# RG flows between Gaussian fixed points

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# Gaussian Fixed points

$$\text{GFP}_k: \qquad Z_k \phi \Box^k \phi$$

- Canonical choice: $k = 1$
- Field classical dimension $[\phi] = \frac{d-2k}{2}$ is fixed by a preferred $\text{GFP}_k$ in perturbation theory
- How are they related in the RG flow?

# Shift invariant scalar model

Shift symmetry $\phi \to \phi + c$ and $\mathbb{Z}^2$ symmetry invariance

$$\Gamma[\phi] = \int d^4x \left[ \frac{1}{2} Z_1 (\partial\phi)^2 + \frac{1}{2} Z_2 (\Box\phi)^2 + \frac{1}{4} g\big((\partial\phi)^2\big)^2 + \cdots \right]$$

Wetterich equation with adaptive Litim Cutoff

$$R_k^{(24)} = Z_1(k^2 - q^2)\theta(k^2 - q^2) + Z_2\,(k^4 - q^4)\theta(k^4 - q^4)$$

# The charts

| U1 | U2 |
| --- | --- |
| • Chart centred in $GFP_1$<br>• $[\phi] = 1$<br>• $Z_1$ reabsorbed in $\phi$<br>$\tilde{g} = \frac{gk^4}{Z_1^2}, \quad \tilde{Z}_2 = \frac{Z_2 k^2}{Z_1}$ | • Chart centred in $GFP_2$<br>• $[\phi] = 0$<br>• $Z_2$ reabsorbed in $\phi$<br>$\hat{\gamma} = \frac{\gamma k^4}{\zeta_1^2}, \quad \hat{\zeta}_2 = \frac{\zeta_2 k^2}{\zeta_1}$ |

$$\hat{\zeta}_1 = \frac{1}{\tilde{Z}_2}, \qquad \hat{\gamma} = \frac{\tilde{g}}{\tilde{Z}_2^2}$$

# Flow equations

**U1**

$$\eta_1 = \frac{8\,\tilde{g}\left(1 + 2\,\tilde{Z}_2\right)}{\tilde{g} + 128\pi^2\left(1 + \tilde{Z}_2\right)^2}$$

$$\beta_{\tilde{g}} \equiv \partial_t \tilde{g} = (4 + 2\eta_1)\tilde{g} + \frac{10 + 20\tilde{Z}_2 - \eta_1}{64\pi^2\left(1 + \tilde{Z}_2\right)^3}\,\tilde{g}^2$$

$$\beta_{\tilde{Z}_2} \equiv \partial_t \tilde{Z}_2 = (2 + \eta_1)\tilde{Z}_2$$

**U2**

$$\eta_2 = 0$$

$$\beta_{\hat{\zeta}_1} = -2\hat{\zeta}_1 - \frac{8\hat{\gamma}\left(2 + \hat{\zeta}_1\right)}{\hat{\gamma} + 128\pi^2\left(1 + \hat{\zeta}_1\right)^2}$$

$$\beta_{\hat{\gamma}} = \frac{\left(2 + \hat{\zeta}_1\right)\left(\hat{\gamma} + 640\pi^2\left(1 + \hat{\zeta}_1\right)^2\right)}{32\pi^2\left(1 + \hat{\zeta}_1\right)^3\left(\hat{\gamma} + 128\pi^2\left(1 + \hat{\zeta}_1\right)^2\right)}\,\hat{\gamma}^2$$

# Fixed points

**U1**

| FP | $\tilde{Z}_{2*}$ | $\tilde{g}_*$ | $\eta_{1*}$ | $\theta_1$ | $\theta_2$ |
|---|---|---|---|---|---|
| $GFP_1$ | 0 | 0 | 0 | -4 | -2 |
| $NGFP_1$ | 0 | -127.6 | -0.90 | 4.40 | -1.10 |
| $NGFP_2$ | 0 | -12505 | 8.90 | 43.60 | -10.90 |
| $NGFP_3$ | -0.6 | -1011 | -2 | -13.84 | 10.84 |

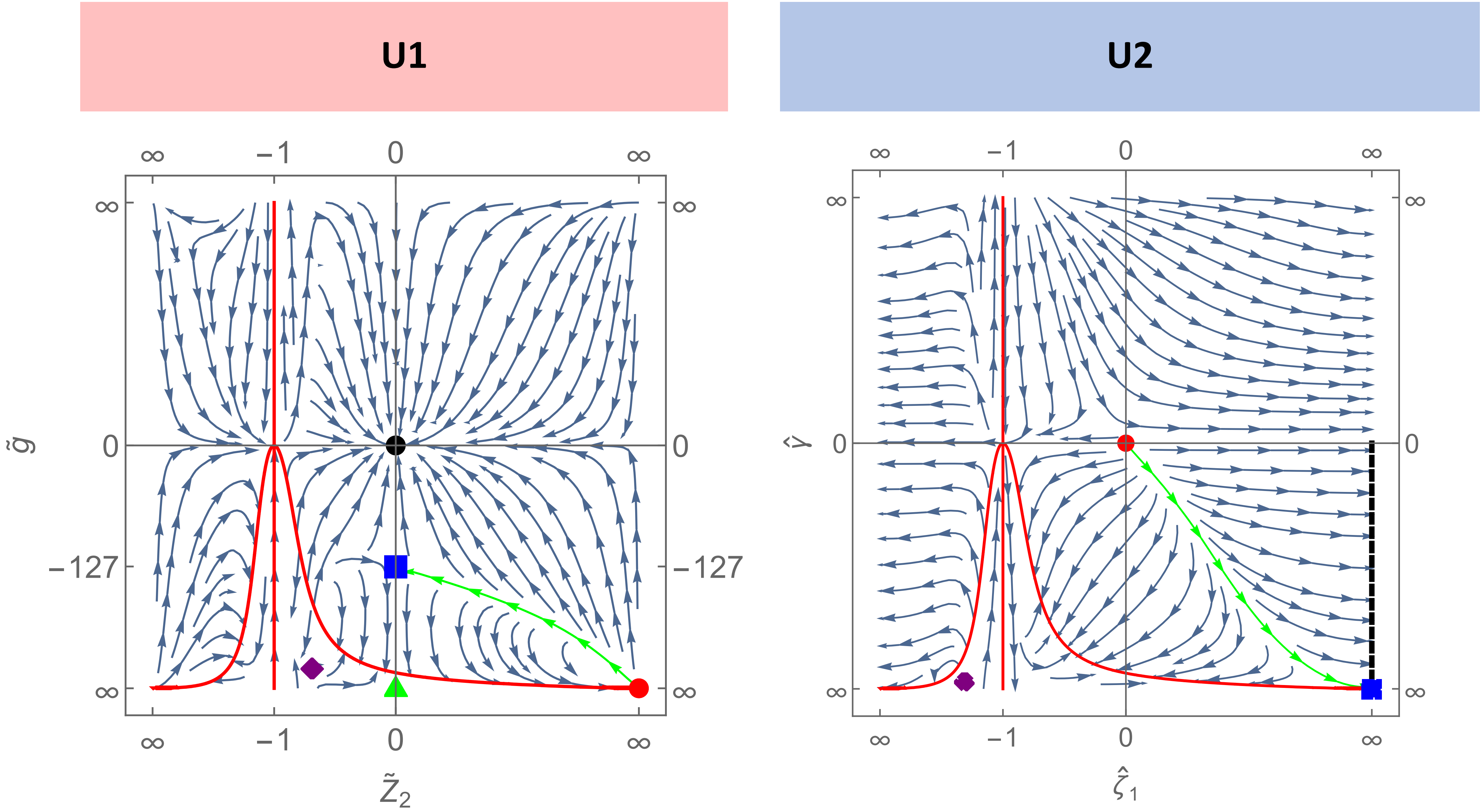
**U2**

| FP | $\hat{\zeta}_{1*}$ | $\hat{\gamma}_*$ | $\eta_{2*}$ | $\theta_1$ | $\theta_2$ |
|---|---|---|---|---|---|
| $GFP_2$ | 0 | 0 | 0 | 2 | 0 |
| $NGFP_3$ | -1.67 | -2807 | 0 | -13.84 | 10.84 |

$NGFP_1$ and $NGFP_2$ known in litterature (De Brito et al. ‘21, Laporte et al. ’21, ‘22)

$g<0$ in NGFPs → negative interaction energy

The two $NGFP_3$ have the same critical exponents and their coordinates in theories space are related by the map showed before



U1
U2
∞
−1
0
∞
∞
0
−127
∞
$\tilde{g}$
$\tilde{Z}_2$
$\hat{y}$
$\hat{\zeta}_1$

# Flow between GFPs

- "Classical" trajectory of free theories $g = 0$ (Rosten '11)
- Perturbative trajectories near to it
- Separatrix in the strongly interacting region

$\tilde{g} \sim -16\pi^2 \tilde{Z}_2$ for large $\tilde{Z}_2$

ends up in $NGFP_1$

out from $GFP_2$ as $\hat{\gamma} \sim -16\pi^2 \hat{\zeta}_1$

$\hat{\gamma} \sim -128\pi^2(2\sqrt{6} - 5)\hat{\zeta}_1^2$ for large $\hat{\zeta}_1$

# Dimensionful couplings

$$\partial_t Z_2 = 0$$

| $\boldsymbol{k \to 0}$ | $\boldsymbol{k \to \infty}$ |
| --- | --- |
| Also $Z_1$ and $g \to const$ | $Z_1 \sim \frac{4 Z_2 k^2}{11 \log k}, \quad g \sim -\frac{64\pi^2 Z_2}{11 \log k}$ |

Identifying $k$ with an external momentum $p$:

| | |
| --- | --- |
| $Z_1 \phi \Box \phi \sim p^2$ dominant | $Z_2 \phi \Box^2 \phi \sim p^4$ dominant |
| IR asymptotic freedom | UV asymptotic freedom |

# Ghost mass

- Higher derivative theories suffer of the existence of ghosts
- $m_{gh}^2 = Z_1/Z_2$
- If we fix $Z_1 = 1$ in the IR limit, the pole mass at $\tilde{Z}_2 = 1$ corresponds to different masses on different trajectories
- On the classical trajectory $m_{gh}^2 = 1/Z_2 = const$
- On the separatrix $m_{gh}^2 \to \infty$ ⇒ decoupled

# Inessential operators

- Different # DOF in different $\mathrm{GFP}_k$
- Essential RG fixes the DOF of the theory
- cannot handle deformations that introduce new DOF
- Starting from $\mathrm{GFP}_l$, $\phi\Box^k\phi$ are inessential for $k > l$
  with redefinition $\delta\phi = \frac{Z_k}{2Z_l}\Box^{k-l}\phi$

# Truncation

- We kept only relevant operators in $\mathrm{GFP}_2$
- Starting with an irrelevant deformation of $\mathrm{GFP}_2$, UV asymptotic freedom is broken
- All irrelevant operator are generated in the IR flow, but it is expected that the classical scaling prevails

# Symanzik model

$$S = \int d^4 x \left[\frac{1}{2} Z_1 (\partial\phi)^2 + \frac{1}{2} Z_0 \phi^2 + \frac{1}{4!} \lambda\phi^4 + \cdots\right]$$

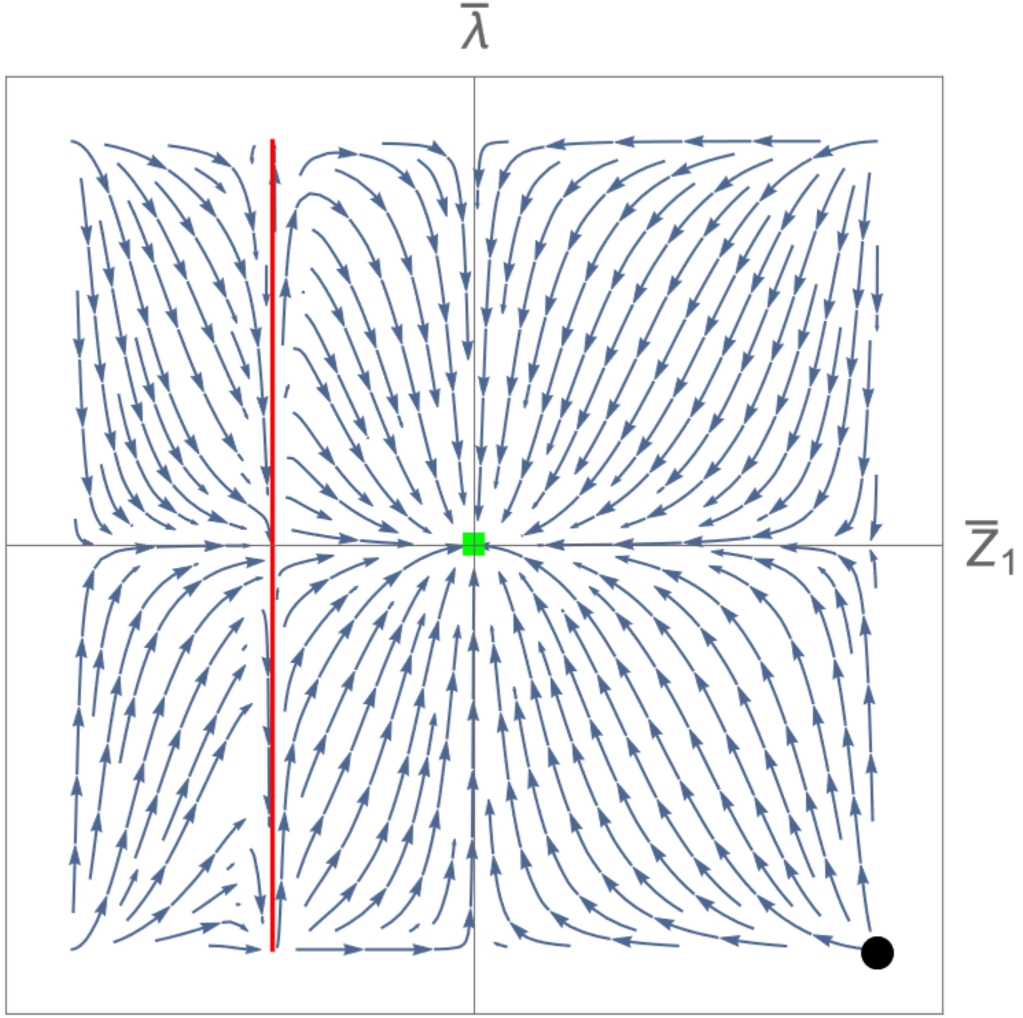
U0

$$\bar{Z}_1 = \frac{Z_1 k^2}{Z_0}, \qquad \bar{\lambda} = \frac{\lambda k^4}{Z_0^2}$$

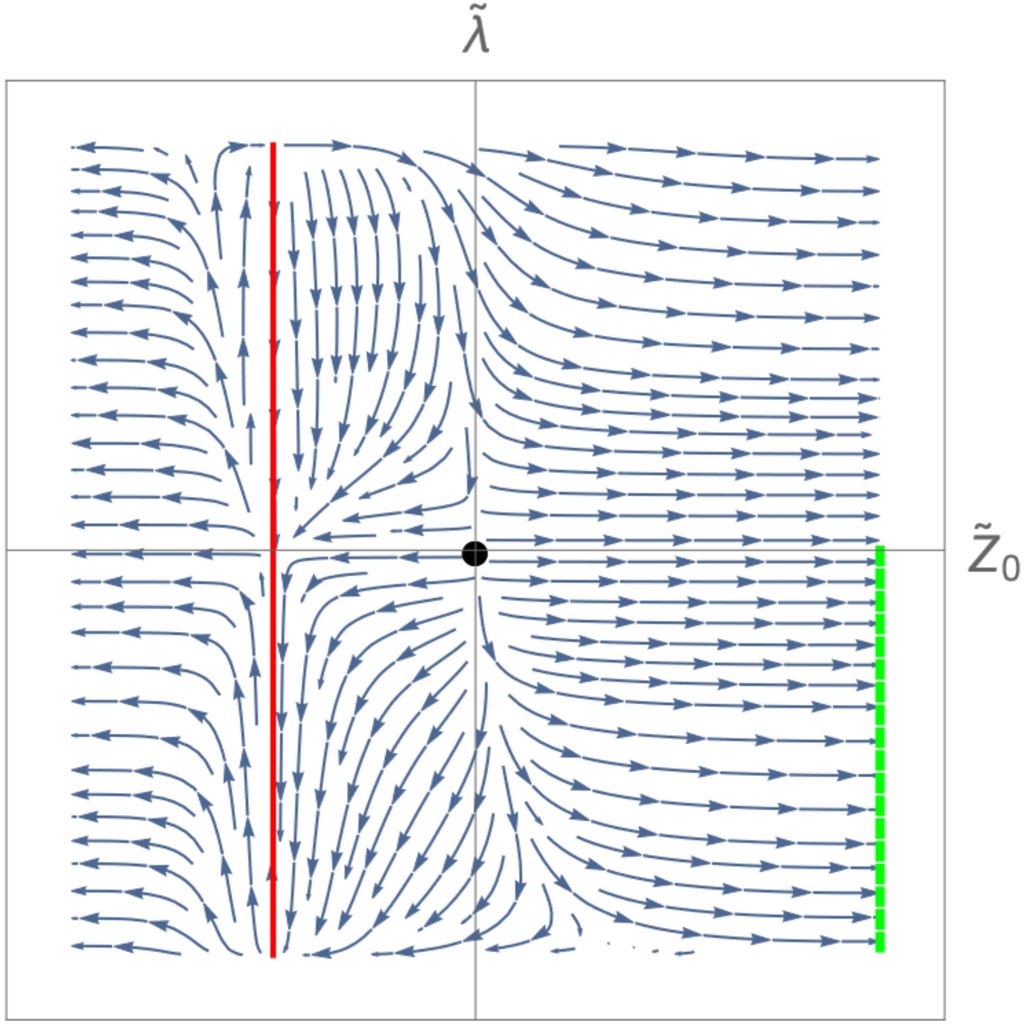
U1

$$\tilde{Z}_0 = \frac{Z_0}{Z_1 k^2}, \qquad \tilde{\lambda} = \frac{\lambda}{Z_1^2}$$

## U0




## U1



# Main features

- From $GFP_1$, well known $\lambda\phi^4$ theory
- Particle freezes out in IR
- $GFP_0$ trivial theory
- EOM: $\phi = 0$ → All operators are inessential

# Conclusions

- Free theory both in IR and UV

- Ghosts and negative interaction energy

- Anomalous dimension matches with classical dimension in GFPs

- Similar flow in $\lambda\phi^4$ theory for negative $\lambda$

# Further developments

- for different $k$ similar fluxes between GFPs.

  In $d = 4$ and $k > 2$, infinite number of relevant operators

  may be manageable with larger symmetries

- Extended truncations (Laporte et al. ‘22)

  we expect higher order kinetic terms to be generated

- NLSM

- Analogies with $R^2$ theories of gravity

Thank you