The Spectral Geometry of de Sitter Space in Asymptotic Safety

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Motivations

1 To understand the dynamical "thinning out" of degrees of freedom in the UV

2 To improve upon (non-rigorous arguments in) RG improvement

Largely ameliorates unitarity/positivity issues

Fractality
$\sim$ 2D theory

The Tool

Gravitational effective average action

A Background Independent and diffeomorphism-covariant continuum approach to quantum gravity

The Idea

Pick a solution of the FRGE:

Running effective action	$\Gamma_k[h_{\mu\nu}, \cdots; \bar{g}_{\mu\nu}]$
↓	
Running self-consistent metrics	$\left(\bar{g}_k^{sc}\right)_{\mu\nu}$
↓	
Running kinetic operators	$\mathscr{K}_k = -\Box_g + \cdots \big|_{g=\bar{g}_k^{sc}}$
↓	
Running spectral problem	$\mathscr{K}_k\, \chi_n(k) = \mathscr{F}_n(k)\, \chi_n(k)$
↓	

Running spectra $\{\mathscr{F}_n(k)\}_{n=1,2,\ldots}$ and eigenfunctions $\{\chi_n(k)\}_{n=1,2,\ldots}$

Spectral flow

Spectral flow

$$k \mapsto \{\mathscr{F}_n(k)\}, \quad \{\chi_n(k)\}$$

encodes information about the **fractal structure of spacetime**

classical, smooth manifolds
say $g_{\mu\nu}^{Minkowski}, g_{\mu\nu}^{deSitter}, \ldots$

Macroscopic metric $\left(\bar{g}^{sc}_{k=0}\right)_{\mu\nu}$

$\downarrow$

Zoom into spacetime's microstructure $\left(\bar{g}^{sc}_{k}\right)_{\mu\nu}$

$\downarrow$

?

Running Gravitational Effective Average Action

↓

Running metrics

↓

Running spectra

Background Independence: $h_{\mu\nu}$-dynamics on all backgrounds

$$\langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} \equiv \bar{g}_{\mu\nu} + \langle \hat{h}_{\mu\nu} \rangle_{\bar{g}}$$

$$\langle \hat{h}_{\mu\nu} \rangle_{\bar{g}} = 0 \iff \langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} = \bar{g}_{\mu\nu} \quad \text{for} \quad \bar{g} = \bar{g}^{sc}_k$$

Self-consistent geometries

$$\frac{\delta}{\delta h_{\mu\nu}(x)} \Gamma_k \left[h; \bar{g} \right] \Big|_{h=0,\ \bar{g}=\bar{g}^{sc}_k} = 0$$

tadpole condition or effective Einstein equation

Generic solutions $\left(\bar{g}^{sc}_k\right)_{\mu\nu}$ will depend on the RG scale k: $k \mapsto \left(\Gamma_k, \left(\bar{g}^{sc}_k\right)_{\mu\nu} \right)$

Remarks

all the expectation values have a nontrivial (indirect) dependence on the background, which is kept completely arbitrary

the dynamics determines the expectation value of the metric s.t. the fluctuations are "as content as possible" about it

Running Effective Average Action

$$\Gamma_k[h;\bar{g}] = \frac{1}{16\pi\, G(k)} \int d^4x \sqrt{-g}\, \Big(R(g) - 2\Lambda(k) \Big)\Big|_{g=\bar{g}+h} + \cdots$$

↓

Running effective field equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda(k) g_{\mu\nu} = 0$$

↓

Running self-consistent metrics

e.g. $g_{\mu\nu} \sim \mathrm{S}^4(L^{sc}(k))$

↓

Running spectra

$$-\Box_{\bar{g}_k^{sc}}\, \chi_{nm}(x;k) \;=\; \mathscr{F}_n(k)\, \chi_{nm}(x;k)$$ eigenvalue problem

$$k \mapsto \big\{ \mathscr{F}_n(k) \big\}$$ spectral flow

Two different types of spectral problems

1 **Rigid background**

"Off-shell"

$$-\Box_{\bar{g}}\, \chi_{nm}[\bar{g}](x) \;=\; \mathscr{F}_n[\bar{g}]\;\chi_{nm}[\bar{g}](x)$$

$\bar{g}_{\mu\nu}$: generic background metric

2 **Self-consistent dynamical background**

"On-shell"

generalized trajectory $\quad k \mapsto \left(\Gamma_k, \bar{g}_k^{sc}\right)$

running D'Alembertian $\quad \Box_k \equiv \Box_{\bar{g}}\Big|_{\bar{g}=\bar{g}_k^{sc}}$

$$-\Box_k\;\chi_{nm}(x;k) = \mathscr{F}_n(k)\;\chi_{nm}(x;k)$$

Cutoff modes (COMs):

$$\chi_{n_{COM(k)}}(x) \quad \text{with}$$

$$\mathcal{F}_n(k)\Big|_{n=n_{COM}(k)} = k^2$$

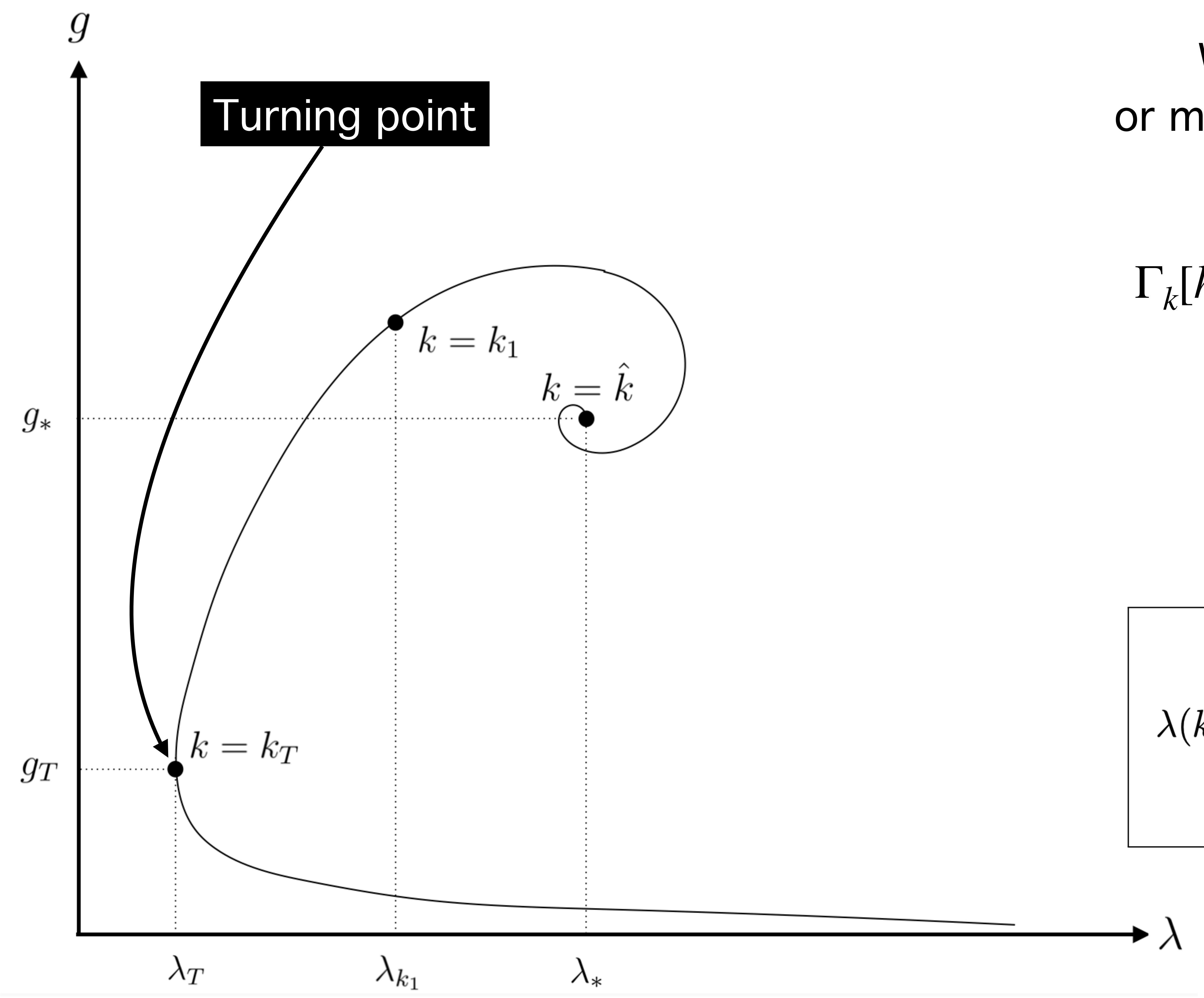
Remarks

The cutoff modes are located precisely at the threshold between "already integrated out at RG scale", and "not yet integrated out" if the fluctuations propagate on a background which is self-consistent at that given k.

COMs are a valuable link between the bare off-shell world under the path integral, and the effective level of the on-shell expectation values.

RG trajectory

Trajectory of the Type IIIa

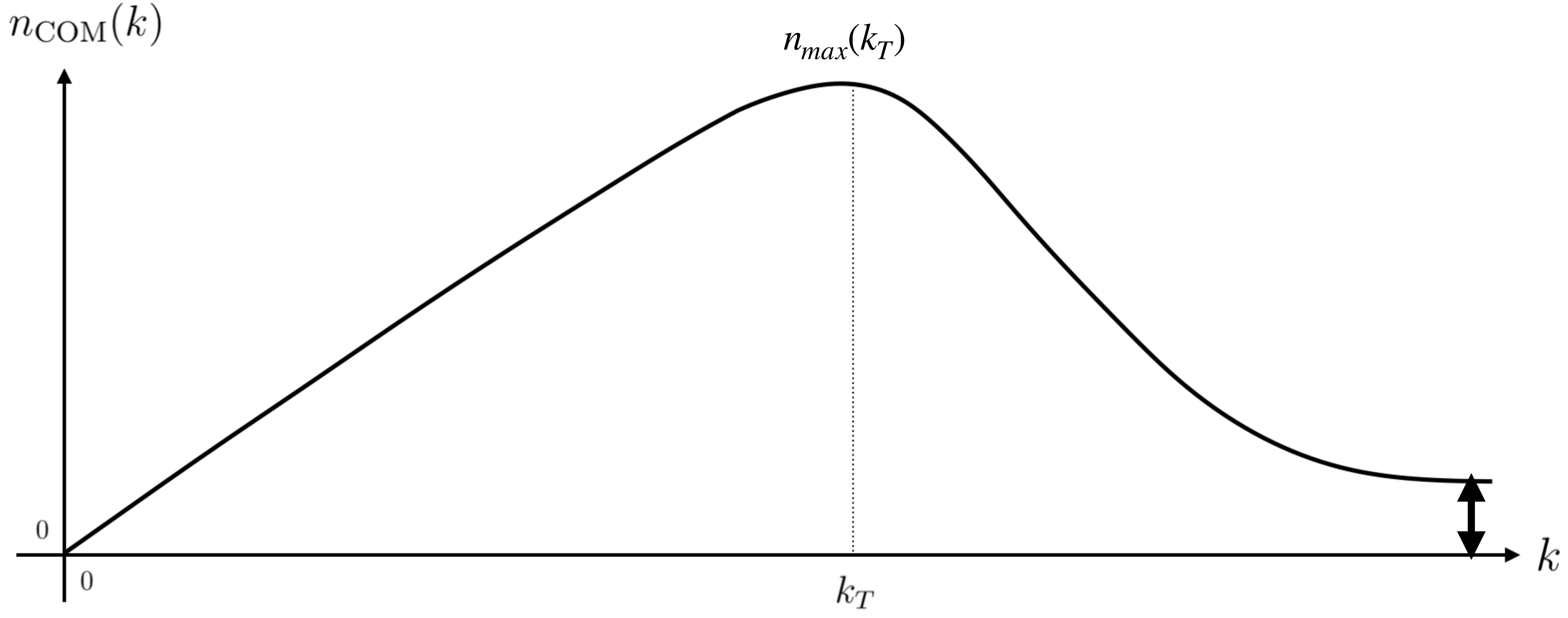


We restrict the analysis to pure quantum gravity,
or matter-coupled gravity in a vacuum dominated regime.

$$\Gamma_k[h;\bar{g}] = \frac{1}{16\pi\, G(k)} \int d^4x \sqrt{-g}\, \Big(R(g) - 2\Lambda(k) \Big)\Big|_{g=\bar{g}+h} + \cdots$$

Caricature trajectory

$$\lambda(k) = \begin{cases} \frac{1}{2}\lambda_T \left[\left(\frac{k_T}{k}\right)^2 + \left(\frac{k}{k_T}\right)^2 \right] & \text{for} \quad 0 \le k \le \hat{k} \quad \text{(semiclassical)} \\ \lambda_* & \text{for} \quad \hat{k} < k < \infty \quad \text{(fixed point)}. \end{cases}$$



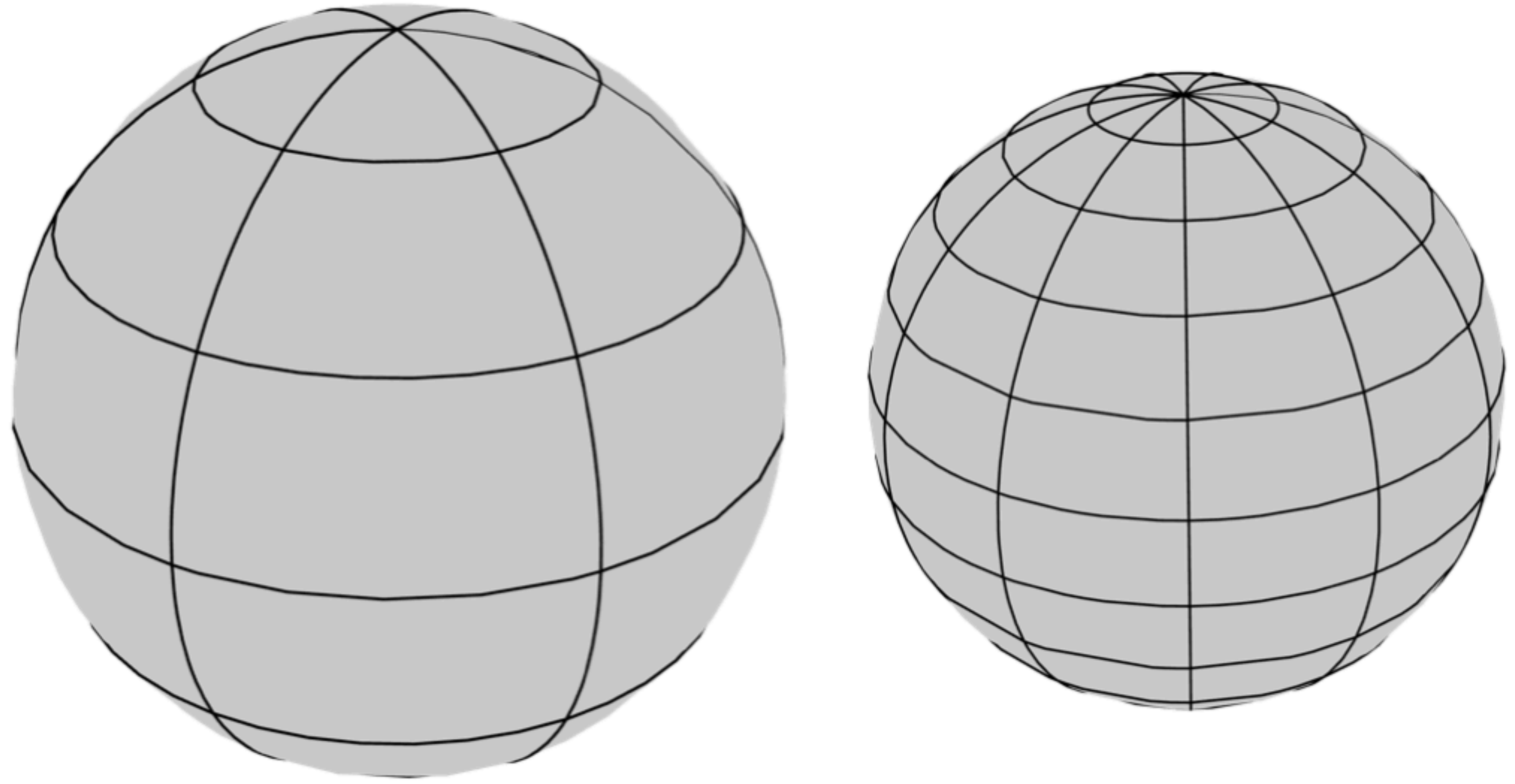
$$S^4(L^{sc}(k))$$

self-consistent spheres

$$\mathscr{F}_n \sim n^2$$

$$n_{COM}(k) \sim k\, L^{sc}(k)$$

Less DOFs in the UV!



Resolution $2\pi / n_{COM}(k)$

Limitations on the distinguishability of spacetime points

Schwindt & Reuter 2005
Pagani & Reuter 2019

Physical interpretation:

	k-dependence of $n_{COM}(k)$	Resolution
$0 \leq k \leq k_T$ classical	increasing $n_{COM} \propto k$ S^4 harmonics of increasing angular momentum	increasing as $2\pi / n_{COM}(k)$ continuously improving resolving power
$k > k_T$ quantum	decreasing $n_{COM} \propto \frac{1}{k}$ rapid shrinking of spacetime	Higher eigenvalue - lower fineness!
$k \to \infty$ UV-FP	$n_{COM} \approx n^*_{COM}$ fixed finite value	Fixed limited resolution: FUZZYNESS Why not an unlimited resolving power in the UV?

This apparent paradox is explained by the rapid shrinking of spacetime caused by the enormous shrinking of $L^{sc}(k)$ for $k \to \infty$.

$$n_{COM}(k) = k\, L^{sc}(k)$$

Background independence

NEW!

Self-consistent Lorentzian spacetimes

DE SITTER SPACE

Rigid de Sitter Space (off-shell)

Conformal coordinates: $$ds^2 = b(\eta)^2 \left[-dt^2 + d\mathbf{x}^2\right] = \frac{-dt^2 + d\mathbf{x}^2}{H^2\,\eta^2}$$

Eigenvalue equation: $$-\Box_{dS_4}\ \chi_{\nu,\mathbf{p}}(\eta,\mathbf{x}) = \mathscr{F}_\nu\ \chi_{\nu,\mathbf{p}}(\eta,\mathbf{x})$$ $$v''_{\nu,p}(\eta) + \left[p^2 - \frac{\nu^2 - 1/4}{\eta^2}\right] v_{\nu,p}(\eta) = 0$$

Eigenfunctions: $$\chi_{\nu,\mathbf{p}}(\eta,\mathbf{x}) = -\eta\ \ v_{\nu,p}(\eta)\ e^{i\mathbf{p}\cdot\mathbf{x}}\,, \qquad v_{\nu,p}(\eta) = \left(p\,|\eta|\right)^{1/2}\left[A_p\,J_\nu\left(p\,|\eta|\right) + B_p\,Y_\nu\left(p\,|\eta|\right)\right]$$

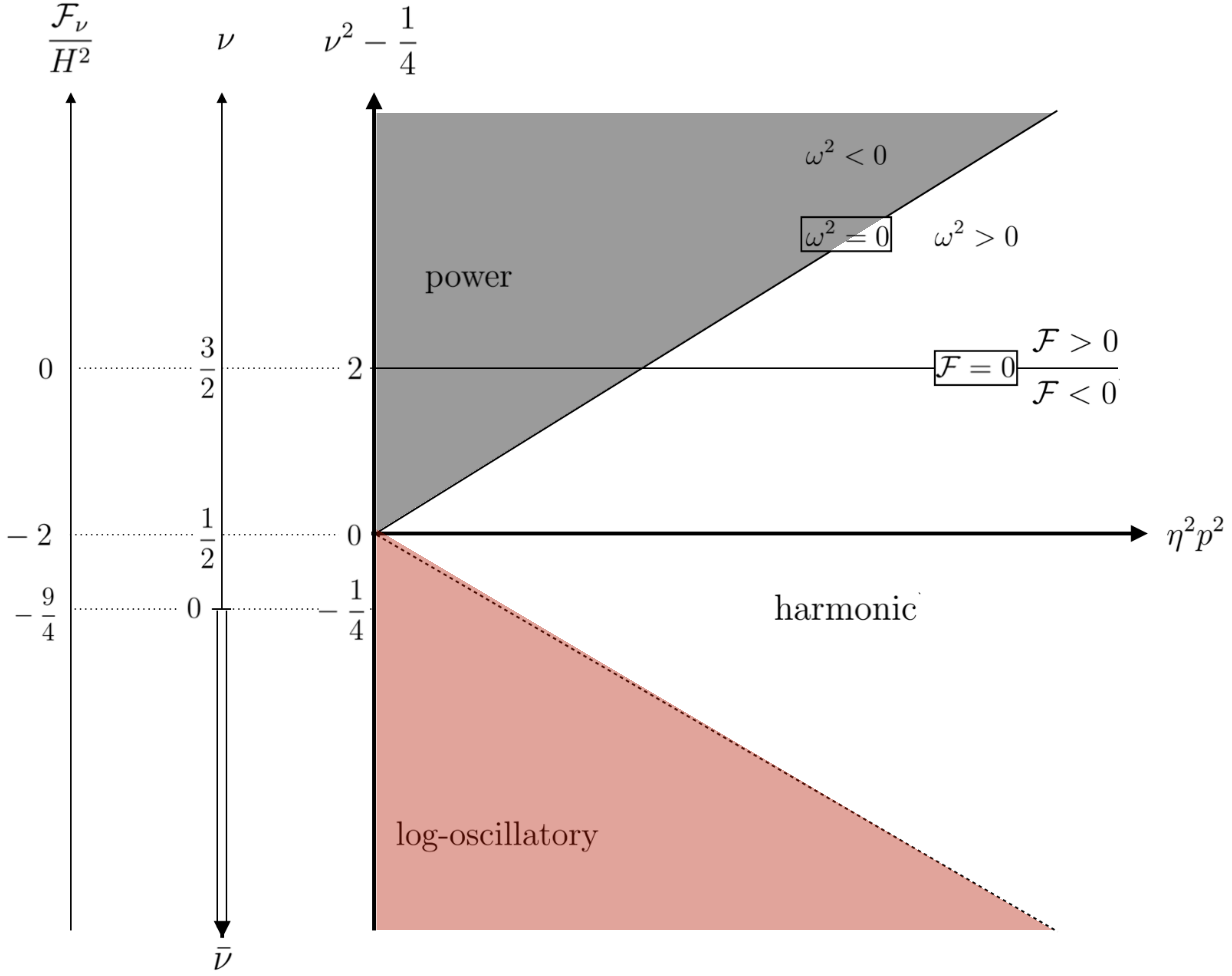
Bessel functions

Eigenvalues: $$\mathscr{F}_\nu = \left(\nu^2 - \frac{9}{4}\right) H^2$$

Type	Eigenvalue	$\mathcal{F}_\nu/\mathbf{H^2}+\mathbf{2}$	Index
spacelike: $\mathcal{F} > 0$	$\mathcal{F}_\nu \in (0,\infty)\ H^2$	$\nu^2 - \frac{1}{4} \in (2,\infty)$	$\nu \in \left(\frac{3}{2},\infty\right)$
null: $\mathcal{F} = 0$	$\mathcal{F}_\nu = 0$	$\nu^2 - \frac{1}{4} = 2$	$\nu = \frac{3}{2}$
timelike: $\mathcal{F} < 0$	$\mathcal{F}_\nu \in \left(-\frac{9}{4},0\right)\ H^2$	$\nu^2 - \frac{1}{4} \in \left(-\frac{1}{4},2\right)$	$\nu \in \left(0,\frac{3}{2}\right)$
	$\mathcal{F}_\nu \in \left(-\infty,-\frac{9}{4}\right)\ H^2$	$\nu^2 - \frac{1}{4} \in \left(-\infty,-\frac{1}{4}\right)$	$i\,\nu \equiv \bar{\nu} \in (0,\infty)$

The ν–p plane

$$v''_{\nu,p}(\eta) + \left[p^2 - \frac{\nu^2 - 1/4}{\eta^2}\right] v_{\nu,p}(\eta) = 0$$



Scale dependent dS_4 solutions (on-shell)

$$H(k) = \sqrt{\frac{\Lambda(k)}{3}}$$

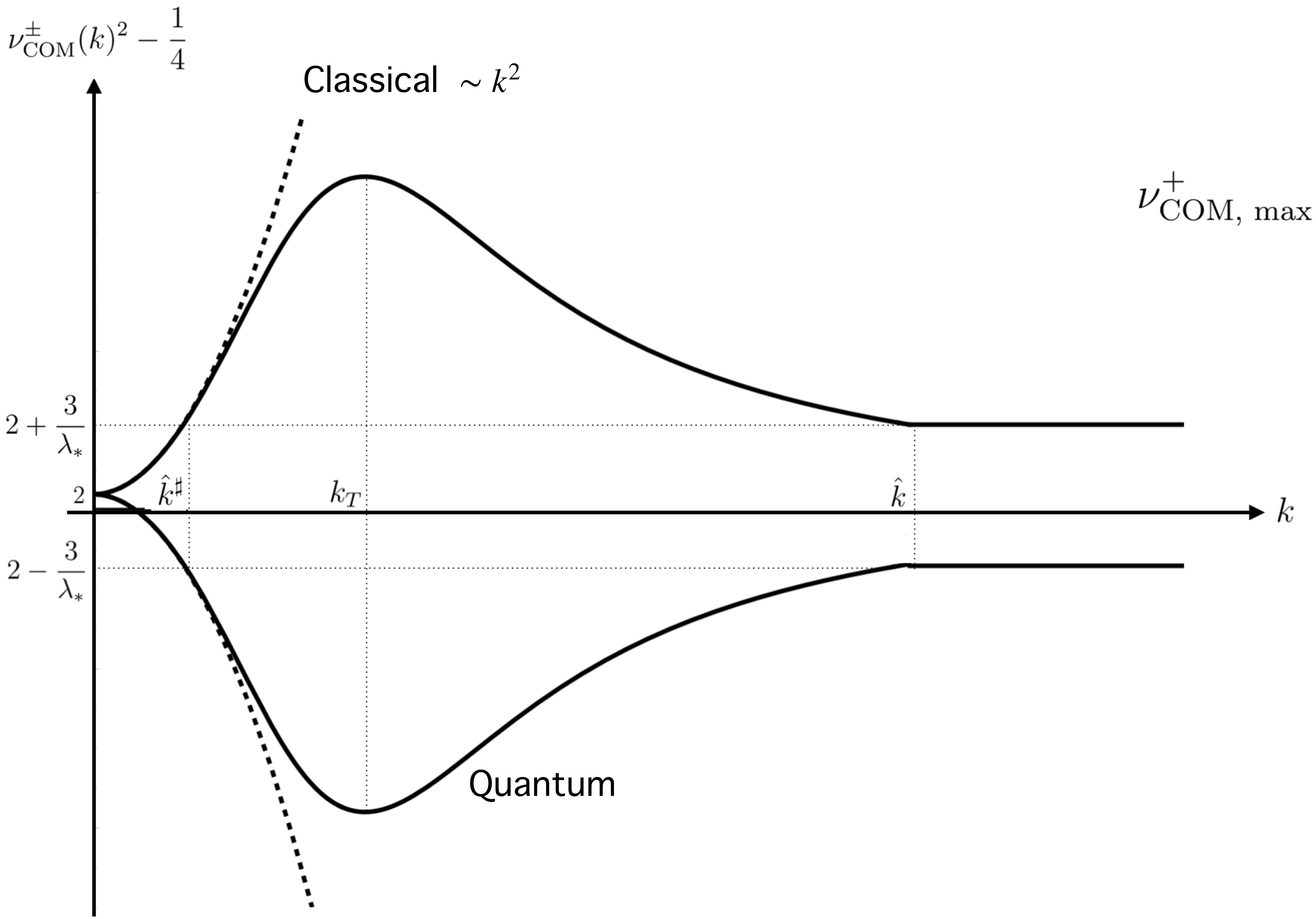
Self-consistent dS background: $$\left(g_k^{sc}\right)^k_{\mu\nu} dx^\mu dx^\nu = \frac{-d\eta^2 + d\mathbf{x}^2}{\eta^2\, H(k)^2}$$

Running eigenvalues: $$\mathscr{F}_\nu(k) = \left(\nu^2 - \frac{9}{4}\right) H(k)^2 = \left(\nu^2 - \frac{9}{4}\right) \frac{\Lambda(k)}{3}$$

spacelike and timelike

Cutoff modes: $$\mathscr{F}_\nu(k)\Big|_{\nu=\nu^\pm_{COM}(k)} = \pm k^2 \quad \Longrightarrow \quad \nu^\pm_{COM}(k)^2 = \frac{9}{4} \pm \frac{3}{\lambda(k)}$$

Evolving COM quantum numbers



$$\nu^{+}_{\rm COM,\ max} = \nu^{+}_{\rm COM}(k_T) \approx \left(\frac{3}{\lambda_T}\right)^{1/2}$$

What can we conclude about the resolution?

EFT and cutoff modes

Effective quantum geometry at scale k

Which are the geometrical features that are displayed by the “on-shell’" mean field configurations?

Are there structures which have a size that is comparable to the length scale at which Γ_k defines a **“good effective field theory”**?

Resolving structures on a time slice: effective spatial geometry

For every fixed time η and scale k the modes possess **unlimited resolving power** for spatial structures on the respective 3D time slice of the dS manifold.

We have no way of controlling the η-dependence of the modes if we use up all our freedom by optimizing the spatial resolution.

Impose conditions on the space of detectable modes inspired by experimental setting

The characteristic COM proper lengths

Proper wavelength

$$L_p(\eta, k) \equiv b_k(\eta)\ \Delta x_p = \frac{2\pi}{|\eta|\ p\ H(k)}$$

Transition wavelength

It is the largest possible **proper** wavelength a cutoff mode can posses in the harmonic regime.

$$L^+_{COM}(k) = \frac{2\pi}{k}\sqrt{\frac{3}{3 + 2\,\lambda(k)}}$$

depends on k, it is independent of η

Consider the ratio:

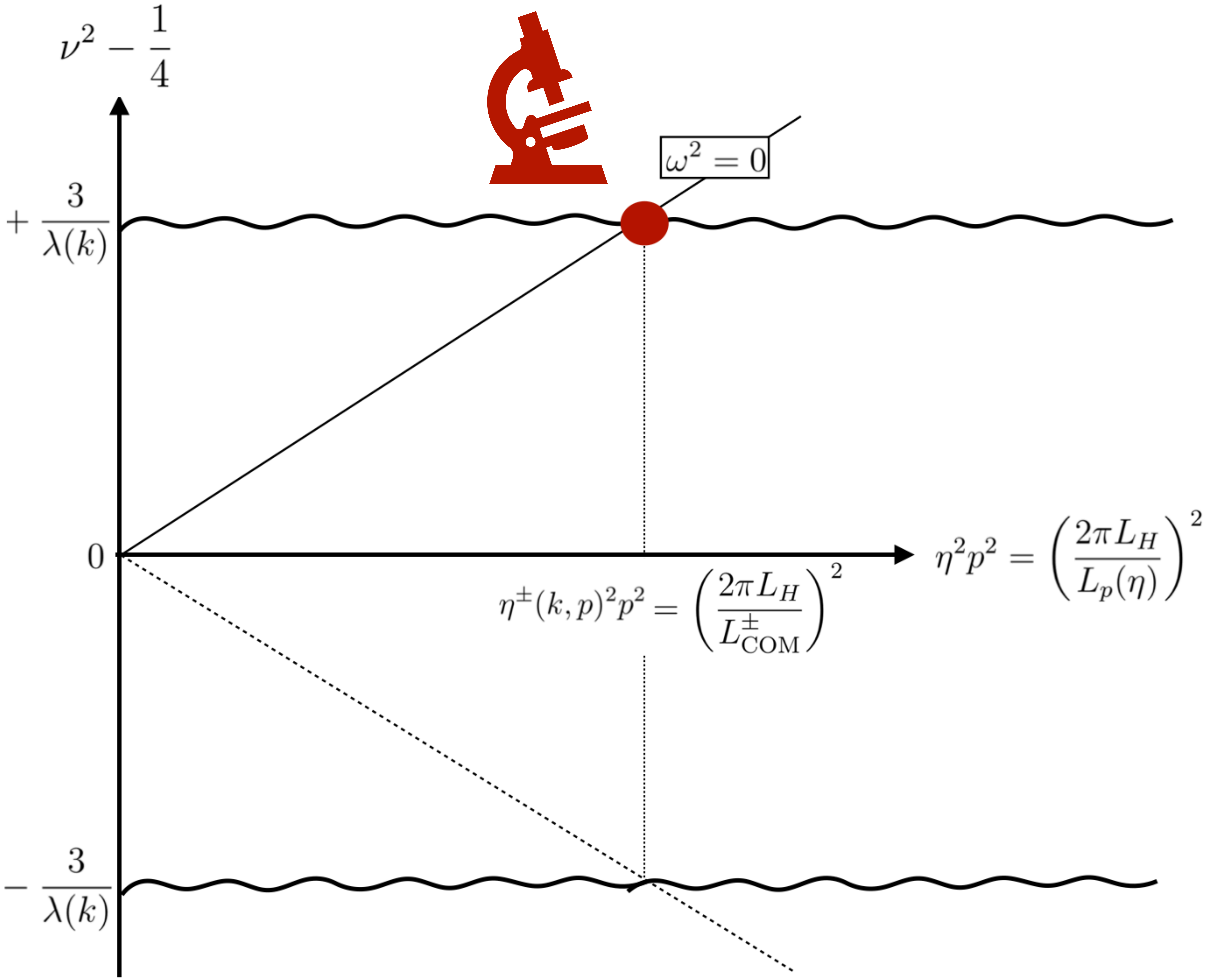
$$\frac{L^+_{COM}(k)}{L_H(k)} = 2\pi\sqrt{\frac{\lambda(k)}{3 + 2\,\lambda(k)}}$$

Near the turning point

$$\left(\frac{L^+_{COM}(k)}{L_H(k)}\right)_{max} \approx \frac{2\pi}{\nu^+_{COM}(k_T)} \approx 2\pi\left[\frac{4}{9}\ G_0\ \Lambda_0\right]^{1/4}$$

$\longrightarrow$ $L^+_{COM}(k) \ll L_H(k)$

Detector model



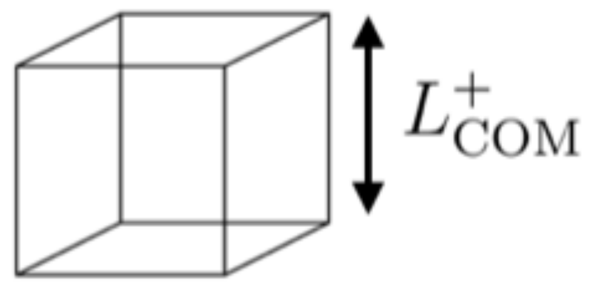
For every k, only η-independent COMs and combinations thereof are registered. All observed structures of field configurations are strictly time independent then.

$$L_p = L^+_{COM}(k)$$

Patterns observed in the Universe should display a maximum size which is significantly smaller than the Hubble radius (**CAUSALITY**).

$$L_p = L^+_{COM}(k) \ll L_H(k)$$

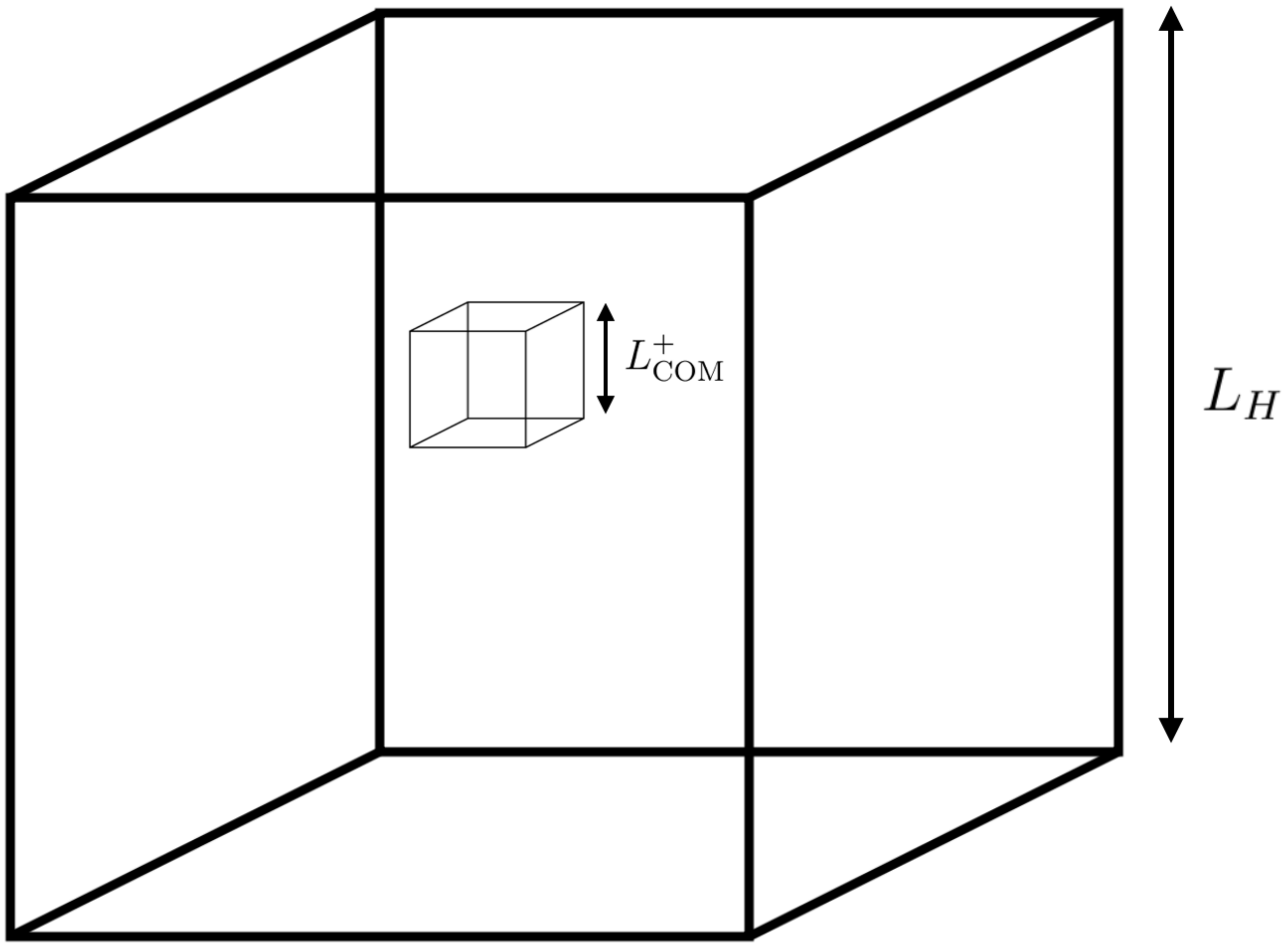
Coherence length



Physics and geometry is well described within a patch by one of the effective field theories $\{\Gamma_k\}_{k\geq 0}$.

Counting boxes

PATCHWORK:
Fragmentation of 3D space



How many of those “COM boxes" would fit into one Hubble volume?

$$N_b(k) = \left(\frac{L_H(k)}{L^+_{COM}(k)}\right)^3 = \frac{1}{(2\pi)^3}\left[2 + \frac{3}{\lambda(k)}\right]^{3/2}$$

$$N_b^{max} = N_b(k_T) \approx \frac{1}{(2\pi)^3}\left[\frac{4}{9}\,\varpi\, G_0\, \Lambda_0\right]^{-3/4}$$

E.g.: $\varpi = O(1), \quad G_0\Lambda_0 \approx 10^{-120}$

$N_b^{max} \approx 10^{90}$

inter-domain entropy

CMBR photons

More than an analogy?

$$\mathcal{S}(T,V)/N(T,V) = 2\pi^4 \mathrm{k_B}/45\zeta(3)$$

$$\mathcal{S} \approx 10^{90}\ \mathrm{k_B}$$

agrees precisely with the inter-domain entropy $N_b(k_T)$

Just a coincidence?

$$N(T,V) = \frac{V}{\left[1.27\ \ \lambda_{peak}(T)\right]^3}$$

$$\lambda_{peak} \approx 1.06\ \mathrm{mm}$$

$$L^+_{COM}(k_T) \approx \left(10^{30}H_0\right)^{-1} \approx \left(10^{-30}m_{Pl}\right)^{-1} \approx 10\ \ \mu m$$

This analogy seems to motivate a scenario in which the CMBR traces out coherent grains of space.

Conclusions

On-shell spectral flow along the functional RG trajectories

1 Fineness and resolving power of the cutoff modes no longer improves when k is increased beyond k_T, it rather deteriorates quite considerably when it approaches the Planck scale, until Asymptotic Safety establishes a constant fixed point value.

Euclidean

limitation on the distinguishability of points in spacetime

microscopic effect

Lorentzian

no analogous restriction for the resolvability of points on the 3D spatial manifold related to the foliation

macroscopic effect

2 nonperturbative quantum gravity-generated vacuum structures of the 3D space, seen as a slice through dS_4

EFT

$$\mathcal{S} \approx 10^{90}\ \mathrm{k_B}$$

Thank you for your attention

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