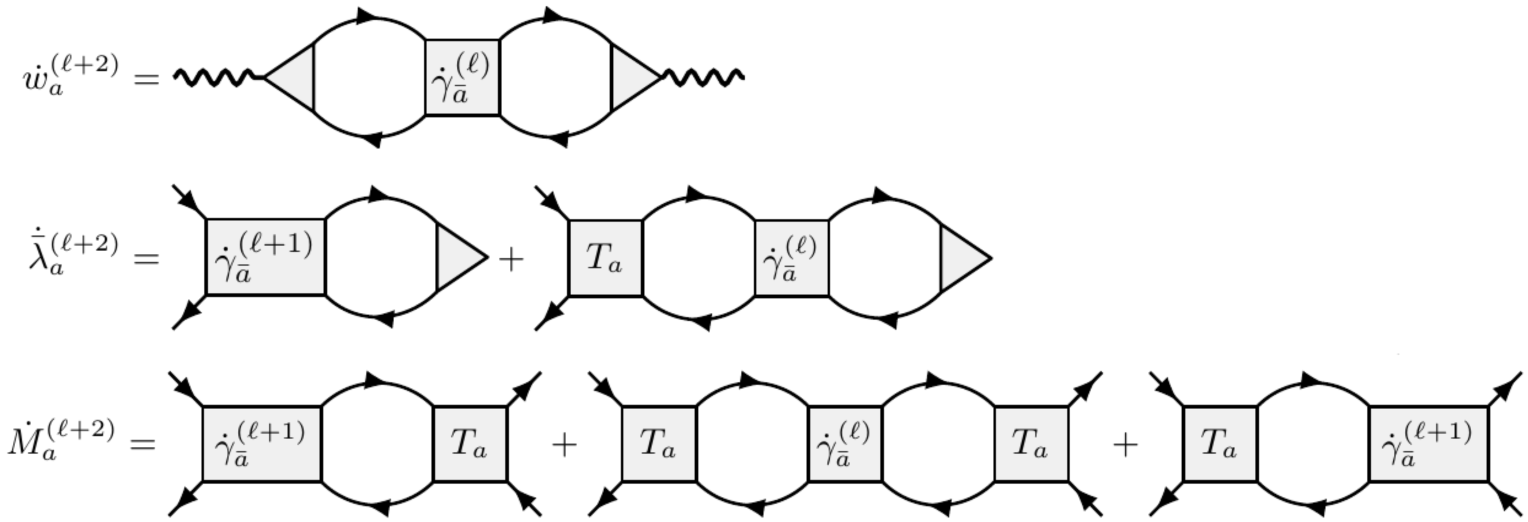
Multiloop flow equations for single-boson exchange fRG

Marcel Gievers

MPQ (Garching) / LMU (Munich)



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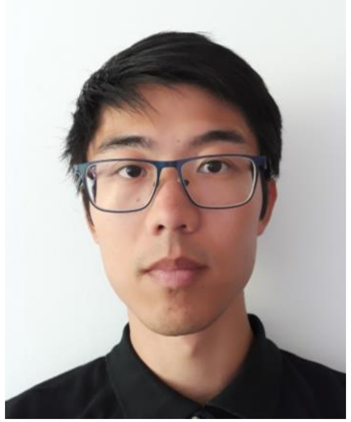
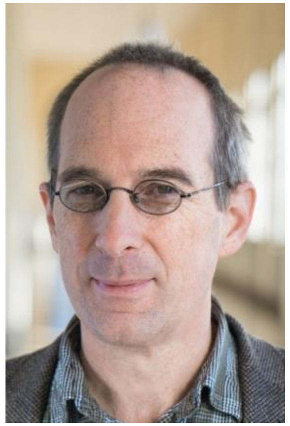


Anxiang Ge
LMU (Munich)

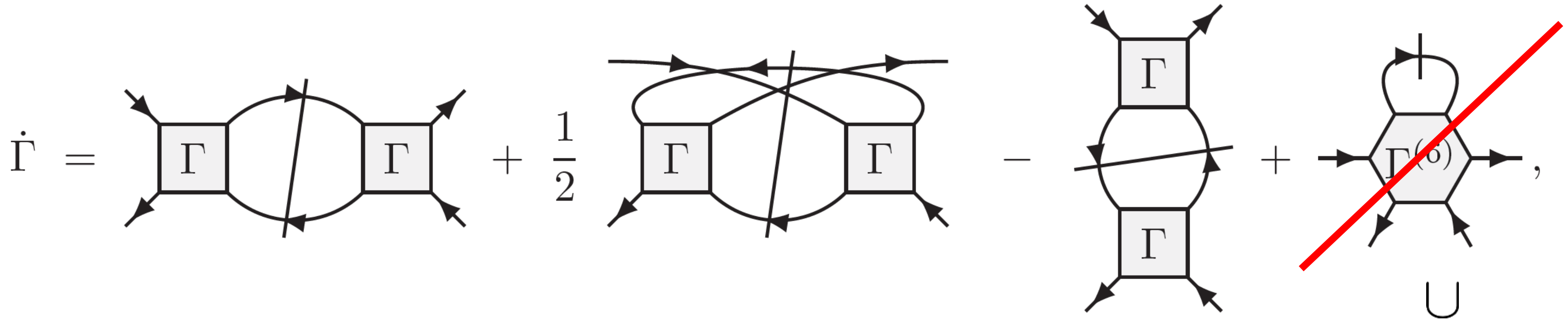


Jan von Delft
LMU (Munich)

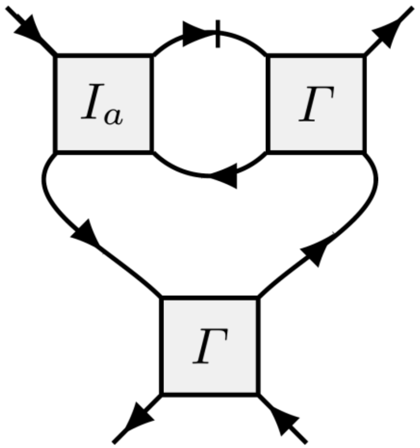
Fabian Kugler
Rutgers U. (NJ, USA)

Background

- **Wetterich equation** → exact formulation, but vertex expansion needed
- **one-loop fRG** → 6-point vertex $\Gamma^{(6)}$ not included



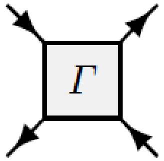
- **multiloop fRG** (mfRG) → 2PR parts of $\Gamma^{(6)}$ included
  - equivalent to parquet formalism
  - regulator independence
  - Mermin-Wagner theorem



Kugler, von Delft PRB, PRL, NJP (2018)

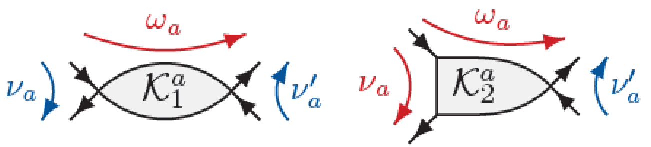
Motivation for single-boson exchange (SBE)

- problem: high-complexity of 4-point vertex $\Gamma = \Gamma_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}(\omega, \nu, \nu')$



→ parametrization in terms of its **asymptotics**

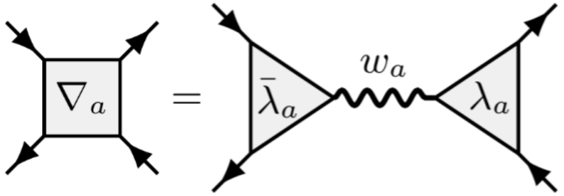
Wentzell, Li, Tagliavini, Taranto, Rohringer, Held, Toschi, Andergassen
arXiv (2016), PRB (2020)



Hubbard-like models:

→ vertex decomposition in **single-boson exchange** (SBE) processes

Krien, Valli, Capone, Lichtenstein, Rohringer, Held, Kauch, Harkov
PRBs (2019, 2020, 2021)



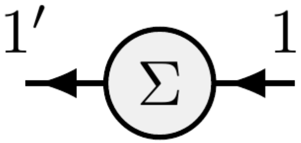
→ **1l fRG in SBE** decomposition in Hubbard model

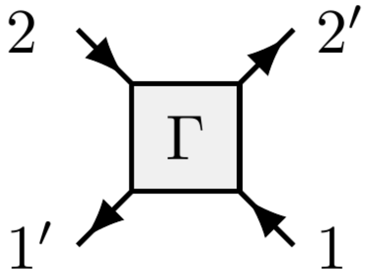
Bonetti, Toschi, Hille, Andergassen, Vilardi PRR (2022)

→ general formulation of SBE, SBE mfRG flow equations

Setting: Fermionic theory

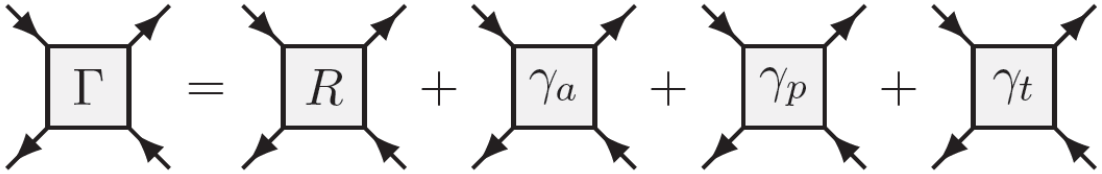
- general fermionic action: $S = -\bar{c}_{1'}[G_0^{-1}]_{1'|1}c_1 - \frac{1}{4}U_{1'2'|12}\bar{c}_{1'}\bar{c}_{2'}c_2c_1$
- constant energy-conserving bare interaction U 
- 1- & 2-particle correlation functions: $G_{1|1'} = -\langle c_1\bar{c}_{1'}\rangle \quad G^{(4)}_{12|1'2'} = \langle c_1c_2\bar{c}_{2'}\bar{c}_{1'}\rangle$
- self-energy Σ and 4-point vertex Γ

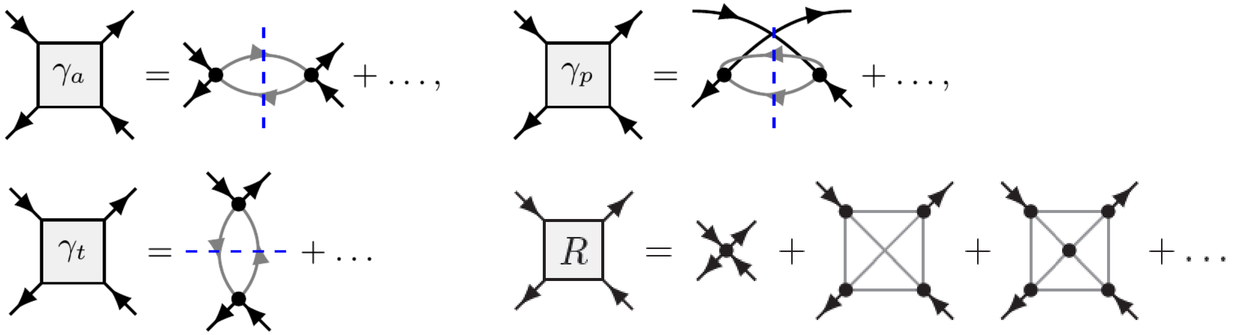
$\Sigma_{1'|1} =$ 

, $\Gamma_{1'2'|12} =$ 

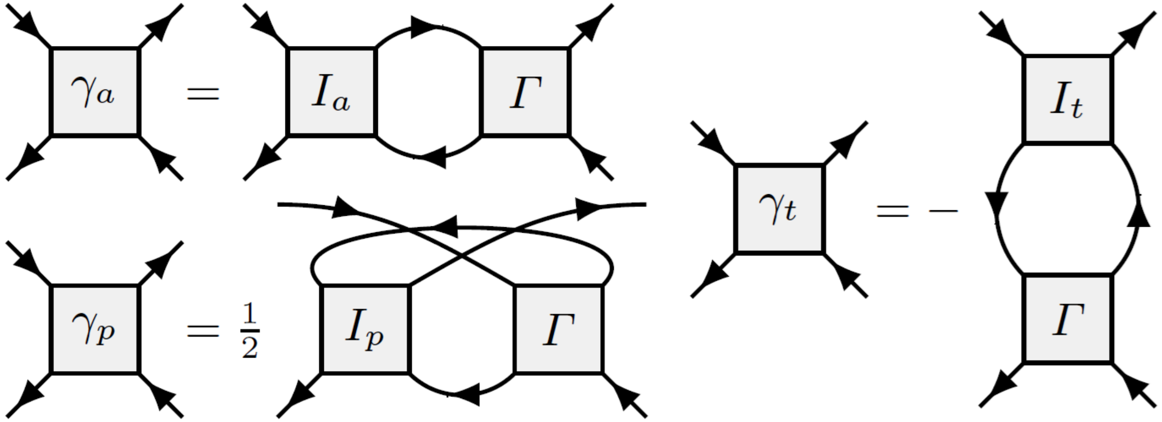
Parquet decomposition

- full 1PI vertex Γ as sum of 2PR vertices γ_r





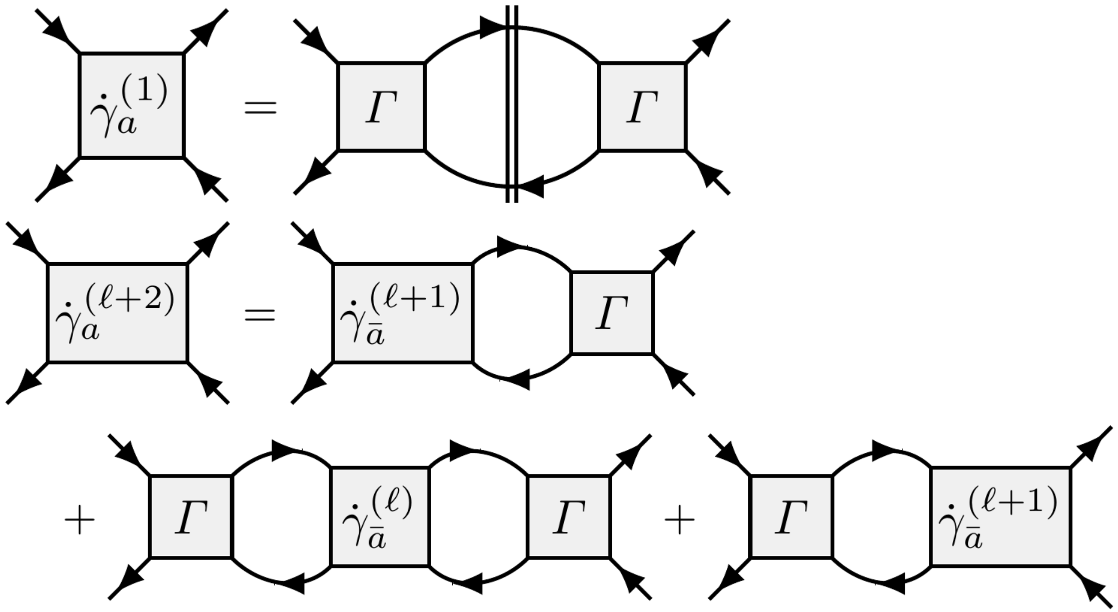
- 2PR vertices γ_r are built from Bethe-Salpeter equations (BSE) $\gamma_r = I_r \circ \Pi_r \circ \Gamma$

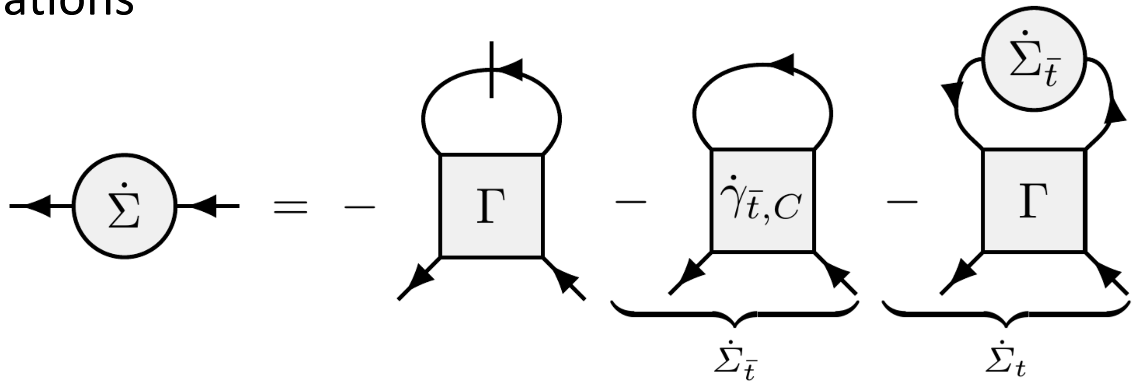


antiparallel ($\overline{ph}$)
parallel (pp)
transversal (ph)

Multiloop fRG

- make bare Green's function scale dependent $G_0 \to G_0(\Lambda) \Rightarrow \Sigma(\Lambda), \Gamma(\Lambda)$
- no flowing 2PI vertex $R \neq R(\Lambda)$
- differentiating BSE leads to multiloop fRG (mfRG) equations

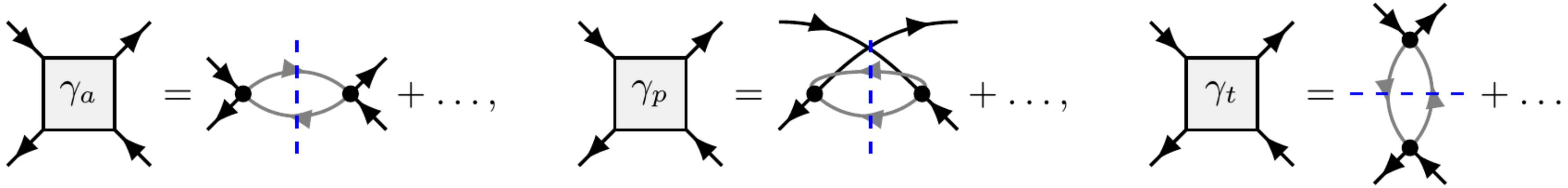




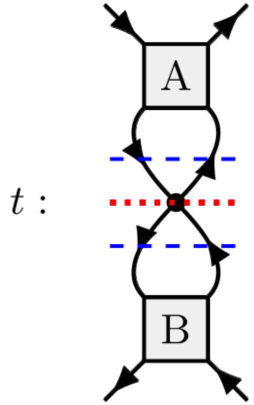
Kugler, von Delft PRB, PRL, NJP (2018)

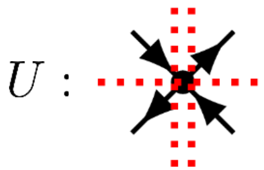
2P reducibility vs. U reducibility

- 2P reducibility: cutting two Green‘s function lines, i.e., a bubble Π_r



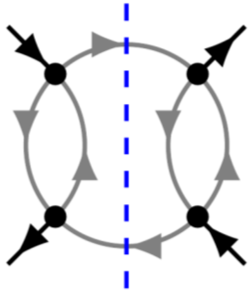
- U reducibility: cutting out a bare interaction vertex U





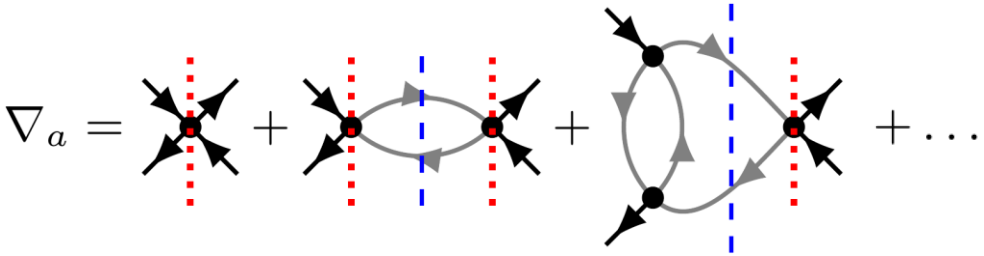
(except for U itself)

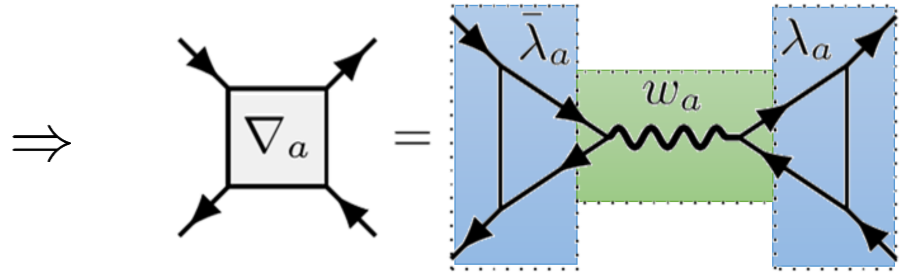
- U reducibility in r $\Rightarrow$ 2P reducibility in r
- but: 2P reducibility in r $\not\Rightarrow$ U reducibility



Single-boson exchange vertex

- U-reducible vertices ∇_r → single-boson exchange





- effective interaction w_r → propagator of an exchange boson

$$w_r = U + U \bullet \chi_r \bullet U$$

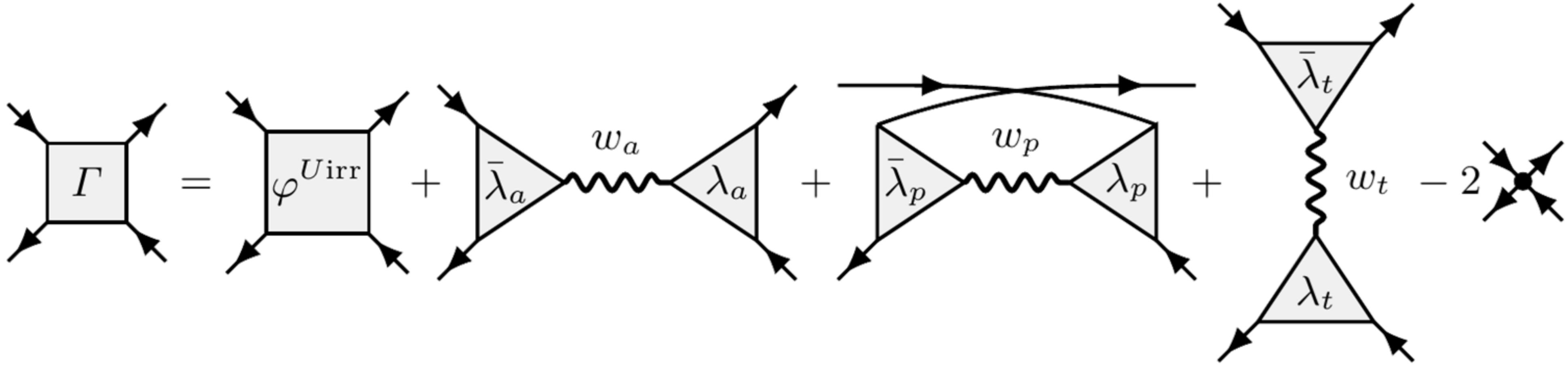
susceptibility

exchange field: $\psi = \sum_\nu \bar{c}\, c$

pairing field: $\phi = \sum_\nu c\, c$

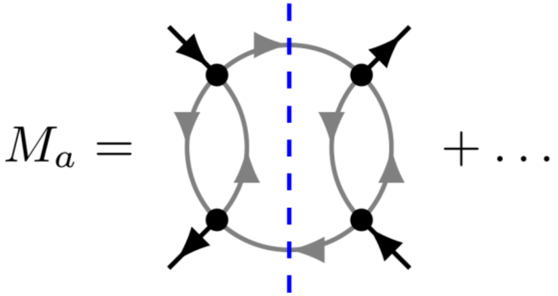
- Hedin vertices $\lambda_r, \bar{\lambda}_r$ → exchange process between one boson and two fermions

SBE Decomposition

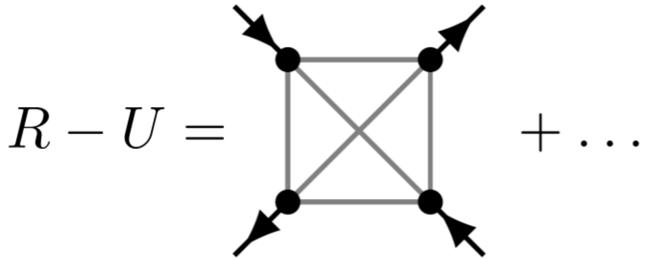


Krien, Valli, Capone, Lichtenstein, Rohringer, Held, Kauch, Harkov PRBs (2019,2020,2021)

- SBE core $\varphi^{U\mathrm{irr}}$:



multi-boson exchange



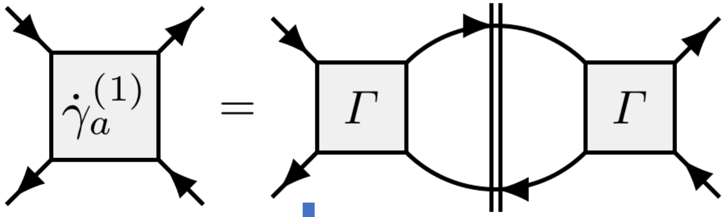
totally irreducible

- closed system of equations: SBE equations (similar to parquet equations)

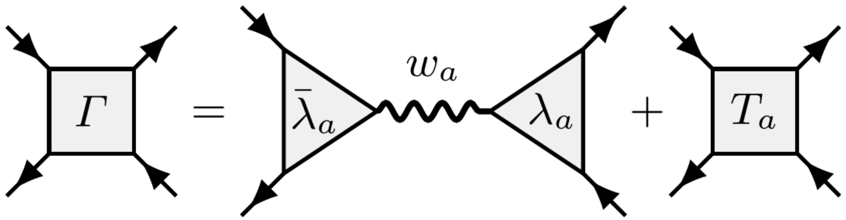
  → SBE mfRG

SBE fRG (one loop)

1-l fRG equation

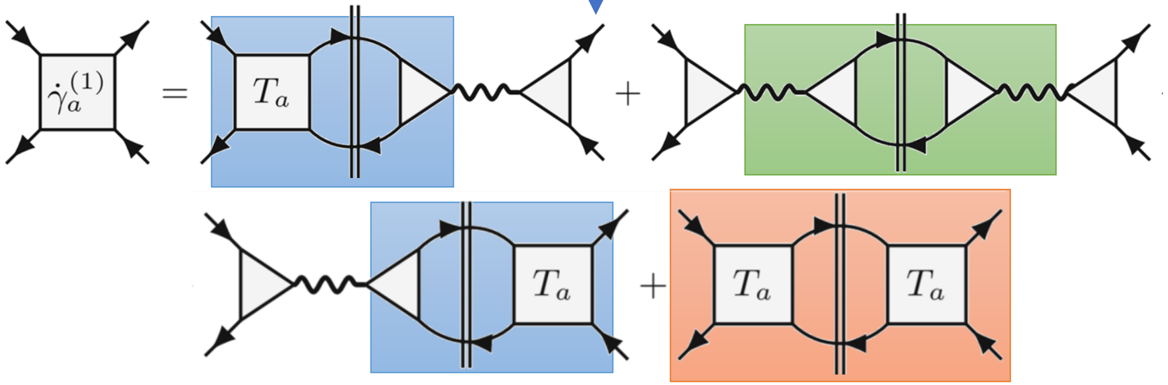


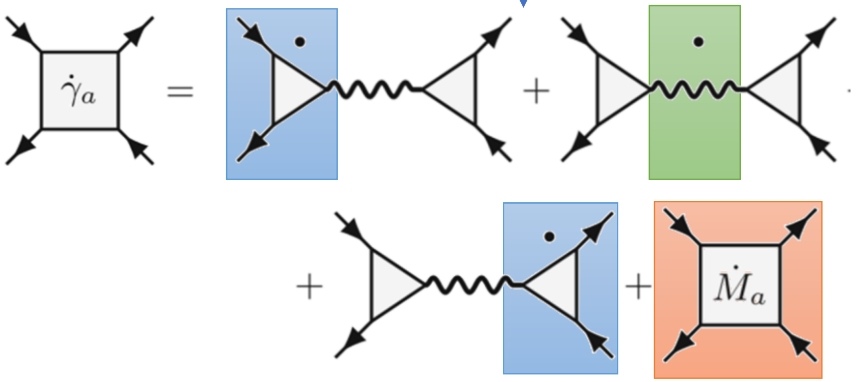
SBE decomposition

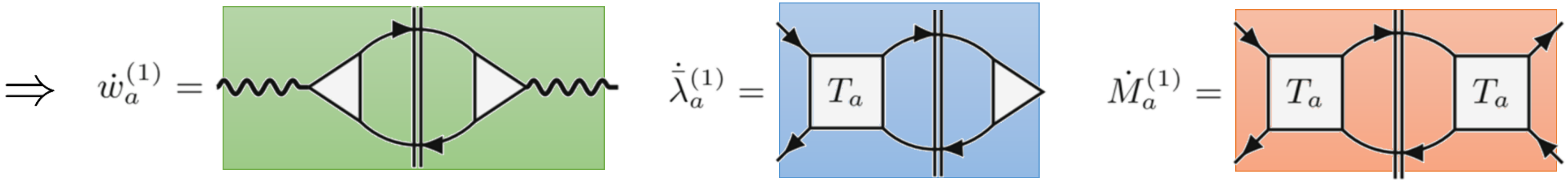


insertion

differentiation

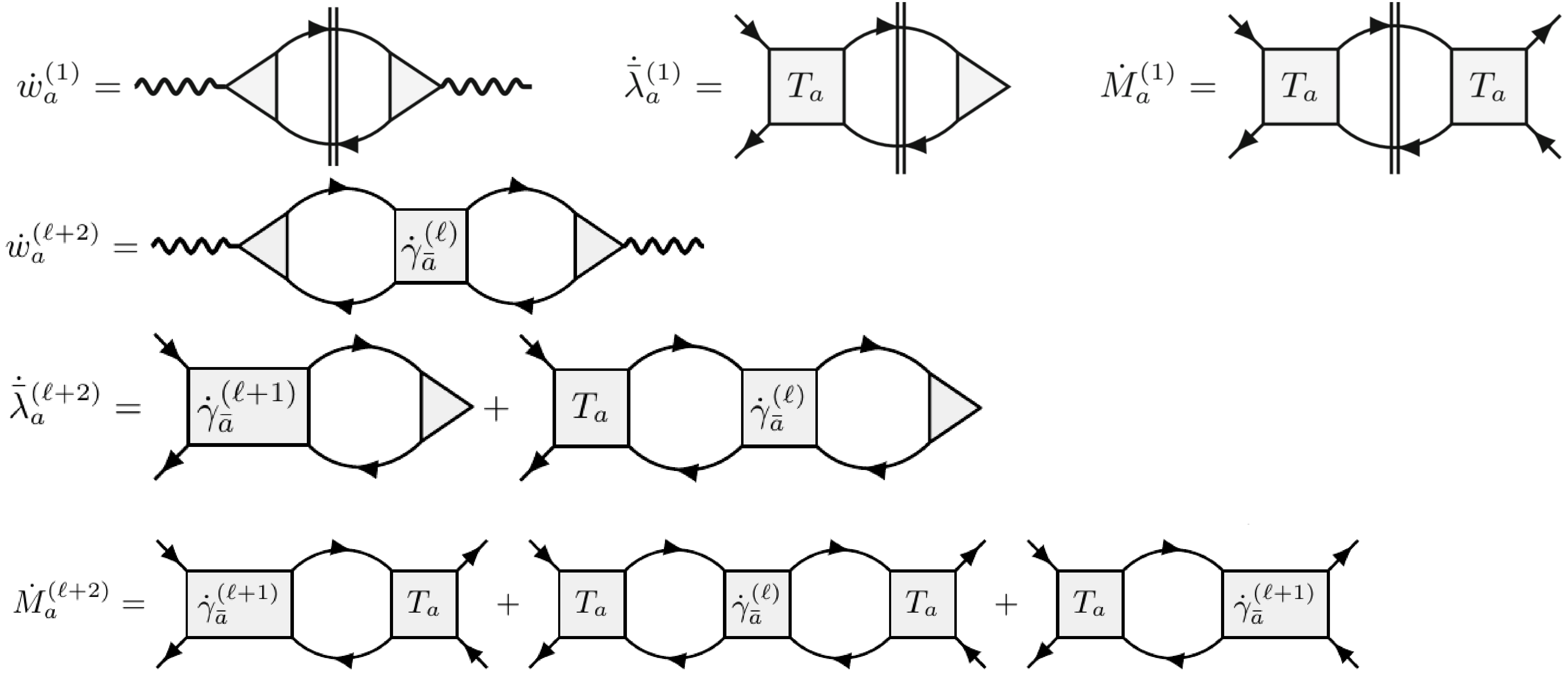






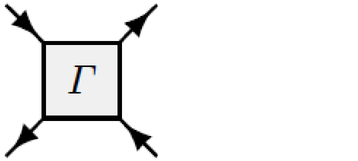
Bonetti et al. PRR (2022)

mfRG for SBE ingredients

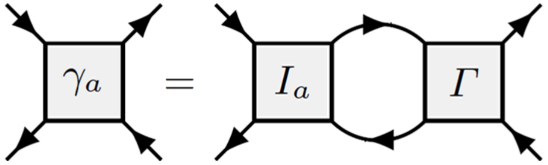


Asymptotic classes vs. SBE ingredients

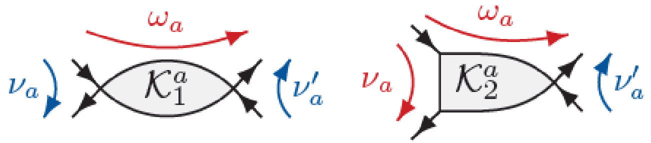
- full vertex: Γ



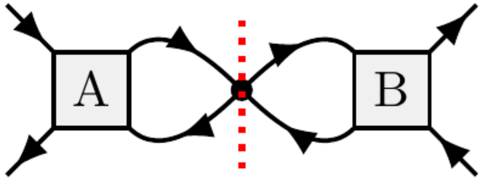
- parquet decomp.: $R + \sum_r \gamma_r$



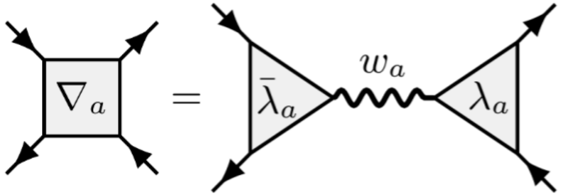
- asymptotic classes: $R + \sum_r [\mathcal{K}_1^r + \mathcal{K}_2^r + \mathcal{K}_{2'}^r + \mathcal{K}_3^r]$



- U reducibility: $R - U + U + \sum_r [\bar{\lambda}_r \bullet w_r \bullet \lambda_r - U + M_r]$



- SBE decomp.: $\varphi^{U\mathrm{irr}} - 2U + \sum_r \bar{\lambda}_r \bullet w_r \bullet \lambda_r$



  - SBE approximation: $\varphi^{U\mathrm{irr}} \to 0$
  - $\mathcal{K}_2$ approximation: $\varphi^{\mathcal{K}_3} = R - U + \sum_r \mathcal{K}_3^r \to 0$ ← includes more diagrams!

Wentzell et al. PRB (2020)

Bonetti et al. PRR (2022)

Advantages of the SBE decomposition

- SBE vertex with transparent physical meaning:

$$\mathcal{K}_3^r = (\bar{\lambda}_r - \mathbf{1}_r) \bullet w_r \bullet (\lambda_r - \mathbf{1}_r) + M_r \qquad \nabla_r = \bar{\lambda}_r \bullet w_r \bullet \lambda_r$$

- at criticality: $\chi_r \sim w_r \gg 1$

$$\mathcal{K}_1^r, \mathcal{K}_2^r, \mathcal{K}_{2'}^r, \mathcal{K}_3^r \gg 1 \qquad w_r \gg \bar{\lambda}_r, \lambda_r, M_r$$

- faster convergence in parquet equations

- extension of cores in frequency space

$$\varphi^{K_3} \qquad \varphi^{U\mathrm{irr}}$$

Harkov, Lichtenstein, Krien PRB (2021)

Fraboulet, Heinzelmann, Bonetti, Al-Eryani, Vilardi, Toschi, Andergassen arXiv (2022)

Outlook

- detailed numeric analysis left as future work
- Numerical renormalization group (NRG) study of single-impurity Anderson model (SIAM): better qualitative agreement for low temperatures & frequencies

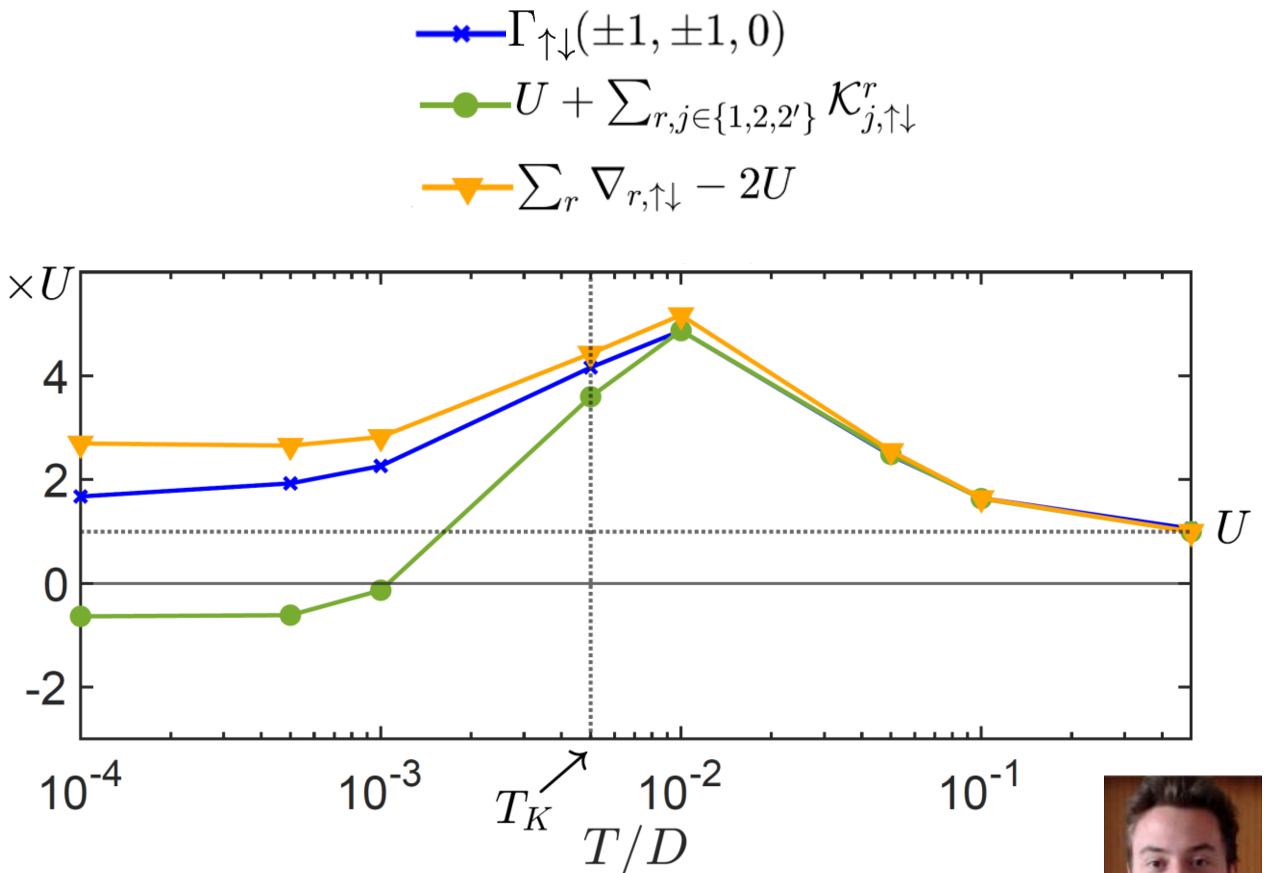
  → promising starting point for DMF²RG

- concrete systems:
  - Keldysh mfRG of SIAM → non-eq. transport

    Anxiang Ge, Elias Walter
  - Keldysh Hubbard model

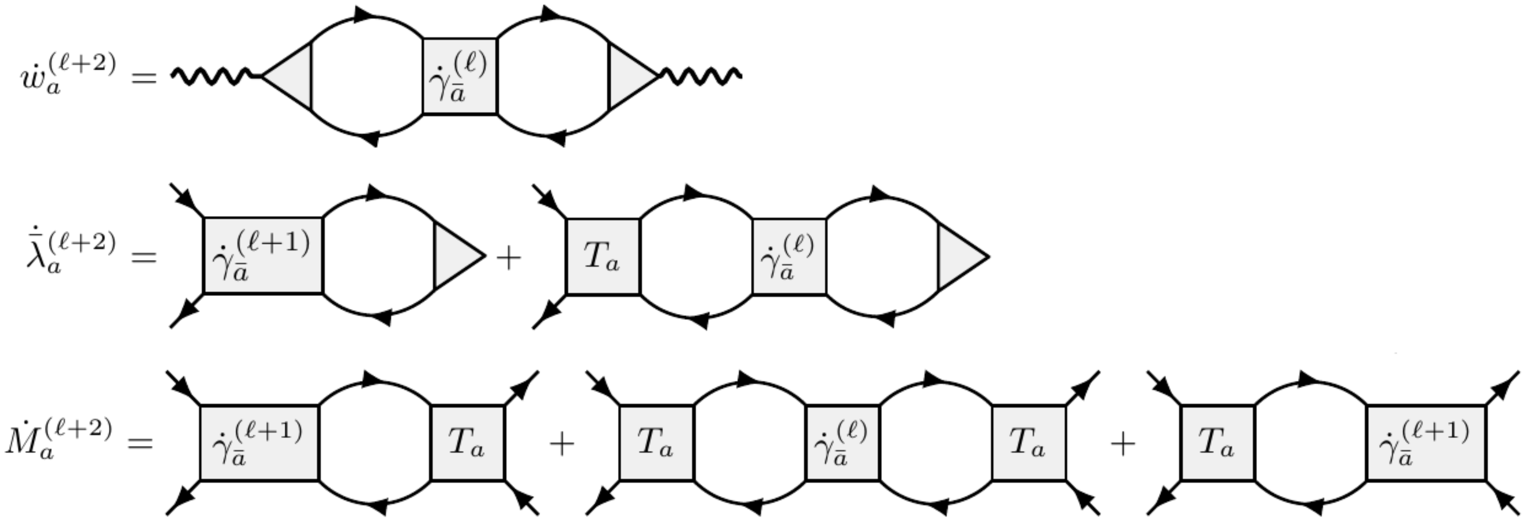
    Nepomuk Ritz
  - Fermi polaron problem

    Marcel Gievers



Johannes Halbinger

Thank you for your attention!



<u>M. Gievers</u>, E. Walter, A. Ge, J. von Delft, F. Kugler EPJ (2022)