Active Matter
A treasure trove of novel universality classes

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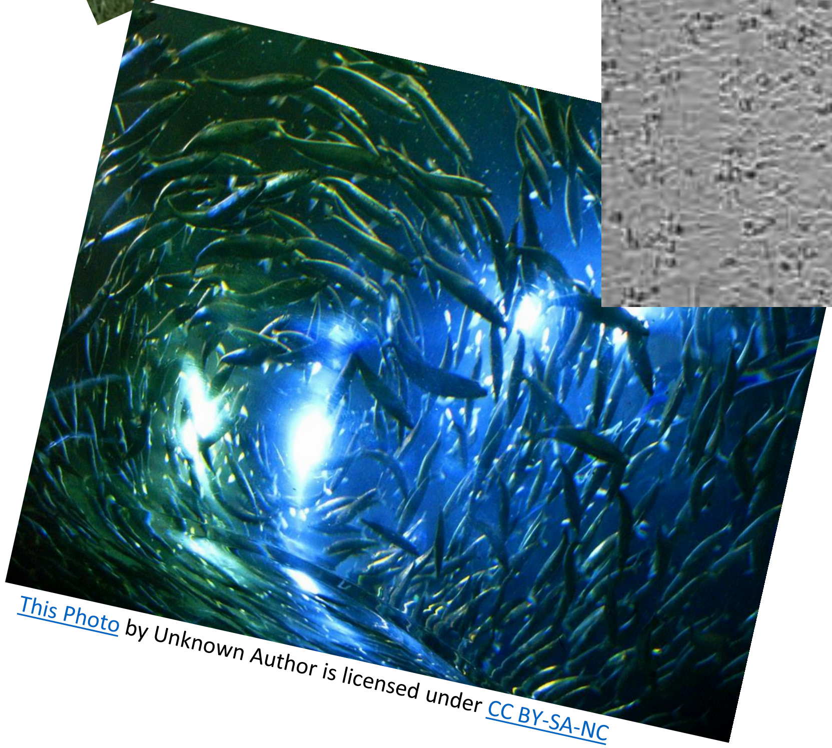

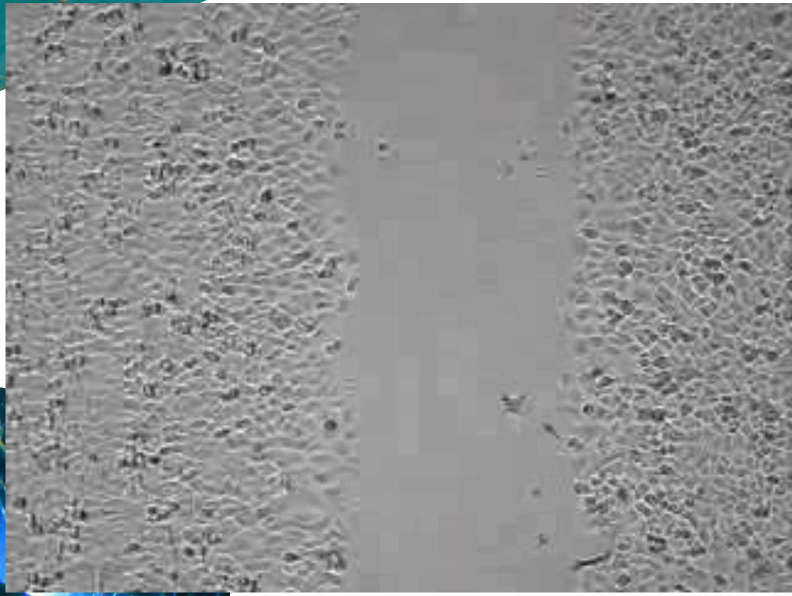

https://www.youtube.com/watch?v=v9xq_GiRXeE

Active matter = A many-body system in which the (microscopic) constituents can exert forces

→ an extreme form of non-equilibrium system !

After this talk, you will know

1. How to study the hydrodynamic behaviour of an active matter system
2. RG analyses have uncovered many novel universality classes in active matter
3. Biophysics is one of the most exciting areas of physics

Acknowledgement

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Ananyo Maitra (CY Cergy Paris)
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Imperial College London

EPSRC
Engineering and Physical Sciences Research Council

BBSRC
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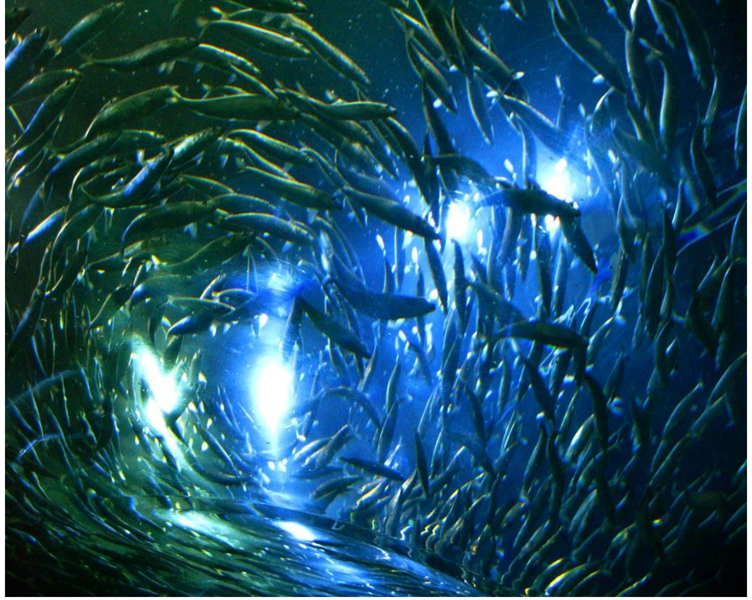
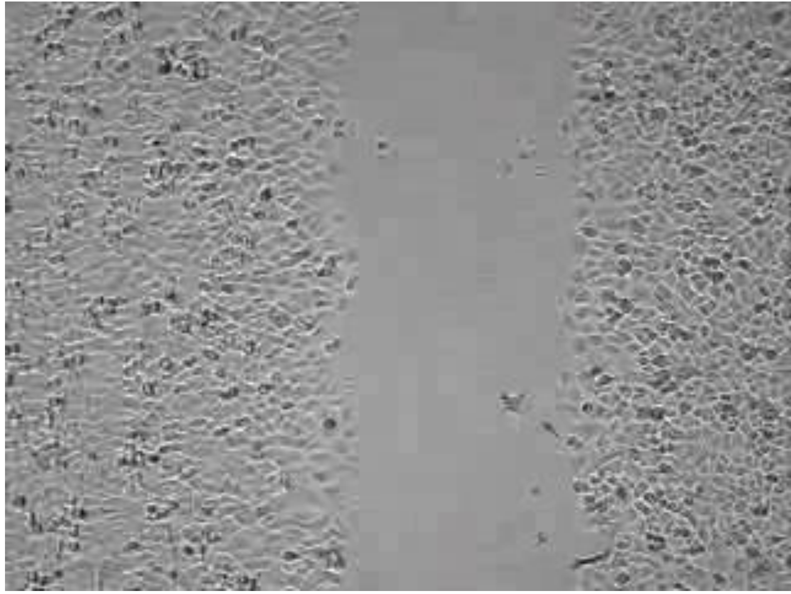




Far from equilibrium systems:

- X no Hamiltonian
- X no Free energy
- X no fluctuation-dissipation

- ✓ symmetries
- ✓ conservation laws
- ✓ constraints / exact identities

Hydrodynamics of incompressible active fluids

Long-time, large-distance behaviour

Only 1 field (hydro. variable) to worry about: velocity $\vec{v}$ (d-dim., vectorial)

Start with the equation of motion (EOM): $\partial_t \vec{v} = \vec{F}$

What is $\vec{F}$? Completely arbitrary except for its respect of

1) Symmetries: translational, rotational, temporal, chiral
2) Conservation law: mass conservation
3) Exact identity: incompressibility → $\nabla \cdot \vec{v} = 0$

Hydrodynamic EOM of incompressible active fluids

$$\partial_t \vec{v} = -\vec{\nabla} P + \overrightarrow{f_0} - \lambda_0 (\vec{v} \cdot \vec{\nabla}) \vec{v} - (a_0 + b_0 v^2) \vec{v} - \mu_0 \nabla^2 \vec{v} + c_0 v^4 \vec{v} + \xi_0 (\nabla^2)^2 \vec{v} + \cdots$$

Gaussian noise

Higher order terms in ∇ & $\vec{v}$

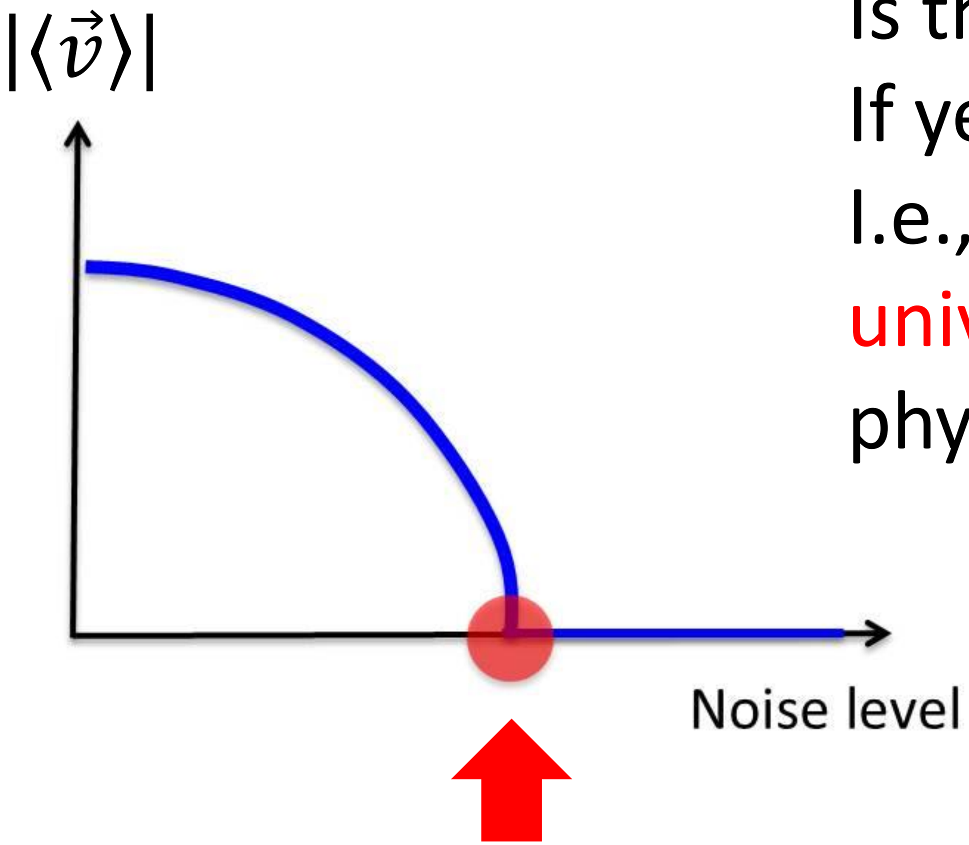
'pressure' (Lagrange multiplier) to enforce incompressibility

Now that we have a model (a.k.a. Toner-Tu model [Toner & Tu (1995) PRL]), what do we do next?

Simplest thing first → mean-field analysis: $(a_0 + b_0 v^2) = 0$

$a_0 < 0$ → ordered phase with non-zero $\vec{v}$: **active** collective motion

Order-disorder critical transition



Critical region

Is the transition really critical?
If yes, is it a new critical phenomenon?
I.e., does it correspond to a new universality class in non-equilibrium physics?

RG analysis of the EOM:

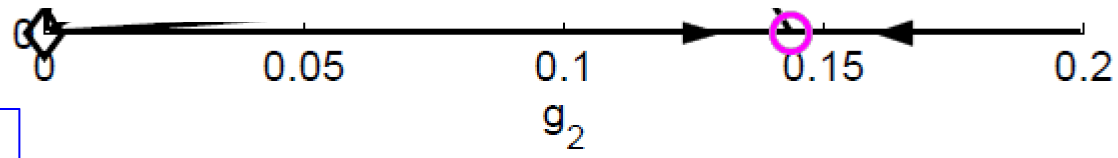
$$\partial_t \vec{v} + \vec{\nabla} P + \overrightarrow{f_l} = -\lambda_l (\vec{v} \cdot \vec{\nabla})\vec{v} - (a_l + b_l v^2)\vec{v} - \mu_l \nabla^2 \vec{v}$$

Two nonlinearities: λ_l and b_l, $l \sim$ level of RG transformation

Methodology:

Dynamic renormalization group + ϵ-expansion at 1-loop

[L Chen, J Toner, CFL (2015) New J. Phys. 17, 042002]



$$g_1(l) \sim \frac{D_l \lambda_l^2}{\mu_l^3};\ g_2(l) \sim \frac{D_l b_l}{\mu_l^2}$$

"Ferromagnets with dipolar interactions"
Aharony and Fisher (1973) Phys. Rev. Lett.

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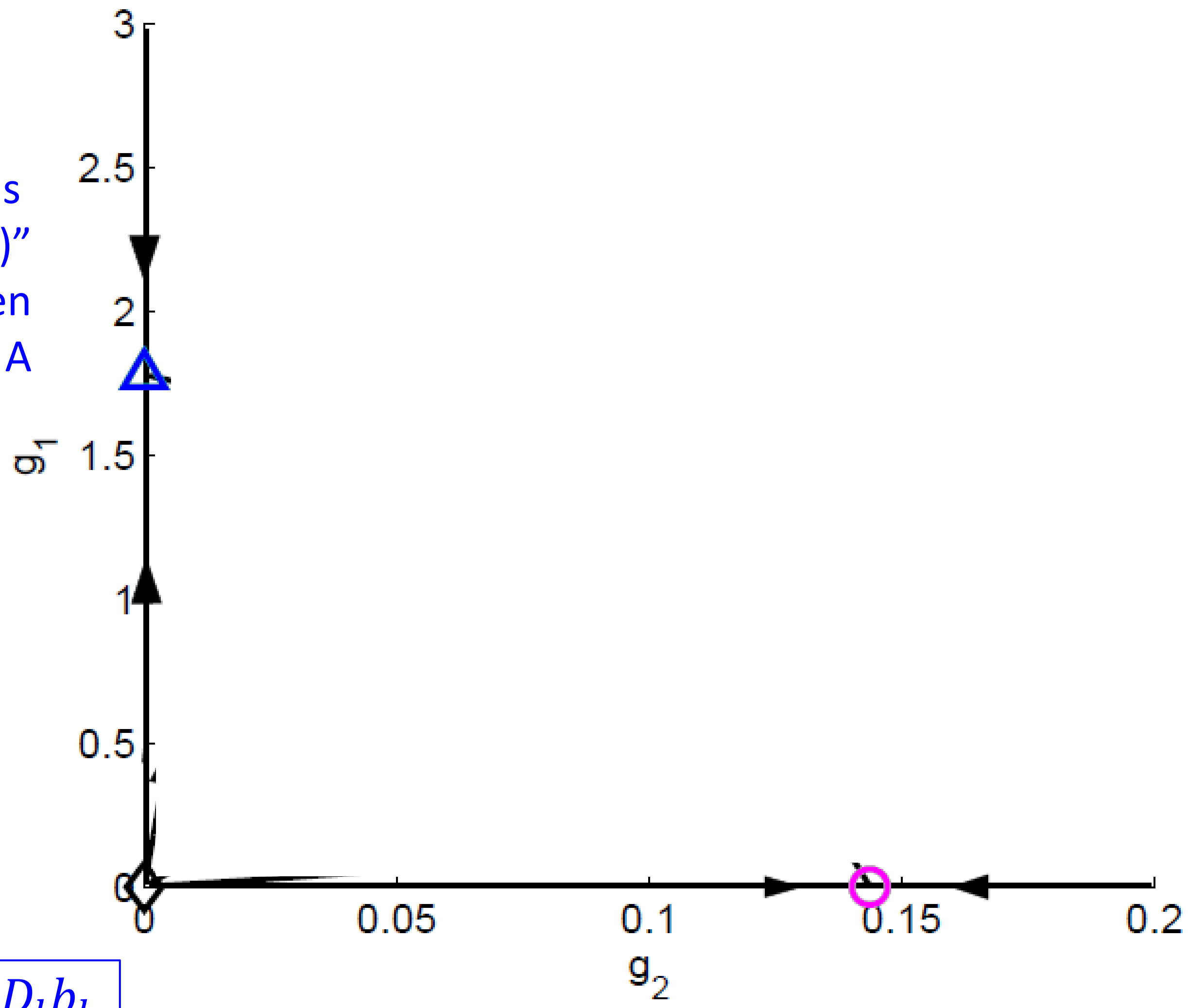
“Randomly stirred fluids (Model B)”
Forster, Nelson & Stephen (1977) Phys. Rev. A



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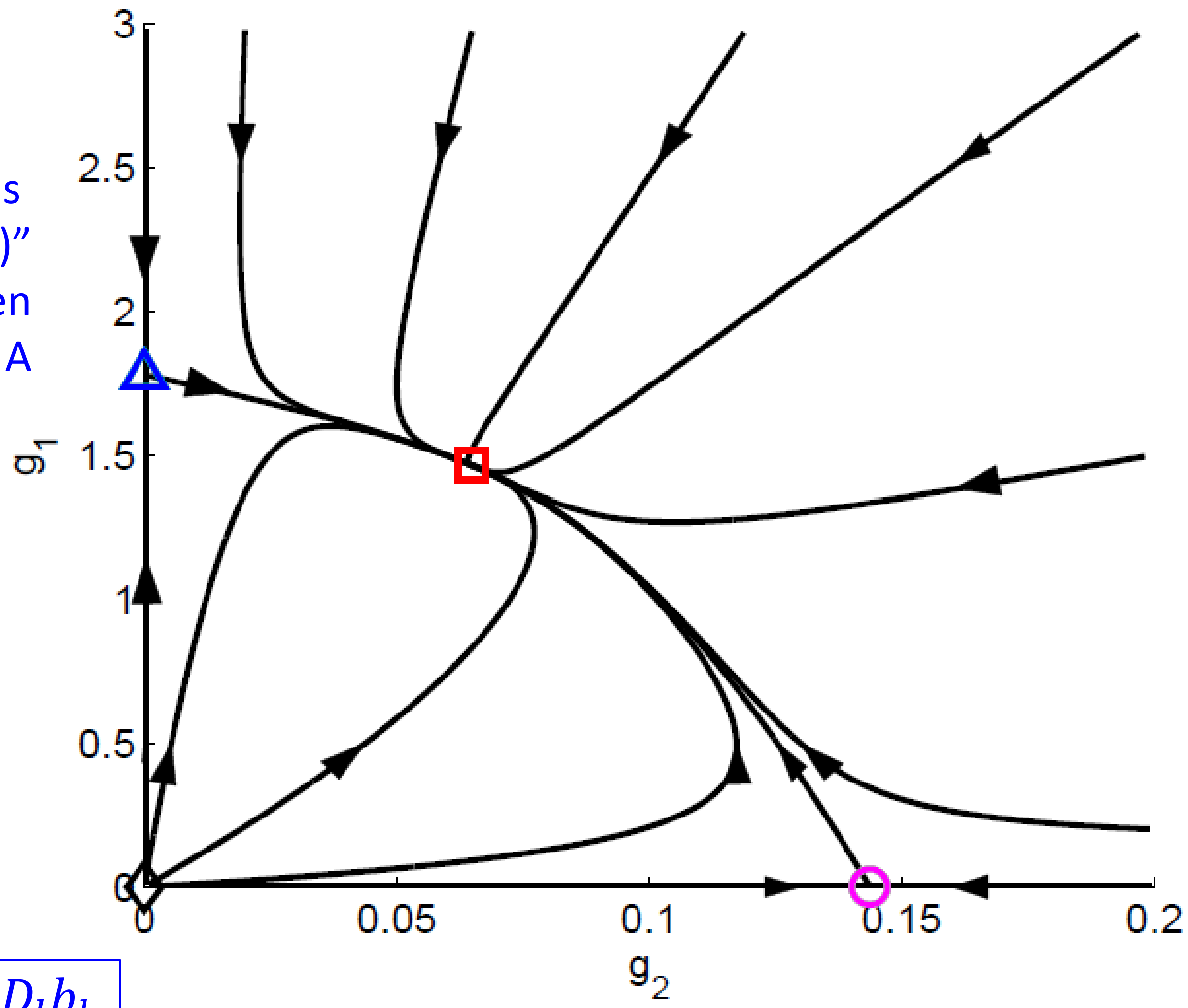

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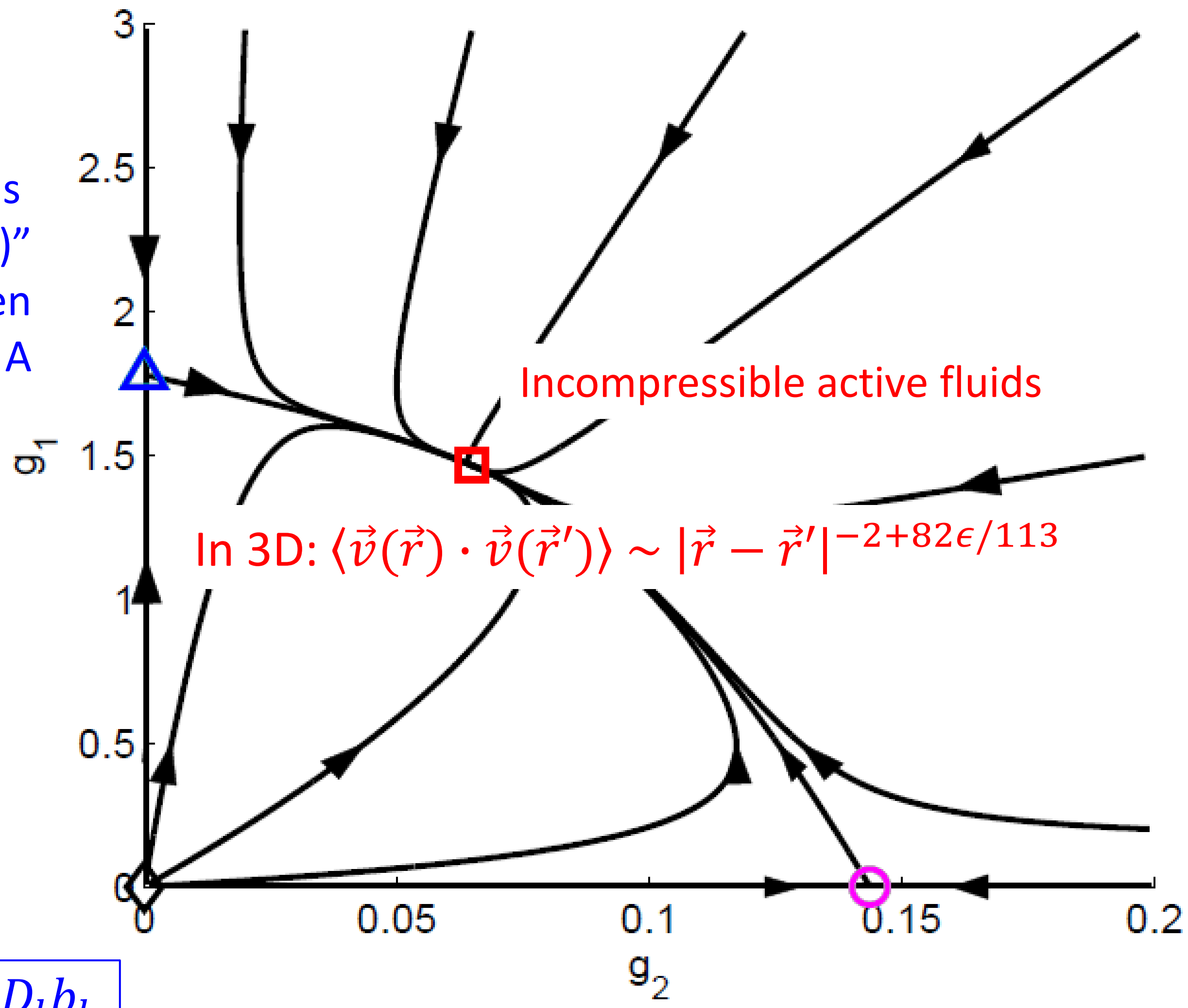

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A treasure trove of novel universality classes (UC)

New order-disorder critical phenomena

- Incomp. active fluids (IAF) [Chen, Toner, CFL (2015) NJP]
- IAF with quenched random field disorder [Zinati, Besse, Tarjus, Tissier (2022) PRE]
- Multicritical phenomena in compressible active fluids – see next talk [Jentsch, CFL, arXiv:2205.01610]

New ordered phases

- IAF for $2 < d < 4$ [Toner & Tu (1995) PRL; Chen, CFL, Toner (2018) NJP]

Surprise: IAF belongs to the (1+1)d Kardar-Parisi-Zhang UC [Chen, CFL, Toner (2016) Nature Comm]

- IAF with quenched random field disorder for $2 \leq d < 5$ [Chen, CFL, Maitra, Toner, arXiv:2202.02865 & arXiv:2203.01892]
- Infinitely compressible active fluids for $2 \leq d < 4$ [Toner (2012) PRL; Chen, CFL, Toner (2020) PRL, PRE]

And many more new UC are expected…

In particular, a key open question is:

What is the universal behaviour of ordered compressible active fluids?

At the beginning, I promised that you will know by now

1. How to study the hydrodynamic behaviour of an active matter system
2. RG analyses have uncovered many novel universality classes in active matter
3. *Biophysics is one of the most exciting areas of physics*

Vision

5 decades ago, technological relevance, experimental advances, and abundance of novel physics propelled condensed matter physics to become the ‘King of Physics’
[Martin (2019) Physics Today]

With its relevance to health and life, experimental advances, and abundance of novel physics, I believe biophysics will become the next ‘King of Physics’