Spectral Functions in Banks-Zaks QCD

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Based on arXiv:2207.14510 with Daniel Litim and Manuel Reichert

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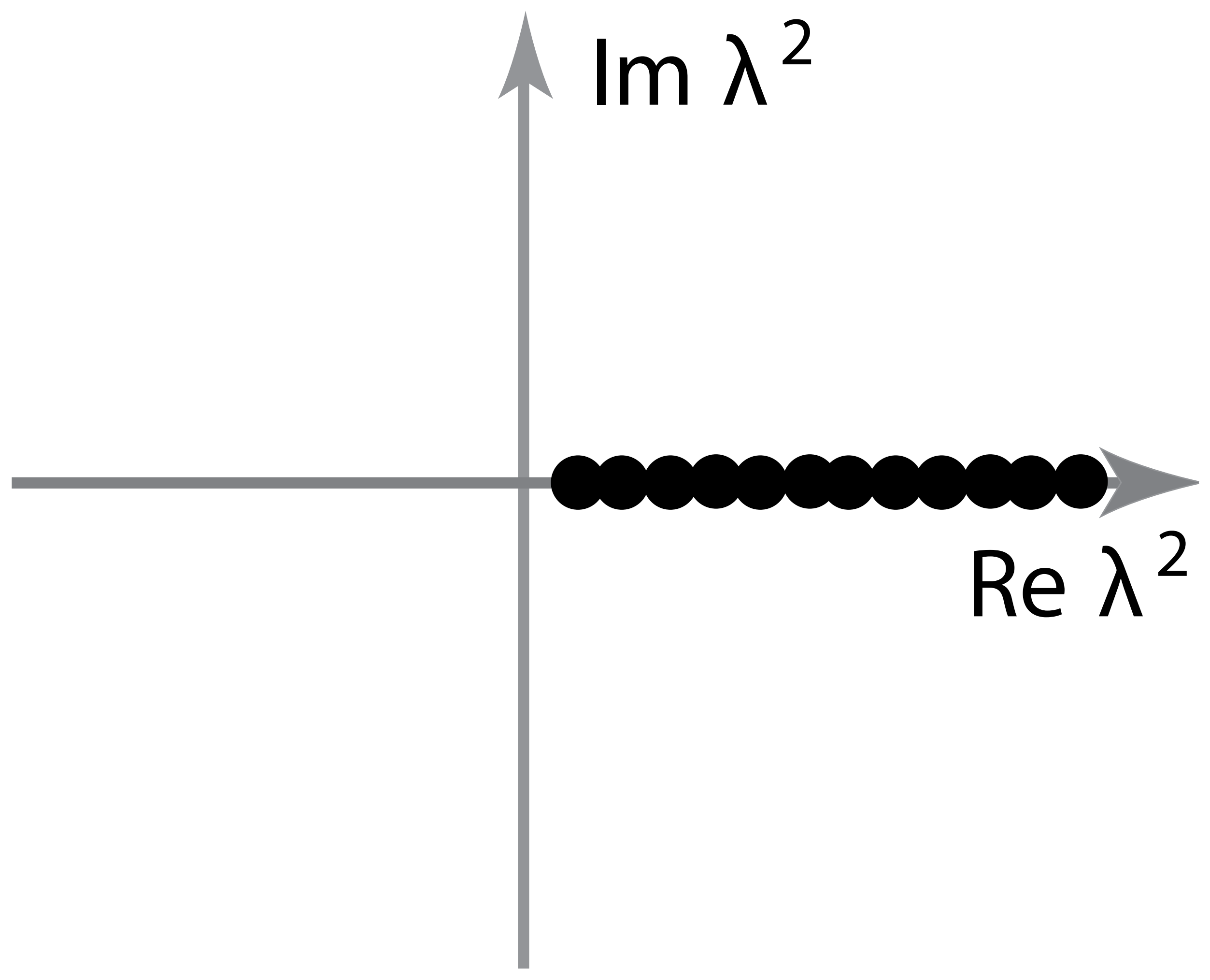
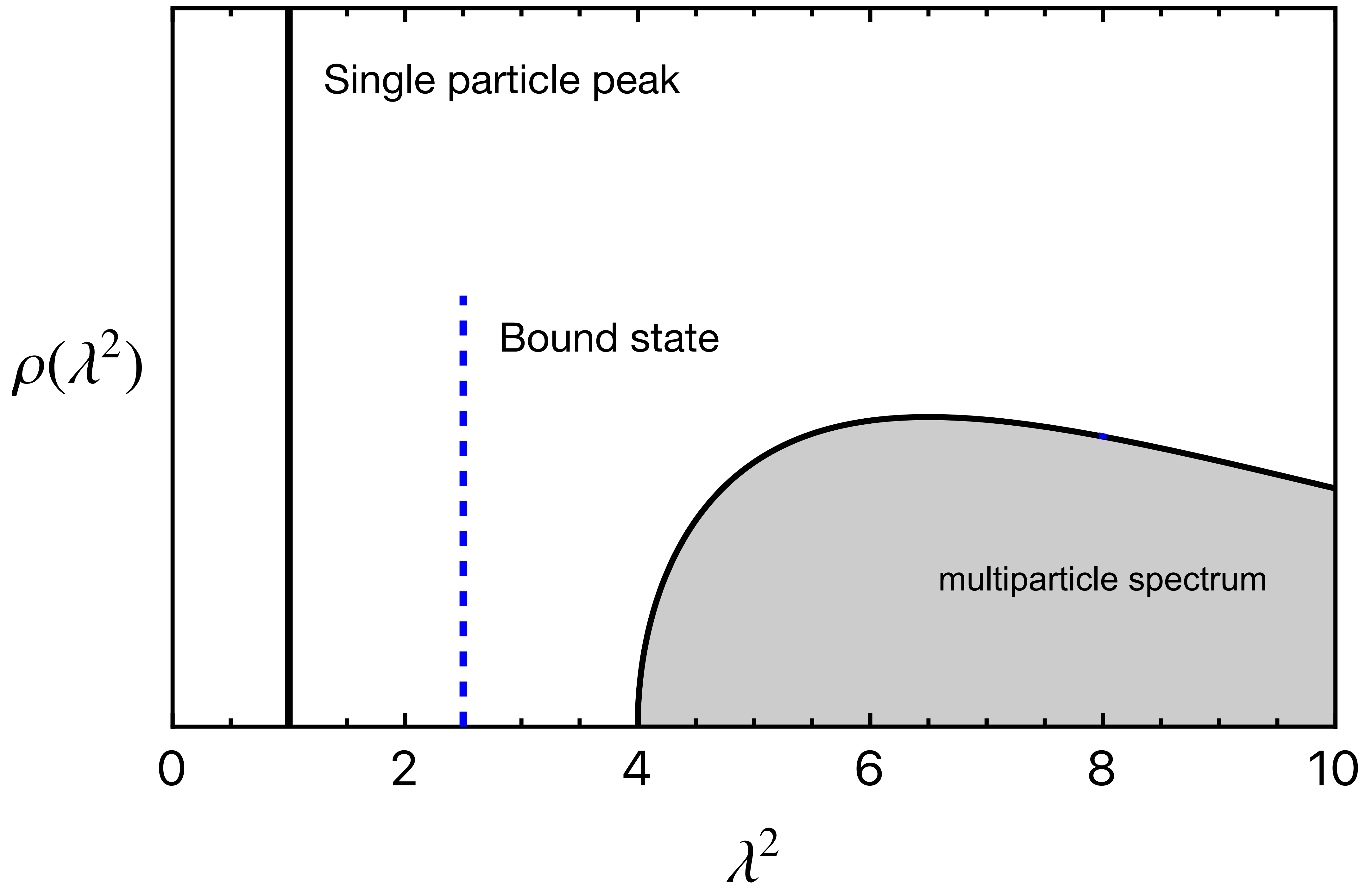
Why Spectral Functions?

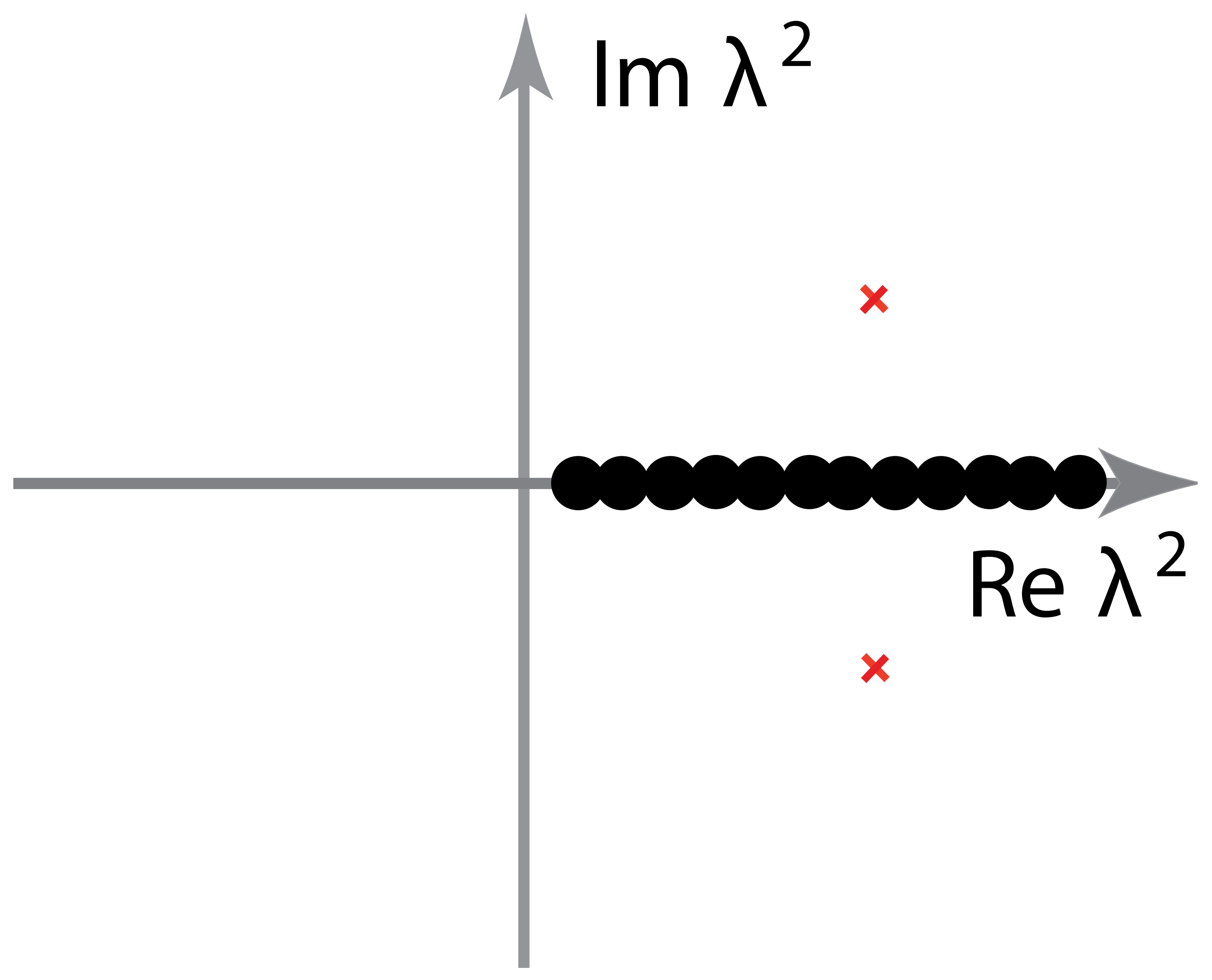
- Källén–Lehmann spectral representations encode analytic properties of propagator
  - $$G(p^2) = \int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \frac{\rho(\lambda^2)}{p^2 - \lambda^2}$$
  - Unitarity requires $\rho(\lambda^2)$ to be positive-definite and normalisable
  - Test Unitarity in Quantum Gravity?





Quantum Gravity and QCD

- Elementary Fields are gauge variant
- Existence of Spectral Representation is not guaranteed
    - Complex Conjugated Poles?
    - Additional Branch Cuts?
- Can we get analytic insights?

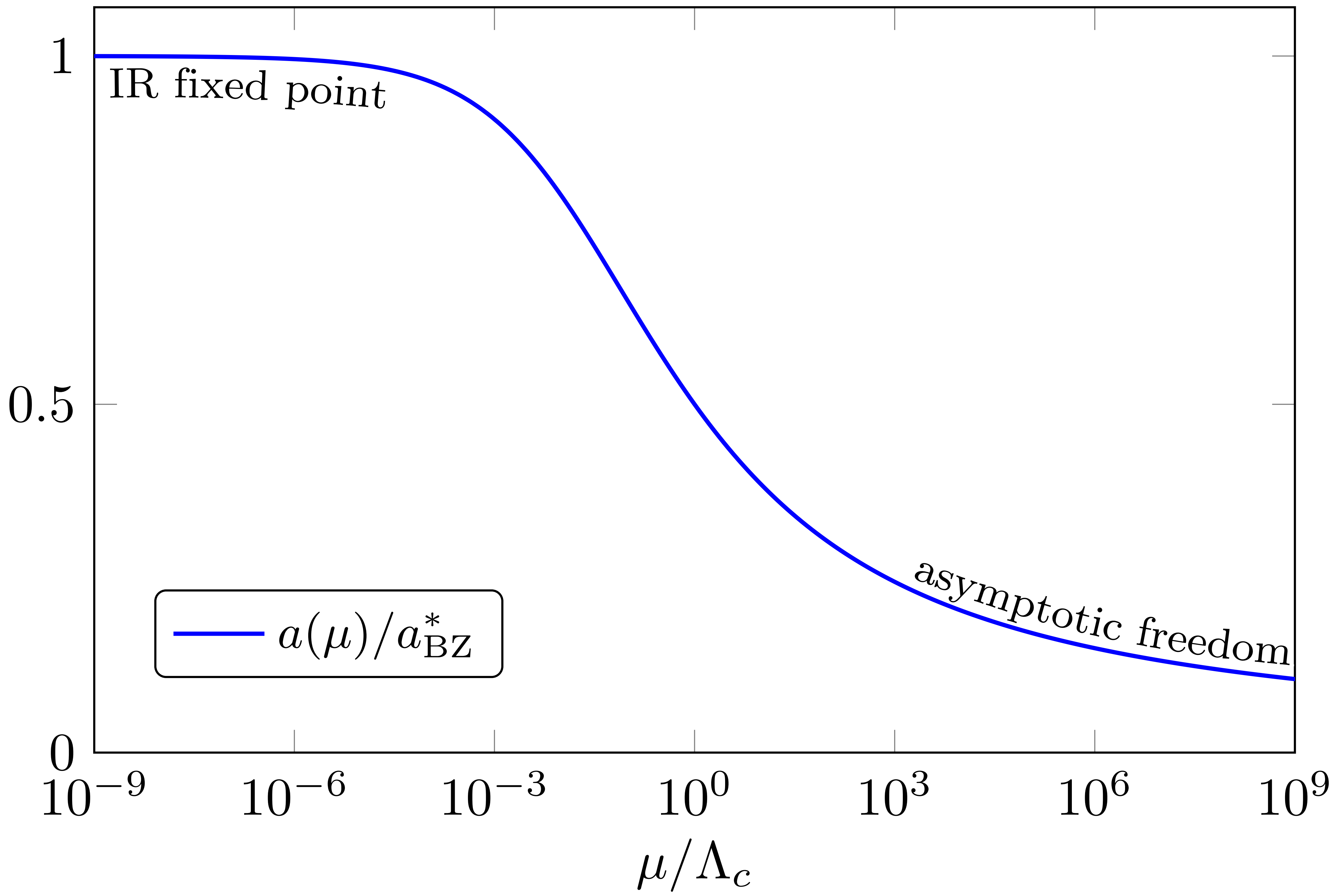


QCD in the Banks-Zaks Phase

- QCD with N_c colours and n_F massless Quarks

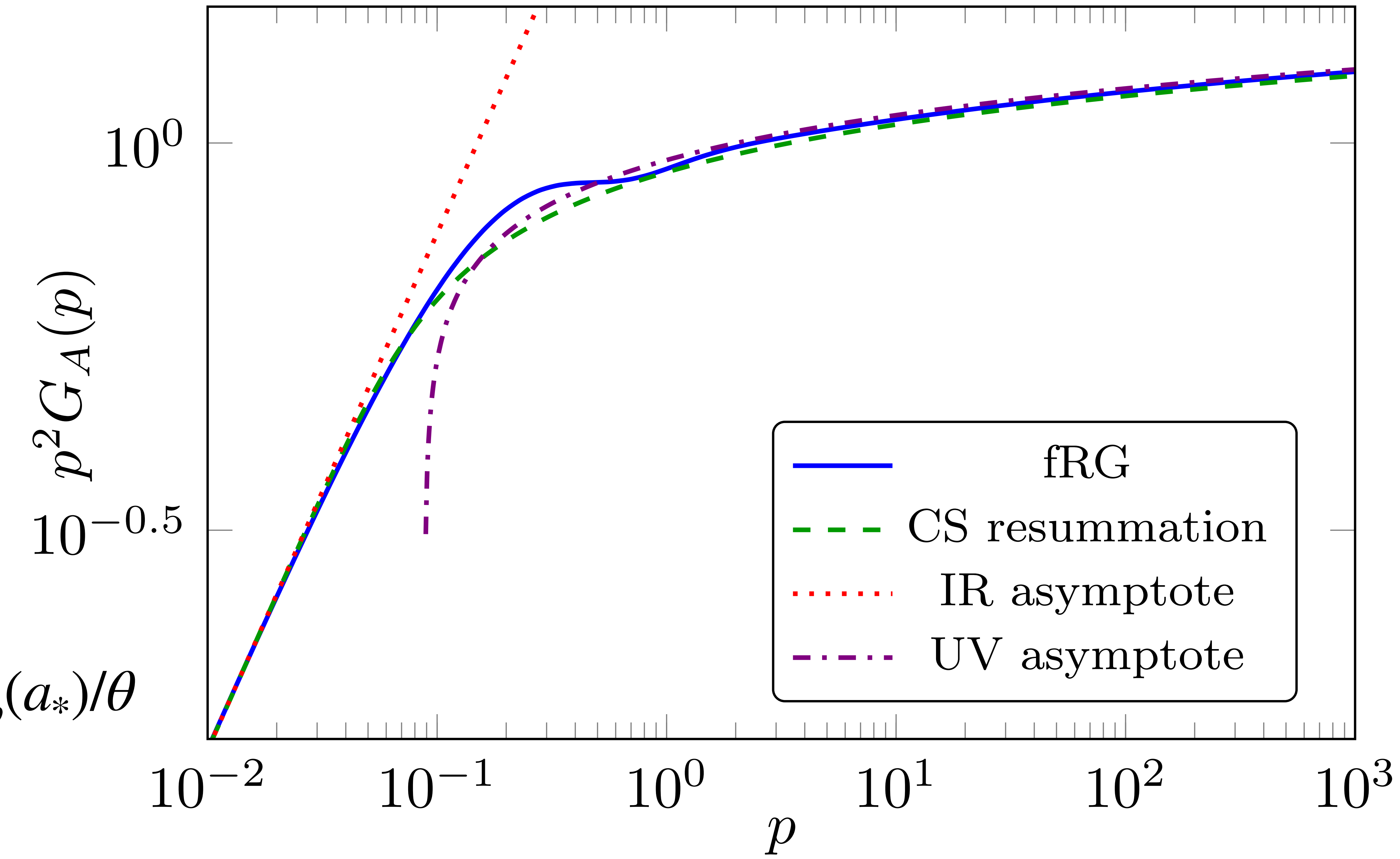
$$\frac{\partial a}{\partial \mu^2} = \beta_1 a^2 + \beta_2 a^3 + \ldots$$

- IR controlled by Banks-Zaks fixed point
  - $a_* = -\frac{\beta_1}{\beta_2} + \ldots$
  - $\beta_1 \propto \varepsilon = \frac{11}{2} - \frac{n_F}{N_c}$



How to obtain the Propagator?

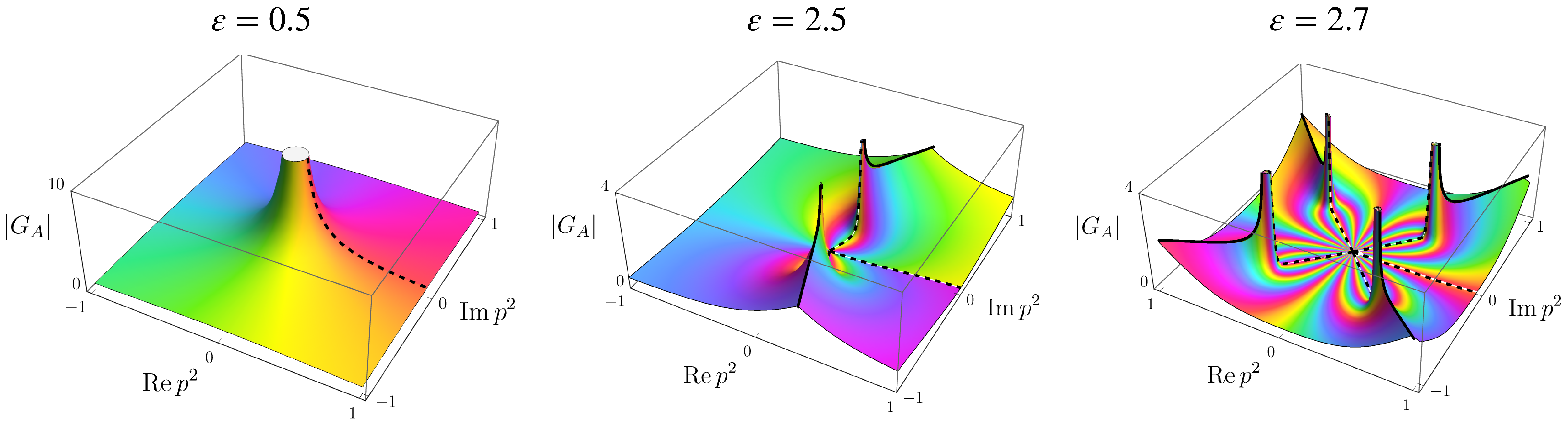
- Perturbation Theory + Callan-Symanzik equation
  - $\log(-p^2/\mu^2)$ need to be resumed
- $G_\phi = \frac{1}{p^2}\frac{\mathcal{N}_\phi}{1+\Pi_\phi^{(1)}\bar{a}+\Pi_\phi^{(2)}\bar{a}^2}\left(\frac{a}{\bar{a}}\right)^{\gamma_\phi^{(1)}/\beta_1}\left(\frac{a-a_*}{\bar{a}-a_*}\right)^{\gamma_\phi(a_*)/\theta}$
  - $\bar{a} = \bar{a}(p^2/\mu^2)$
- Euclidean Propagator agrees with FRG



Gluon propagator at space like momenta

Properties in the Complex Plane

- Propagators inherit analytic properties of running coupling
  - Gauge-invariant branch cuts of $\bar{a}(p^2/\mu^2)$ show up in propagator



Existence Guaranteed for small ε?

- $$G_\phi = \frac{1}{p^2} \frac{\mathcal{N}_\phi}{1 + \Pi_\phi^{(1)} \bar{a} + \Pi_\phi^{(2)} \bar{a}^2} \left(\frac{a}{\bar{a}} \right)^{\gamma_\phi^{(1)}/\beta_1} \left(\frac{a - a_*}{\bar{a} - a_*} \right)^{\gamma_\phi(a_*)/\theta}$$

- In IR $G_\phi \propto \dfrac{N_\phi^{\mathrm{IR}}}{p^2} \left(-\dfrac{p^2}{\mu^2} \right)^{-\gamma_\phi(a_*)}$

    - **For $\gamma_\phi(\mathbf{a}_*) > \mathbf{0}$, divergence in IR prevents KL representation to exist!**

- Existence depends on gauge

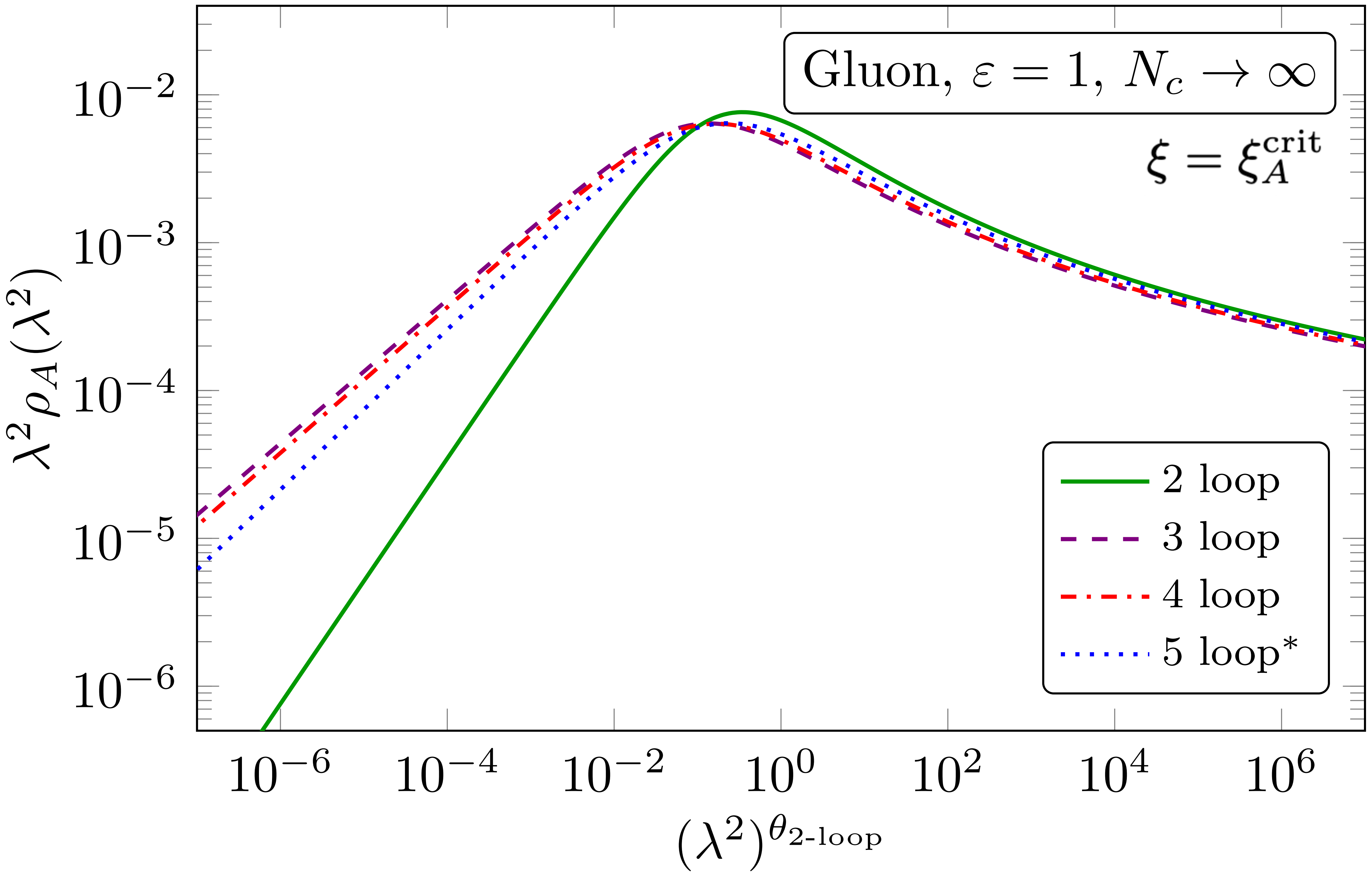
    - Gluons: $\xi > -3 + 0.267\varepsilon + \dots$

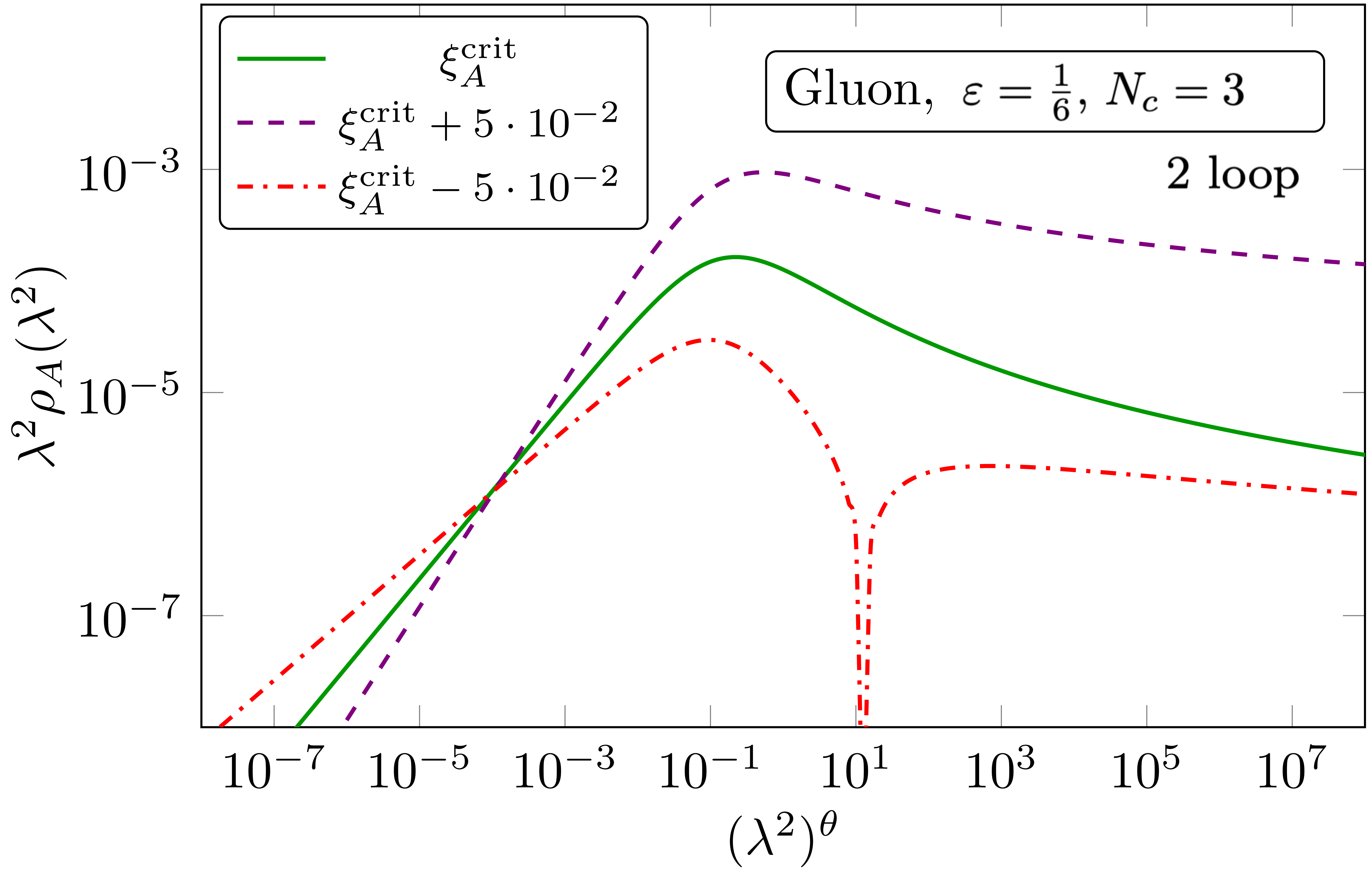
    - Quarks: $\xi > 0.0145\varepsilon^2 + \dots$

Norm of Spectral Function

- $G_\phi = \dfrac{1}{p^2} \dfrac{\mathcal{N}_\phi}{1 + \Pi_\phi^{(1)} \bar{a} + \Pi_\phi^{(2)} \bar{a}^2} \left(\dfrac{a}{\bar{a}}\right)^{\gamma_\phi^{(1)}/\beta_1} \left(\dfrac{a - a_*}{\bar{a} - a_*}\right)^{\gamma_\phi(a_*)/\theta}$
    - Norm determined by UV behaviour!
- $\displaystyle\int_0^\infty \frac{\mathrm{d}\lambda^2}{\pi} \rho_\phi(\lambda^2) = \begin{cases} 0 & \gamma_\phi^{(1)}/\beta_1 < 0, \quad \xi < \xi_\phi^{\text{crit}}, \\ 1 \qquad \text{if} & \gamma_\phi^{(1)}/\beta_1 = 0, \quad \xi = \xi_\phi^{\text{crit}}, \\ \infty & \gamma_\phi^{(1)}/\beta_1 > 0, \quad \xi > \xi_\phi^{\text{crit}}. \end{cases}$
- Gluons: $\xi_A^{\text{crit}} = -3 + \dfrac{4}{3}\varepsilon$ Quarks: $\xi_\psi^{\text{crit}} = 0$

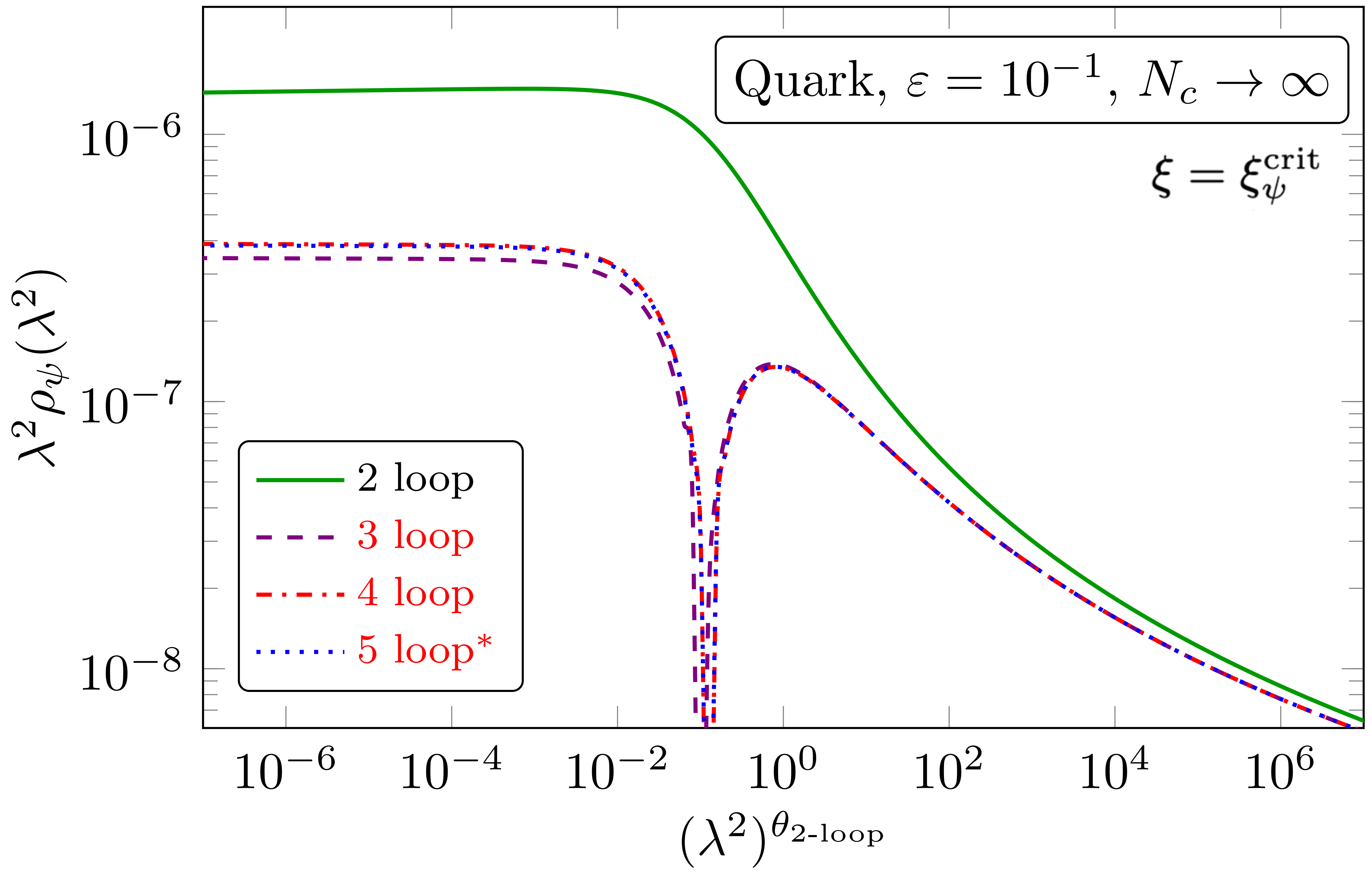
Gluon Spectral Function

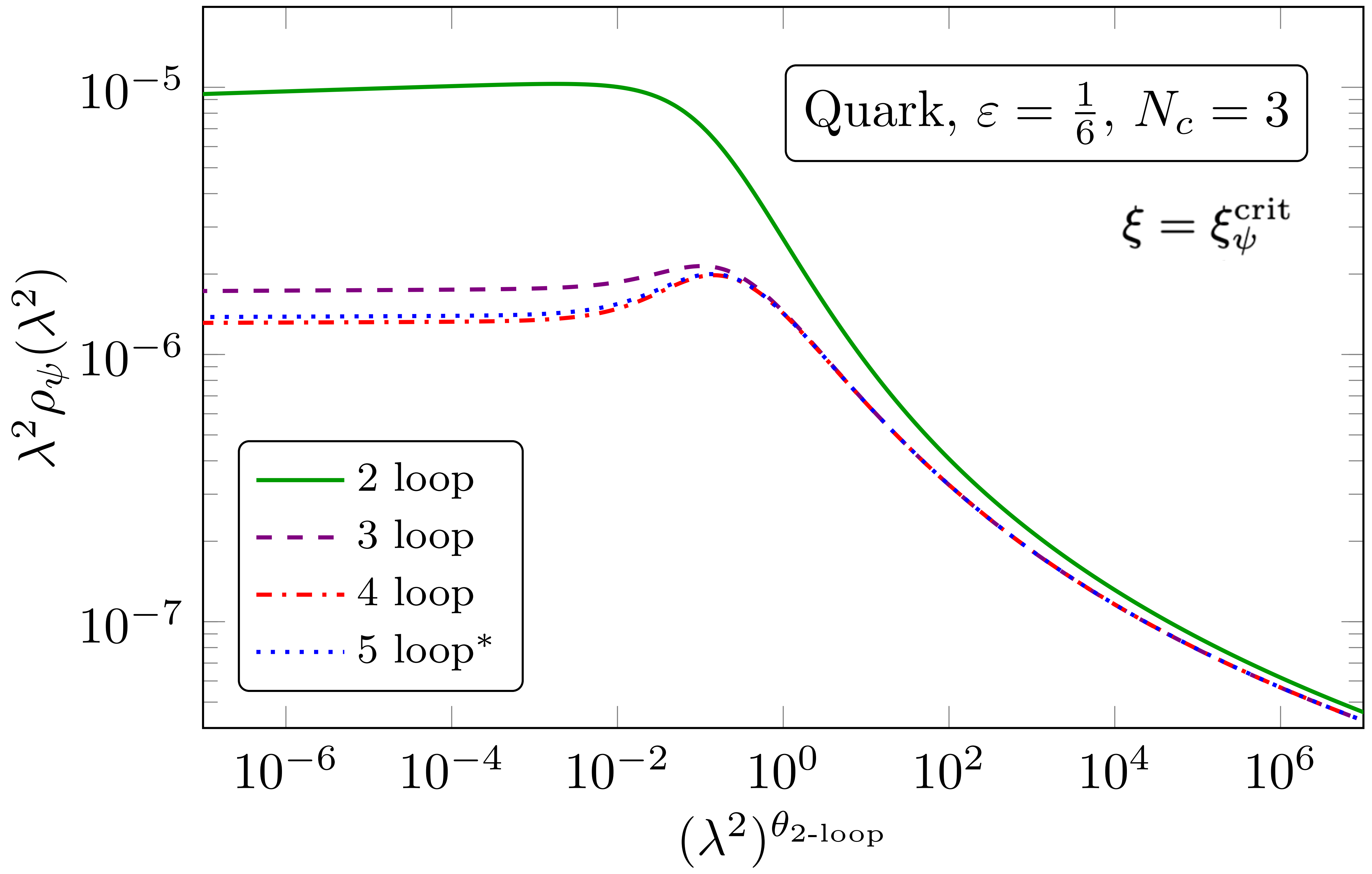




- Positive definite and normalisable Gluon spectral function exists
- Stable for rather large ε

Quark Spectral Function





- Normalisable Quark spectral function does not exist
- Less stable than gluons

Conclusions

- We studied Spectral Functions analytically in Banks-Zaks QCD
  - Different Effects can abolish the KL representation:
    - Branch cuts of the running coupling
    - Diverging Behaviour of the Propagator in the IR
  - Large Parameter Regime where positive-definite KL representation exists!
- Open Questions:
  - Spectral functions in asymptotically safe theory?
  - Bound states?