ERG 2022, Berlin, 25-29 July 2022

talk by Peter Kopietz, Frankfurt

Quantum discontinuity fixed point and renormalization group flow of the SYK model

R. Smit, D. Valentinis, J. Schmalian, PK, Phys. Rev. Research 3, 033089 (2021)

Outline:

- introduction to SYK model
- FRG calculation
- global RG flow
- discontunuity fixed point

Outlook:

- new result on spin FRG for frustrated spin systems

  see also talk by Andreas Rückriegel in Thursday

Sachdev-Ye-Kitaev (SYK) model

- Sachdev, Ye (1993)

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$

$$P(J_{ij}) \sim \exp[-J_{ij}^2/(2J^2)]$$

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Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

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(Received 22 December 1992)

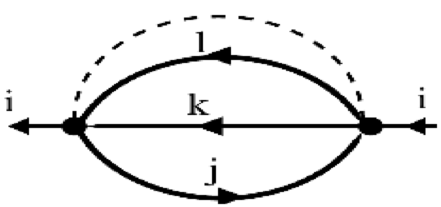
We examine the spin-S quantum Heisenberg magnet with Gaussian-random, infinite-range exchange interactions. The quantum-disordered phase is accessed by generalizing to $SU(M)$ symmetry and studying the large M limit. For large S the ground state is a spin glass, while quantum fluctuations produce a spin-fluid state for small S. The spin-fluid phase is found to be generically gapless—the average, zero temperature, local dynamic spin susceptibility obeys $\bar{\chi}(\omega) \sim \ln(1/|\omega|) + i(\pi/2)\mathrm{sgn}(\omega)$ at low frequencies.

- Kitaev (2015): Talks at KITP: a simple model for quantum holography

- toy model for some phenomena in high-energy physics and general relativity
  finite ground state entropy; same information scambling as black holes;
  holographic principle in string theory

- toy model for non-Fermi liquid behavior in condensed matter

- exactly solvable for large N (only melon-diagrams)
  controlled solution of strongly coupled field theory



- can be generalized to include phonons, superconductivity

complex SYK model

- hamiltonian $$\mathcal{H} = -\mu \sum_i c_i^\dagger c_i + \sum_{i<j,k<l} J_{ij,kl} c_i^\dagger c_j^\dagger c_k c_l$$

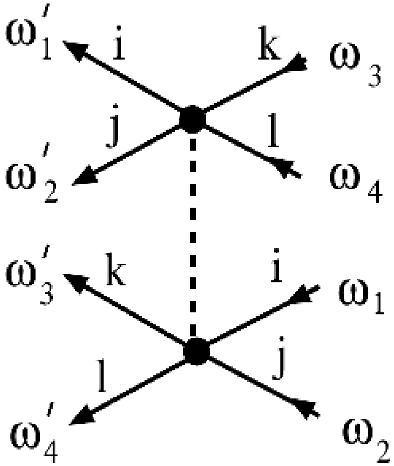
- random interactions with Gaussian probability distribution

$$\mathcal{P}[J_{ij,kl}] = \frac{N^3}{2\pi J^2} \exp\left[-\frac{N^3 |J_{ij,kl}|^2}{2J^2}\right] \qquad \langle J_{ij,kl}\rangle = 0, \quad \langle |J_{ij,kl}|^2\rangle = 2J^2/N^3$$

- disorder averaging generates fermionic 8-point vertex

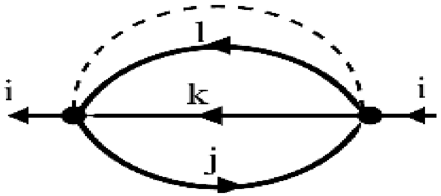
$$\langle \mathcal{Z}\rangle = \int \mathcal{D}[c,\bar{c}] e^{-S_2 - S_8} \qquad S_2 = \sum_i \int_0^\beta d\tau \bar{c}_i(\tau)[\partial_\tau - \mu] c_i(\tau)$$

$$S_8 = -\frac{J^2}{N^3} \sum_{i<j,k<l} \int_0^\beta d\tau \bar{c}_i(\tau)\bar{c}_j(\tau) c_k(\tau) c_l(\tau) \int_0^\beta d\tau' \bar{c}_k(\tau')\bar{c}_l(\tau') c_i(\tau') c_j(\tau')$$



exact self-energy for large N

- for $N \to \infty$ self-energy determined by melon diagram + Dyson equation



$$\Sigma(\tau) = -J^2 G^2(\tau) G(-\tau) \qquad G^{-1}(\omega) = i\omega + \mu - \Sigma(\omega).$$

- non-Fermi liquid solution ($|\omega| \ll J$)

$$\Sigma_{\mathrm{NFL}}(\omega) = \mu - \frac{J A(\theta,\omega)}{\pi^{1/4}} \left|\frac{\omega}{J}\right|^{\eta} \qquad \eta = 1/2$$

$$A = \cos^{1/4}(2\theta)\, e^{-i\,\mathrm{sign}(\omega)\left(\theta - \frac{\pi}{2}\right)}$$

$$n = 1/2 + \theta/\pi + \sin(2\theta)/4$$

propagator $G(\tau) \propto \tau^{-1/2}$ implies finite ground state entropy

- another solution of self-consistency equation for T=0: $\Sigma_{\mathrm{IV}}(\omega) = 0$

describes integer valence (IV) state: $n = 1$ or $0.$

phase diagram

Smit, Valentinis, Schmalian, PK, PRR 2021,
see also Azeyanagi, Ferrari, Schaposnik, PRL 2018

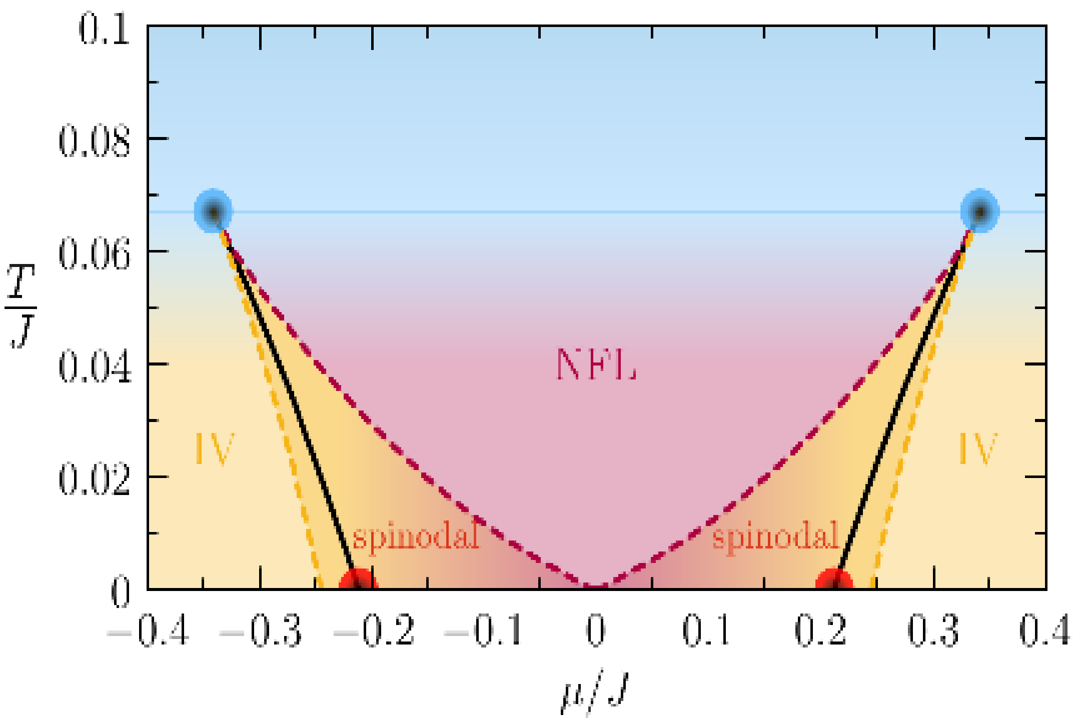


Figure 2: Phase diagram of the SYK model as a function of chemical potential μ and temperature T obtained from the numerical solution of the large-N self-consistency equation for the self-energy; see also Ref. [34]. For $T = 0$ and $|\mu| \leq \mu_* = 0.212J$, the non-Fermi liquid (NFL) phase with anomalous dimension $\eta = 1/2$ is the stable solution. For $\mu = \pm\mu_*$ (red dots) there are first order quantum phase transitions from $n \approx 0.76$ or 0.24 to integer-valence (IV) phases $n = 1$ or 0, respectively. Between the spinodal lines (dashed) both phases are locally stable. The transition terminates at critical points $(\mu_c, T_c) = (\pm 0.34, 0.067)J$ (blue dots).

Questions:

- How does underlying global RG flow look like?
- What are properties of RG fixed point describing quantum first-order transition?

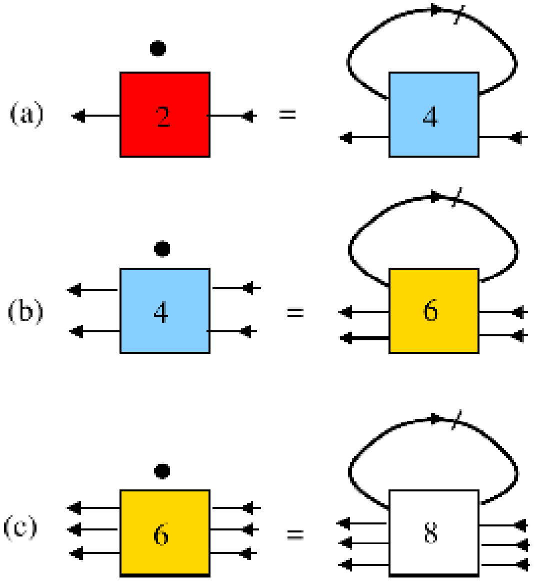
FRG flow equations for SYK @ large N

- Large-N truncation of flow equations for site averaged vertex functions: evolution due to change of cutoff Λ

$$\partial_\Lambda \Sigma_\Lambda(\omega_1) = \int \frac{d\omega_2}{2\pi}\, \dot{G}_\Lambda(\omega_2)\, \Gamma^{(4)}_\Lambda(\omega_1,\omega_2)$$

$$\partial_\Lambda \Gamma^{(4)}_\Lambda(\omega_1,\omega_2) = \int \frac{d\omega_3}{2\pi}\, \dot{G}_\Lambda(\omega_3)\, \Gamma^{(6)}_\Lambda(\omega_1,\omega_2,\omega_3)$$

$$\partial_\Lambda \Gamma^{(6)}_\Lambda(\omega_1,\omega_2,\omega_3) = \int \frac{d\omega_4}{2\pi}\, \dot{G}_\Lambda(\omega_4)\, \Gamma^{(8)}_0(\omega_1,\omega_2,\omega_3,\omega_4)$$



- wave-function renormalization Z_Λ :

$$\Sigma_\Lambda(\omega) = \Sigma_\Lambda(0) + (1 - Z_\Lambda^{-1}) i\omega + \mathcal{O}(\omega^2) \qquad \eta_\Lambda = \frac{\Lambda \partial_\Lambda Z_\Lambda}{Z_\Lambda} = \Lambda \partial_\Lambda \ln Z_\Lambda.$$

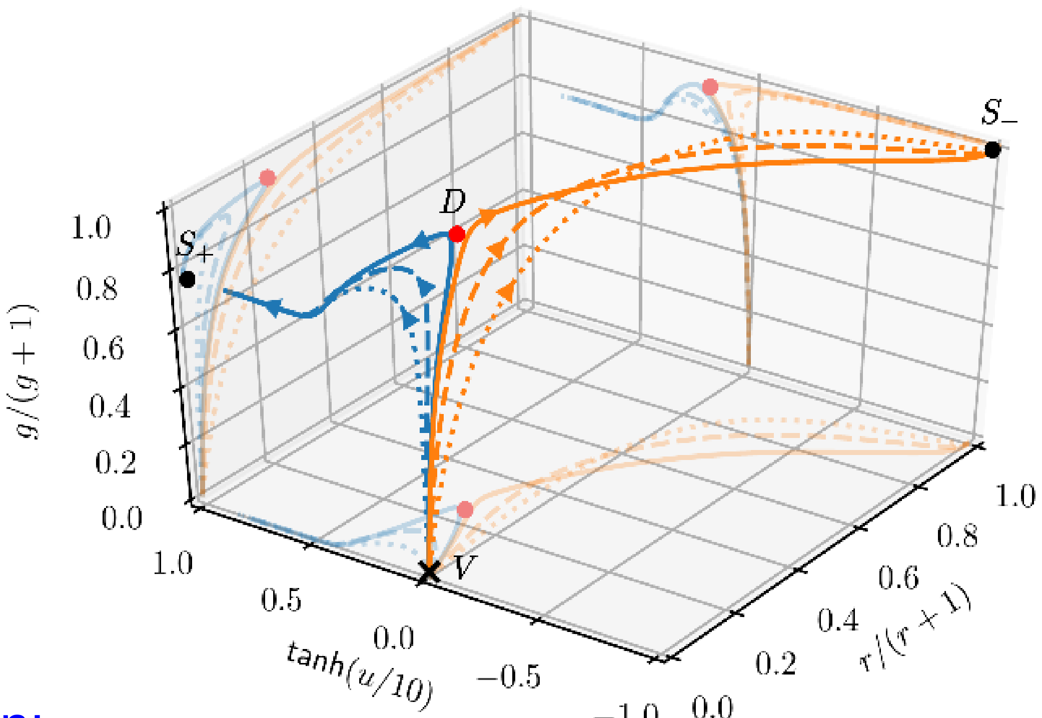
- rescaled dimensionless couplings:

$$r_l = \tilde{\mu}_l^2 = \left[\frac{Z_\Lambda(\mu - \Sigma_\Lambda(0))}{\Lambda}\right]^2 \qquad u_l = \tilde{\Gamma}^{(4)}_\Lambda(0) = \frac{Z_\Lambda^2}{\Lambda}\Gamma^{(4)}_\Lambda(0). \qquad g_l = \frac{J^2 Z_\Lambda^4}{\Lambda^2}.$$

global RG flow of SYK

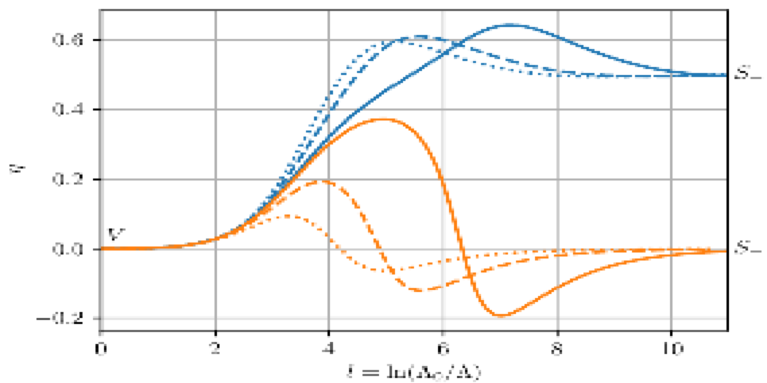
- flow equations for rescaled couplings
  4 fixed points:

$$\begin{aligned}\partial_l r_l &= 2(1-\eta_l)r_l - \frac{2}{\pi}\frac{r_l u_l}{1+r_l}\\ \partial_l u_l &= (1-2\eta_l)u_l + \frac{4}{\pi}\frac{g_l}{1+r_l}\left[\frac{1}{1+r_l}-\frac{3}{4}\right]\\ \partial_l g_l &= 2(1-2\eta_l)g_l\end{aligned}$$



- scale-dependent anomalous dimension:

$$\eta_l = \Lambda Z_\Lambda \int \frac{d\omega}{2\pi}\dot{G}_\Lambda(\omega_2)\left.\frac{\partial\Gamma^{(4)}_\Lambda(\omega_1,\omega_2)}{\partial(i\omega_1)}\right|_{\omega_1=0}$$



- $\eta = 1/2$ at fixed points D and S_+

quantum disontinuity fixed point

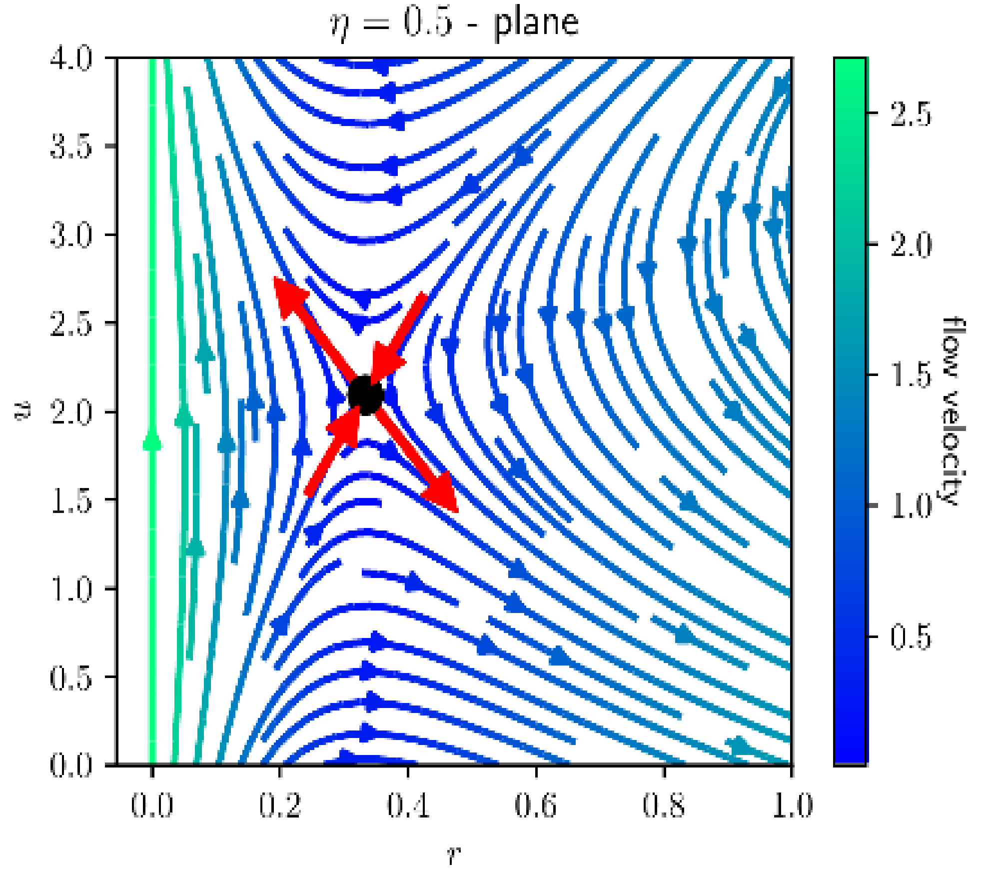
- discontinuity fixed points represent first order phase transitions in RG (Nienhuis, Nauenberg 1975, Fisher 1982)
  - all rescaled couplings finite at D
  - linearized RG flow around D gives scaling exponents

    $\lambda_+ \approx 0.987 \qquad \lambda_- \approx -0.737$

  - scaling of singular part of density

    $$n_{\text{sing}}(\mu) = A^{1/\lambda_+} |\mu - \mu^*|^{\frac{1-\lambda_+}{\lambda_+}}$$

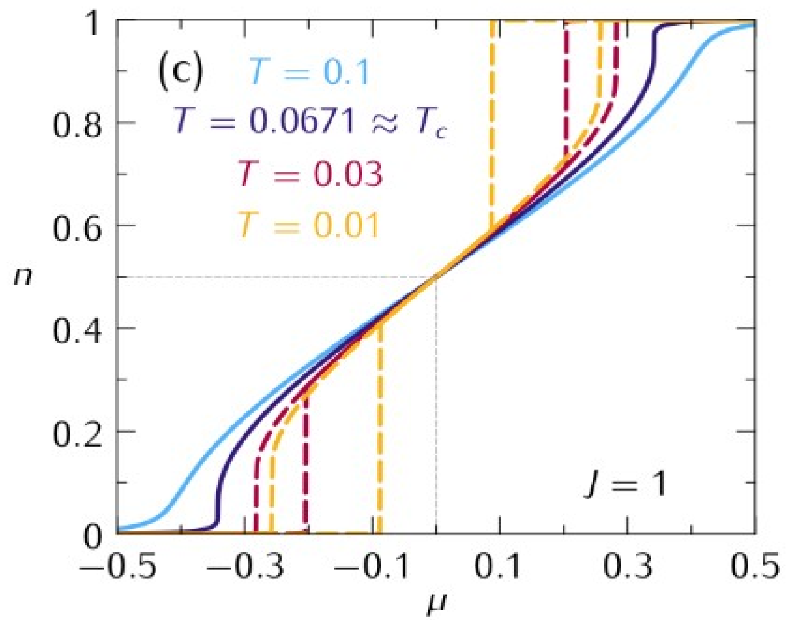
  - $\lambda_+ \approx 1$ implies discontinuity
  - consistent with numerics



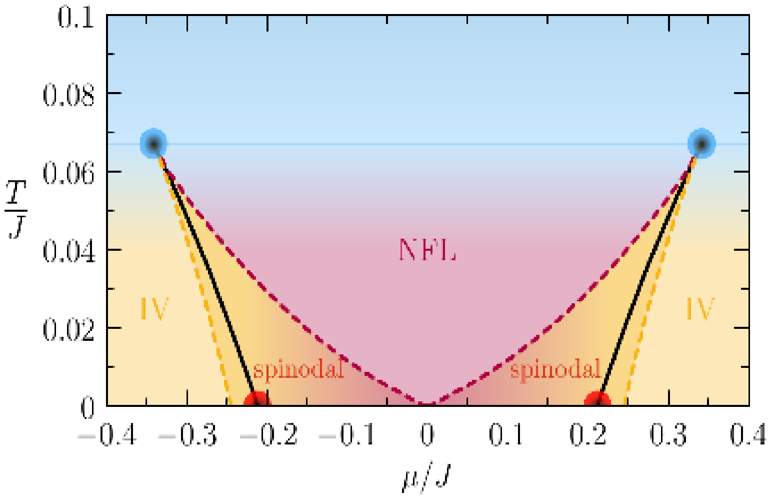
quantum first order phase transitions

- numerical solution of large-N Dyson-Schwinger equations confirm first order transition

- new feature of quantum first order transition: fermionic excitations have anomalous dimension $\eta = 1/2$



- different from classical first order transitions



summary and outlook

- quantum first order transitions can be different from classical counter-parts
- controlled calculation for SYK model: discontinuity fixed point with

  rules out effective description with classical Ising model! $\eta = 1/2$

- challenges: generalization to finite temperature
  SYK+phonons, superconductivity

remember ERG 2018:

PHYSICAL REVIEW B **99**, 060403(R) (2019)

Rapid Communications

Exact renormalization group for quantum spin systems

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(Received 2 August 2018; revised manuscript received 15 January 2019; published 8 February 2019)

We show that the diagrammatic approach to quantum spin systems developed in a seminal work by Vaks, Larkin, and Pikin [Sov. Phys. JETP **26**, 188 (1968)] can be embedded in the framework of the functional renormalization group. The crucial insight is that the generating functional of the time-ordered connected spin correlation functions of an arbitrary quantum spin system satisfies an exact renormalization group flow equation which resembles the corresponding flow equation of interacting bosons. The $SU(2)$ spin algebra is implemented via a nontrivial initial condition for the renormalization group flow. Our method is rather general and offers a different nonperturbative approach to quantum spin systems.

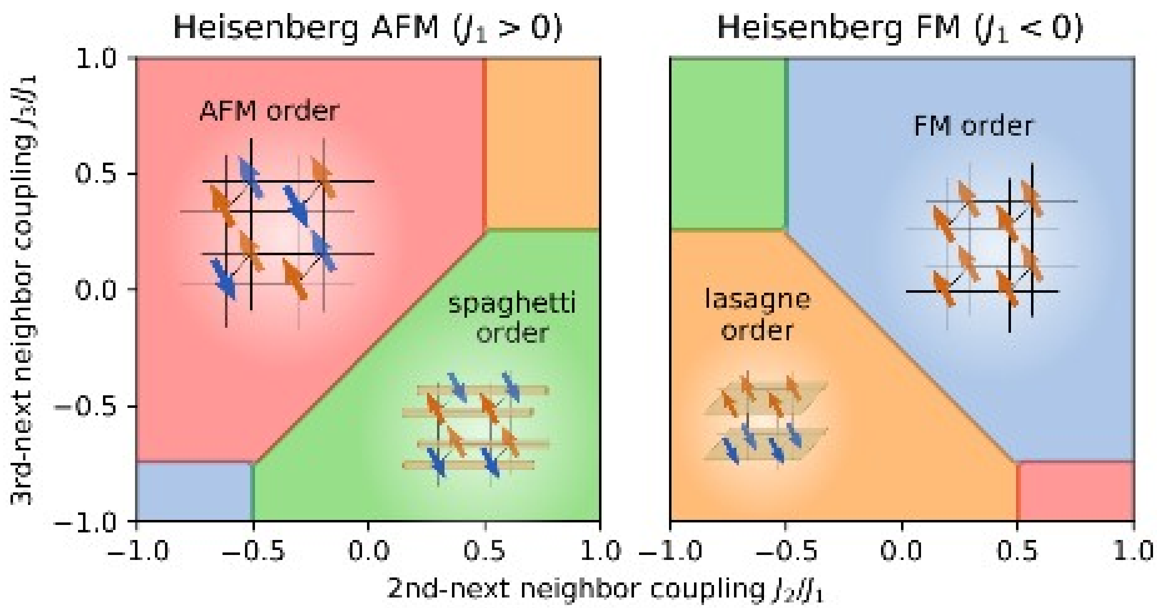
serious alternative to pseudofermion FRG !

recent progress:

- Dissipative spin dynamics in hot quantum paramagnets
  PRB 2021, with D. Tarasevych
- SFRG for dimerized quantum spin systems
  PRB 2022, with A. Rückriegel, J. Arnold, R. Goll, see talk by A. Rückriegel on Thursday
- SFRG for frustrated J1J2J3 quantum Heisenberg magnet
  in preparation, with D. Tarasevych, S. Keupert, V. Mitsiiouanou, and A. Rückriegel

benchmark calculations for J1J2J3 Heisenberg magnet on a cubic lattice:

$$\mathcal{H} = J_1 \sum_{\langle ij \rangle_1} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle ij \rangle_2} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_3 \sum_{\langle ij \rangle_3} \boldsymbol{S}_i \cdot \boldsymbol{S}_j.$$



- static level-1 truncation produces results comparable with 1-loop PFRG

$$G_\Lambda(\boldsymbol{k}) = \frac{1}{J_\Lambda(\boldsymbol{k}) + \Sigma_\Lambda(\boldsymbol{k})}$$

$$\partial_\Lambda \Sigma_\Lambda = -\frac{5}{6} U_0 \frac{T}{N} \sum_{\boldsymbol{q}} \frac{\partial_\Lambda J_\Lambda(\boldsymbol{q})}{[J_\Lambda(\boldsymbol{q}) + \Sigma_\Lambda]^2}$$

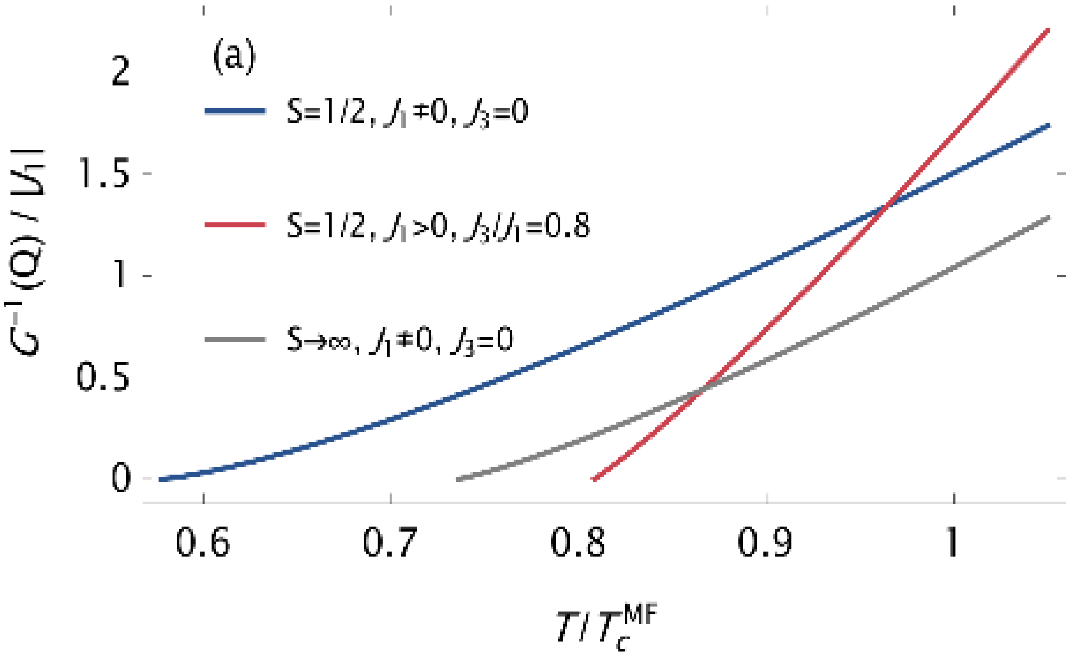
$$U_0 = -T \frac{b_3}{b_1^4}$$

$$b_1 = \frac{S(S+1)}{3} \qquad b_3 = -\frac{(2S+1)^4 - 1}{120}$$

			T_c/T_c^{MF}			rel. error / %	
S	J_1	J_3/J_1	switch	Litim	benchmark	switch	Litim
$1/2$	< 0	0	0.651	0.568	0.559	16.5	1.6
$1/2$	> 0	0	0.651	0.568	0.629	3.5	9.7
$1/2$	> 0	0.8	0.807	0.787	0.808	0.1	2.6
∞	$\neq 0$	0	0.766	0.725	0.722	6.1	0.4

benchmark calculations for J1J2J3 Heisenberg magnet beyond static level-1

- static level-2 truncation produces true RG fixed point (missed in PFRG)



			T_c/T_c^{MF}			rel. error / %	
S	J_1	J_3/J_1	switch	Litim	benchmark	switch	Litim
1/2	< 0	0	0.577	0.525	0.559	3.2	6.1
1/2	> 0	0	0.577	0.525	0.629	8.3	16.5
1/2	> 0	0.8	0.808	0.787	0.808	0.0	2.6
∞	$\neq 0$	0	0.736	0.700	0.722	1.9	3.0

- results for finite S can be improved by including dynamic spin fluctuations

- evidence for spin-liquid phase in maximally frustrated regime of J1J2J3 model (agrees with one-loop PFRG, disagrees with multi-loop PFRG)