Timelike properties of QCD from functional methods

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arXiv: 2203.09333, 2206.10232, in preparation

in collaboration with J. Dolgner, F. Ihssen, J. M. Pawlowski, J. Wessely, N. Wink & the fQCD collaboration

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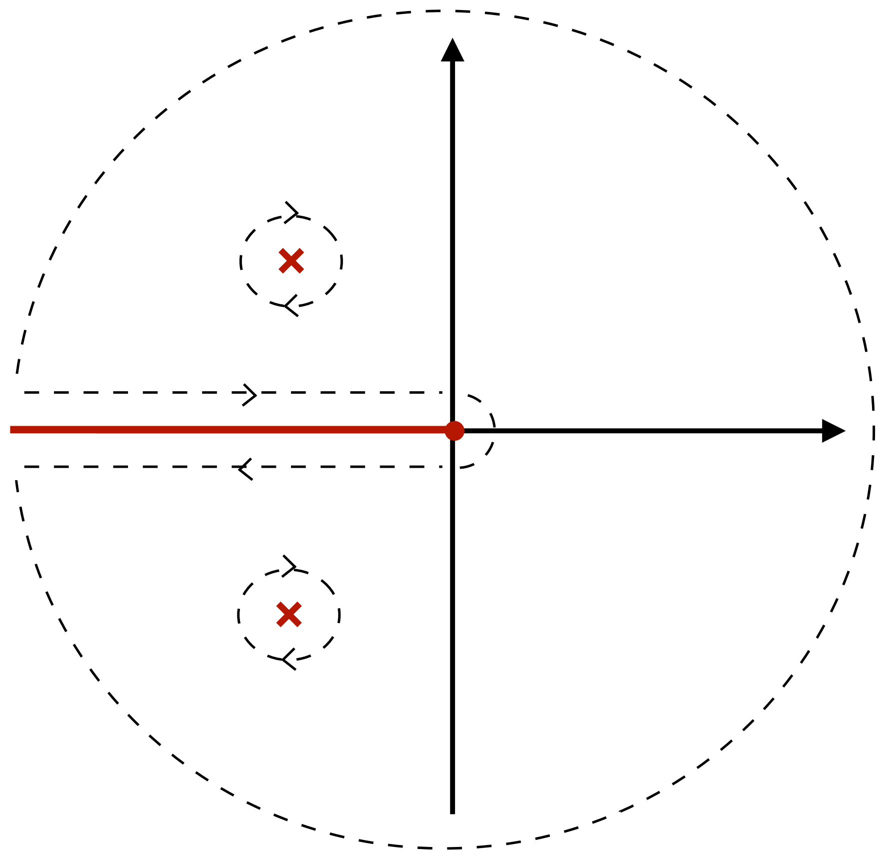
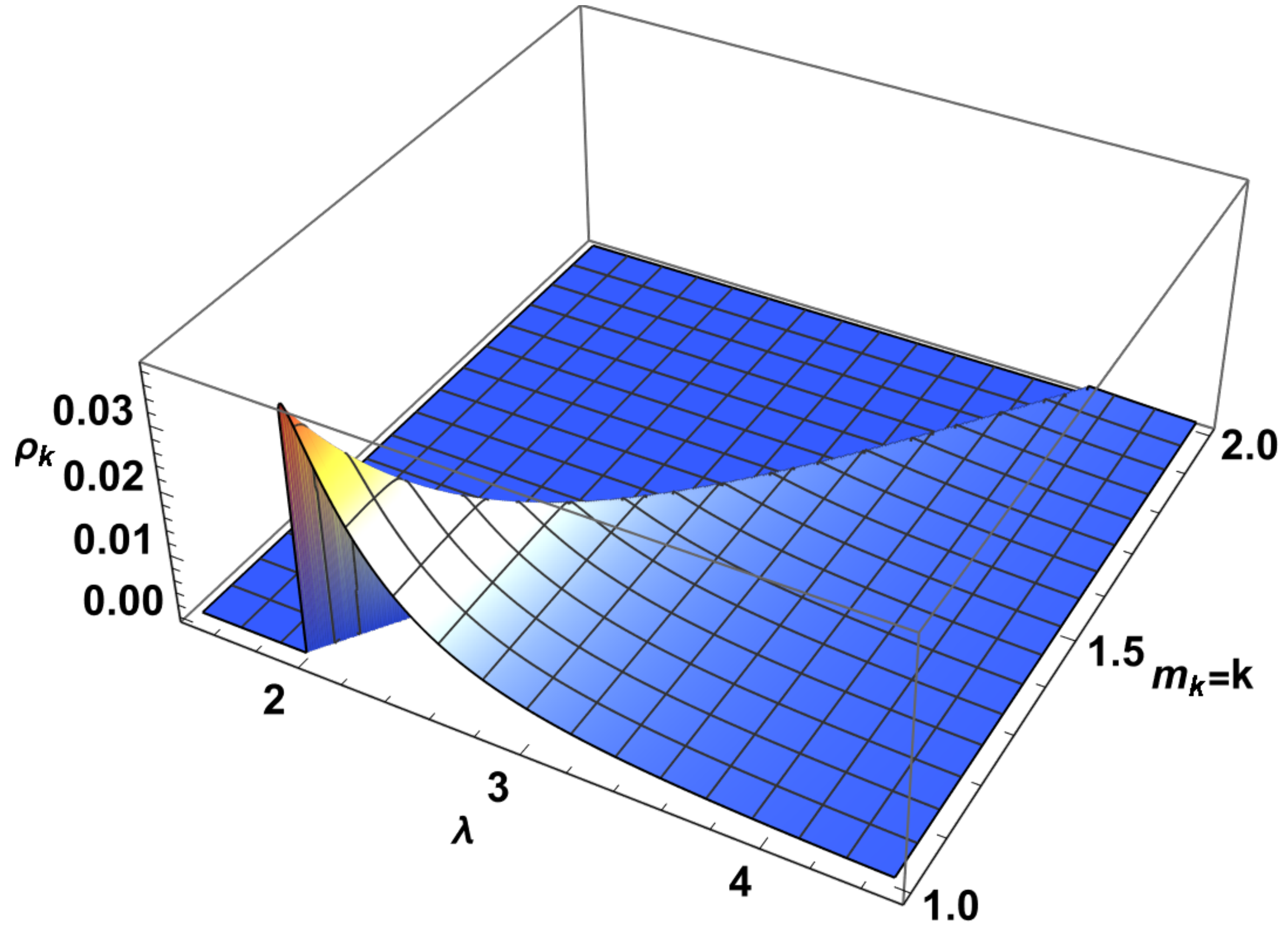
Outline

- Motivation: Transport coefficients
- Complex structure of Yang-Mills theory
- Spectral flows in the scalar theory
- Quark spectral function in QCD





Real-time QFT

- **Dynamics** and **time-like** physics need **real-time**
  (transport coefficients, non-equilibrium phenomena, bound states, …)
- Strongly correlated low-energy regime → **non-perturbative** techniques
  (Functional methods (fRG, DSE, BSE, n-PI), lattice theory)

Challenge: map from Euclidean back to Minkowski

- no analytic access
- **cheaper**

Numerical reconstruction
e.g. Bayesian reconstruction, Padé

- **analytic access**
- more expensive

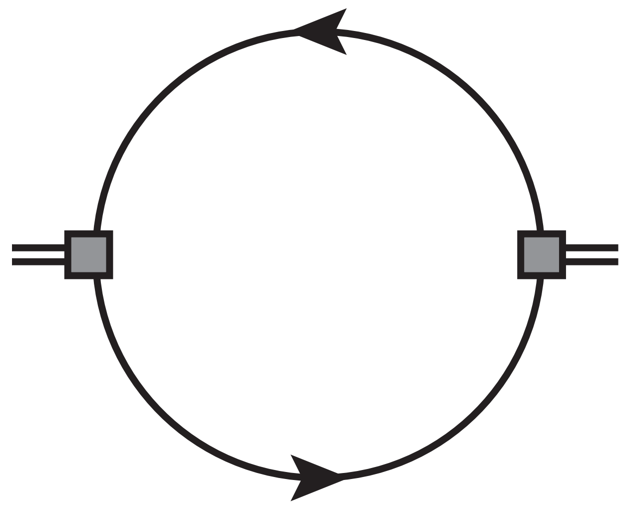
Direct computation
on real momentum axis

Heavy quark diffusion coefficient

- Heavy quark diffusion coefficient via **Kubo relations:**

$$\mathscr{D}_s = \lim_{\omega\to 0} \frac{\sigma(\omega, \mathbf{p}=0)}{\omega \chi_q \pi} \quad , \qquad \sigma(\omega, \mathbf{p}) = \frac{1}{\pi}\int \mathrm{d}t\, e^{i\omega t} \int \mathrm{d}^3x\, e^{i\mathbf{x}\mathbf{p}} \langle [J_i(t,\mathbf{x}), J_i(0,0)] \rangle$$

- Spectral function σ has simple, **exact** diagrammatic expression:

$\sigma(\omega, \mathbf{p}) \quad =$  $+$ 2-loop

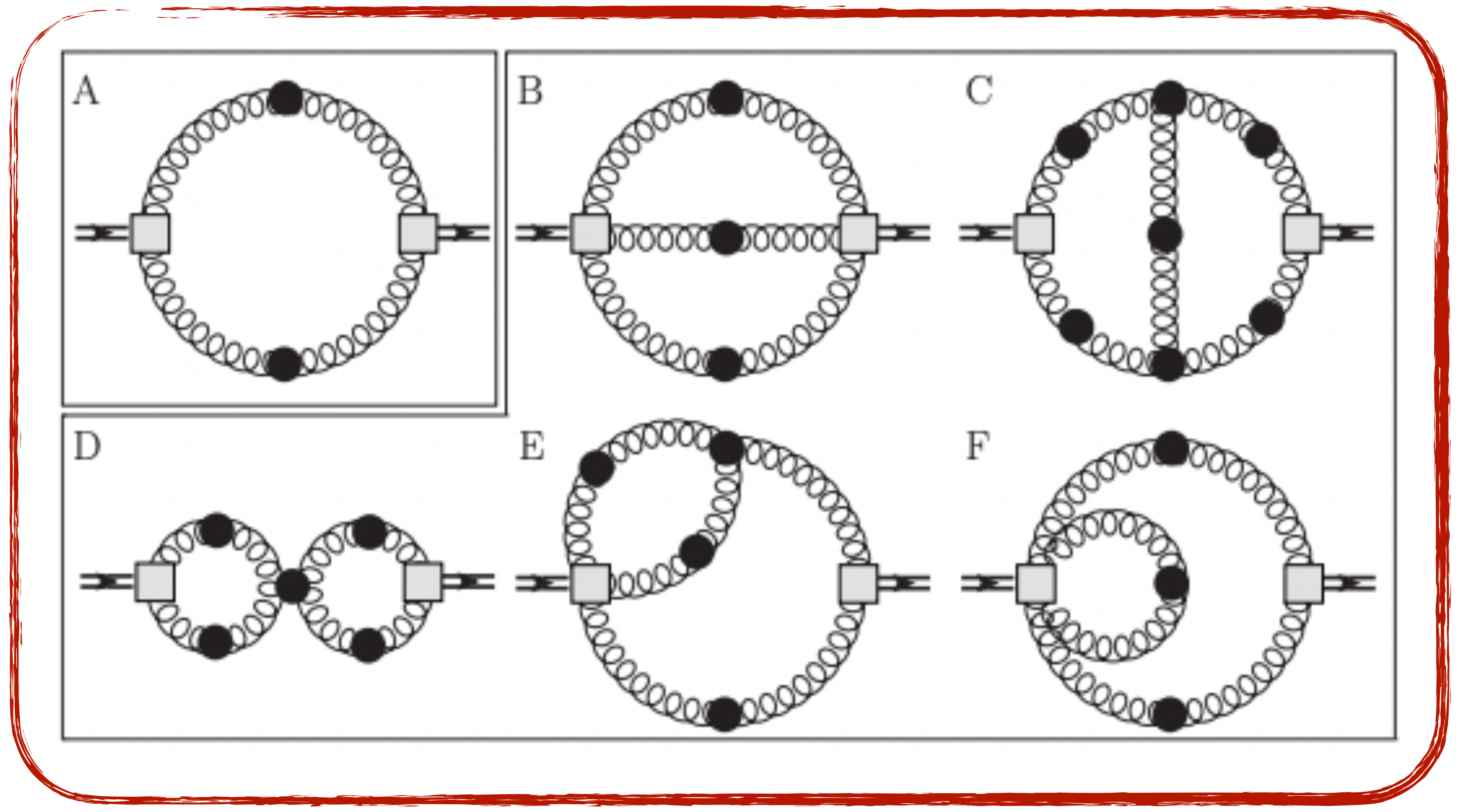
only need **quark propagator** in **real-time**

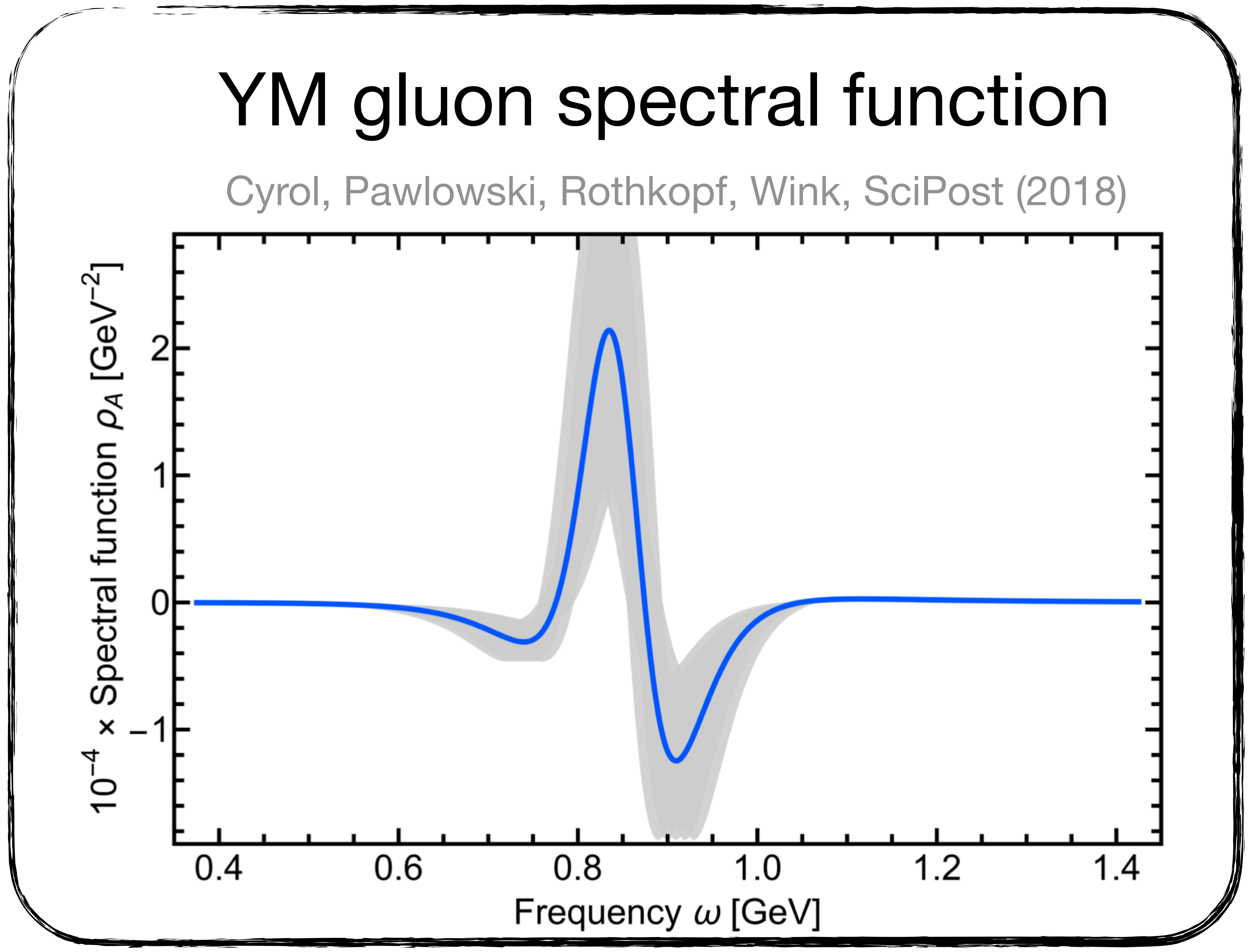
Shear viscosity of Yang-Mills

- Shear viscosity from Kubo relations

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{\rho_{\pi\pi}(\omega, \mathbf{p} = 0)}{\omega} \quad \text{with} \quad \rho_{\pi\pi}(\omega, \mathbf{p}) = \int \frac{d^4x}{(2\pi)^4} e^{-i\omega x_0 + i\mathbf{p}\mathbf{x}} \langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$$

- Diagrammatic expressions for $\langle [\pi_{ij}(x), \pi_{ij}(0)] \rangle$

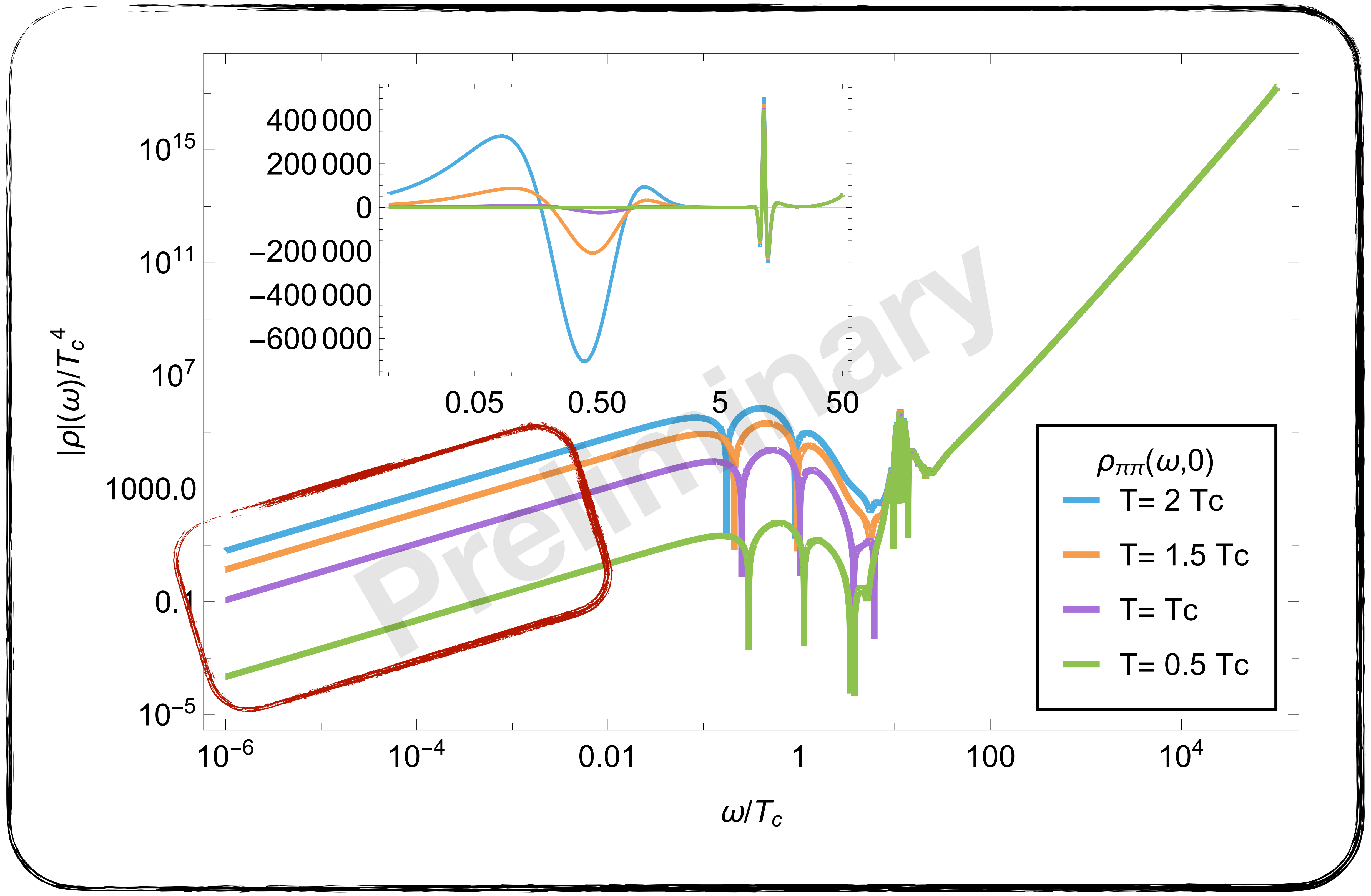




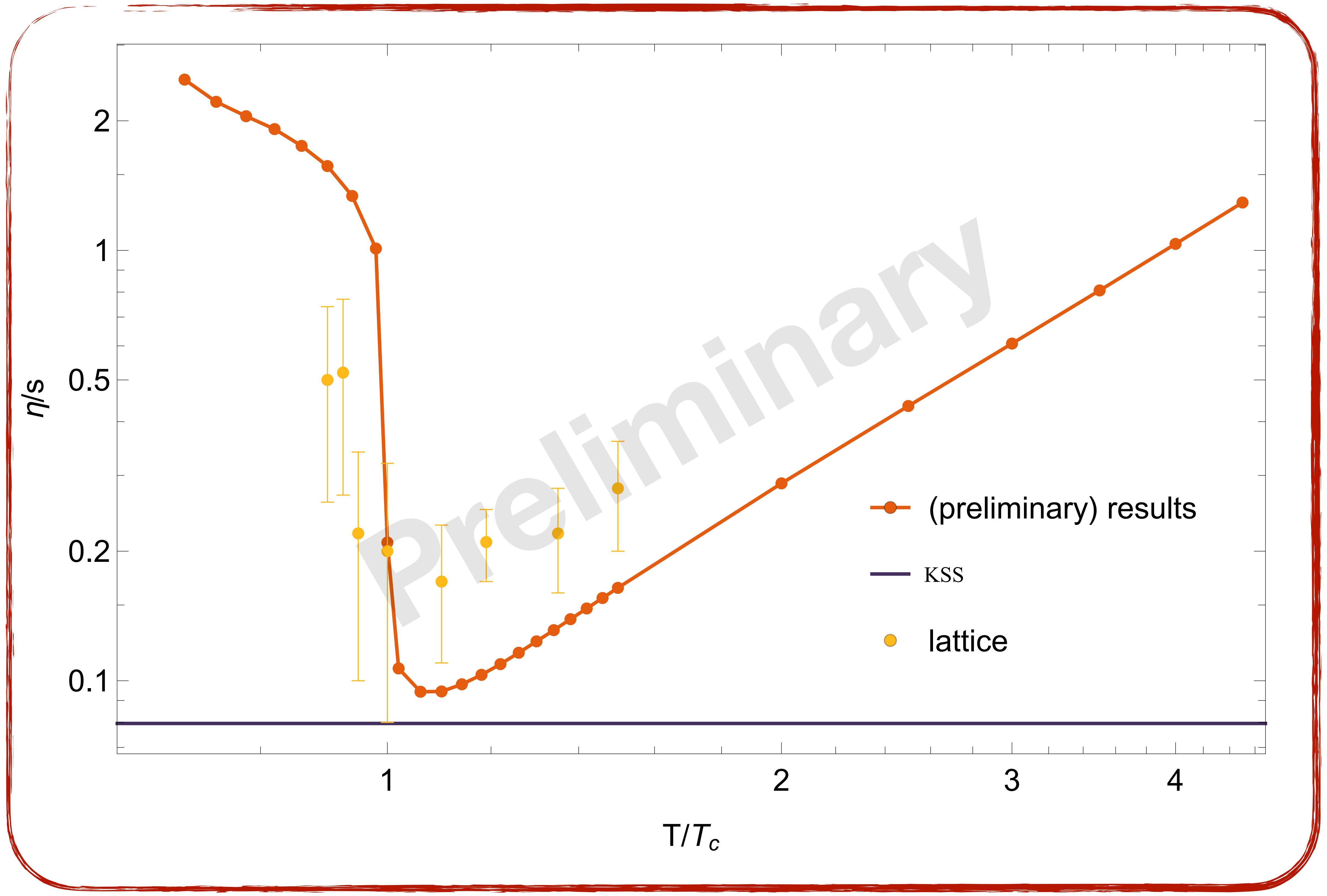
M. Haas, L. Fister, J. M. Pawlowski, 1308.4960
N. Christiansen, M. Haas, J. M. Pawlowski, N. Strodthoff, 1411.7986

EMT spectral function and shear viscosity

TT-EMT spectral function $\rho_{\pi\pi}$



Shear viscosity

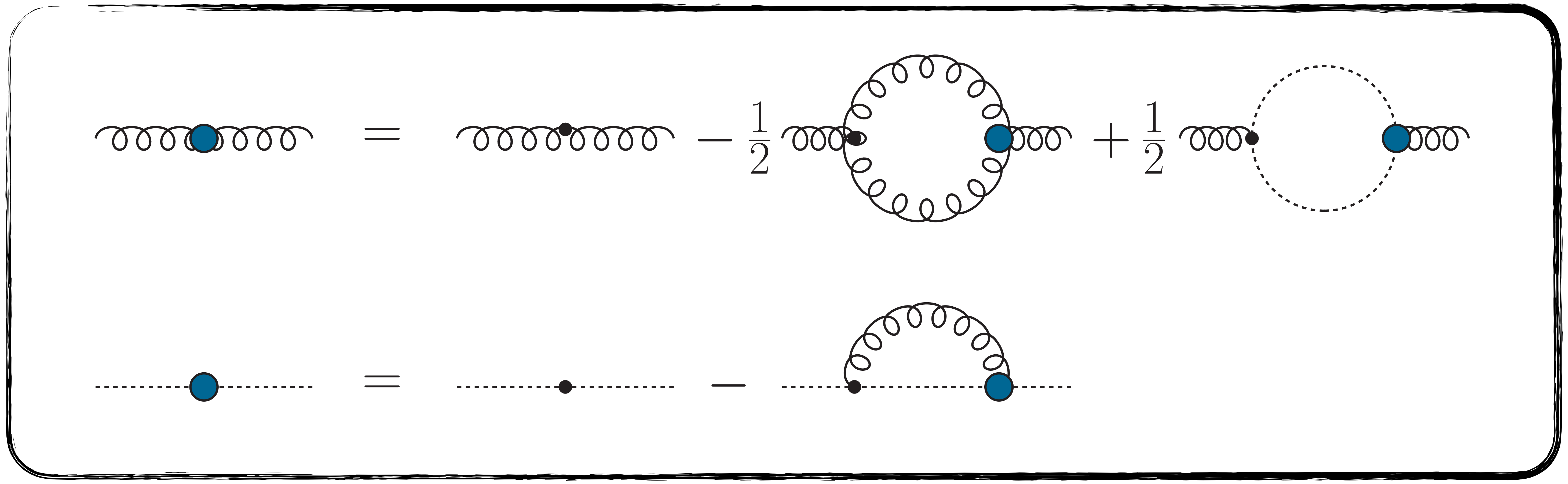


can calculate Euclidean EMT correlator → fix T- dep. normalisation with lattice data

J. Dolgner, JH, J.M. Pawlowski, N. Wink, in preparation

The spectral Dyson-Schwinger equation in YM

- **Spectral DSEs** allow for computing propagators directly **real-time**
- Here: Landau gauge Yang-Mills theory



- Källén-Lehmann **spectral representation**

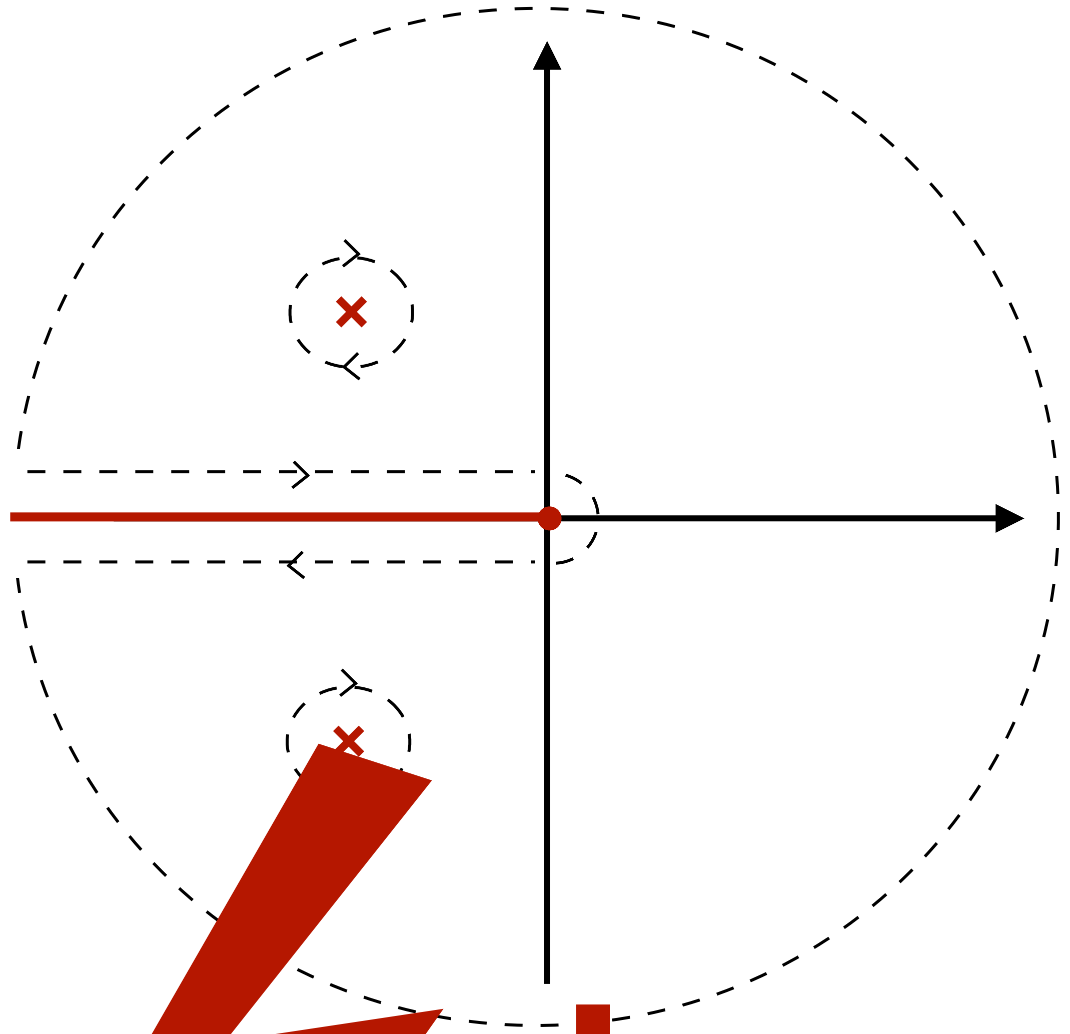
$$G_\phi(p) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda\, \rho_\phi(\lambda)}{p^2 + \lambda^2} \quad \text{with} \quad \rho_\phi(\omega) = 2\, \mathrm{Im}\, G_\phi\big(-\mathrm{i}(\omega + \mathrm{i}0^+)\big)$$

⇝ Calculate **diagrams** at **real frequencies**

Scalar theory: JH, J. M. Pawlowski, N. Wink, 2006.09778
Ghost: JH, J. Papavassiliou, J. M. Pawlowski, N. Wink, 2103.16175

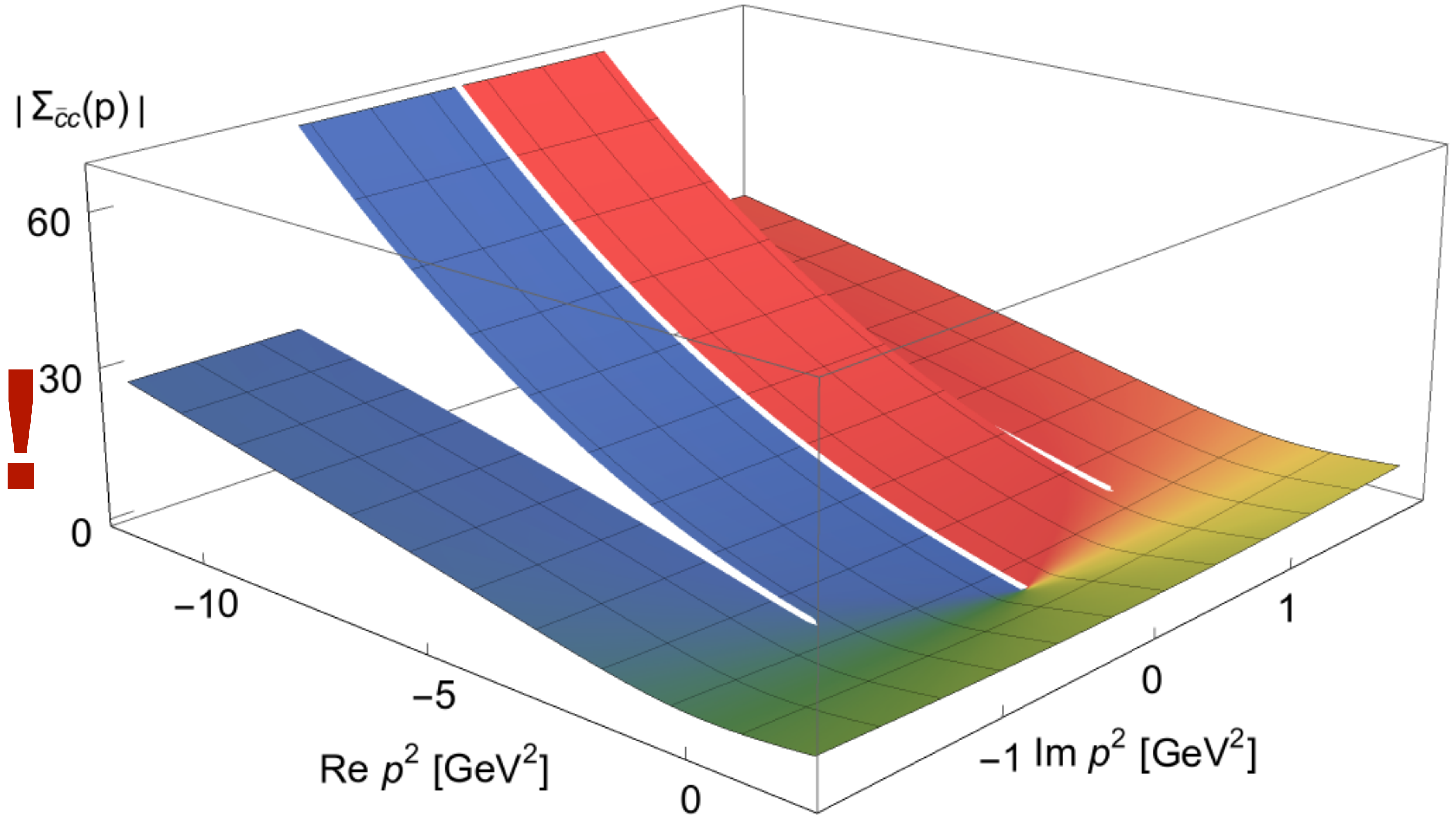
Complex structure of YM with bare vertices

Complex-conjugate poles gluon propagator

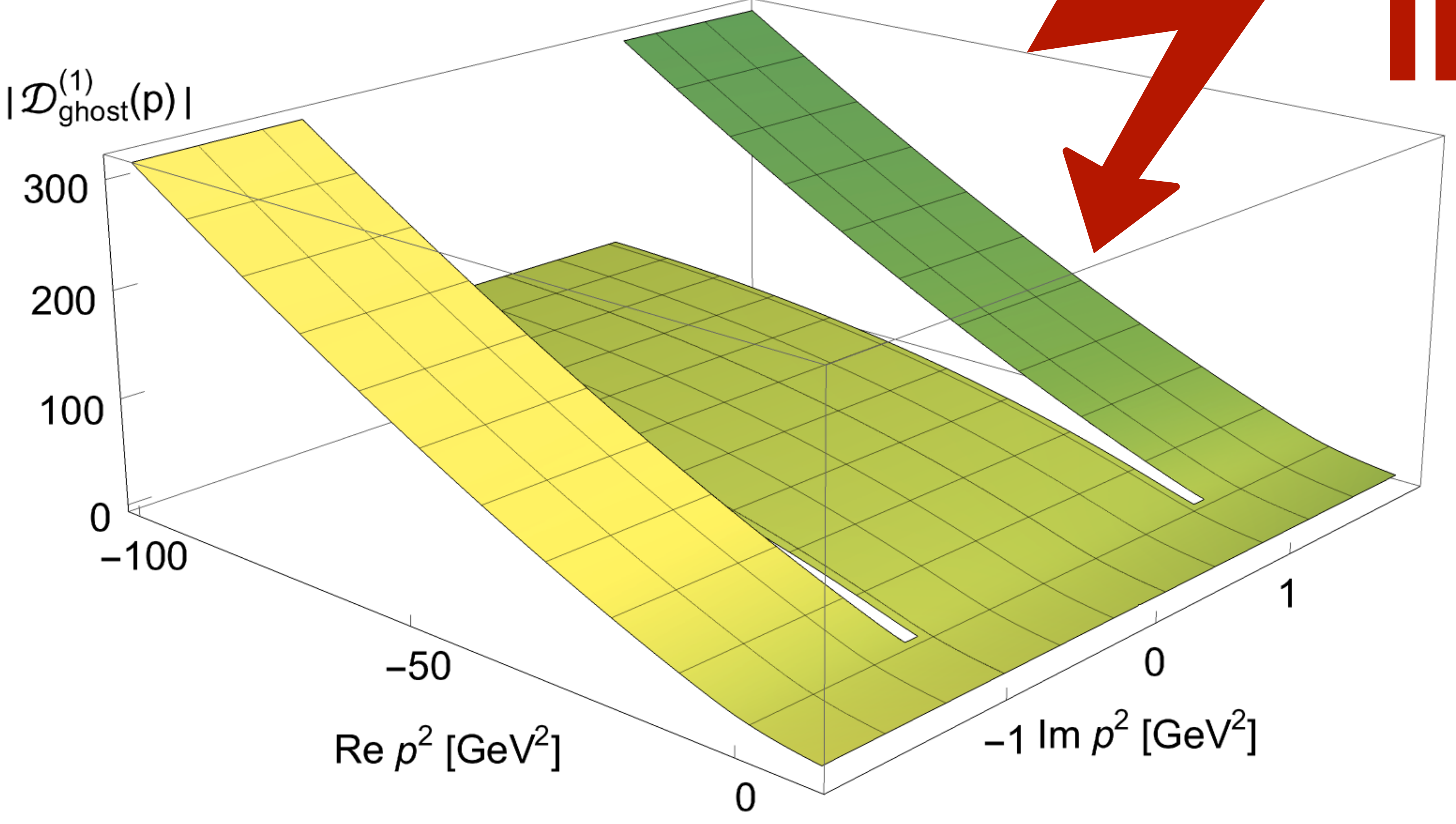


FULLY ANALYTIC

Violation of **ghost spectral representation**



Inconsistent!



Additional branch cuts

in gluon propagator

JH, J.M. Pawlowski, N. Wink, 2203.09333

Renormalised spectral flows

Going timelike with flow equations?

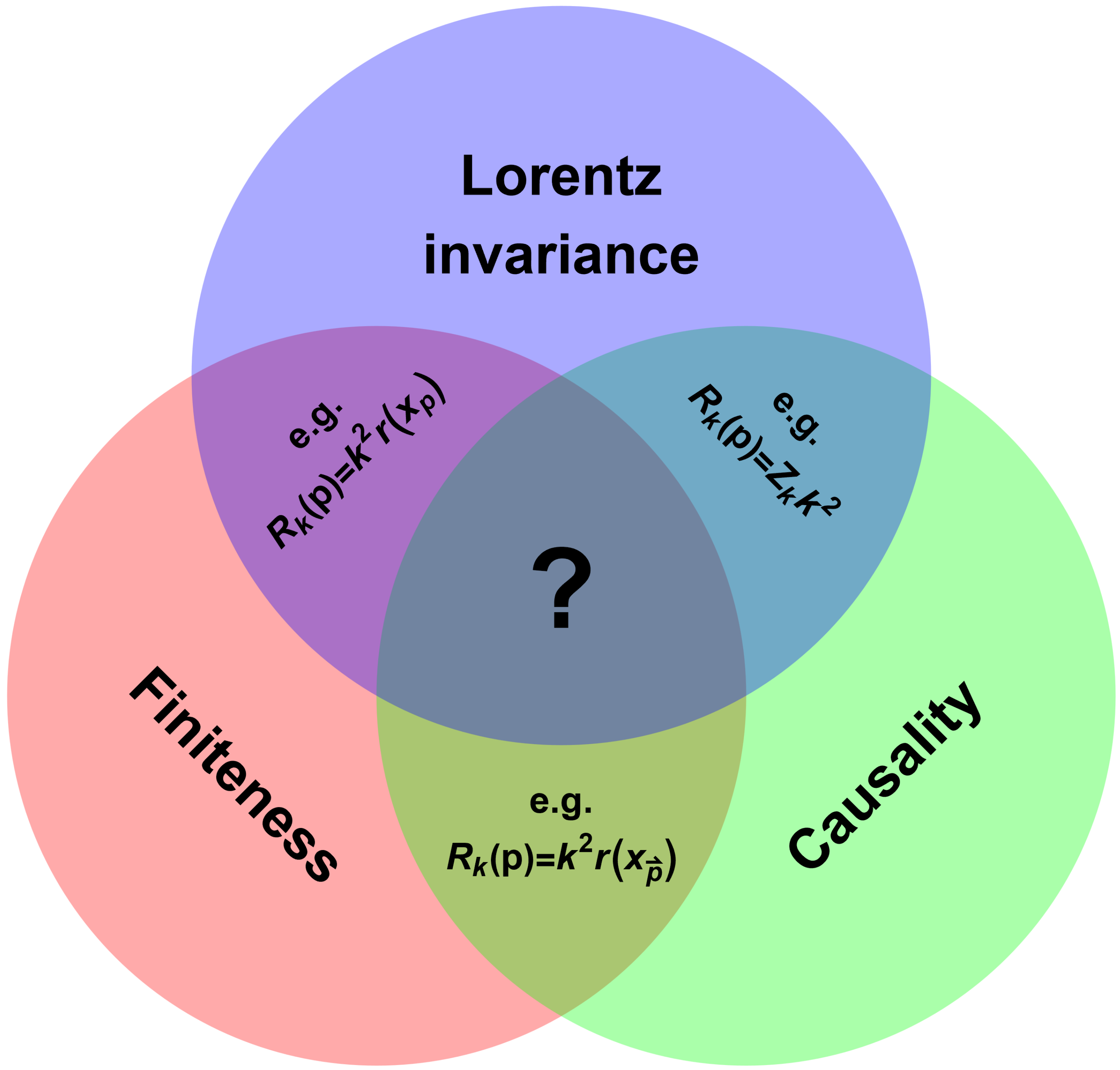
Callan-Symanzik (CS) regulator $R^{\phi}_{\mathrm{CS},k} = Z_\phi \, k^2$

→ **causal** & **Lorentz invariant**

Callan-Symanzik flow equation:

$$\partial_t \Gamma_k[\phi] = \mathrm{Tr}\, G_\phi[\phi]\, k^2 - \boxed{\partial_t S_{\mathrm{ct},k}[\phi]}$$

Regulator trinity



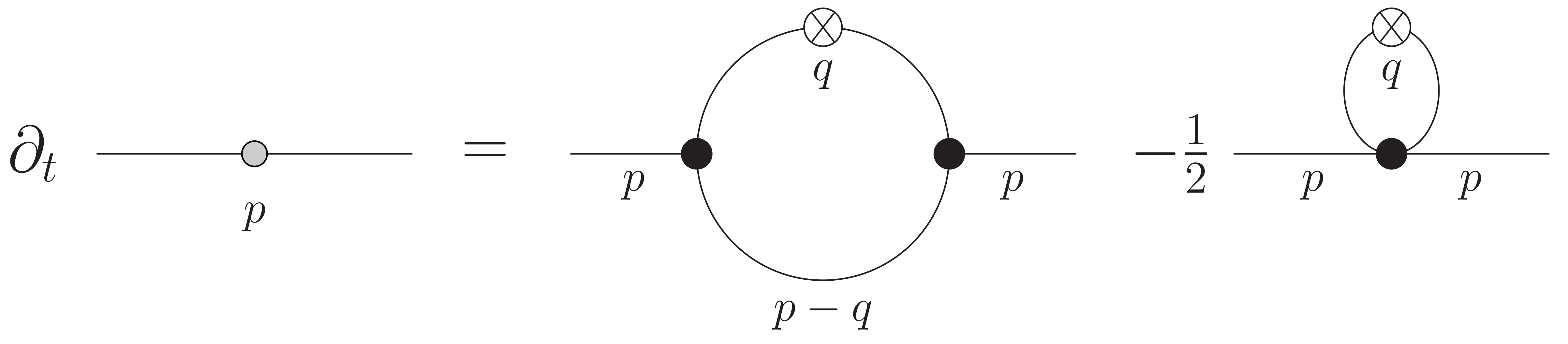
- Counter term action S_{ct} guarantees for **finiteness** and carries **flowing renormalisation conditions**

JH, fQCD collaboration, 2206.10232

Spectral fRG in the scalar theory

poster by Jonas Wessely

CS flow for **propagator** in the broken phase**:**



$-\quad \mathrm{diags}\,(p^2 = -k^2)$

on-shell renormalisation!

- Spectral representation

$$G_\phi(p) = \int_0^\infty \frac{d\lambda}{\pi} \frac{\lambda\, \rho_\phi(\lambda)}{p^2 + \lambda^2}$$

Evaluate at **real frequencies**

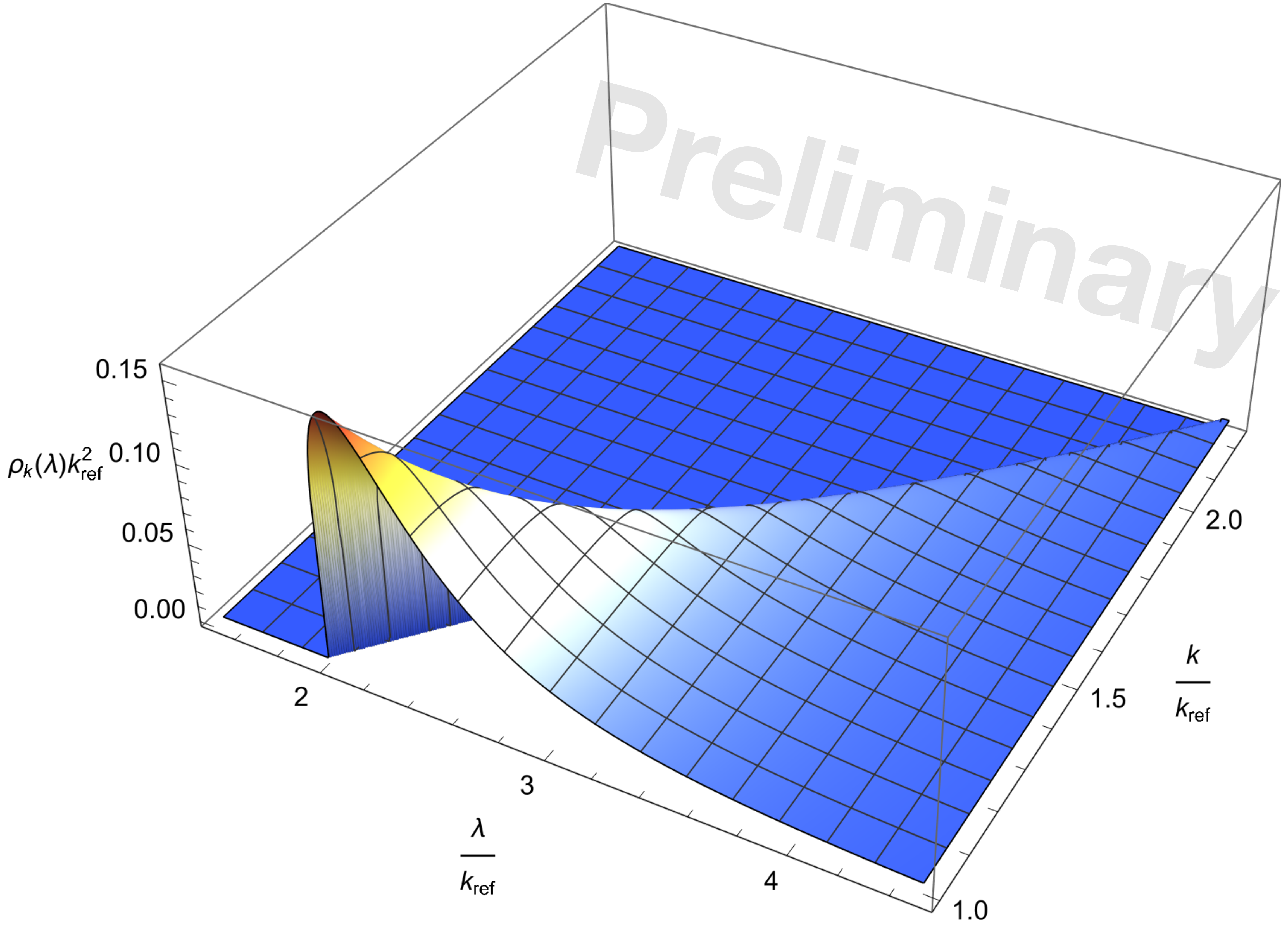
F. Ihssen, JH, J. M Pawlowski, **J. Wessely,** N. Wink, in preparation

Spectral function in theory space

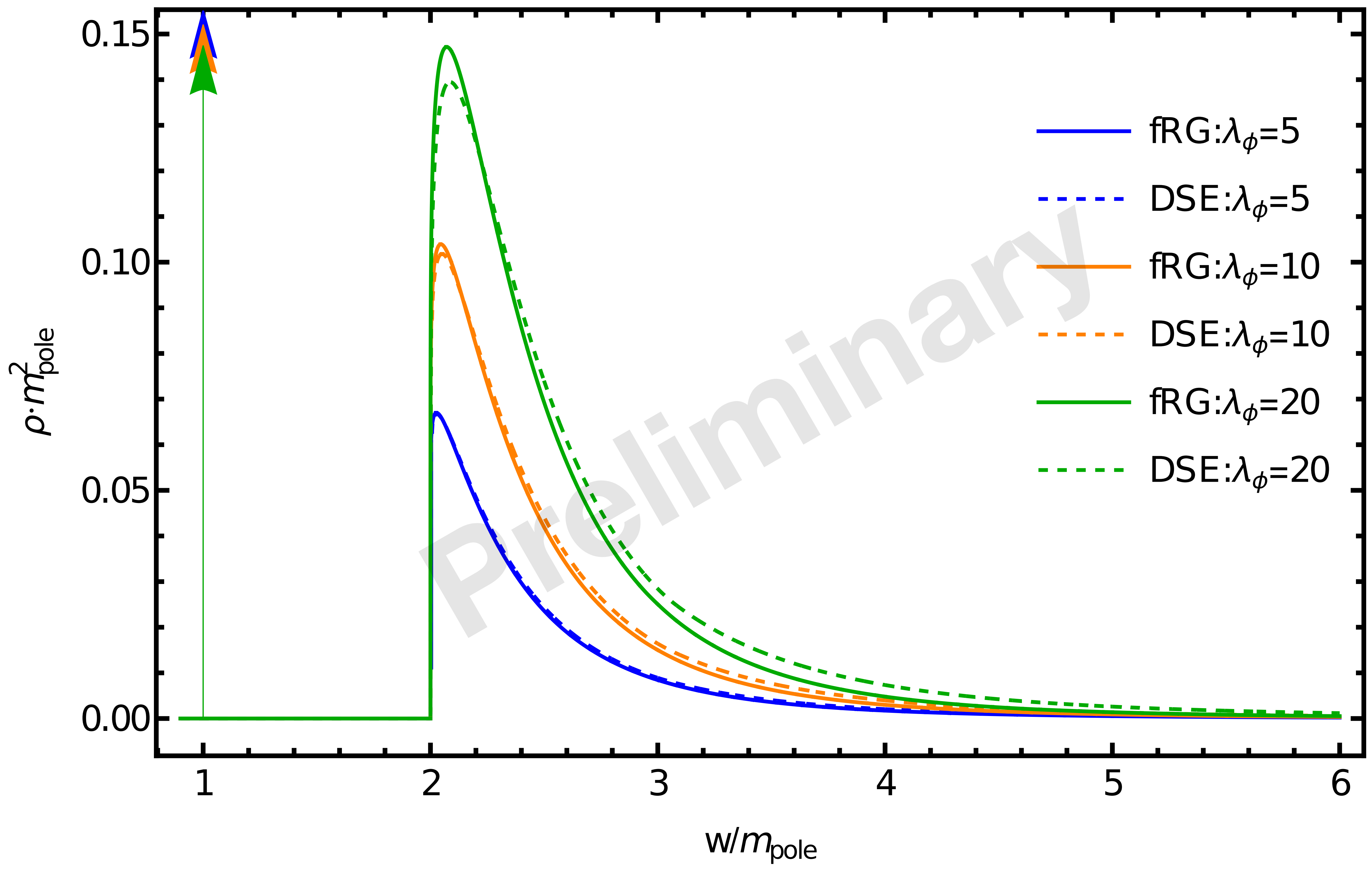
poster by Jonas Wessely

Gravity: talk by Manuel Reichert

Spectral function in theory space



Comparison with DSE

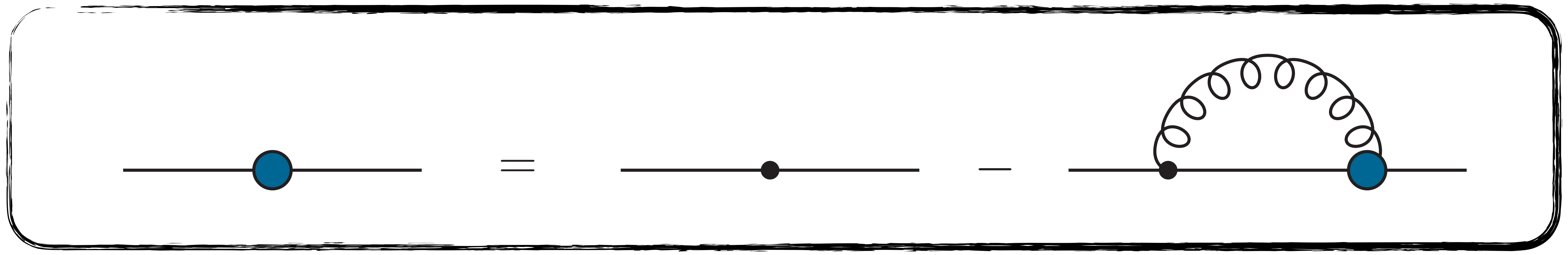


F. Ihssen, JH, J. M Pawlowski, **J. Wessely,** N. Wink, in preparation

Quark propagator from the spectral DSE

Quark propagator

$$G_q(p) = \frac{1}{Z(p)} \frac{1}{\mathrm{i}\not{p} + M(p)}$$



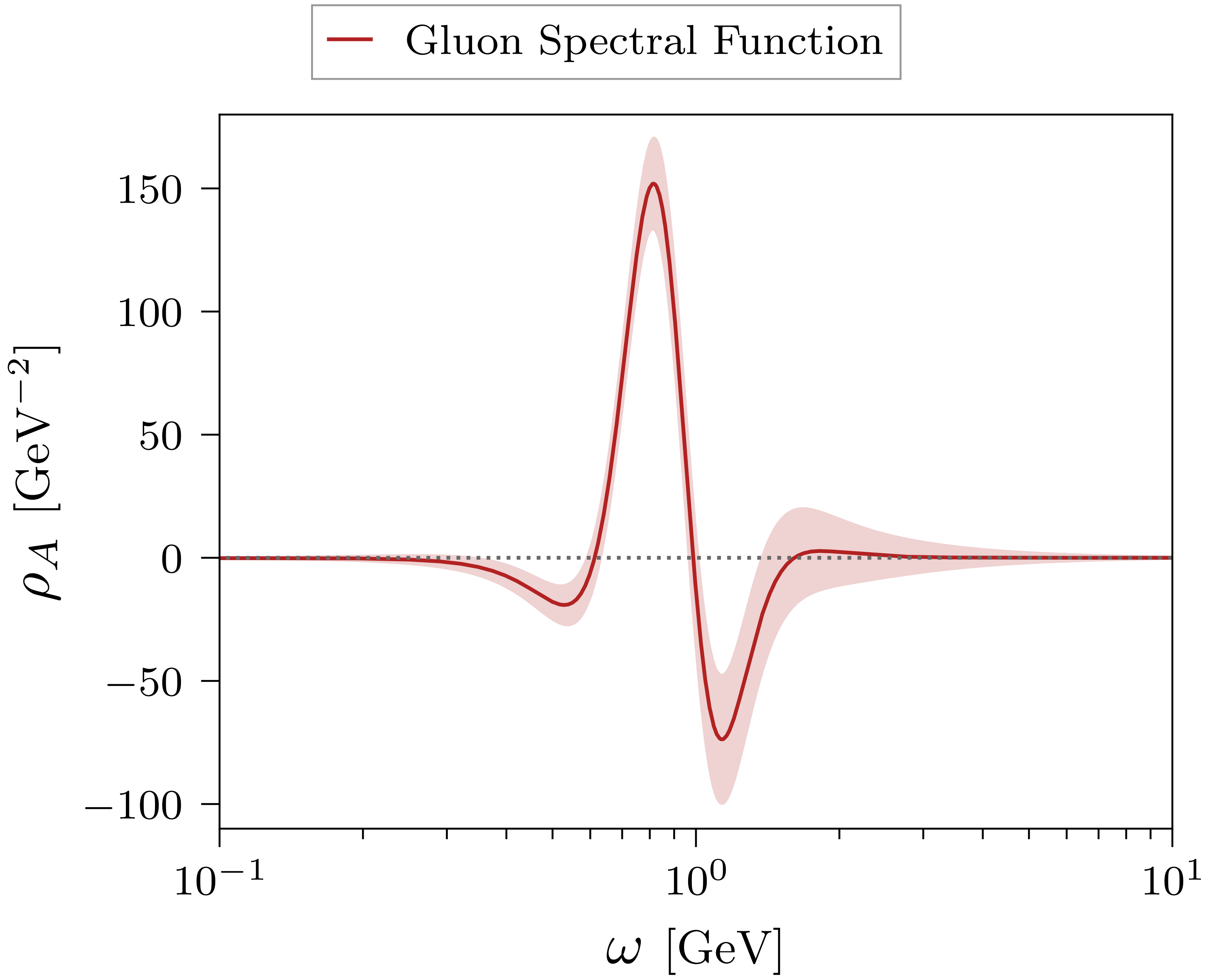
JH, J.M. Pawlowski, J. Rodriguez-Quintero J. Turnwald, J. Urban, N. Wink, S. Zafeiropoulos, 2107.13464

Quark spectral function has two tensor structures:

$$\rho_q(\omega) = \rho_D(\omega)\gamma_0 + \rho_M(\omega)$$

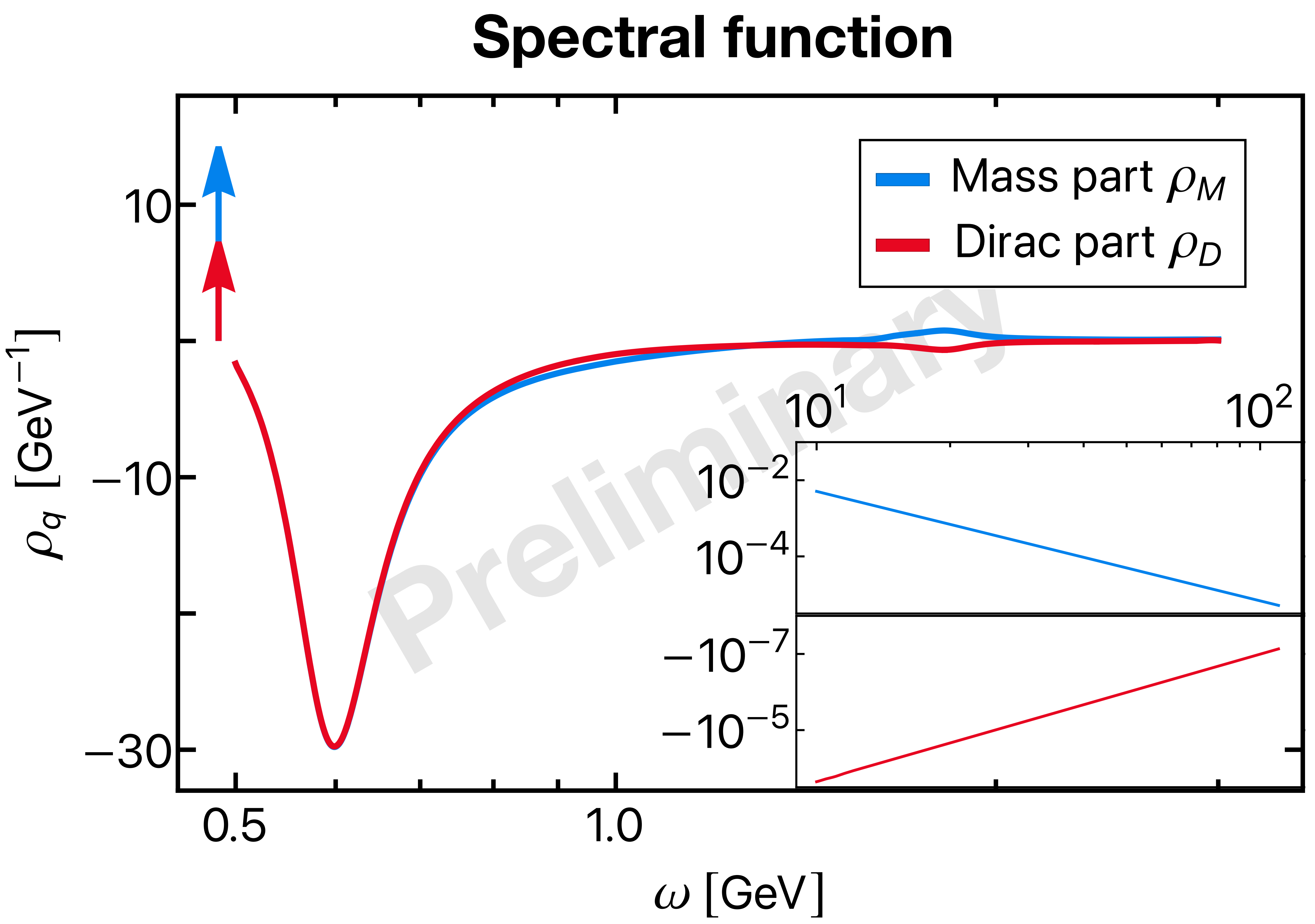
Input:

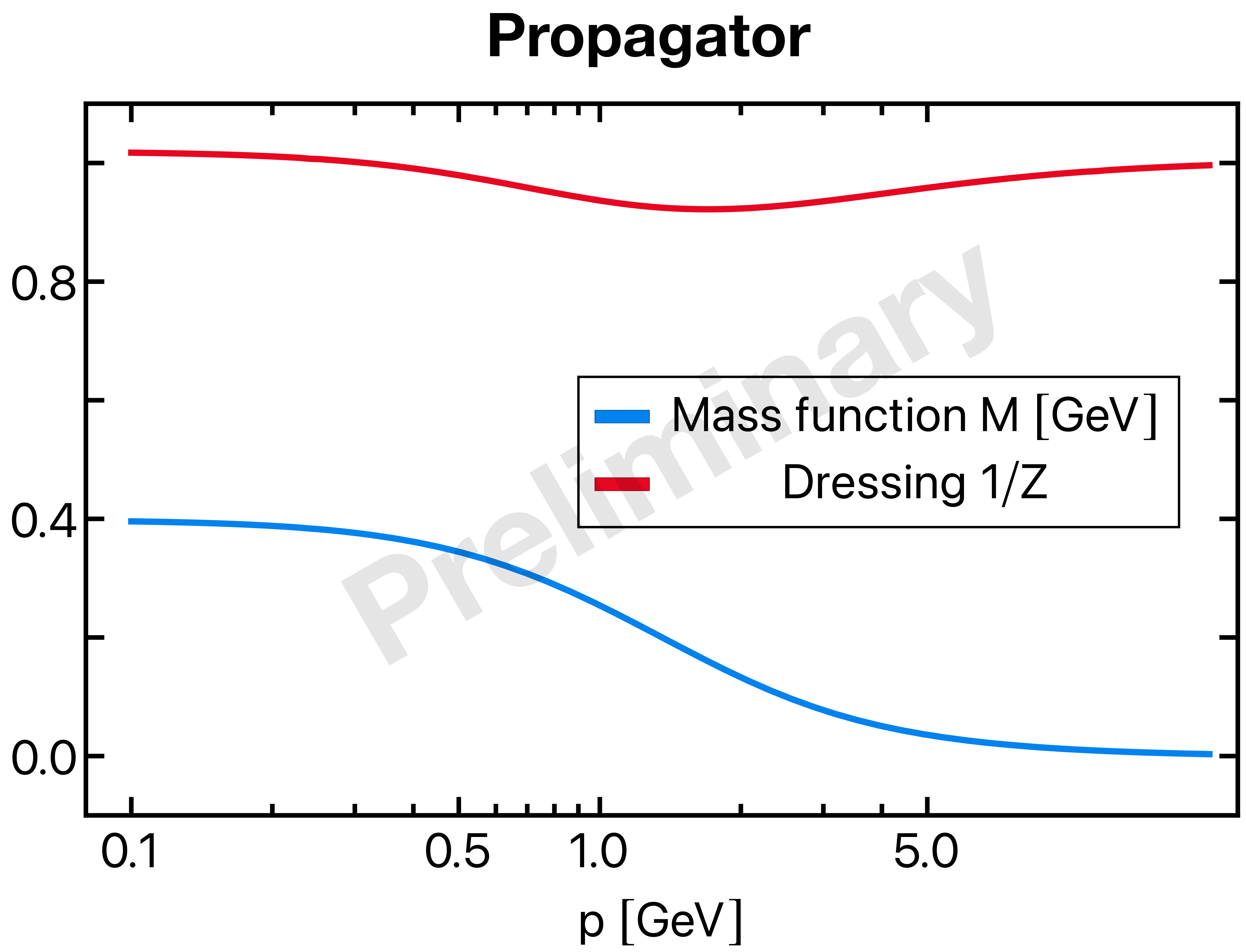
- Gluon spectral function from Gaussian process reconstruction of 2+1f lattice QCD data
- Classical quark-gluon vertex



Quark spectral function in QCD

Quark spectral function shows **quasi-particle peak** and **negative scattering tail**





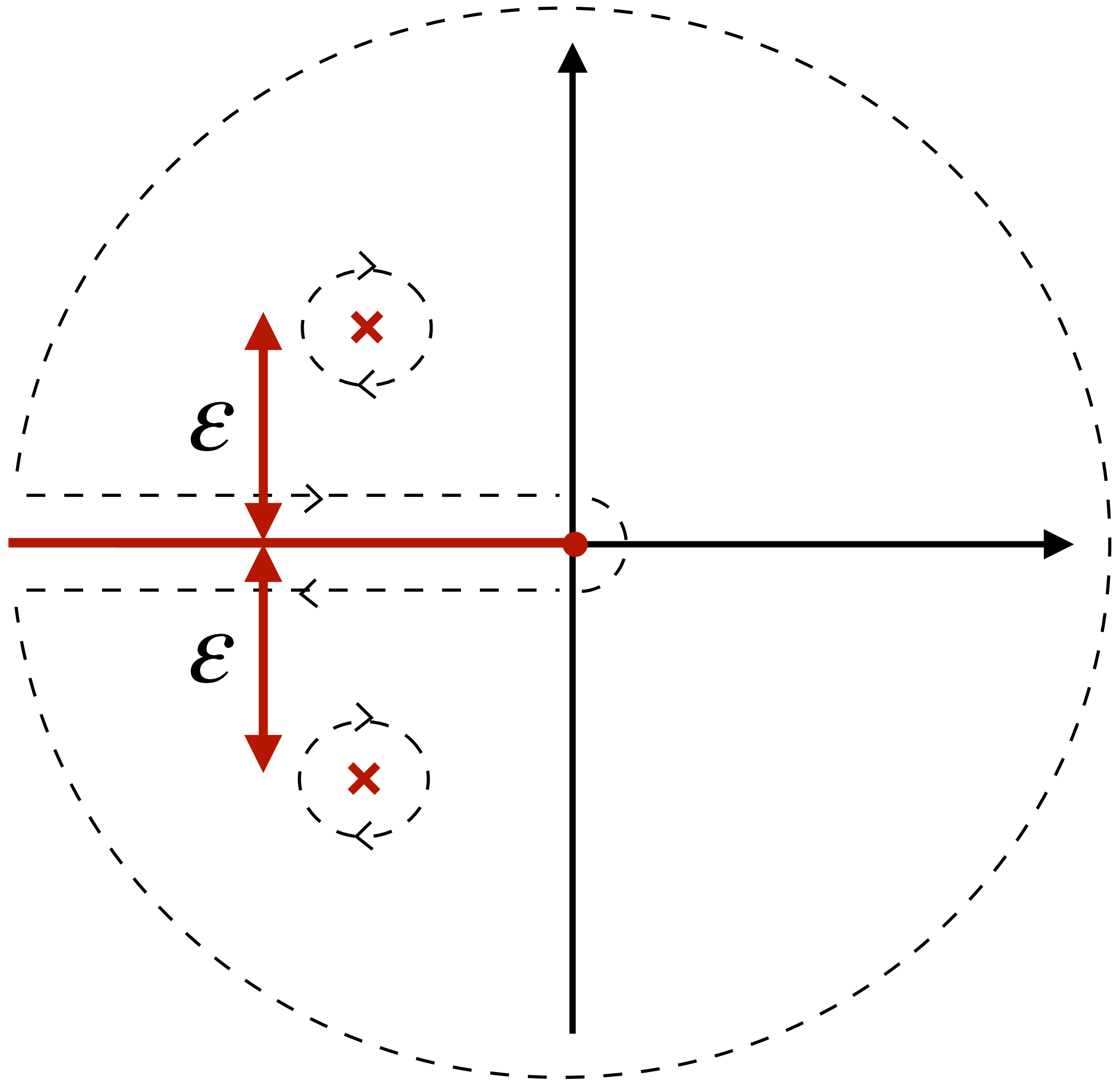
JH, J.M. Pawlowski, N. Wink, in preparation

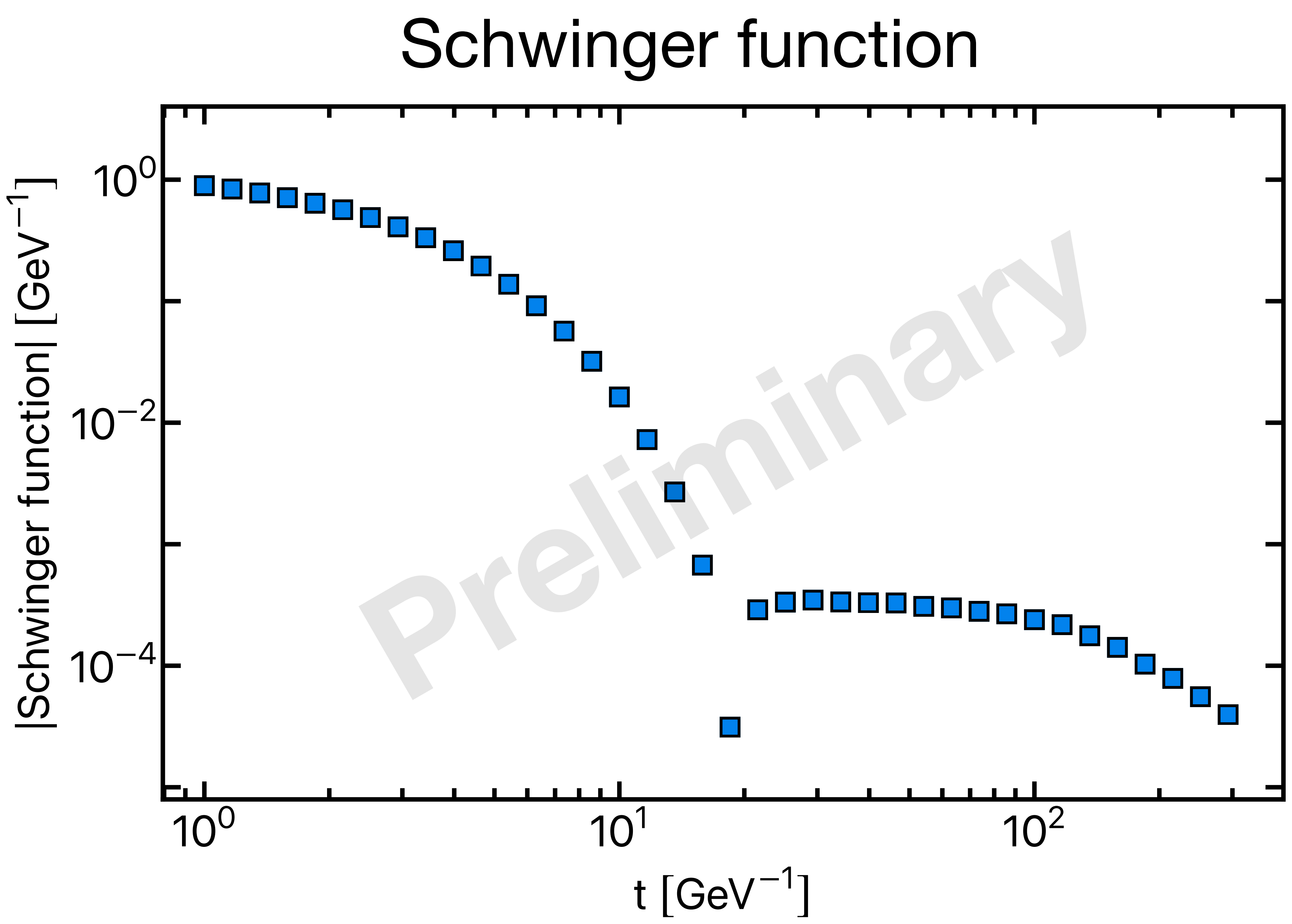
Complex structure of the quark propagator

A close look at the analytic structure reveals complex-conjugate poles at

$$m_{cc} \approx m_{\text{pole}} + \mathrm{i}\varepsilon \quad \text{and} \quad \varepsilon \ll 1$$

⟿ truncation artifact?





JH, J.M. Pawlowski, N. Wink, in preparation

Wrap up

- Complex properties of Yang-Mills disfavour complex-conjugate pole solutions
- Spectral functions in scalar theory from renormalised spectral flows
- QCD quark spectral function from spectral DSE

To come:

- QCD transport coefficients
- Quark spectral function at finite T
- Spectral Bethe-Salpeter equations