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Critical behaviour at thermal m-axial Lifshitz point and stability of the FFLO superfluid phases

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Microscopic Theory

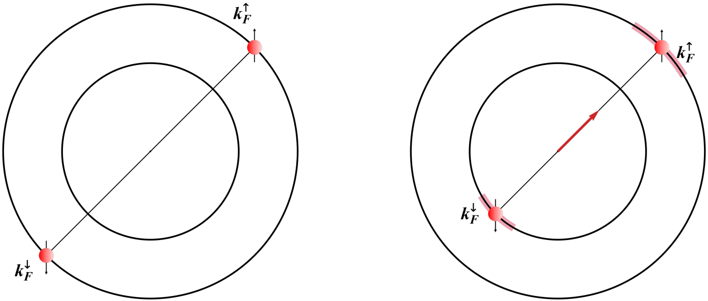
$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k},\sigma} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{k}+\frac{\mathbf{q}}{2}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}+\frac{\mathbf{q}}{2}\downarrow} \hat{c}_{-\mathbf{k}'+\frac{\mathbf{q}}{2}\downarrow} \hat{c}_{\mathbf{k}'+\frac{\mathbf{q}}{2}\uparrow}$$

$$\xi_{\mathbf{k},\sigma} = \epsilon_{\mathbf{k},\sigma} - \mu - \sigma h$$

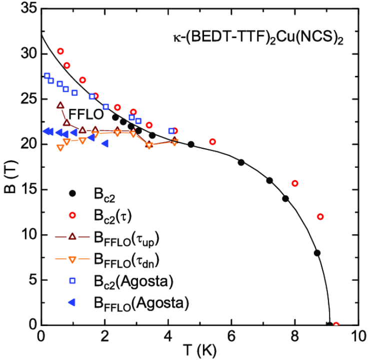
BCS state

FFLO state

Q = $k_F^{\uparrow} - k_F^{\downarrow}$



Indirect evidence in superconductors



Wosnitza, J. *Crystals* **2018**, 8, 183

Main difficulties

- Orbital effects
- Impurities

Ultracold atoms

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k},\sigma} \hat{c}^{\dagger}_{\mathbf{k}\sigma} \hat{c}_{\mathbf{k}\sigma} + \frac{g}{V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{k}+\frac{\mathbf{q}}{2}\uparrow} \hat{c}^{\dagger}_{-\mathbf{k}+\frac{\mathbf{q}}{2}\downarrow} \hat{c}_{-\mathbf{k}'+\frac{\mathbf{q}}{2}\downarrow} \hat{c}_{\mathbf{k}'+\frac{\mathbf{q}}{2}\uparrow}$$

$$\xi_{\mathbf{k},\sigma} = \epsilon_{\mathbf{k},\sigma} - \mu - \sigma h$$

$$\Downarrow$$

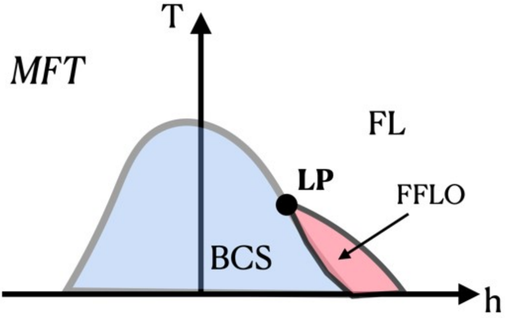
FFLO superconductor

- Zeeman splitting
- h - magnetic field

FFLO superfluid

- population imbalance in Fermi mixtures of ultracold atoms
- $h = \frac{\mu_\uparrow - \mu_\downarrow}{2}$

Mean Field phase diagram



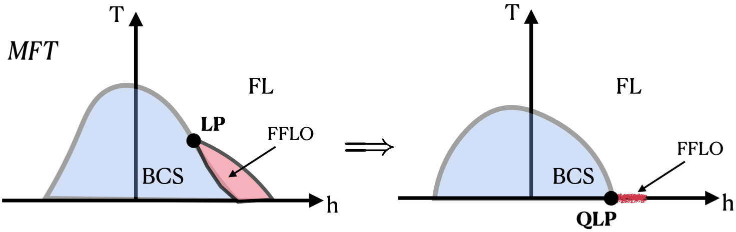
$$S_{eff} = \int \mathrm{d}^m x_\parallel \int \mathrm{d}^{d-m} x_\perp \left[U(\bar{\phi}) + \frac{1}{2} Z_\perp (\nabla_\perp \bar{\phi})^2 + \frac{1}{2} \rho_0 (\nabla_\parallel \bar{\phi})^2 + \frac{1}{2} Z_\parallel (\triangle_\parallel \bar{\phi})^2 \right]$$

$$U[\bar{\phi}] = \frac{1}{2} a |\bar{\phi}|^2 + \frac{1}{4} u |\bar{\phi}|^4 + O(|\bar{\phi}|^6)$$

Including fluctuations...

Isotropic Lifshitz point

$$m = d \implies d_{\text{low}} = 4$$

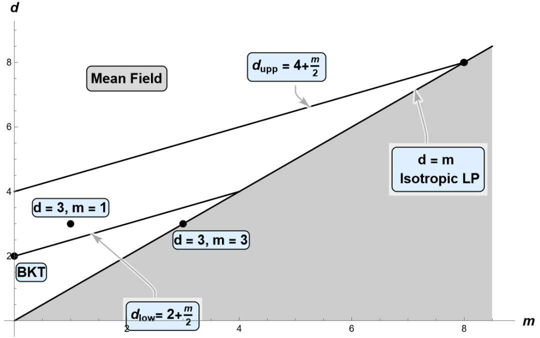


Lifshitz Point - properties

Effective action close to the Lifshitz point

$$S_{eff} = \int \mathrm{d}^m x_{\parallel} \int \mathrm{d}^{d-m} x_{\perp} \left[U(\bar{\phi}) + \frac{1}{2} Z_{\perp} (\nabla_{\perp} \bar{\phi})^2 + \frac{1}{2} \rho_0 (\nabla_{\parallel} \bar{\phi})^2 + \frac{1}{2} Z_{\parallel} (\triangle_{\parallel} \bar{\phi})^2 \right]$$

$$U[\bar{\phi}] = \frac{1}{2} a |\bar{\phi}|^2 + \frac{1}{4} u |\bar{\phi}|^4 + O(|\bar{\phi}|^6)$$



LPA

The effective average action in the local potential approximation (LPA):

$$\Gamma_k[\phi] = \int \mathrm{d}^d x \Big[U_k(\bar{\phi}) + \frac{1}{2} Z_\perp (\nabla_\perp \bar{\phi})^2 + \frac{1}{2} Z_\parallel (\triangle_\parallel \bar{\phi})^2 \Big]$$

Plug into the Wetterich equation:

$$\partial_k U_k(\rho) = \frac{1}{2} \int_q \partial_k R_k(\vec{q}) \big[G_\sigma(\vec{q}, \rho) + (N-1) G_\pi(\vec{q}, \rho) \big],$$

where

$$G_\sigma(\vec{q}, \rho) = \frac{1}{Z_\perp \vec{q}_\perp^2 + Z_\parallel (\vec{q}_\parallel^2)^2 + U_k'(\rho) + 2\rho U_k''(\rho) + R_k(\vec{q})}$$

$$G_\pi(\vec{q}, \rho) = \frac{1}{Z_\perp \vec{q}_\perp^2 + Z_\parallel (\vec{q}_\parallel^2)^2 + U_k'(\rho) + R_k(\vec{q})} \quad .$$

LPA - fixed point solution

Introduce dimensionless variables:

$$\vec{q}_\perp = k\ \tilde{\vec{q}}_\perp \qquad \vec{q}_\parallel = (Z_\perp/Z_\parallel)^{\frac{1}{4}}\ k^{\frac{1}{2}}\ \tilde{\vec{q}}_\parallel$$

$$\rho = Z_\perp^{\frac{m}{4}-1} Z_\parallel^{-\frac{m}{4}}\ k^{d-\frac{m}{2}-2}\ \tilde{\rho} \qquad U_k(\rho) = (Z_\perp/Z_\parallel)^{\frac{m}{4}}\ k^{d-\frac{m}{2}}\ \tilde{u}_k(\tilde{\rho})$$

and consider the cutoff of the form:

$$R_k(\vec{q}) = Z_\perp k^2\ r\left[\tilde{\vec{q}}_\perp^{\,2} + (\tilde{\vec{q}}_\parallel^{\,2})^2\right] .$$

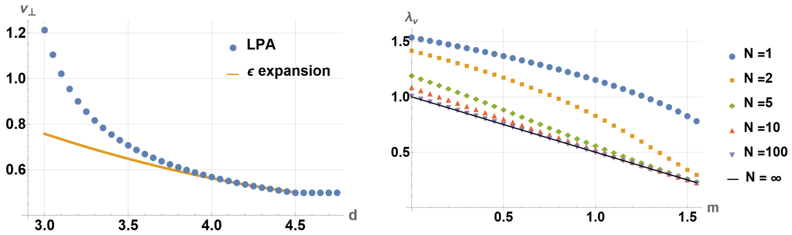
The LPA flow equation for dimensionless potential:

$$\partial_k \tilde{u}_k = -(d-\frac{m}{2})\tilde{u}_k - (2-d+\frac{m}{2})\tilde{\rho}\tilde{u}_k' + \mathcal{V}_{d,m}\int_0^\infty \mathrm{d}y\ y^{\frac{d}{2}-\frac{m}{4}-1}\big(r(y) - y r'(y)\big) \left[\frac{1}{y+\tilde{u}_k'+2\tilde{\rho}\tilde{u}_k''+r} + \frac{N-1}{y+\tilde{u}_k'+r}\right]$$

LPA - results

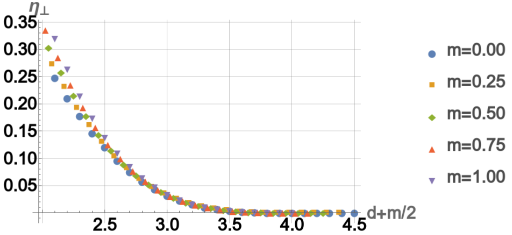
The critical behavior at the m-axial Lifshitz point is fully equivalent to the one at the O(N) critical point with reduced dimensionality.

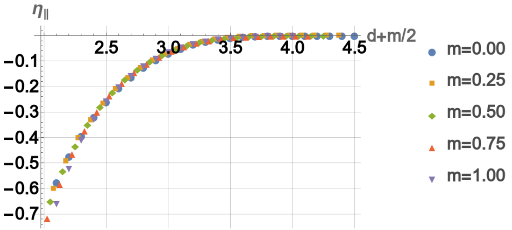
$$\nu(d + \frac{m}{2}, m) = \nu(d, 0)$$



Constrained LPA'

$$\Gamma_k[\phi] = \int \mathrm{d}^d x \Big[U_k(\bar{\phi}) + \frac{1}{2} Z_{\perp,k} (\nabla_\perp \bar{\phi})^2 + \frac{1}{2} Z_{\parallel,k} (\triangle_\parallel \bar{\phi})^2 \Big]$$





Summary

- Analytical correspondence between the anisotropic and isotropic models at the LPA level: $d_{\text{eff}} = d - \frac{m}{2}$.
- Numerical approximate correspondence between the anisotropic and isotropic models at the LPA' level.
- Suppression of the Lifshitz point for: $d_{\text{eff}} < 2$, which can influence the phase diagram of the Fermi imbalanced gases at $T \neq 0$.