau, \boldsymbol{x})\, \delta[\boldsymbol{p} - a(\tau)\, m\, \boldsymbol{u}(\tau, \boldsymbol{x})]$$

- Shell-crossing [Uhlemann '18]



- Including velocity dispersion [AE & Floerchinger '19]

$$f(\tau, \boldsymbol{x}, \boldsymbol{p}) \propto \rho(\tau, \boldsymbol{x}) \exp\left\{ -\frac{1}{2} \left(\frac{p_i}{am} - u_i \right) (\sigma^{-1})_{ij} \left(\frac{p_j}{am} - u_j \right) \right\}$$

Cosmological functional renormalisation group

- Bare action

$$S[\psi,\hat\psi] = -\mathrm{i}\int \hat\psi_a\, E_a(\psi) + \mathrm{i}\int \hat\psi_a\, \varPsi_a^{\mathrm{in}} + \frac{1}{2}\int \hat\psi_a\, P_{ab}^{\mathrm{in}}\, \hat\psi_b$$

- Regulate spectrum of initial fluctuations

$$P_{ab}^{\mathrm{in}}(\boldsymbol{q}) \mapsto P_{k,ab}^{\mathrm{in}}(\boldsymbol{q}) = \theta(k-q)\, P_{ab}^{\mathrm{in}}(\boldsymbol{q})$$

[Matarrese & Pietroni ’07; Floerchinger, Garny, Tetradis & Wiedemann ’17]

- Functional renormalisation group

$$\partial_k \varGamma_k = \tfrac{1}{2}\,\mathrm{Tr}\Big[\Big[\varGamma_k^{(2)} + (P_k^{\mathrm{in}} - P^{\mathrm{in}})\Big]^{-1}\cdot \partial_k P_k^{\mathrm{in}}\Big]$$

Finding closure

- Infinite hierarchy of coupled flow equations

$$\partial_k \Gamma_k^{(n)} = \mathrm{Func}\left[\Gamma_k^{(m \leq n+2)}\right]$$

- Closing the flow equations
  - Symmetries and related Ward identities, e.g. extended Galilean invariance [Canet, Delamotte & Wschebor '15, '16]
  - Truncated expansion schemes, e.g. derivative/vertex expansion [Floerchinger, Garny, Tetradis & Wiedemann '17; Floerchinger, Garny, Katsis, Tetradis & Wiedemann '19]

Extended Galilean invariance

- Time-gauged translation $x_i \mapsto x_i + \epsilon_i(\tau)$

$$\delta_\epsilon S = -\,\mathrm{i} \int_{\tau,\boldsymbol{x}} \hat{u}_i(\tau,\boldsymbol{x}) \Big[\ddot{\epsilon}_i(\tau) + \mathcal{H}\dot{\epsilon}_i(\tau) \Big]$$

- Related Ward identities

$$\left[\sum_{l=1}^{m} \theta(\tau - \tau_l)\, \mathrm{i}\, q_{l,i} + \dots \right] \Gamma^{(m,n)}_{k,a_1\dots b_n}(\dots) = \Gamma^{(m+1,n)}_{k,\boldsymbol{u}_i a_1\dots b_n}(\tau,\mathbf{0};\dots)$$

- Closing the two-point function flow equation*

$$\lim_{|\boldsymbol{q}|\to\infty} \partial_k \hat{\Gamma}^{(2)}_k(\boldsymbol{q}) = \mathrm{Func}\Big[\hat{\Gamma}^{(2)}_k \Big]$$

[AE & Floerchinger ’22]

Time-local effective dynamics

- Effective action ansatz

$$\Gamma_k = -\mathrm{i} \int \hat{\Psi}_a \, E_{k,a}(\Psi) + \mathrm{i} \int \hat{\Psi}_a \, Q_{k,a} + \tfrac{1}{2} \int \hat{\Psi}_a \, H_{k,ab} \, \hat{\Psi}_b$$

- Time-local first-order effective dynamics

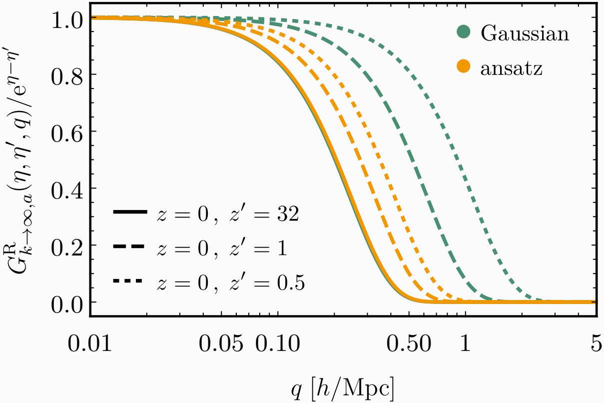
$$E_{k,a}(\Psi) = \partial_\tau \Psi_a + \Omega_{k,ab} \Psi_b + I_{k,a}(\Psi)$$

- Expansion of inverse propagator in small frequencies

$$\mathrm{i}\, \hat{\Gamma}_k^{(1,1)}(\tau, \tau', \boldsymbol{q}) = \int_C \frac{\mathrm{d}s}{2\pi\,\mathrm{i}} \, \mathrm{e}^{s(\tau - \tau')} \left[s \, \delta_{ab} + \Omega_{k,ab}(\tau, \boldsymbol{q}) \right]$$

[AE & Floerchinger (in prep.)]

Mean linear response function



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Scale-dependent fields

- Flow of continuity equation

$$\partial_\tau \rho + \underbrace{\partial_i(\rho u_i) + \partial_i \partial_i F_k(\rho, u_j, \sigma_{jl})}_{\partial_i[\rho \tilde{u}_{k,i}]} = 0$$

- Flow of Cauchy momentum equation*

$$\underbrace{(\partial_\tau + \mathcal{H} + u_j \partial_j) u_i + \frac{1}{\rho} \partial_j(\rho \sigma_{ij}) + \partial_j F_{ij,k}}_{(\partial_\tau + \mathcal{H} + \tilde{u}_j \partial_j)\tilde{u}_i + \partial_j(\rho \tilde{\sigma}_{ij})/\rho} + \partial_i \phi = 0$$

- Flow equation for scale-dependent fields

$$\tilde{\Gamma}_k[\Phi] = \Gamma_k[\Psi_k[\Phi]]$$

[Wetterich ’96; Gies & Wetterich ’02; Floerchinger & Wetterich ’09]

Conclusion and outlook

- Generic need for non-perturbative methods to address small-scale dark matter physics
- Functional renormalisation group particularly suited to make use of symmetries and Ward identities
- Further investigation of sensible truncations schemes by guidance of conservation laws