Towards a microscopic understanding of macroscopic properties of strong-interaction matter

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CRC-TR 211

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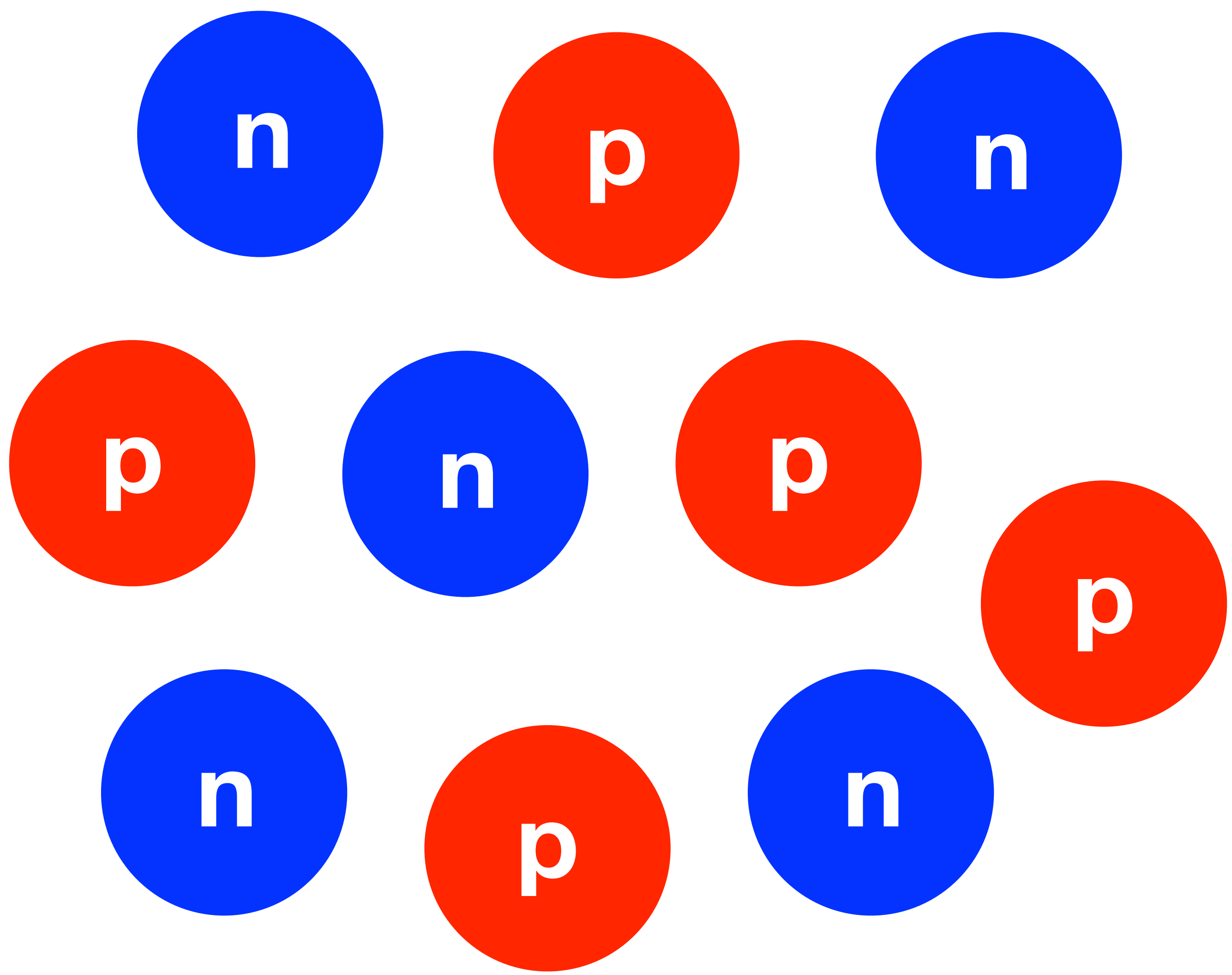
SFB 1245

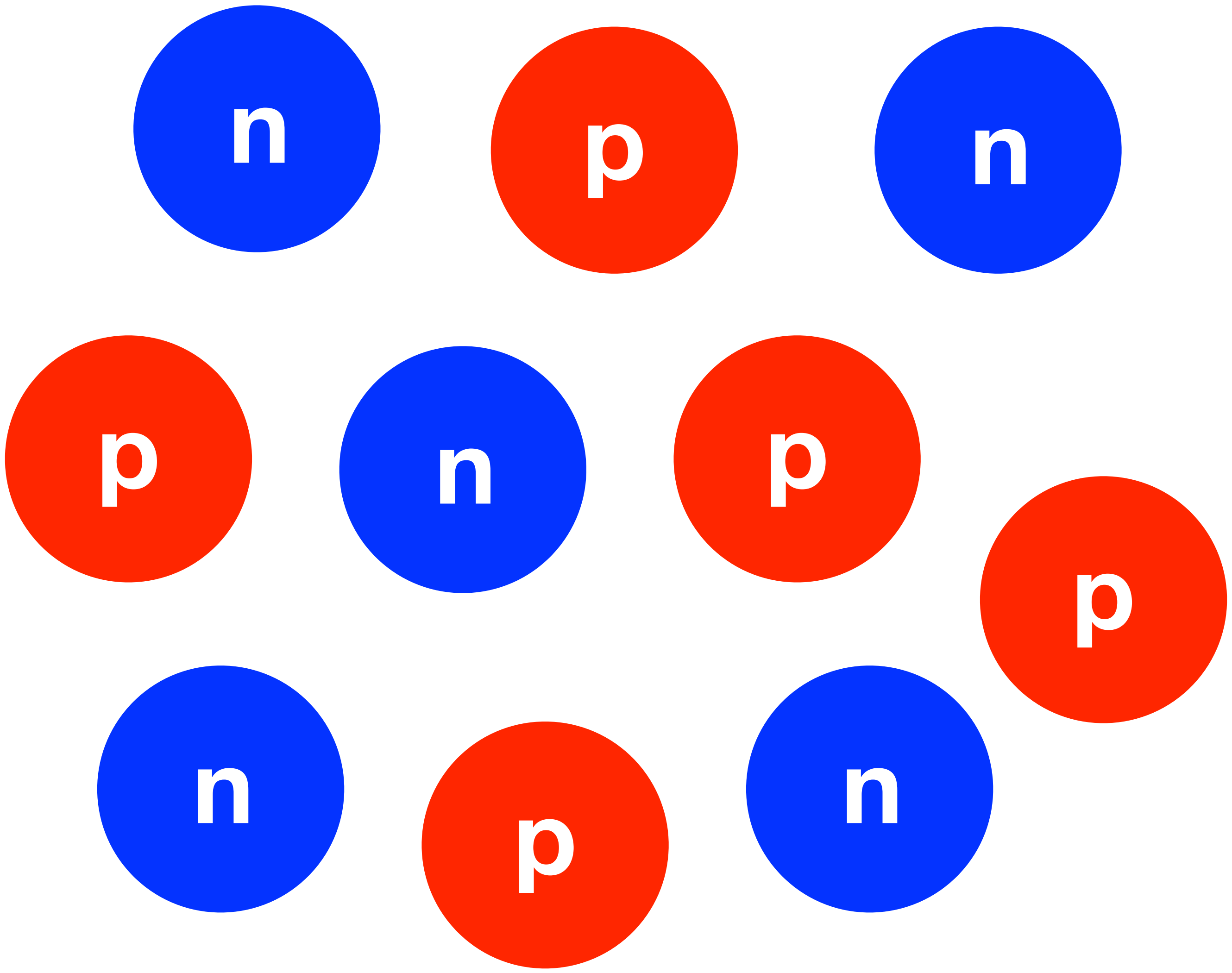
EL EM EN TS

Strong-interaction matter: what is it?

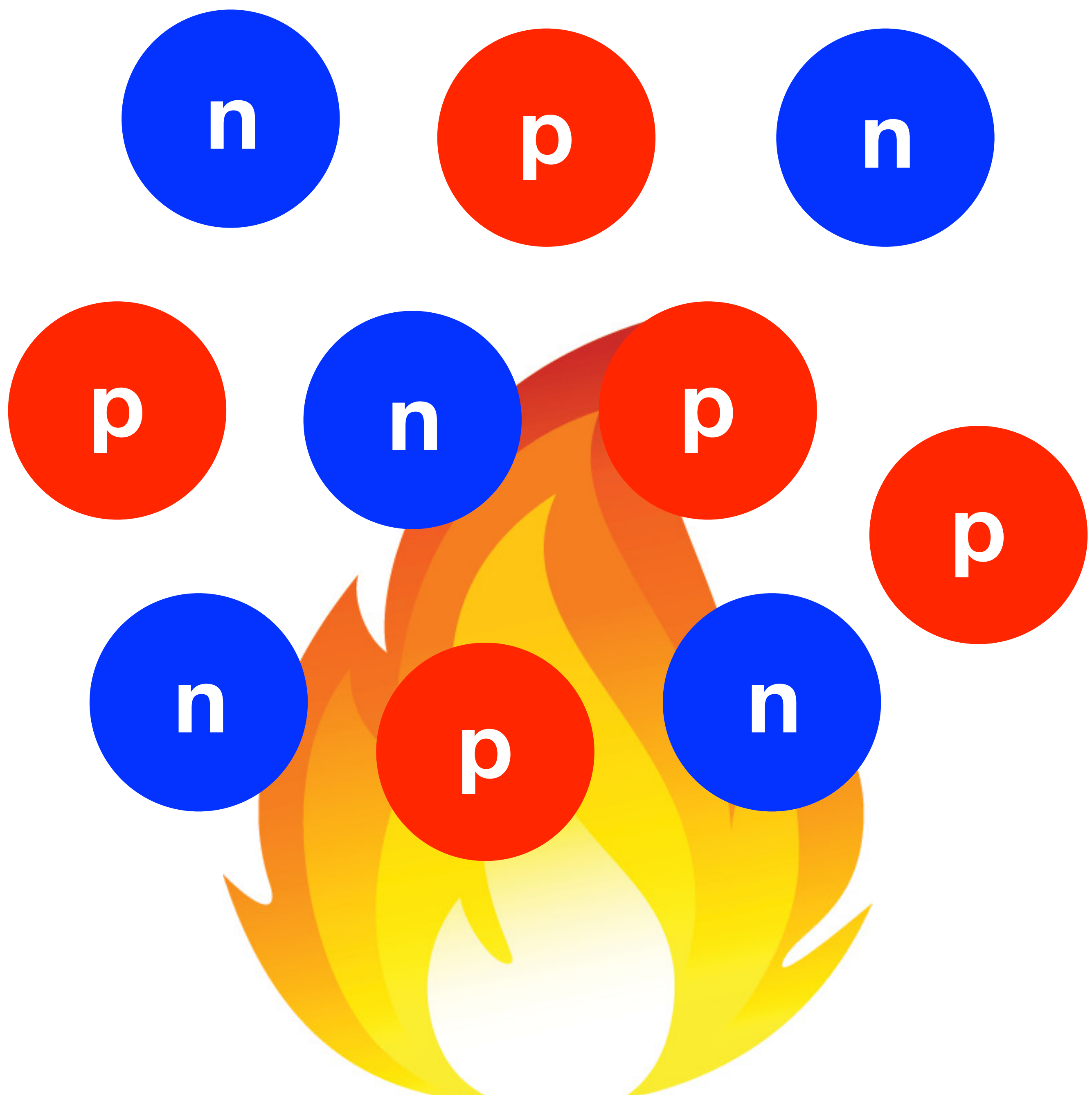
Strong-interaction matter: “one form of appearance”



Strong-interaction matter: what can you do with it?



Strong-interaction matter: you can heat it up



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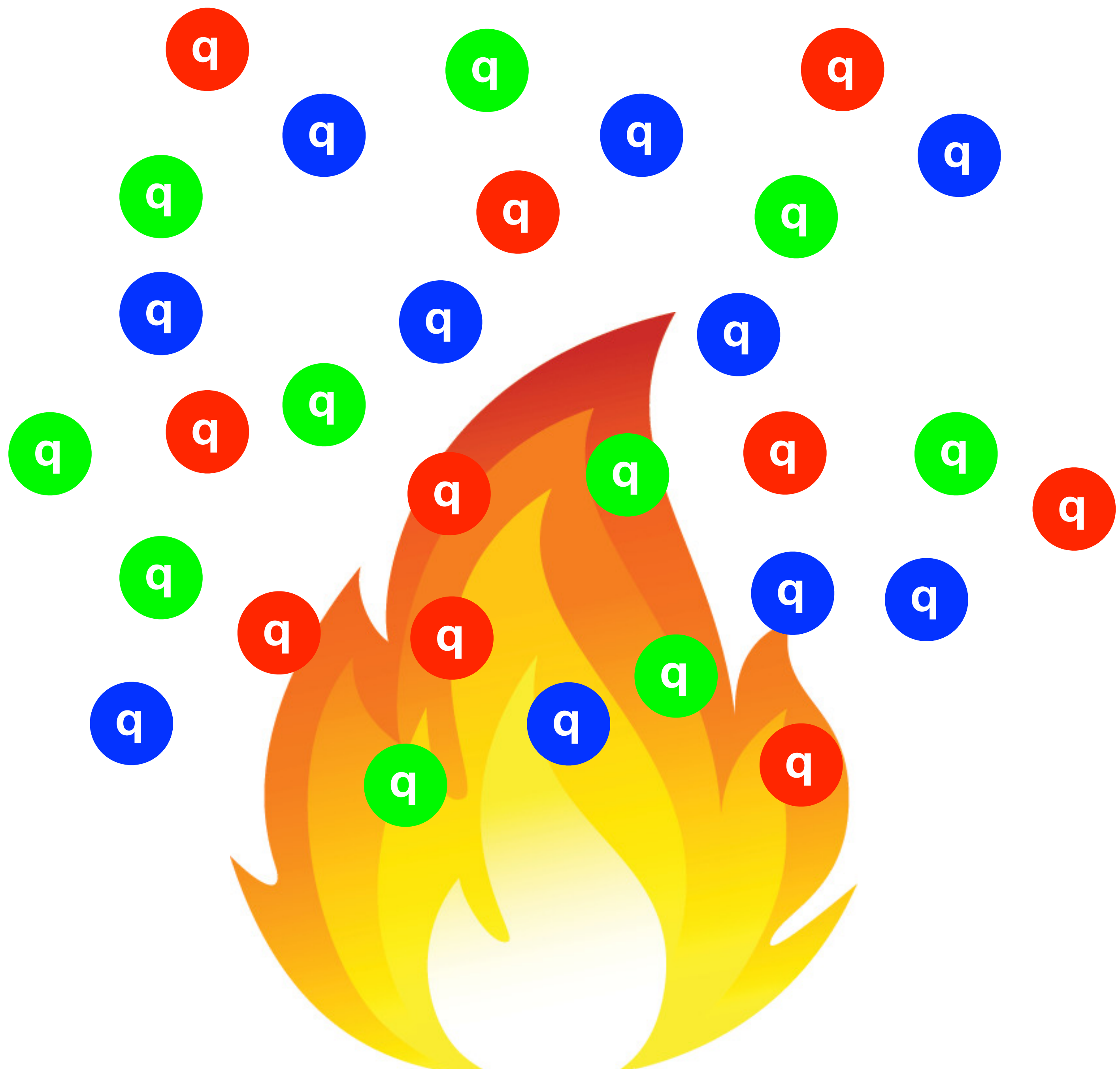

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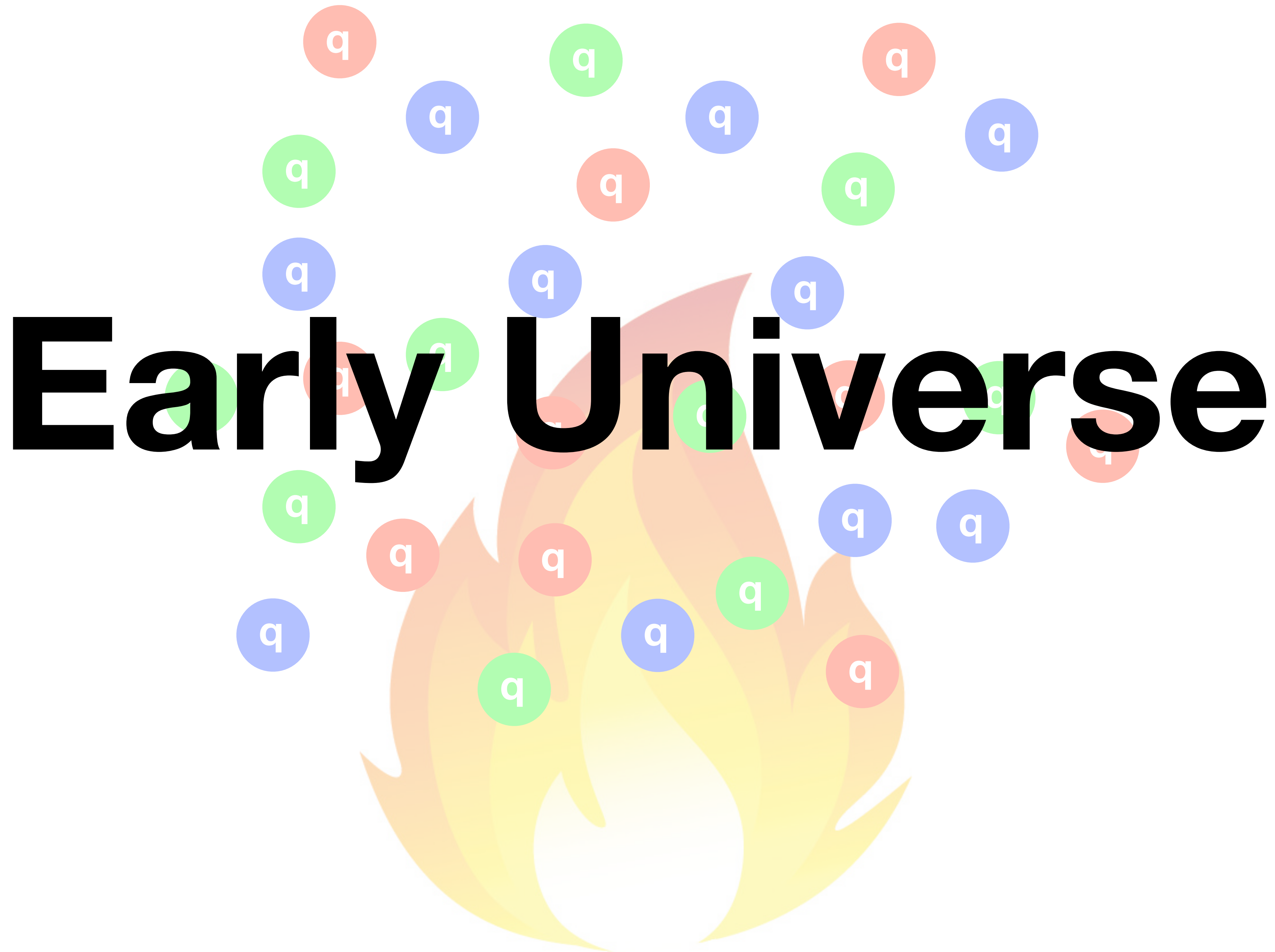
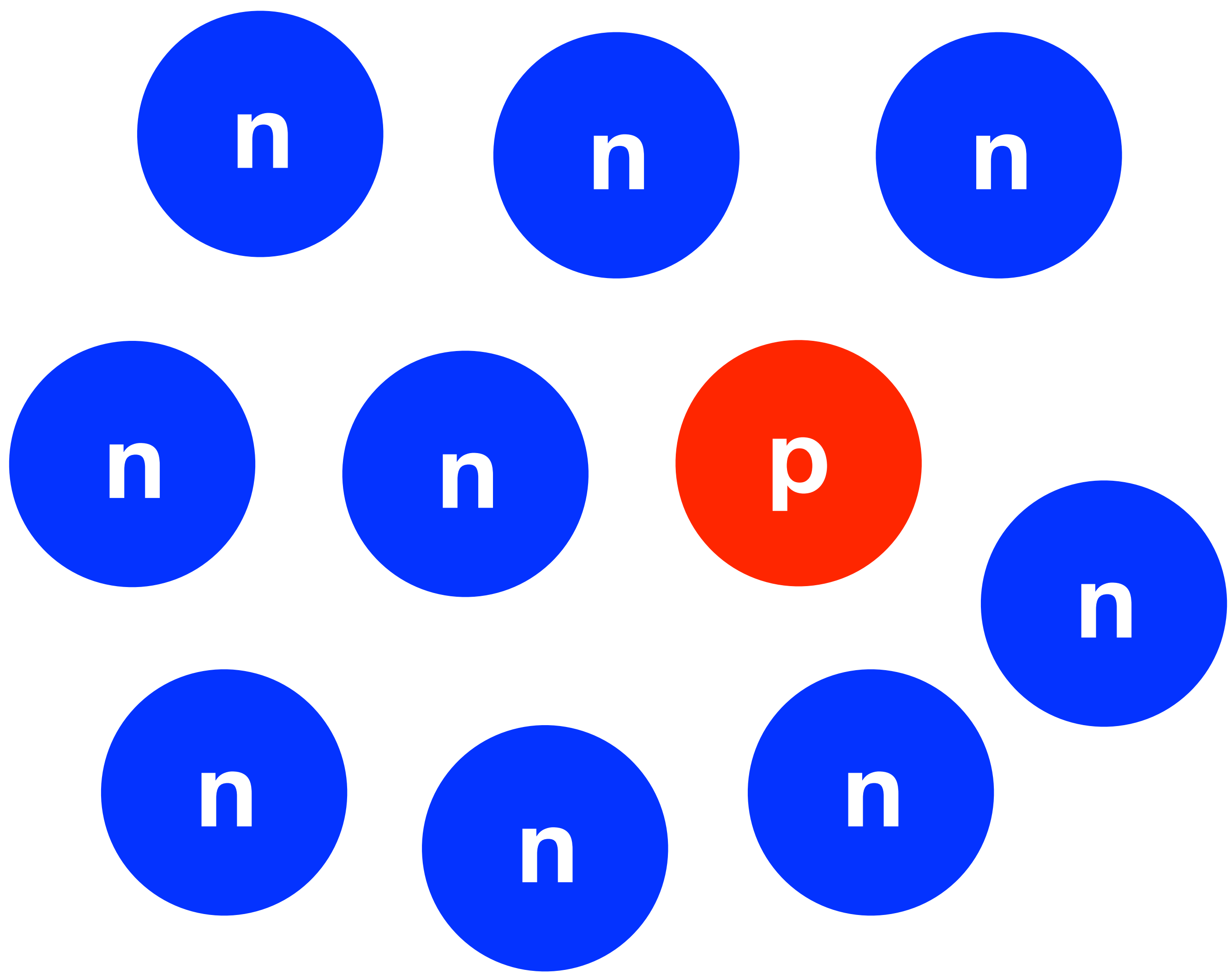
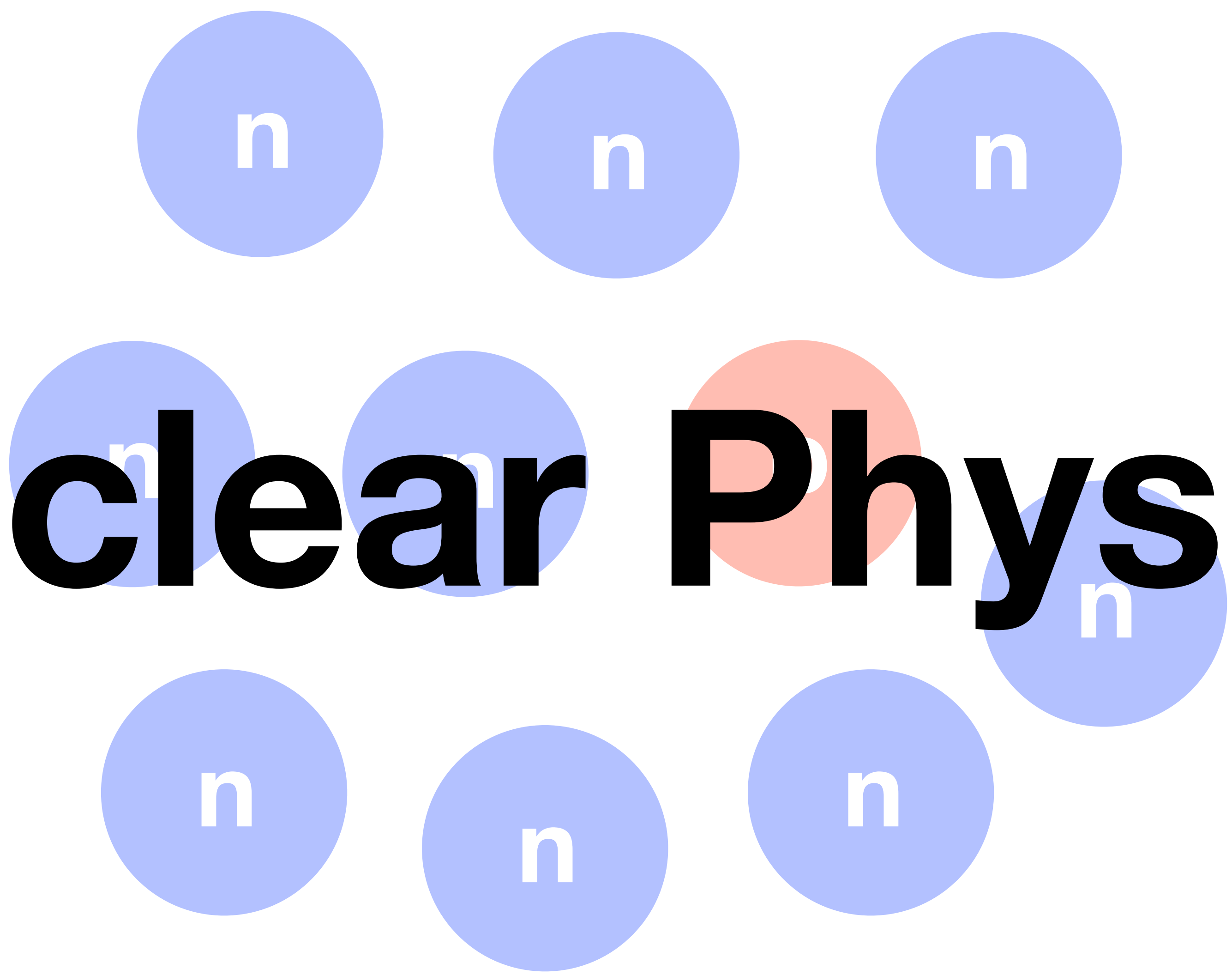


Strong-interaction matter: you can “polarize” it

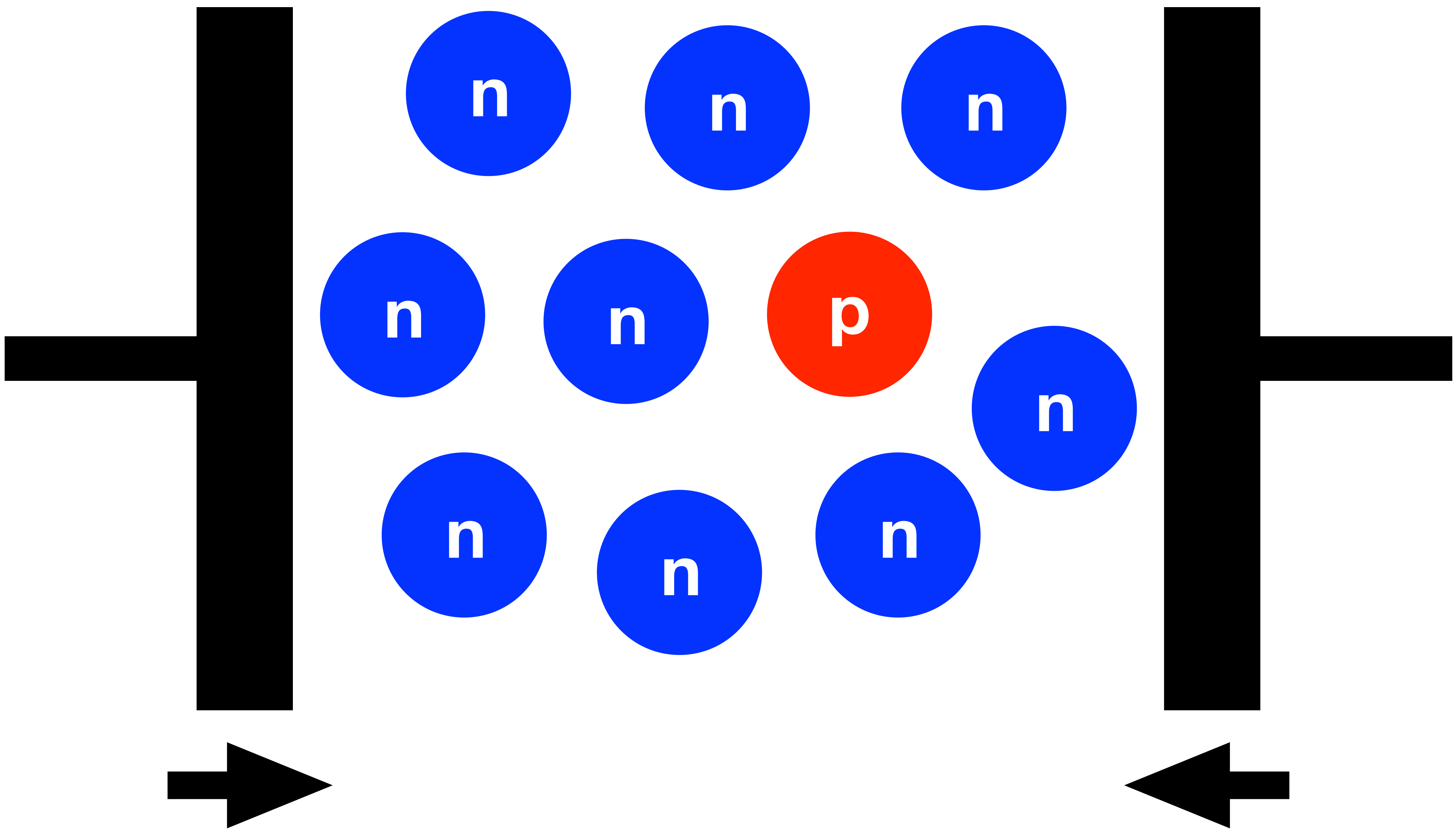


Strong-interaction matter: why "polarizing" it?

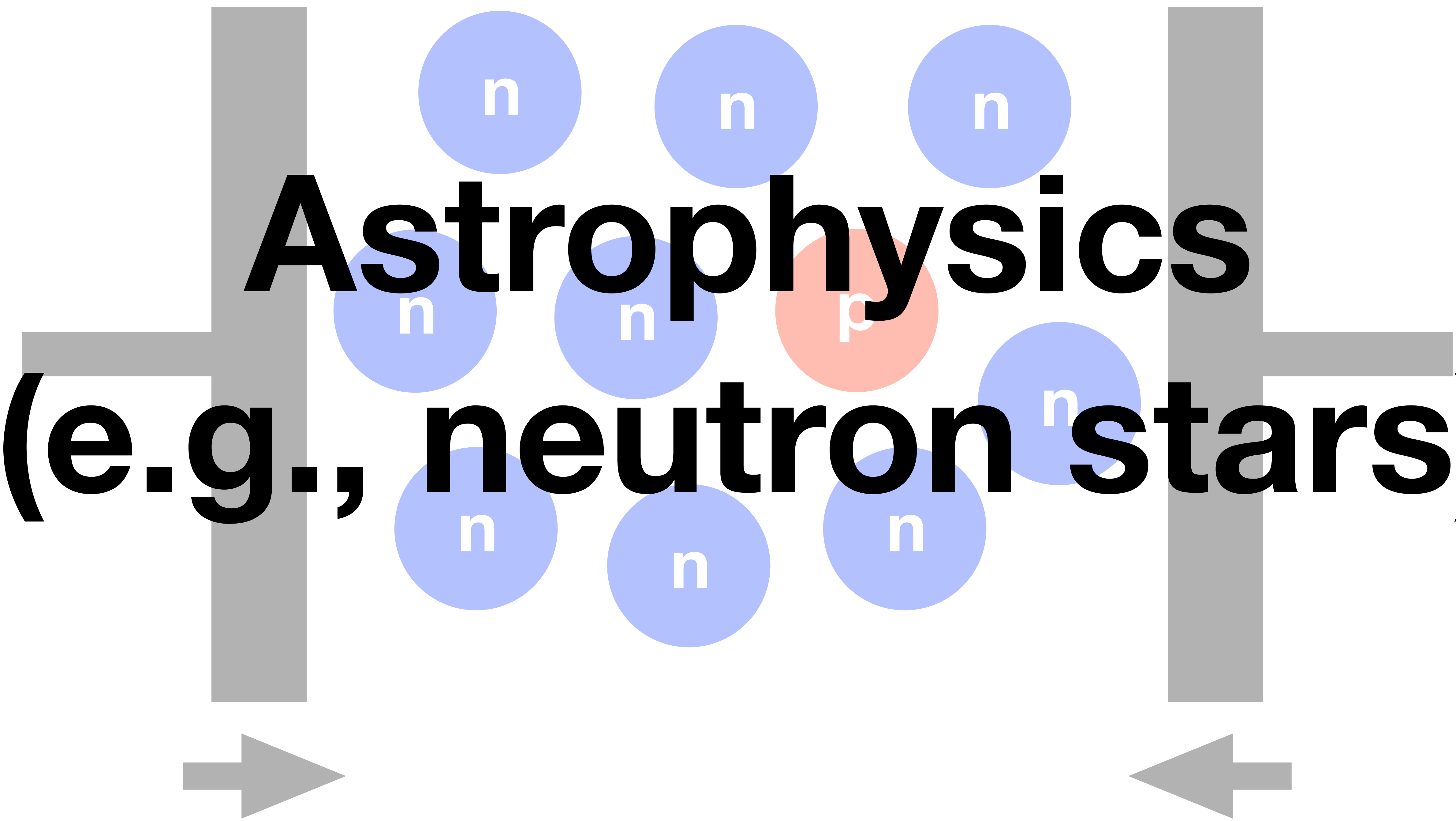


Nuclear Physics

Strong-interaction matter: you can then compress it

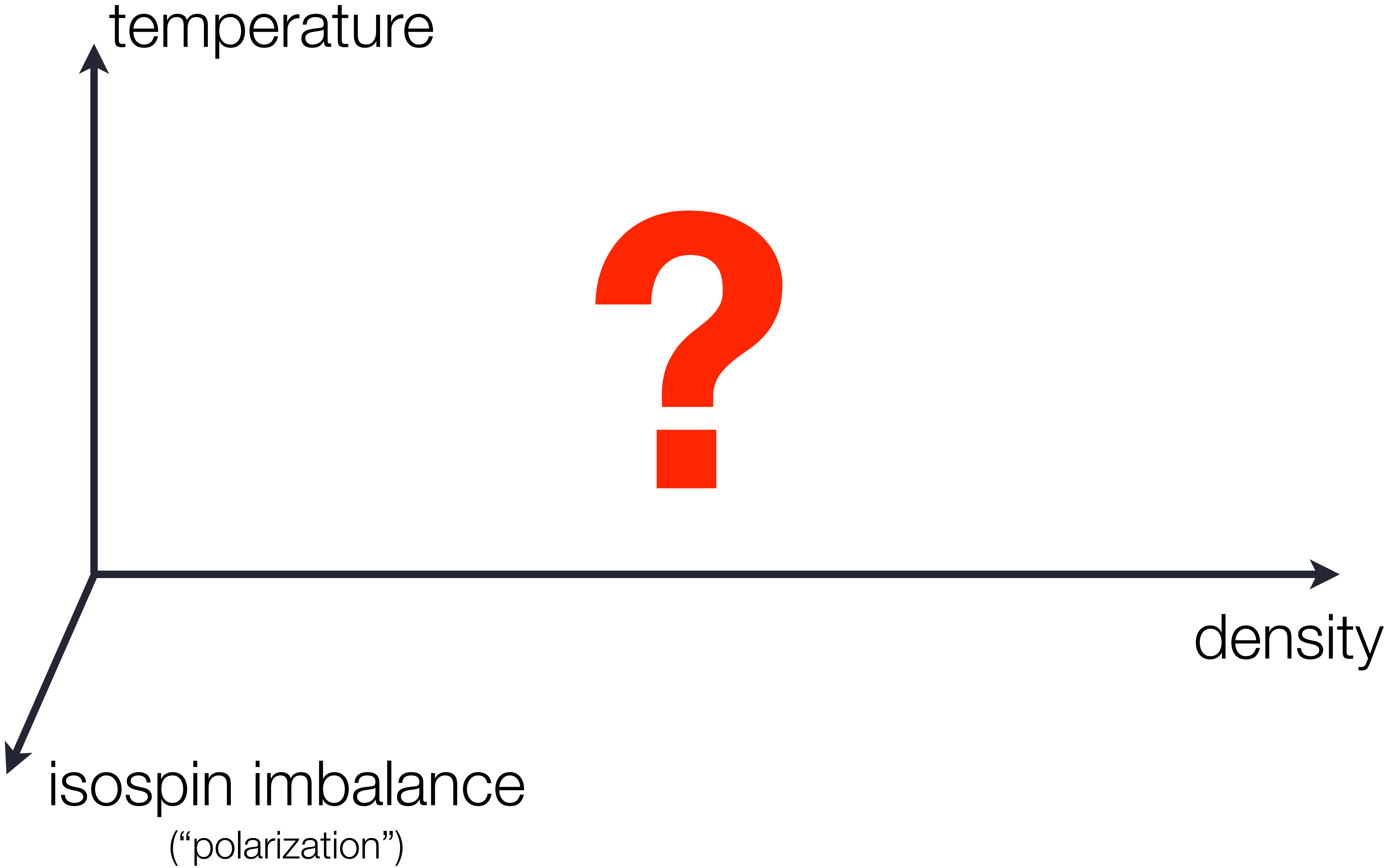


Strong-interaction matter: why compressing it?



Astrophysics (e.g., neutron stars)

Phases of strong-interaction matter?



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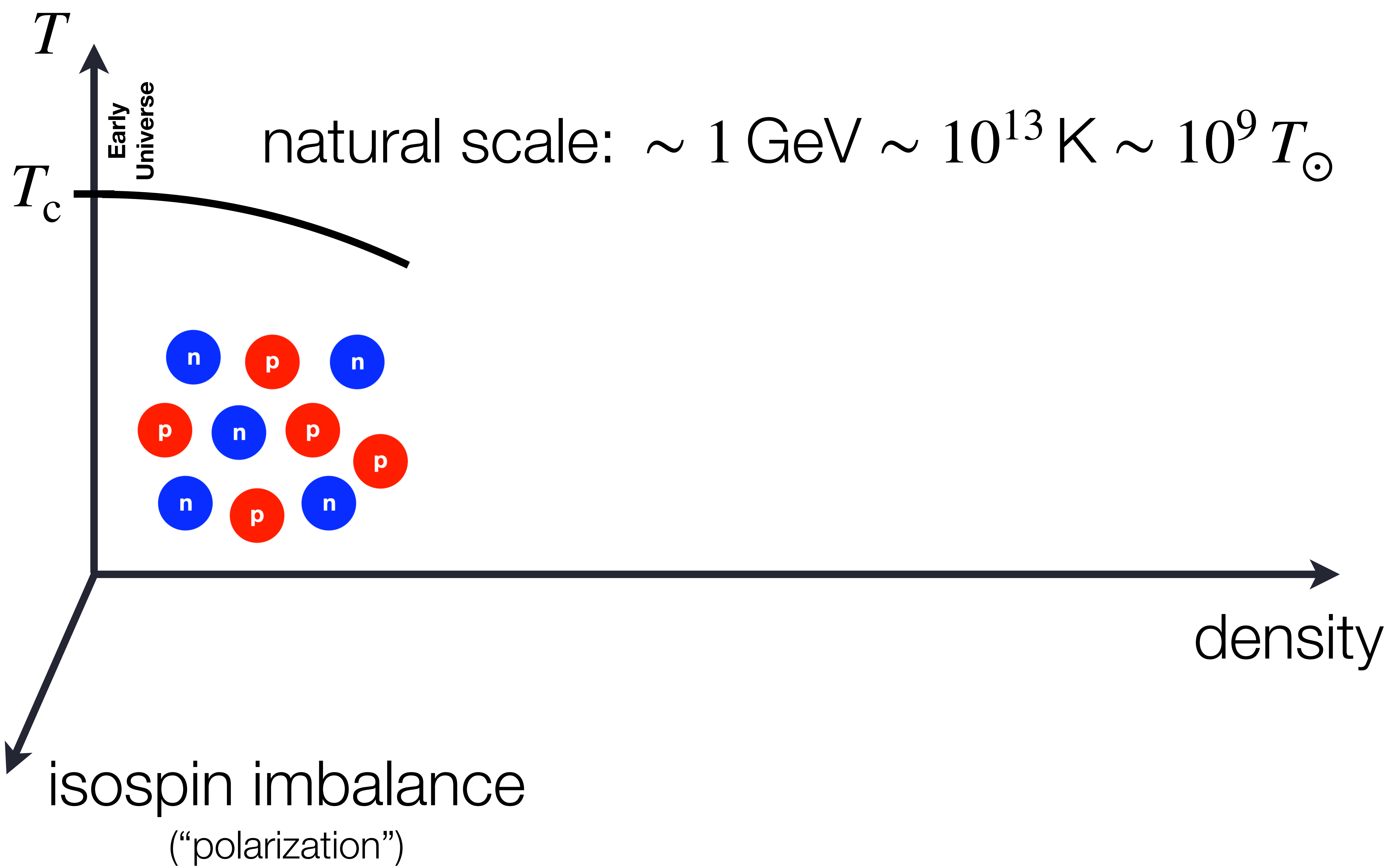

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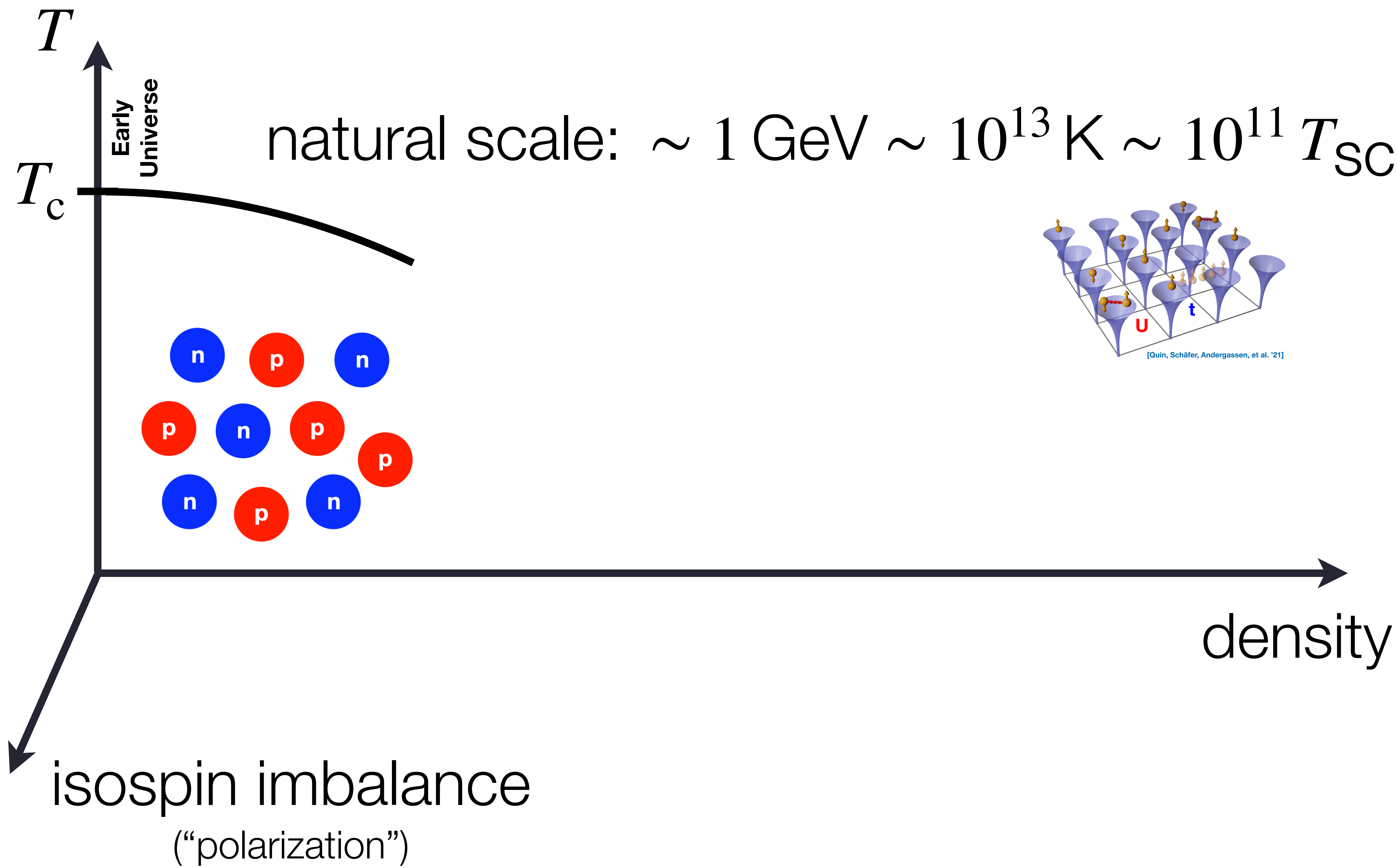

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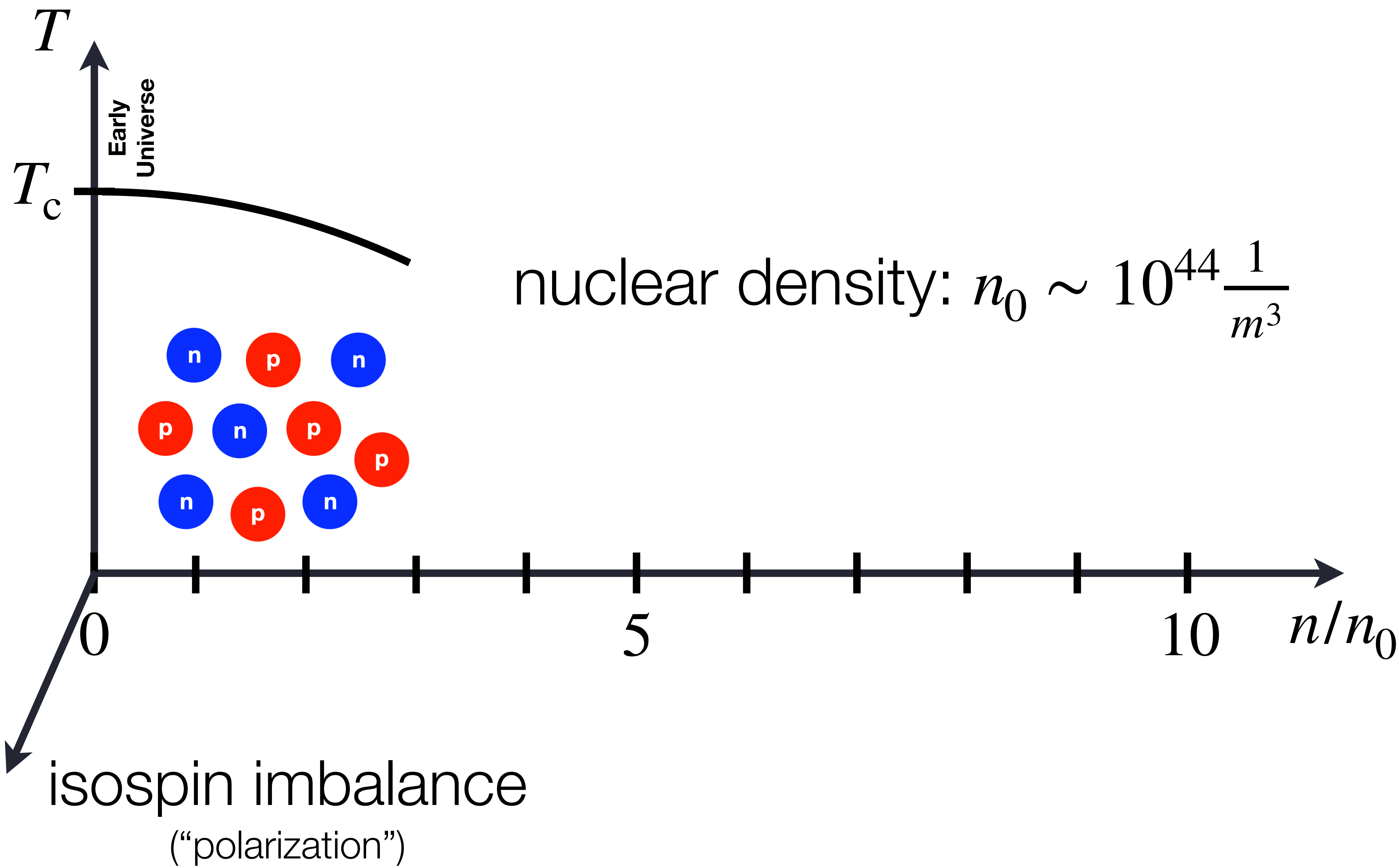

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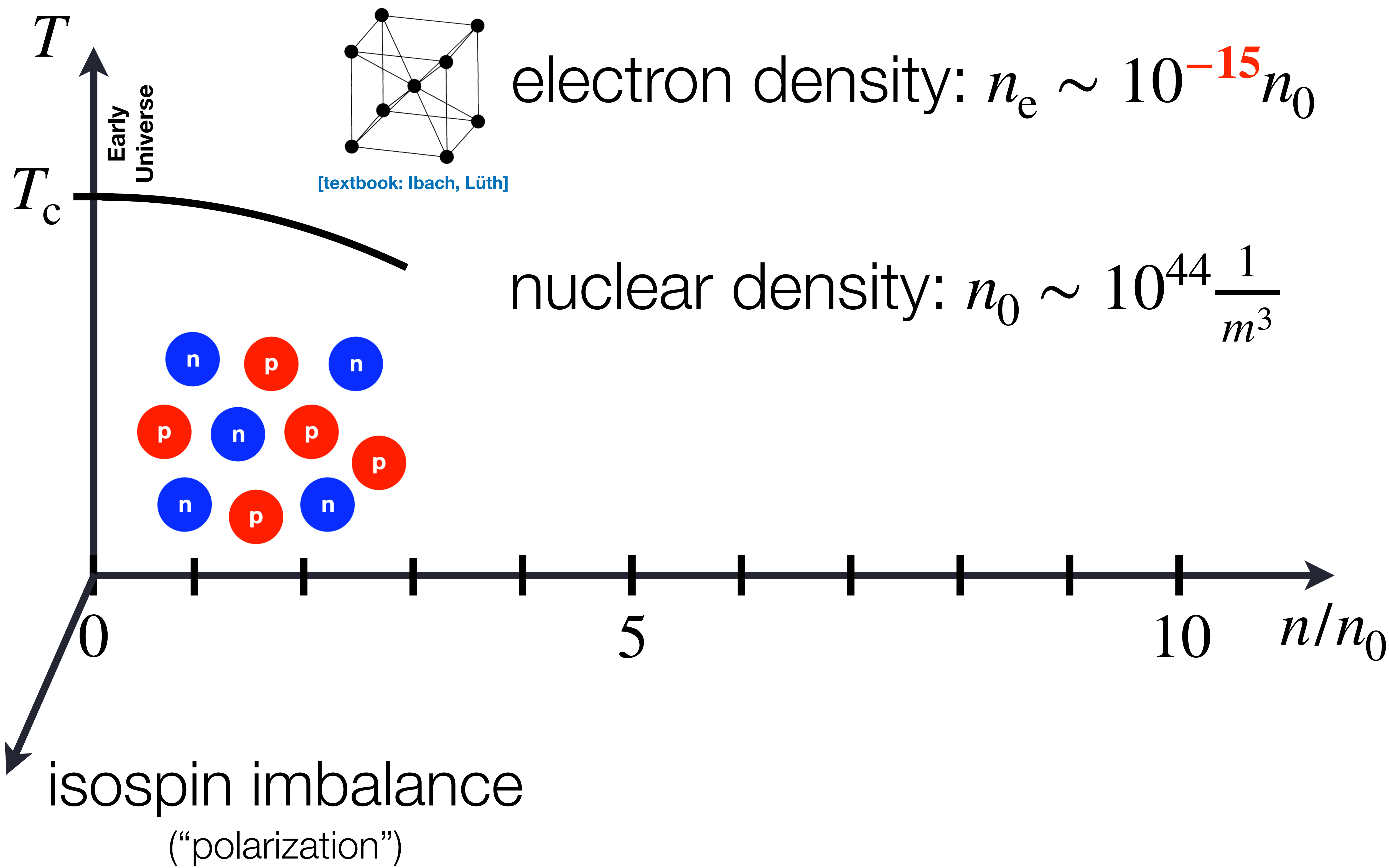
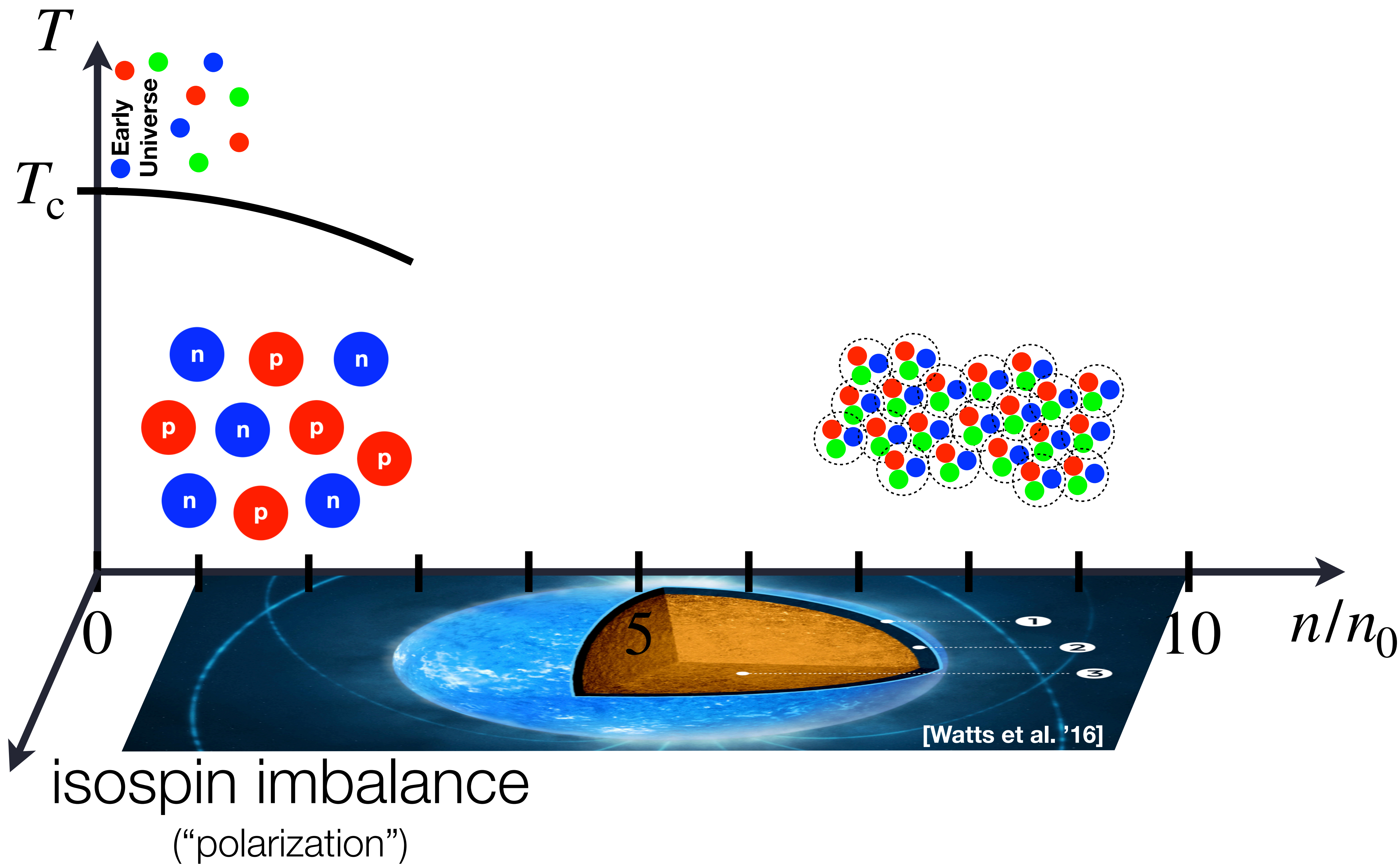


electron density: $n_e \sim 10^{-15} n_0$

nuclear density: $n_0 \sim 10^{44} \frac{1}{m^3}$

Phases of strong-interaction matter?



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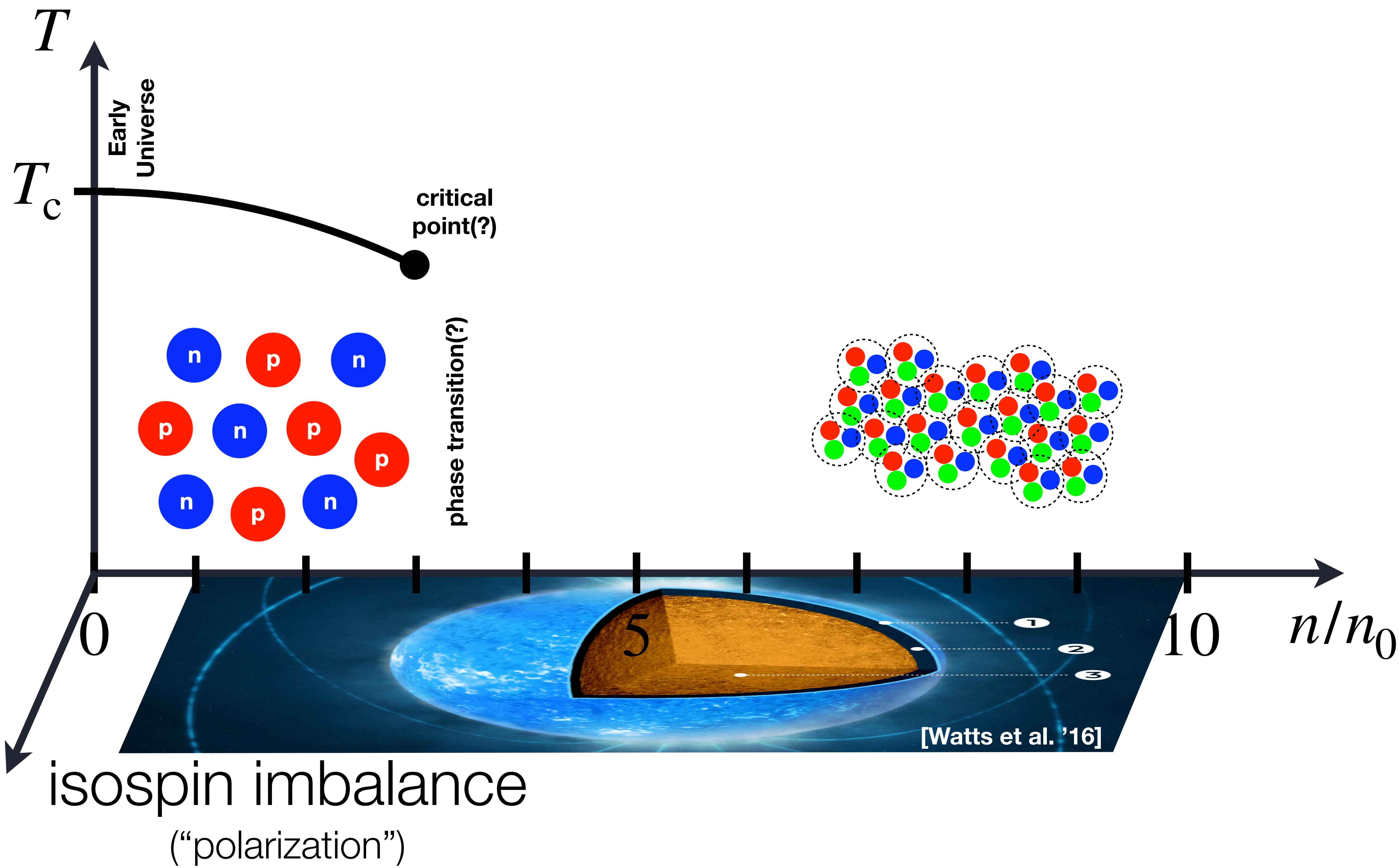

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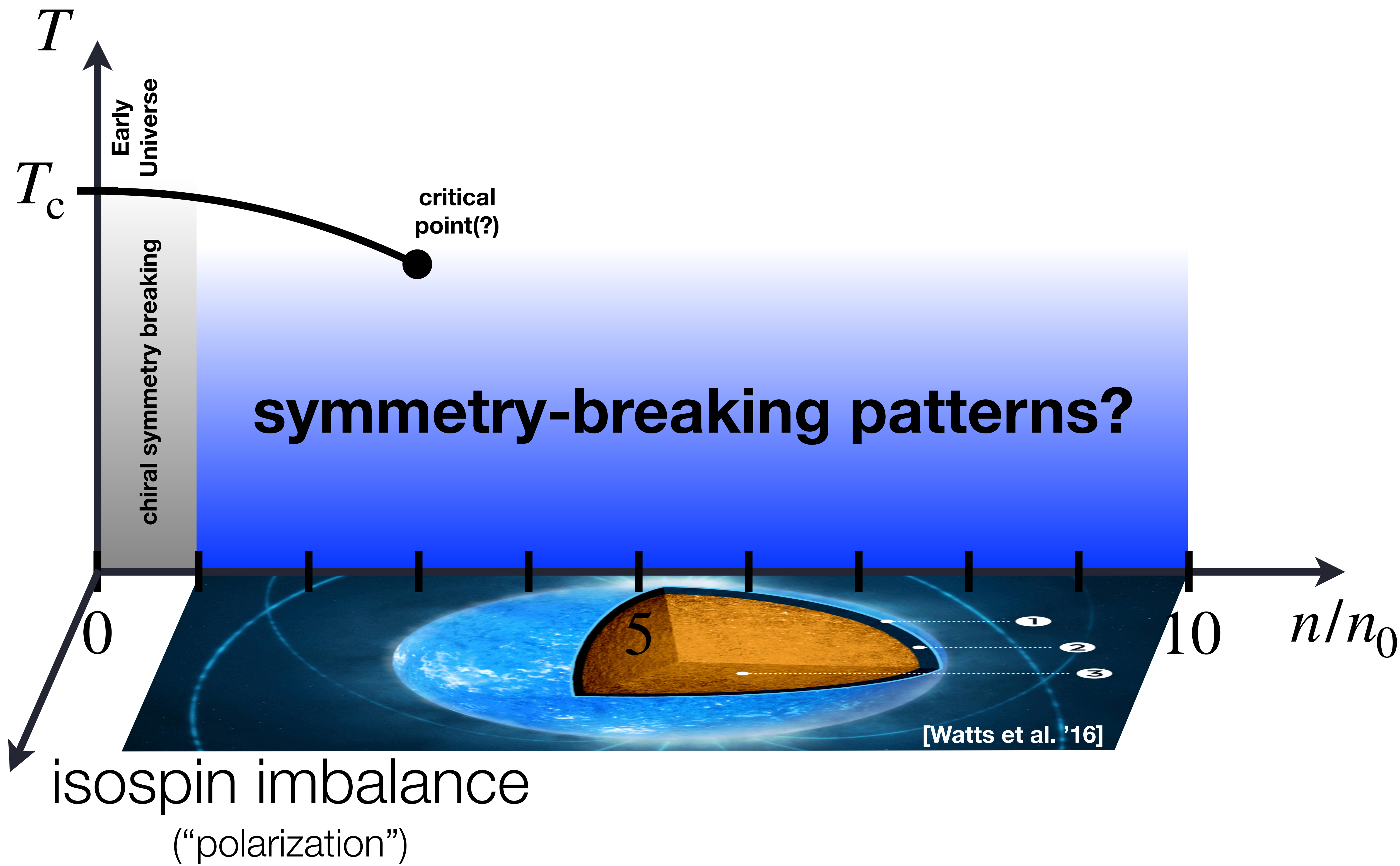


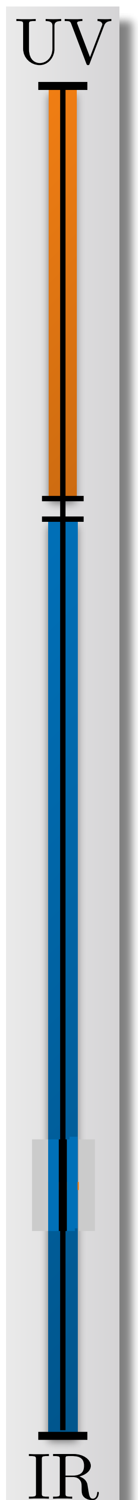
Theory of the strong interaction (QCD)

$$S = \int_0^{\overset{\text{inverse temperature}}{\beta}} \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \underbrace{\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}}_{\text{gluons}} + \overbrace{\bar{\psi} \left(\mathrm{i}\partial\!\!\!/ + \underbrace{g_s A\!\!\!/}_{\text{quark-gluon coupling}} - \mathrm{i}\underbrace{\mu}_{\text{chemical potential}}\gamma_0 \right) \psi}^{\text{quarks}} \right\}$$

Functional Renormalization Group

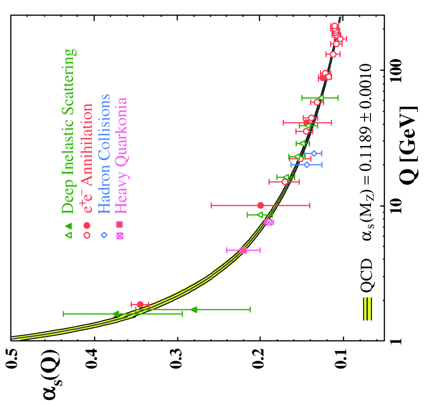
[Wetterich ’92]

Λ



“resolution scale”

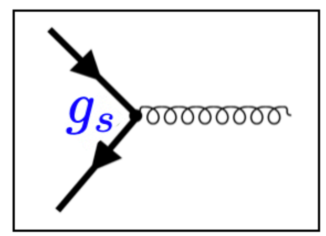
$k \sim Q$



0

- $$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + g_s A\!\!\!/)\psi \right\}$$

  input parameter: strong coupling $g_s(\Lambda)$



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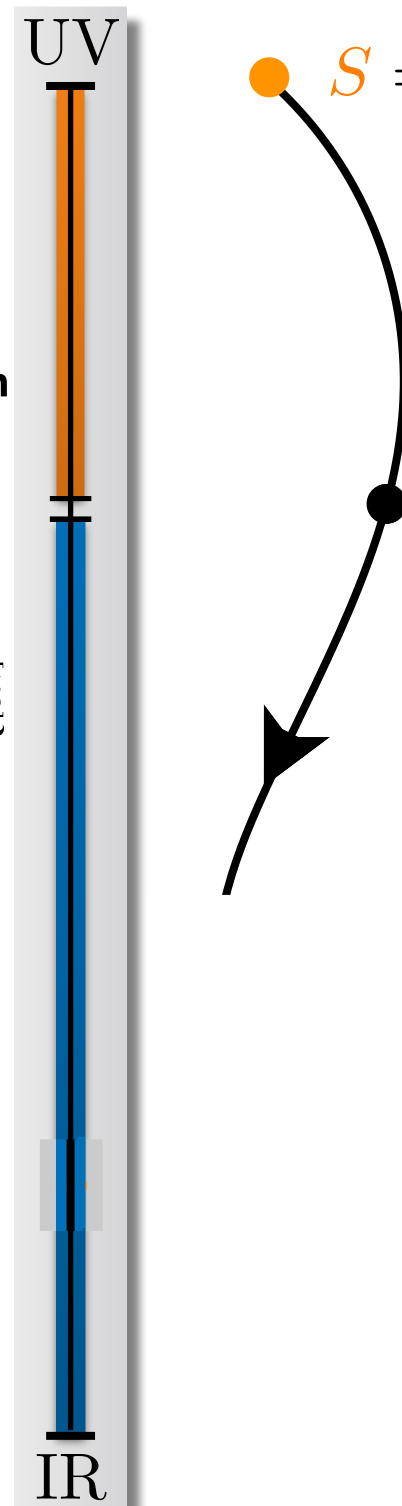

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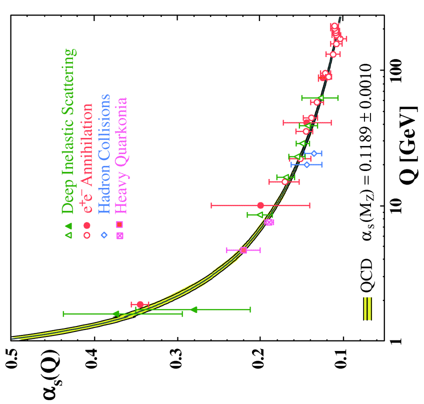

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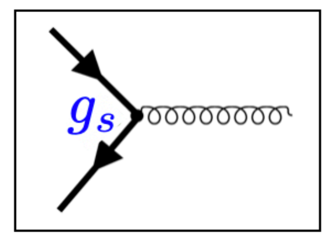
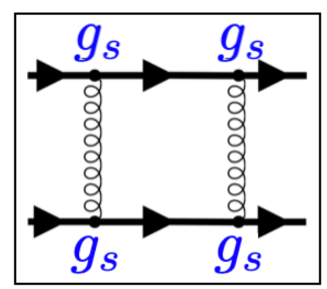
Γ_k

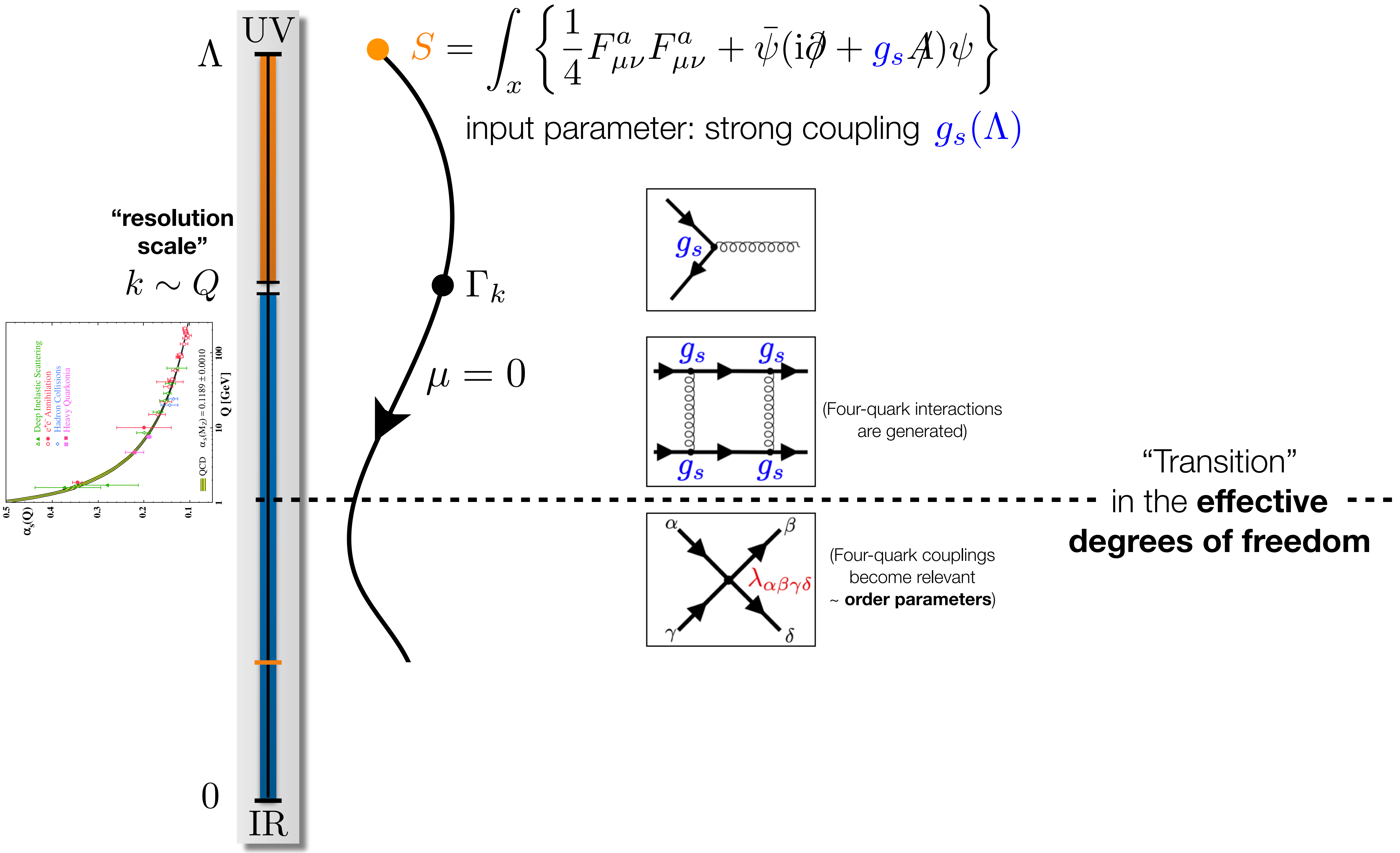




(Four-quark interactions are generated)

Functional Renormalization Group: Vacuum

[Wetterich '92]



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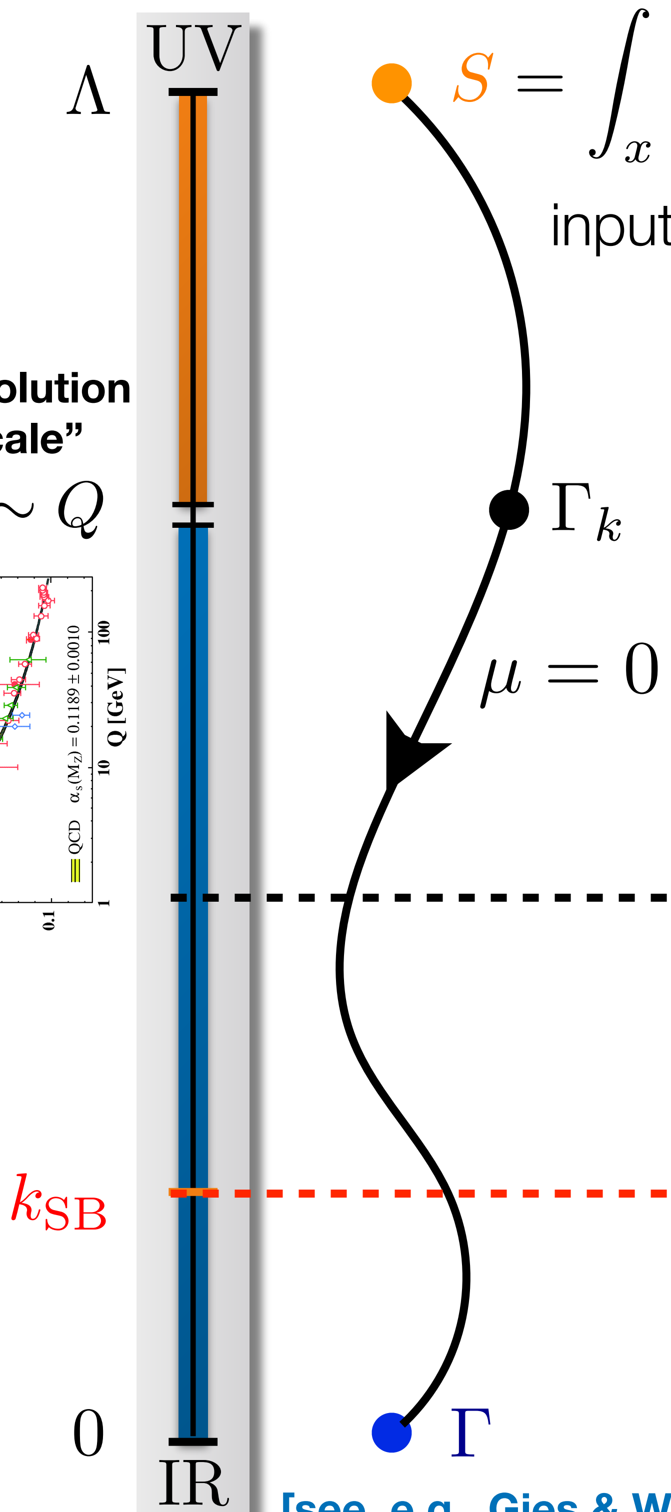
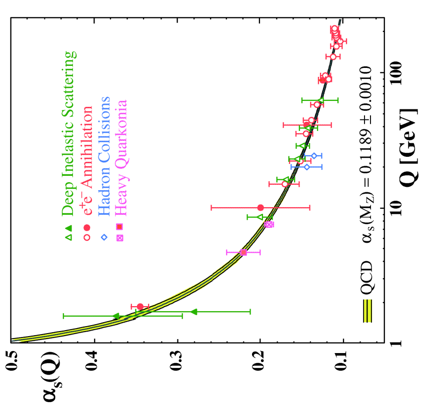
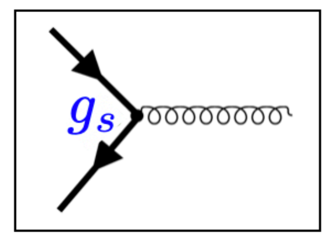
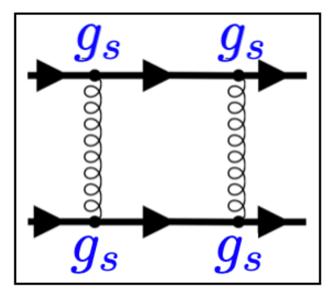


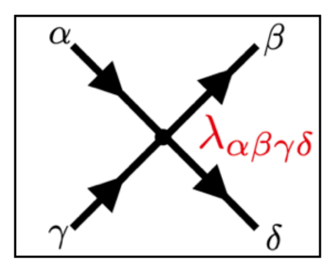




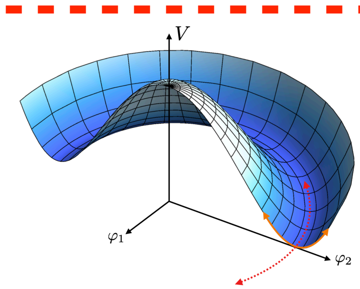


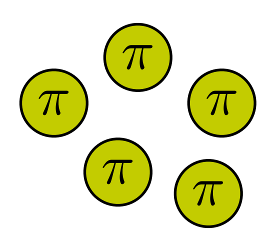
(Four-quark interactions are generated)

"Transition" in the **effective degrees of freedom**



(Four-quark couplings become relevant ~ **order parameters**)





spontaneous **chiral symmetry breaking**
(signalled by a corresponding strong four-quark channel)

[see, e.g., Gies & Wetterich '02; Gies & Jaeckel '05; JB & Gies '05; JB, Fister, Pawlowski, Rennecke '14; Mitter, Pawlowski, Strodthoff '14]

How many four-quark channels? Symmetries!

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[Gies & Jaeckel ’05; JB & Gies ’05, ’06; Mitter, Pawlowski, Strodthoff ’14; JB, Leonhardt, Pospiech’ 17]

symmetry	**group**
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\mathrm{U}_A(1)$
Poincare	✓
parity	✓
charge conjugation	✓
# of channels (pointlike limit)	**4**

Fierz-complete set

How many four-quark channels? Symmetries!

[JB, Leonhardt, Pospiech' 17]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\cancel{\mathrm{U}_A(1)}$
Poincare	✗
parity	✓
charge conjugation	✗
# of channels (pointlike limit)	**10**

Fierz-complete set

How many four-quark channels? Symmetries!

[JB, Leonhardt, Pospiech’ 17; JB, Leonhardt, Pawłowski, Rosenblüh ’20]

symmetry	group
color	$\mathrm{SU}(N_c)$
chiral	$\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$
vector	$\mathrm{U}_V(1)$
axial	$\cancel{\mathrm{U}_A(1)}$

Poincare	✗
parity	✓
charge conjugation	✗

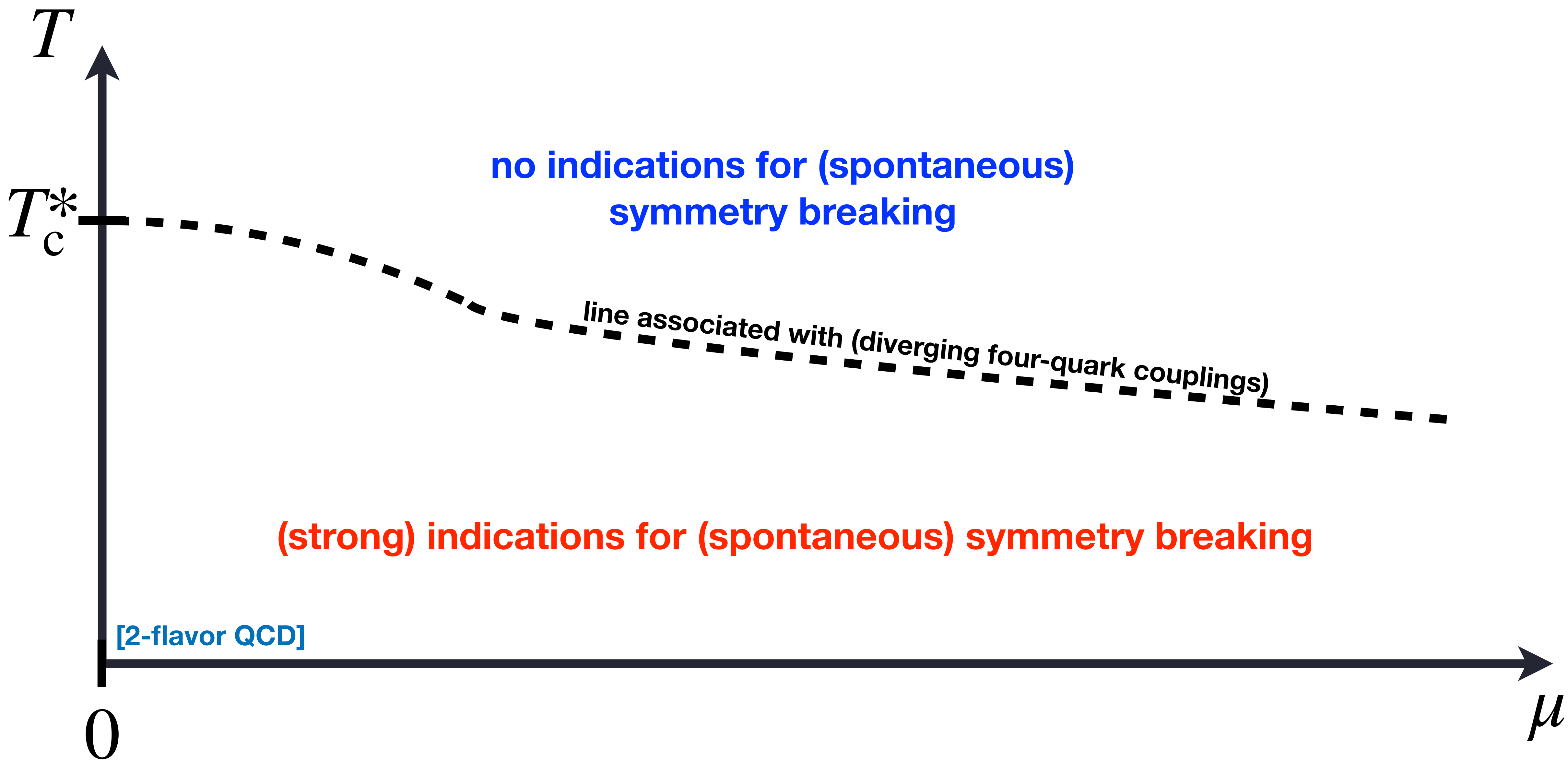
# of channels (pointlike limit)	32

Fierz-complete set

2+1-flavor QCD

Symmetry-breaking patterns

[JB, Leonhardt, Pospiech '17 & '20]



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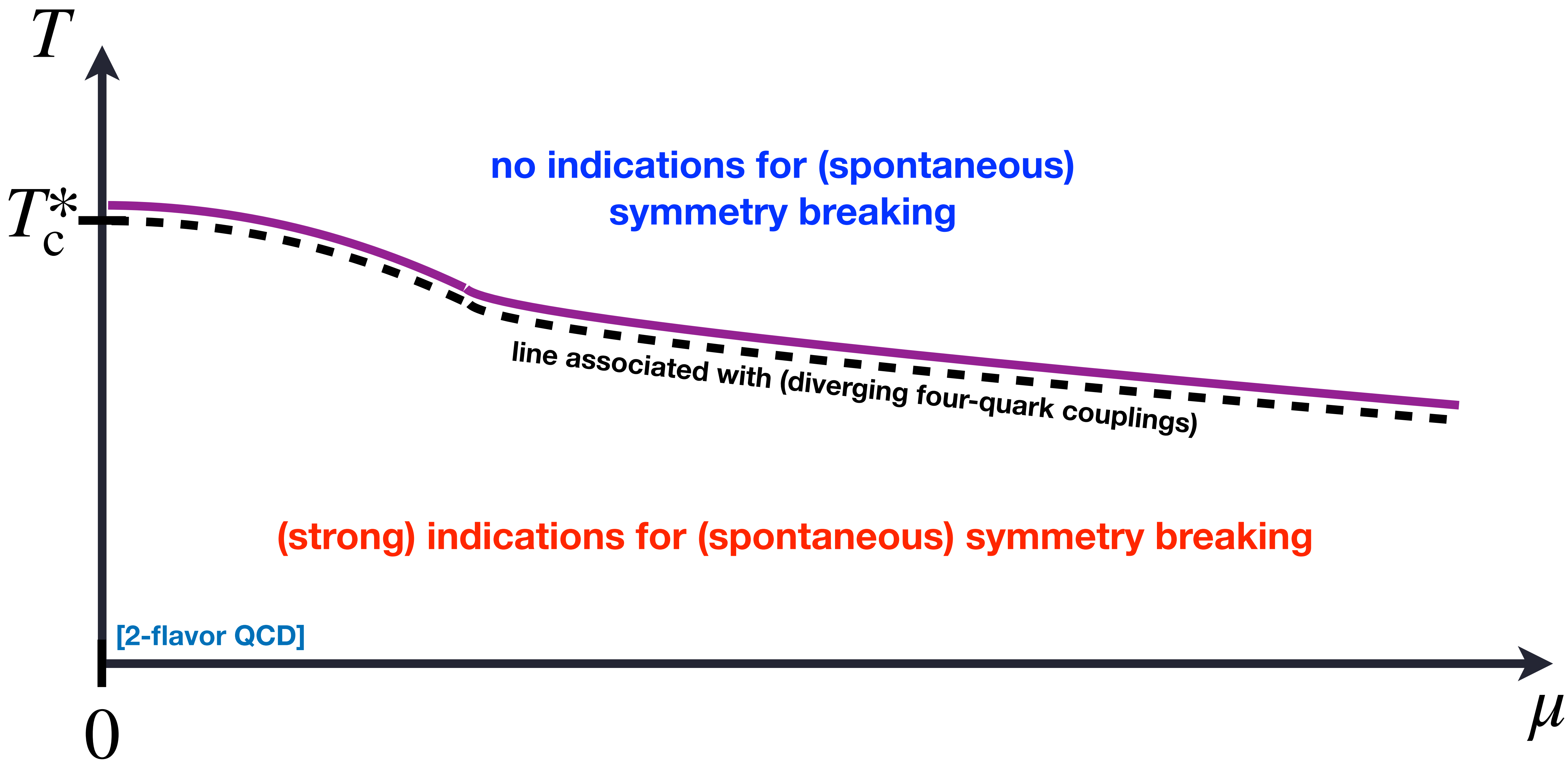

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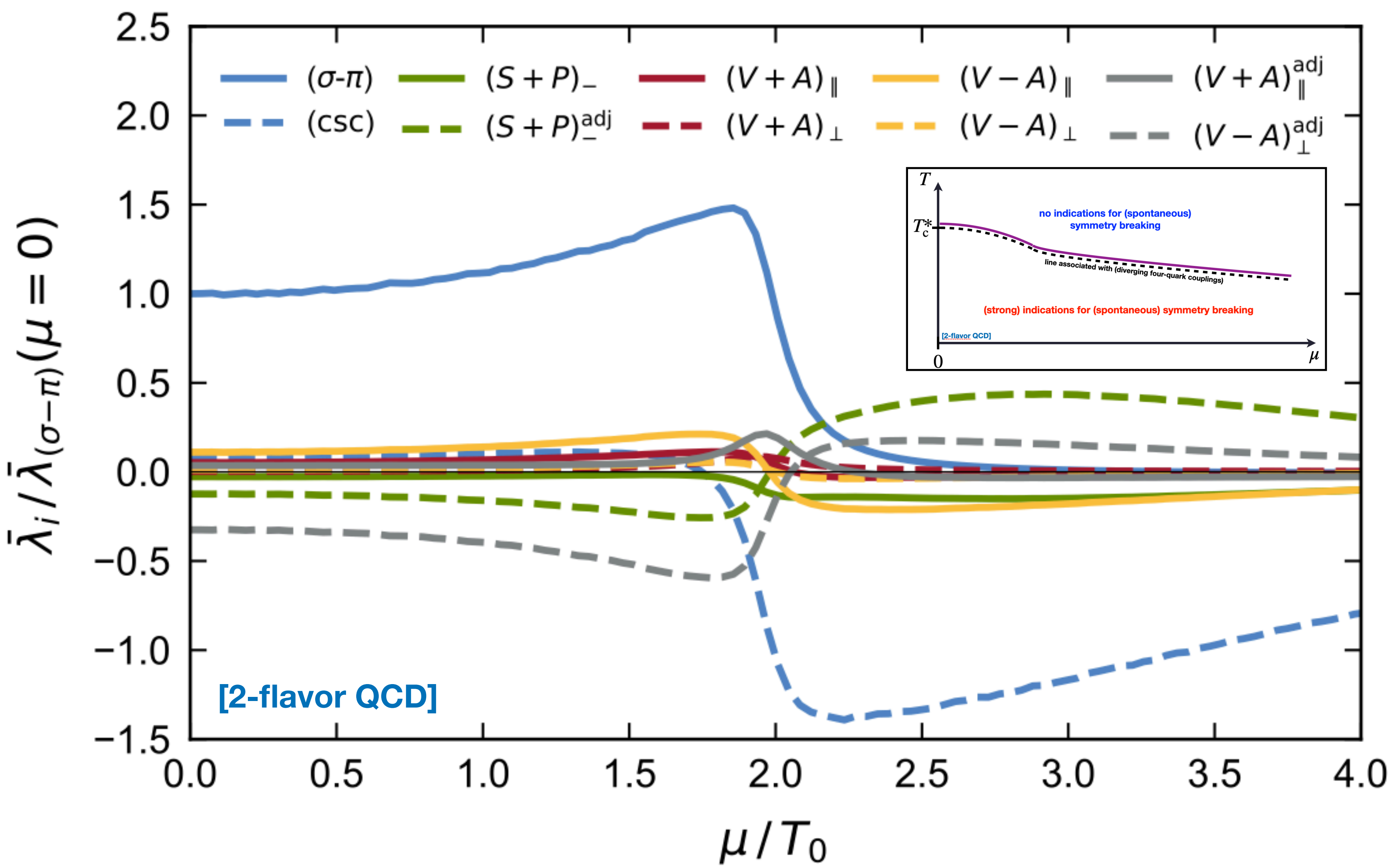

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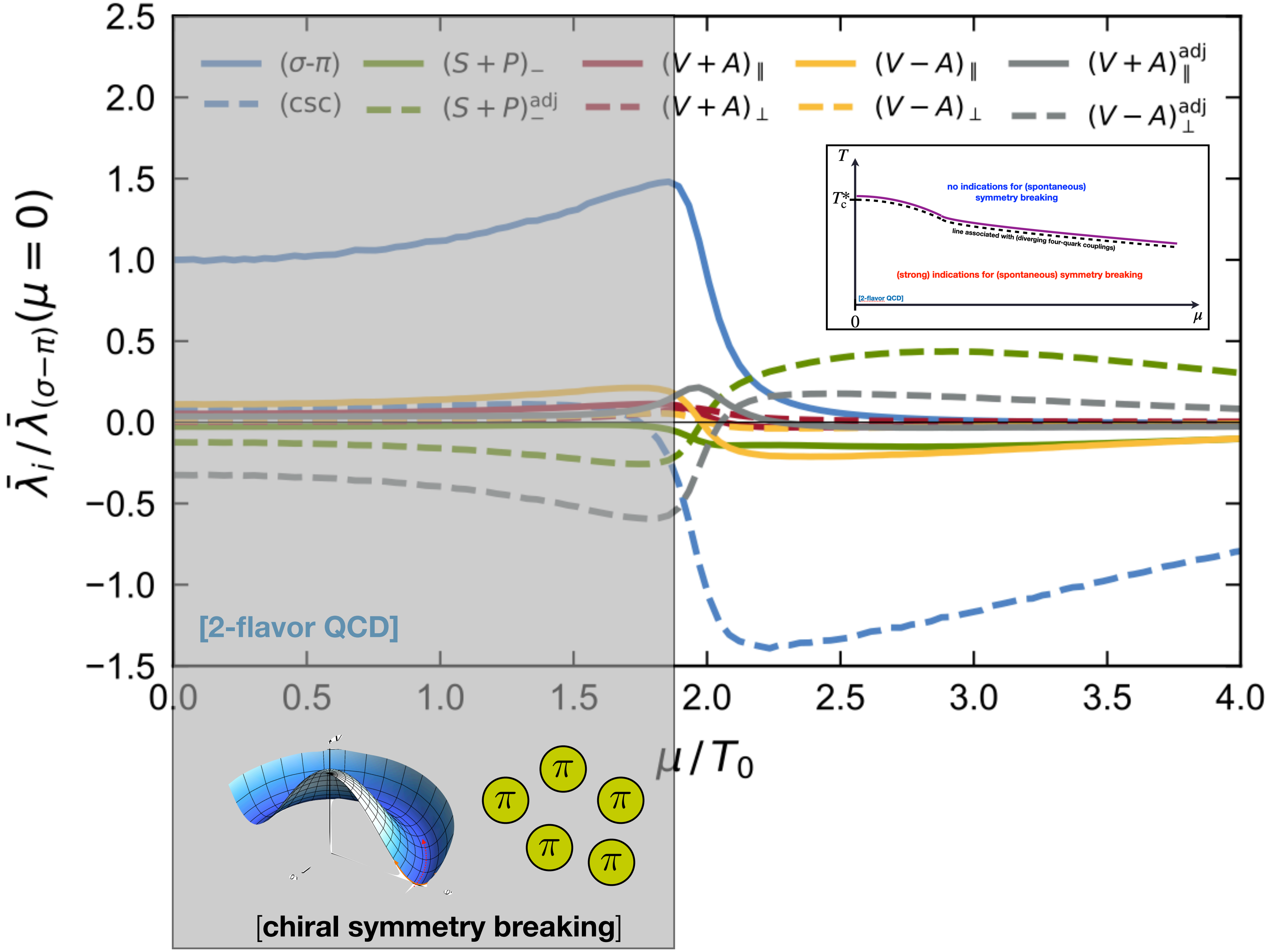

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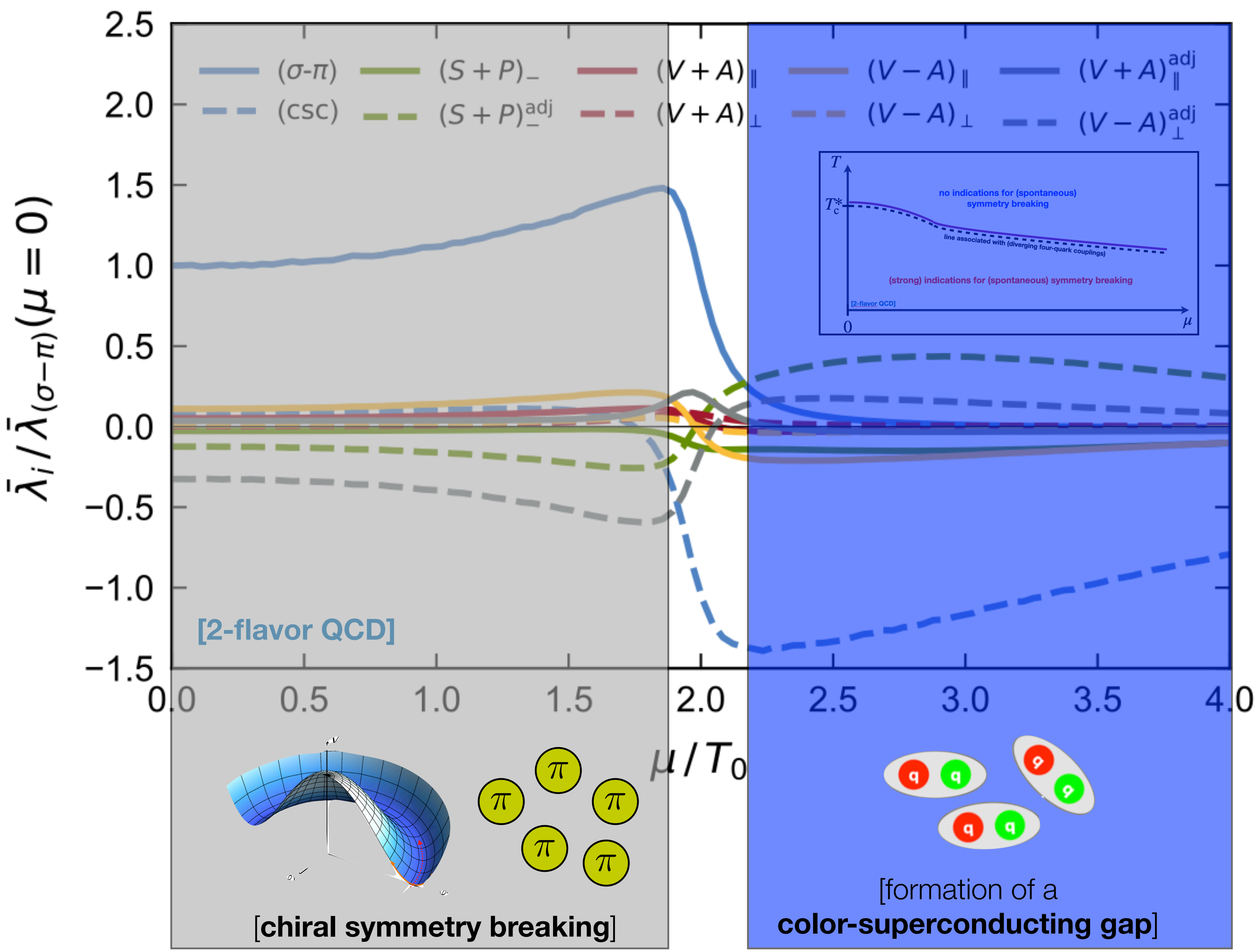

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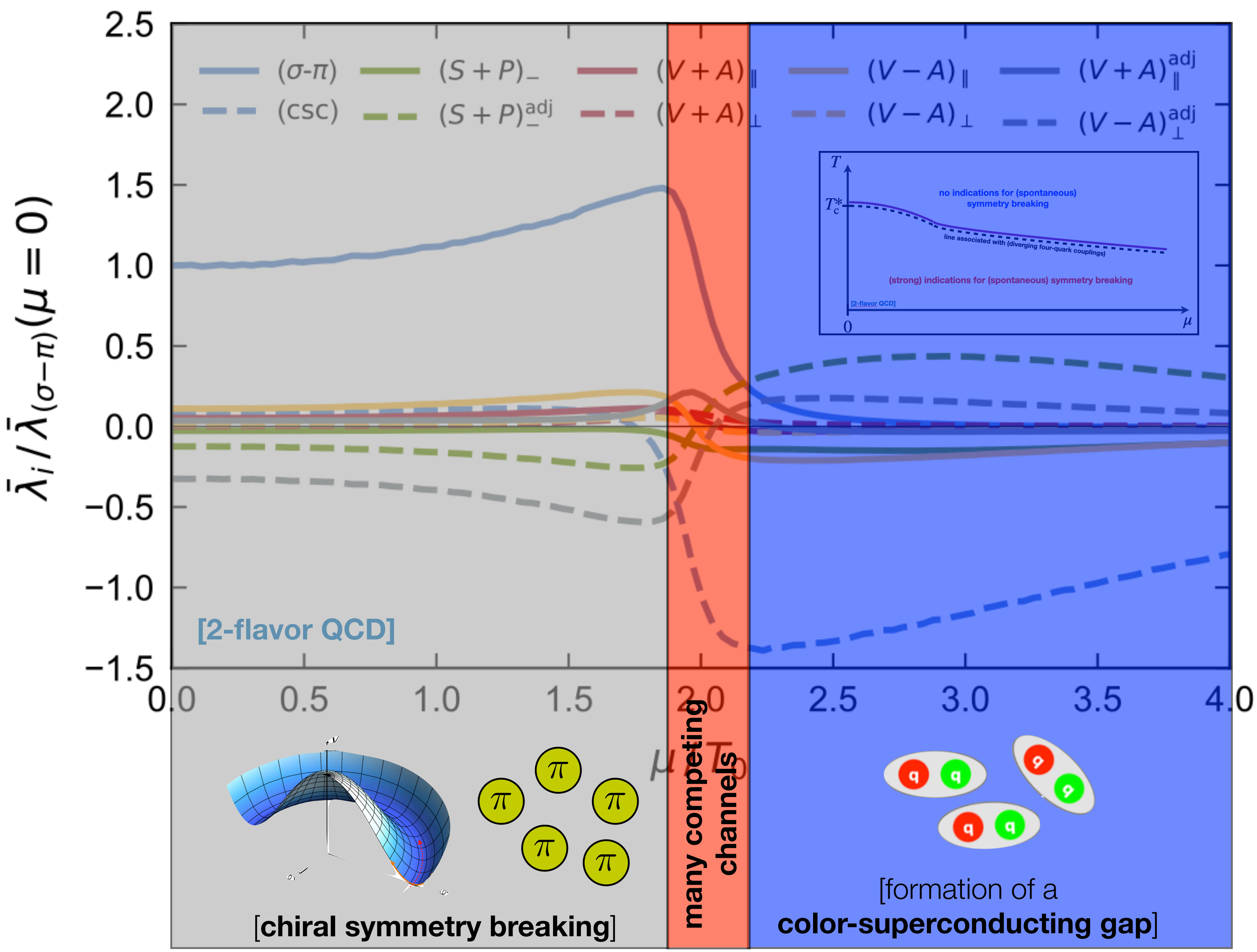

Symmetry-breaking patterns

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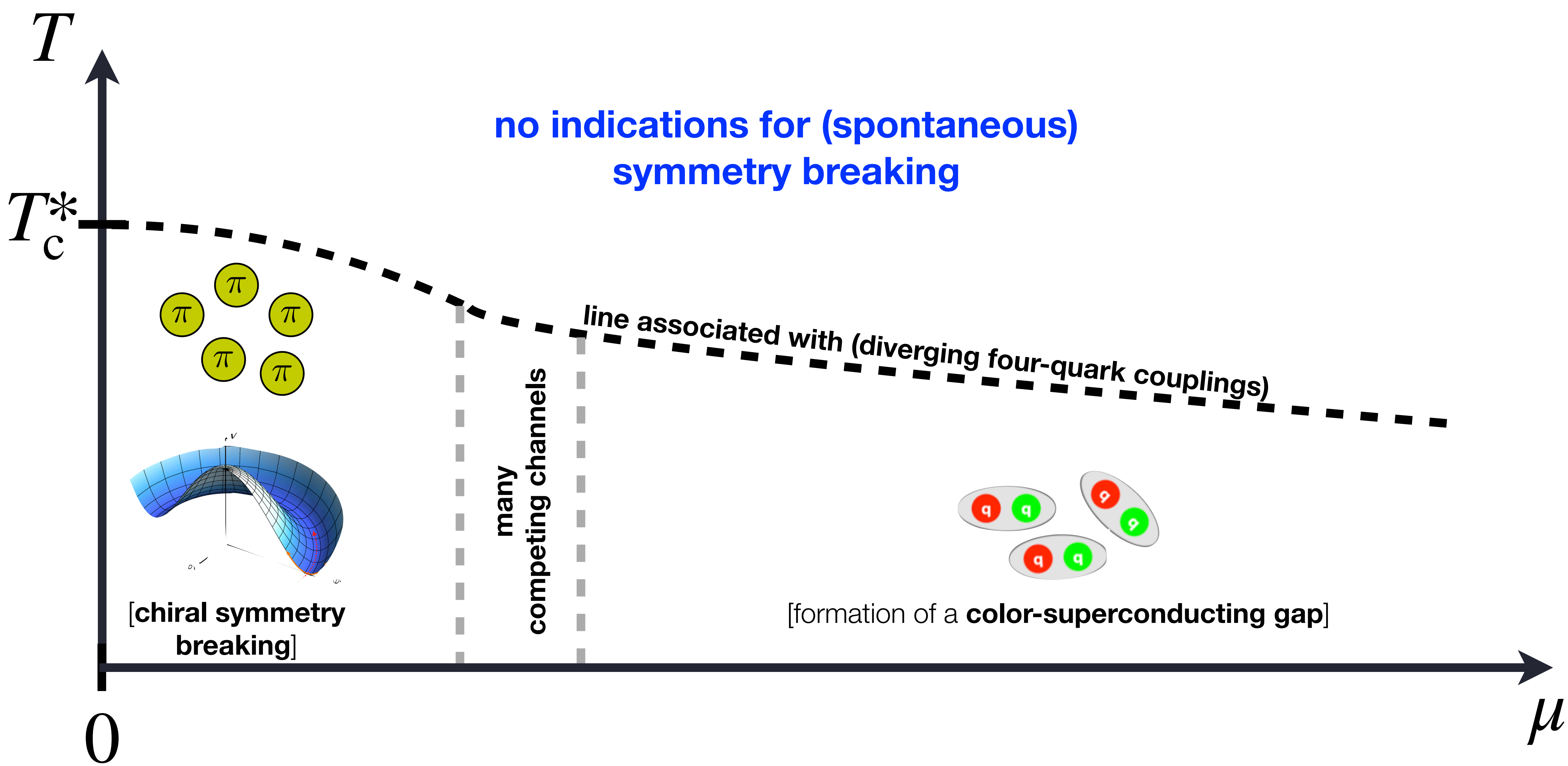
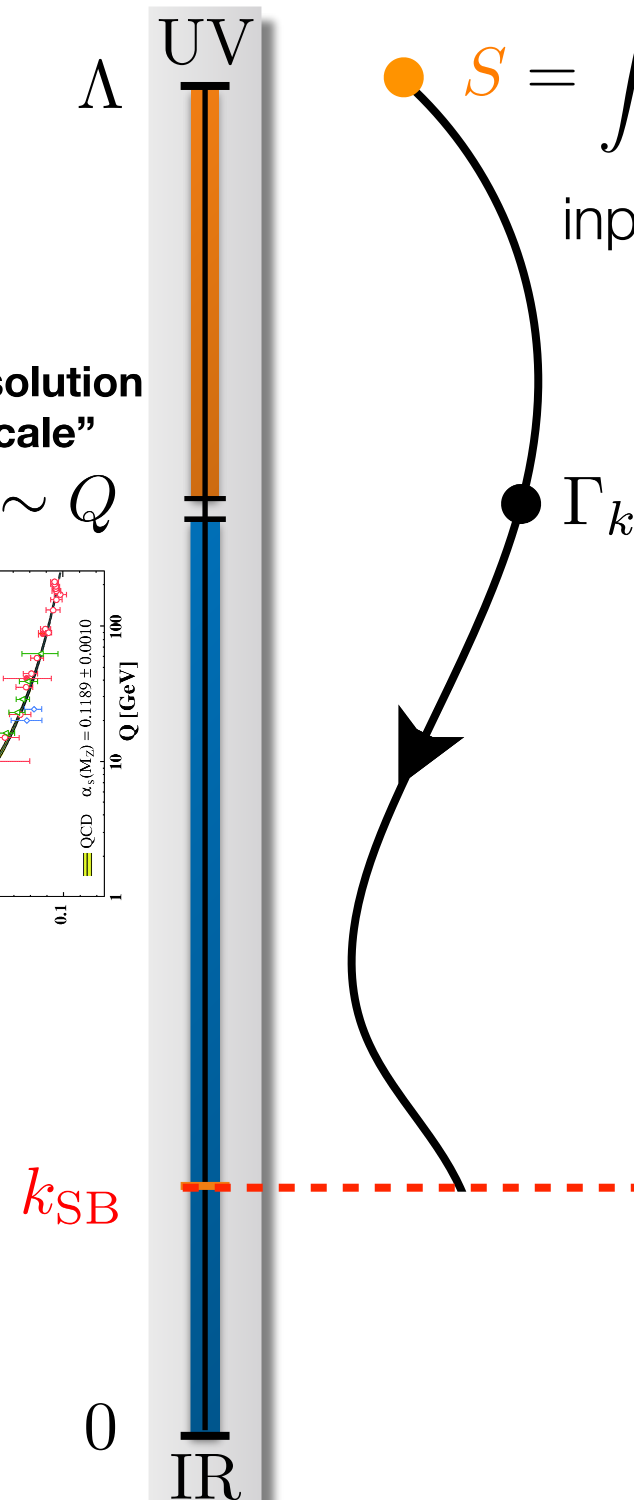


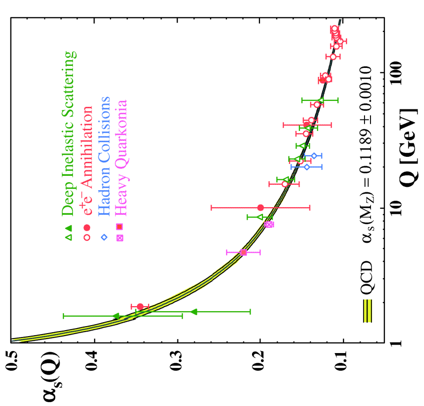
Functional Renormalization Group: Vacuum

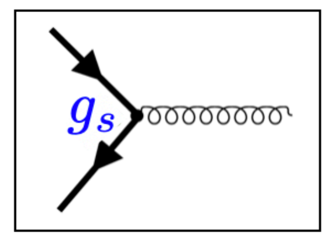
[Wetterich '92]

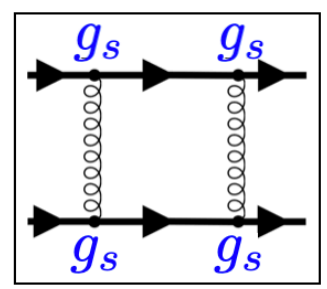
$$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + g_s A\!\!\!/)\psi \right\}$$

input parameter: strong coupling $g_s(\Lambda)$

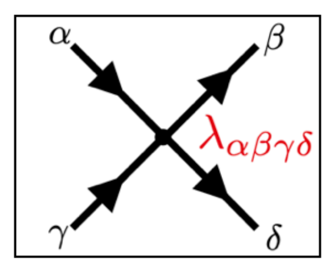




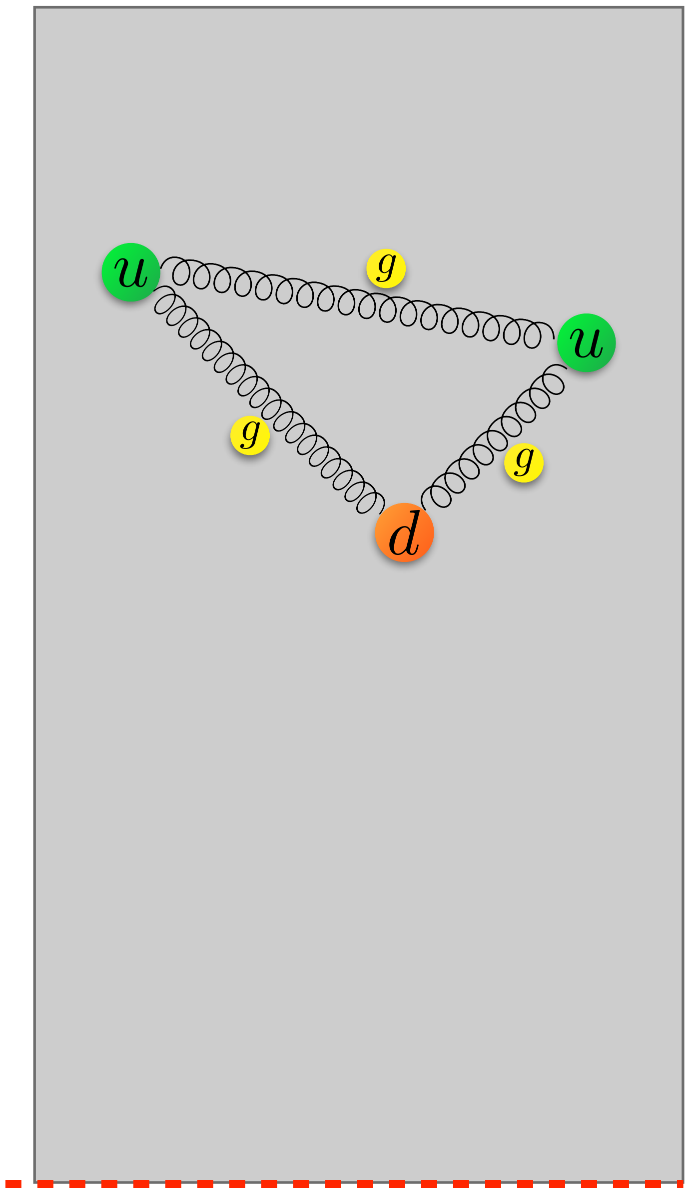




(Four-quark interactions are generated)



(Four-quark couplings become relevant ~ **order parameters**)



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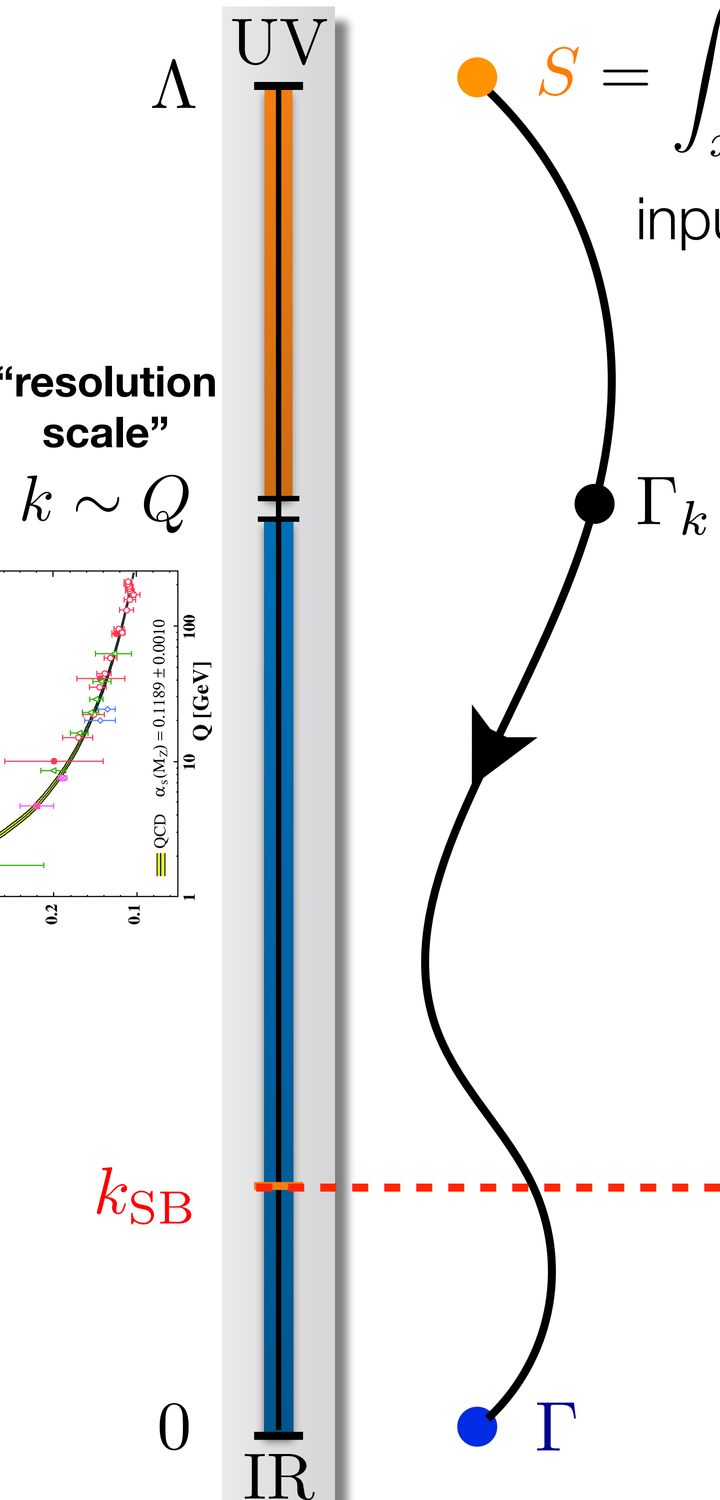

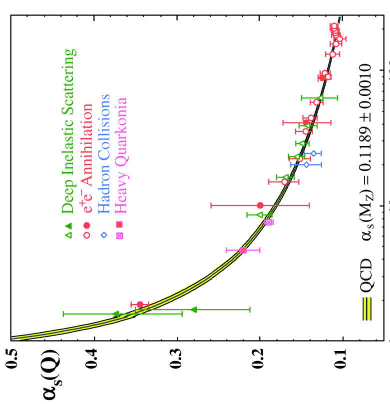

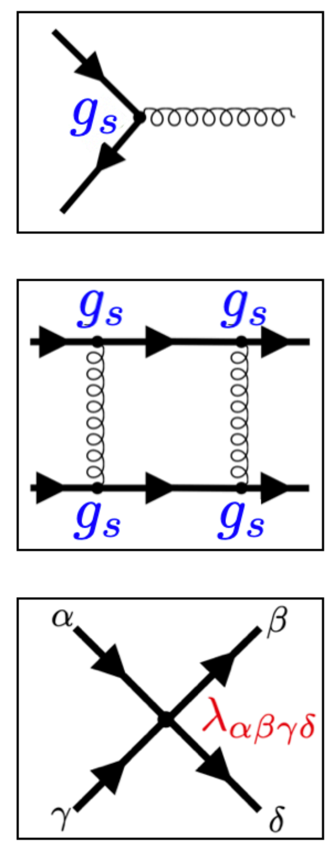

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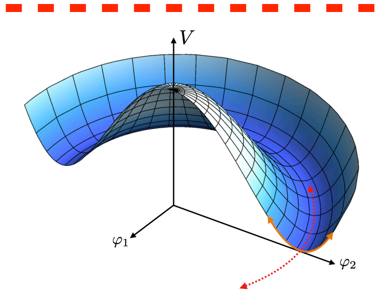
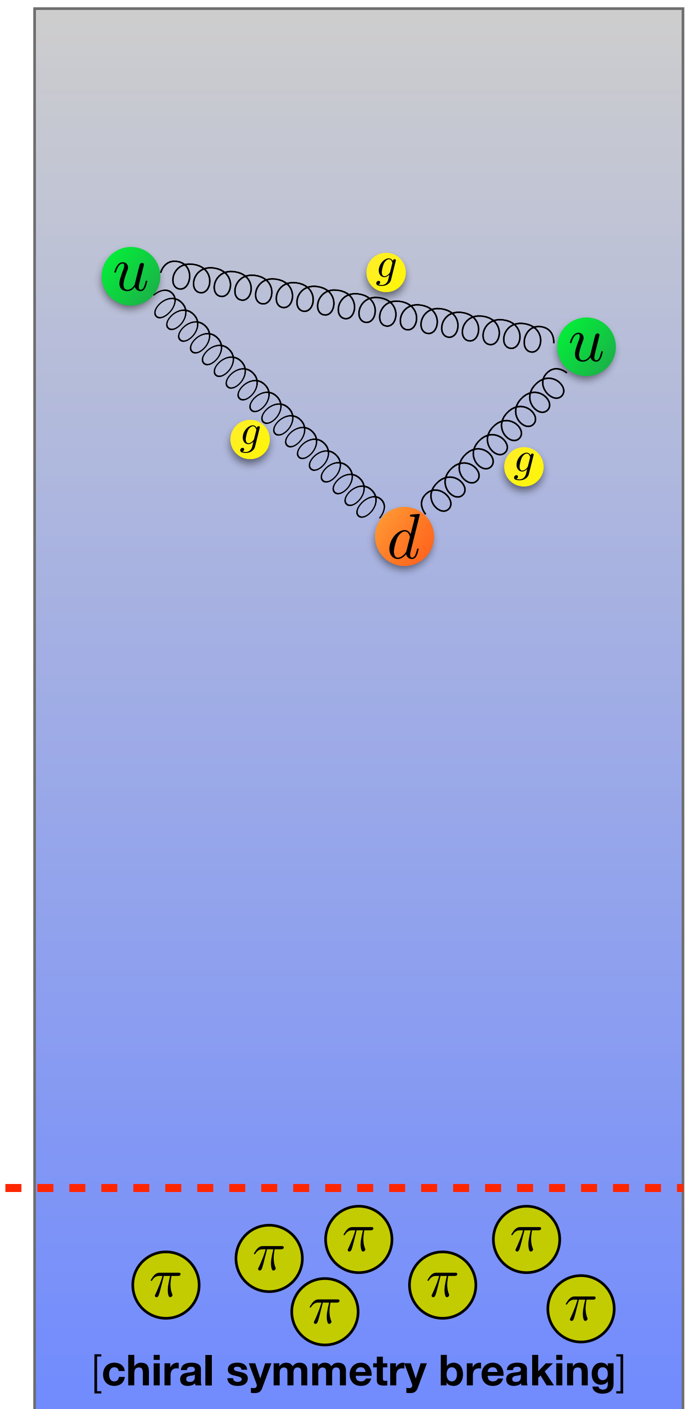
(Four-quark couplings become relevant ~ **order parameters**)





Continuous transition in the degrees of freedom

Functional Renormalization Group: High density

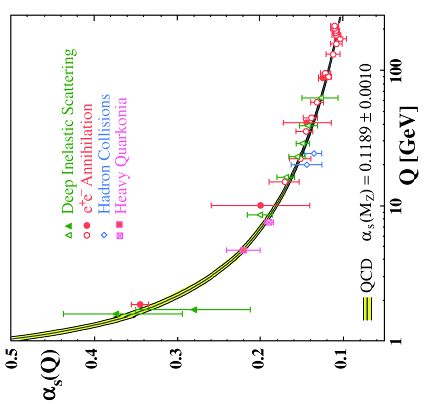
[JB & Schallmo '21]

$$S = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + g_s A\!\!\!/)\psi \right\}$$

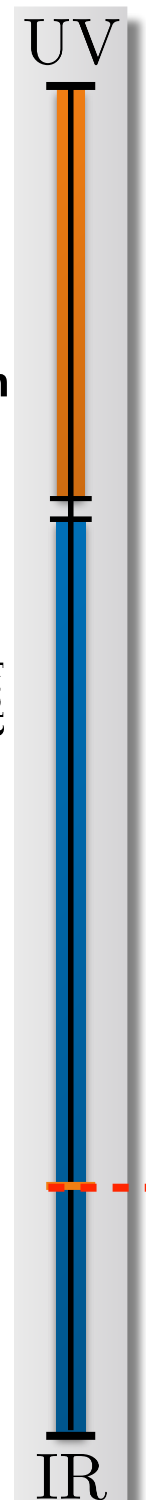
input parameter: strong coupling $g_s(\Lambda)$

"resolution scale"

$k \sim Q$

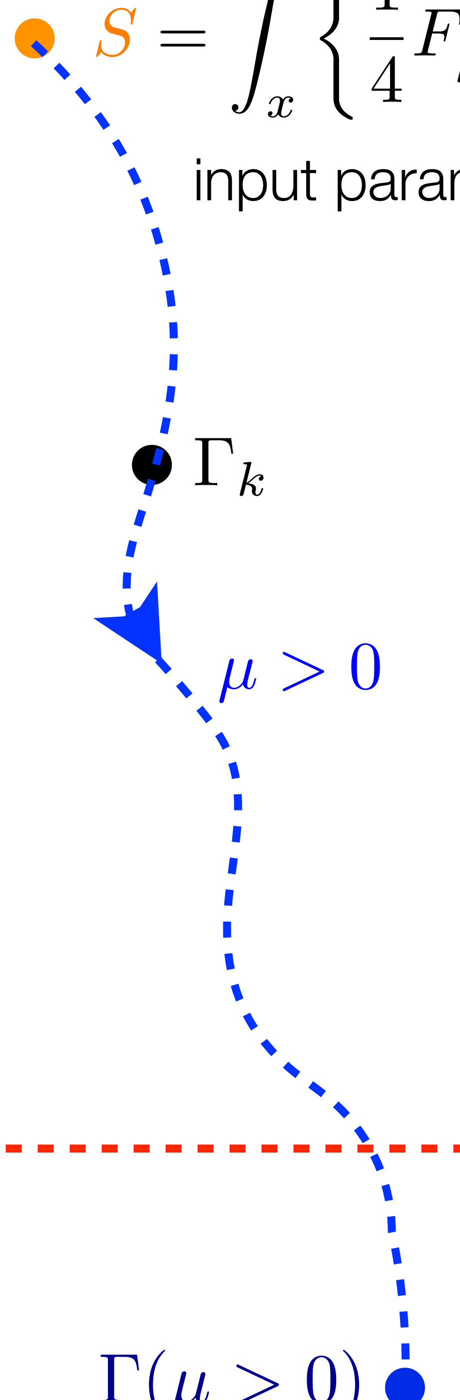


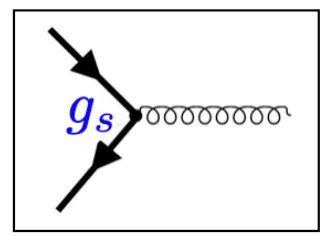
Λ

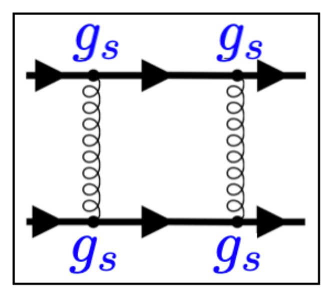


k_{SB}

0







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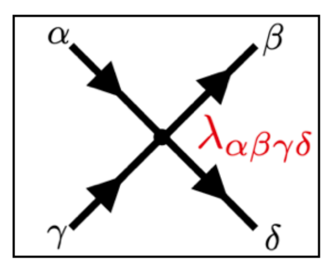

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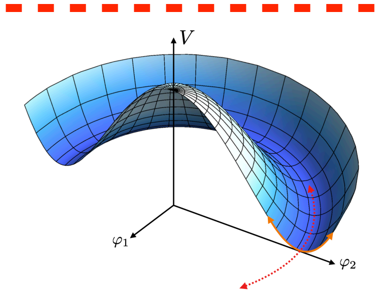

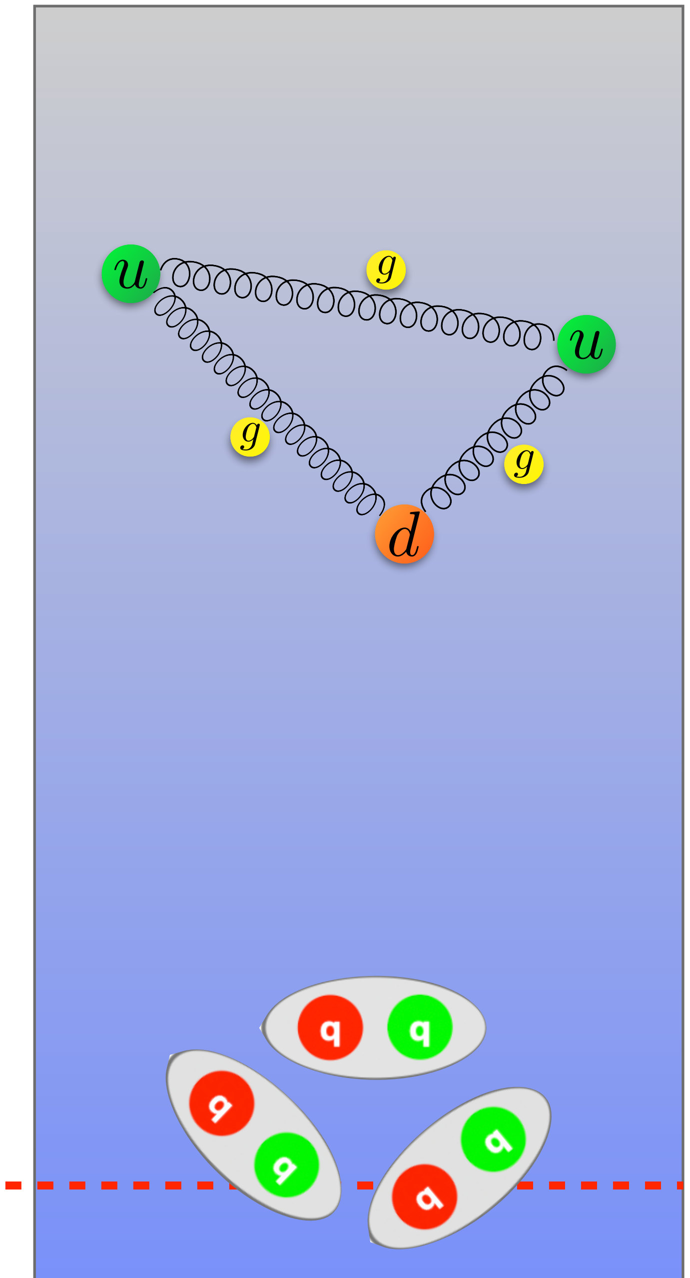

[formation of a **color-superconducting gap**]

Continuous transition in the degrees of freedom

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Renormalization flow of bound states

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(Received 1 August 2001; published 5 February 2002)

A renormalization group flow equation with a scale-dependent transformation of field variables gives a unified description of fundamental and composite degrees of freedom. In the context of the effective average action, we study the renormalization flow of scalar bound states which are formed out of fundamental fermions. We use the gauged Nambu–Jona-Lasinio model at weak gauge coupling as an example. Thereby, the notions of a bound state or fundamental particle become scale dependent, being classified by the fixed-point structure of the flow of effective couplings.

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Universality of spontaneous chiral symmetry breaking in gauge theories

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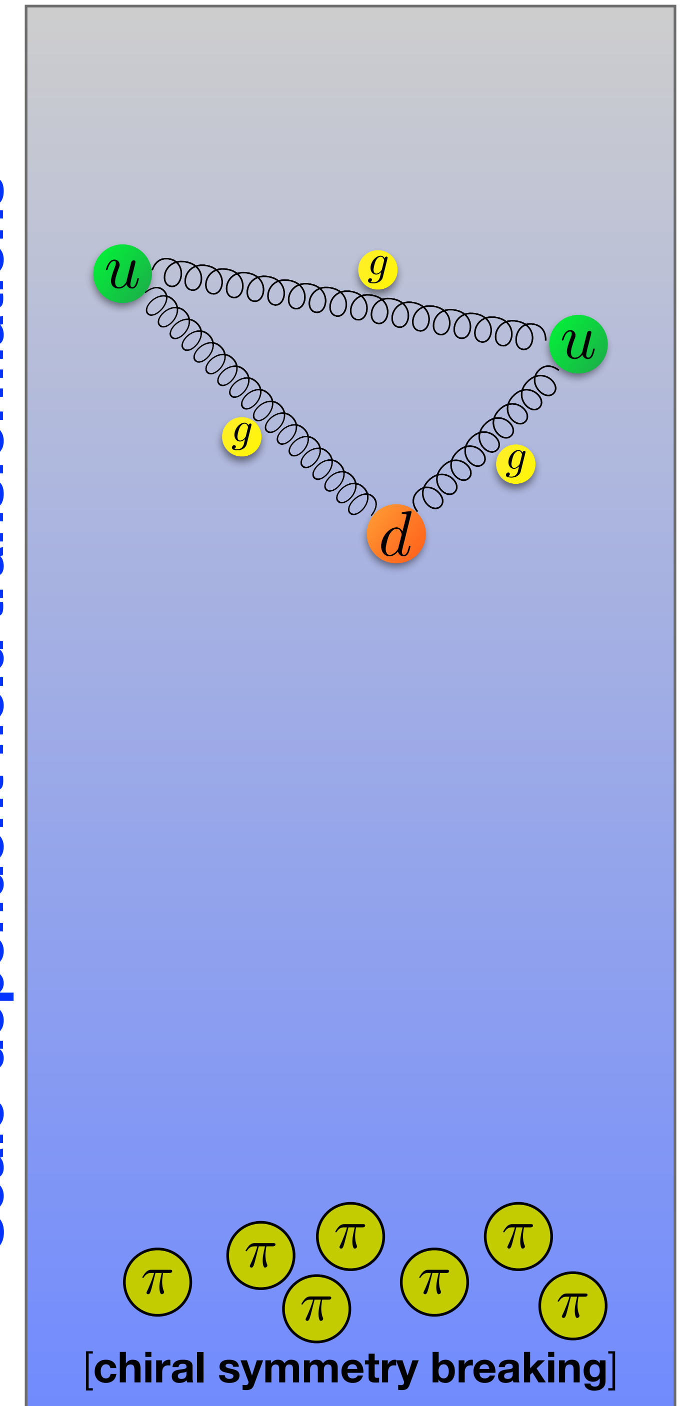
(Received 30 September 2002; published 13 January 2004)

We investigate one-flavor QCD with an additional chiral scalar field. For a large domain in the space of coupling constants, this model belongs to the same universality class as QCD, and the effects of the scalar become unobservable. This is connected to a "bound-state fixed point" of the renormalization flow for which all memory of the microscopic scalar interactions is lost. The QCD domain includes a microscopic scalar potential with minima at a nonzero field. On the other hand, for a scalar mass term m^2 below a critical value m_c^2, the universality class is characterized by perturbative spontaneous chiral symmetry breaking which renders the quarks massive. Our renormalization group analysis shows how this universality class is continuously connected with the QCD universality class.

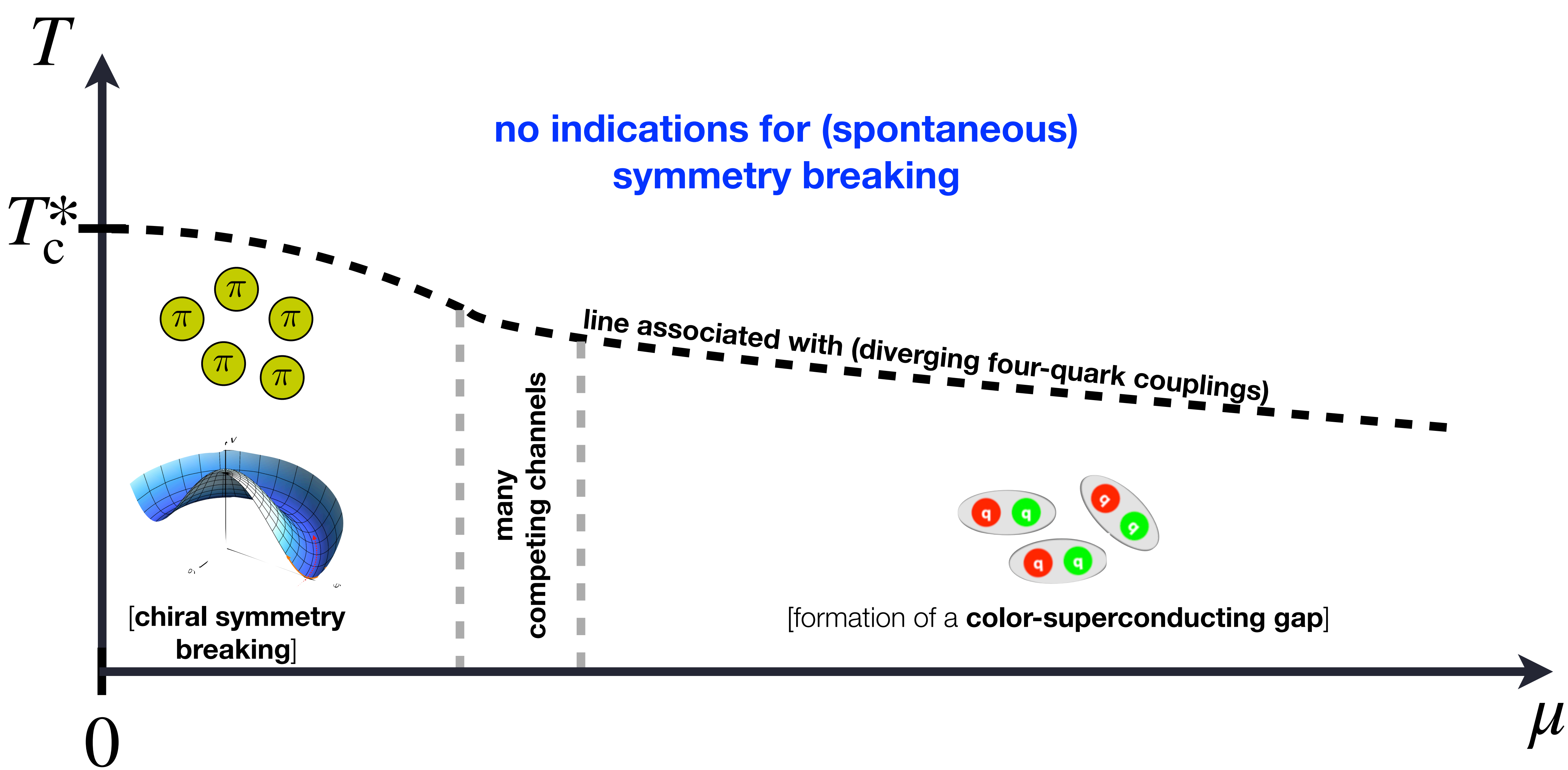
Scale-dependent field transformations



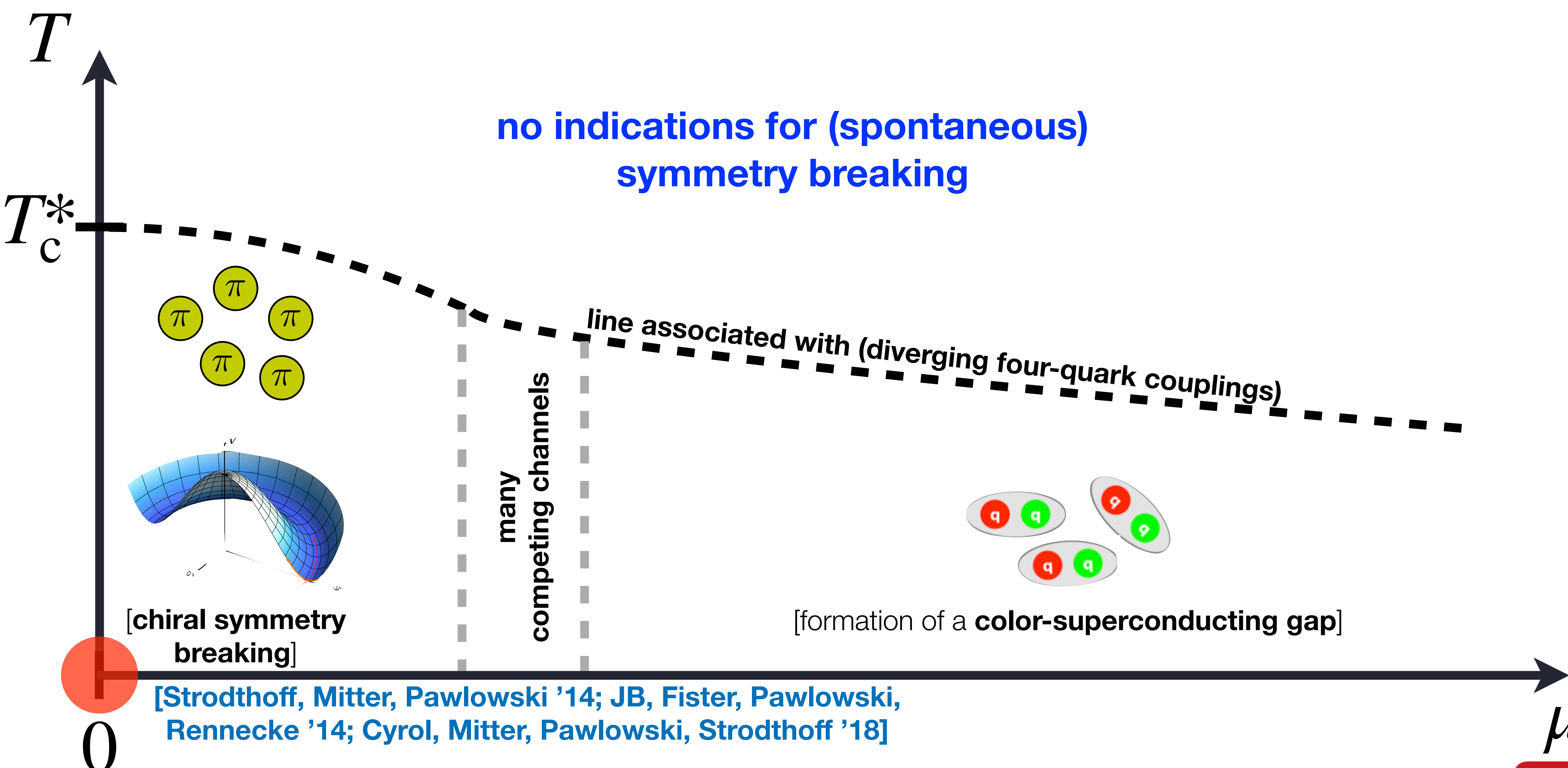
Continuous transition in the degrees of freedom

[see also Pawlowski '05; Floerchinger & Wetterich '09; JB, Fister, Pawlowski, Rennecke '14; Strodthoff, Fukushima, Pawlowski '21]

Phase structure



Phase structure: Vacuum



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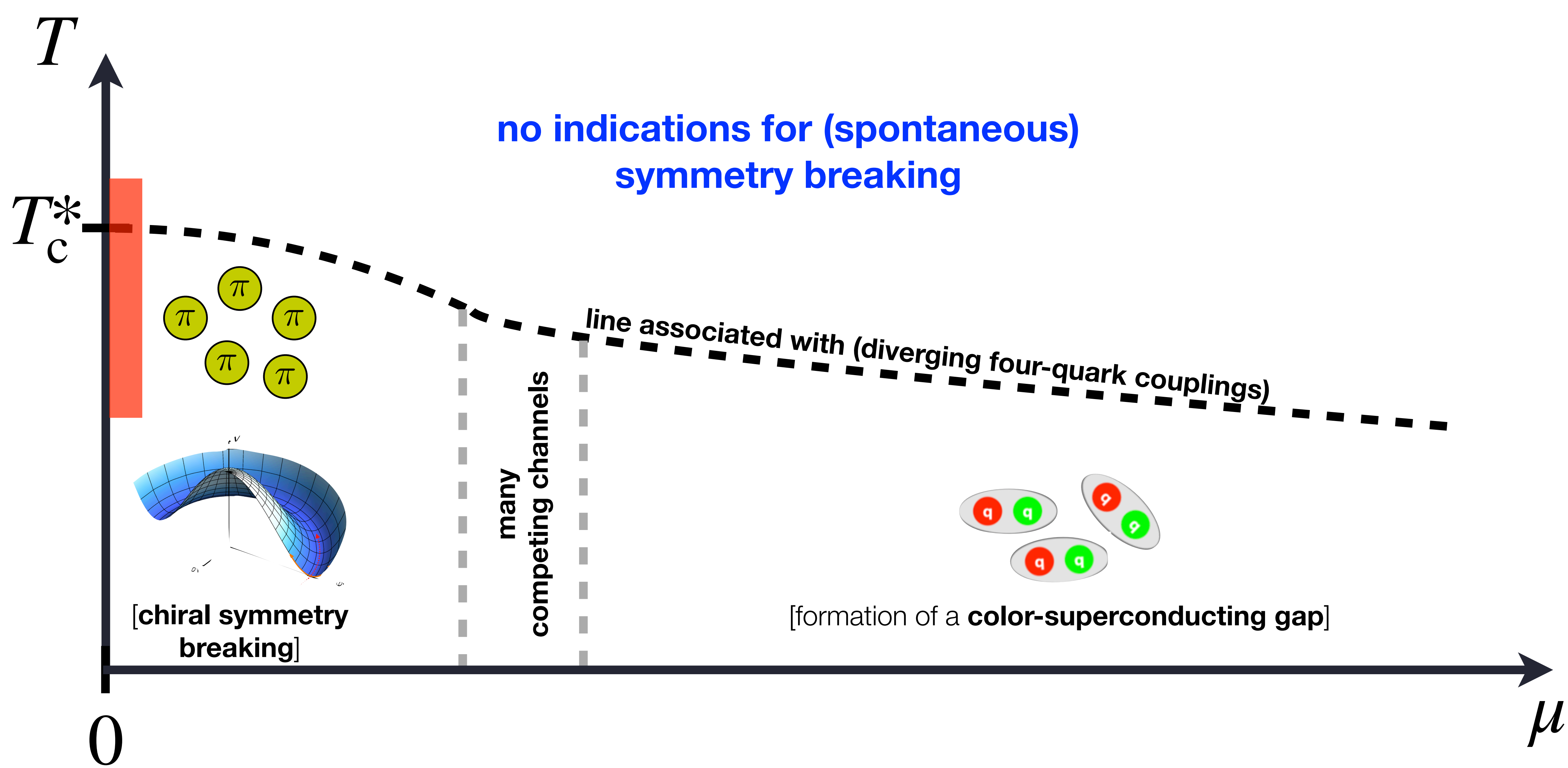
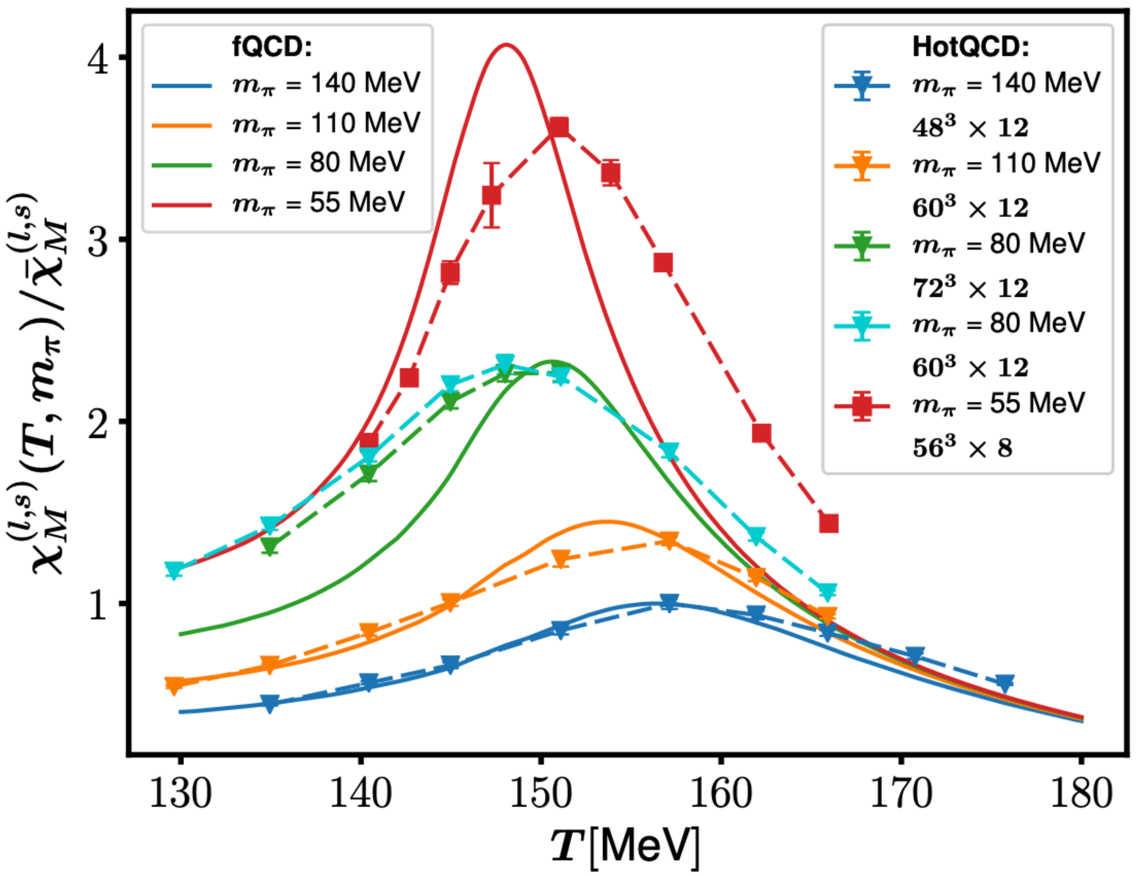
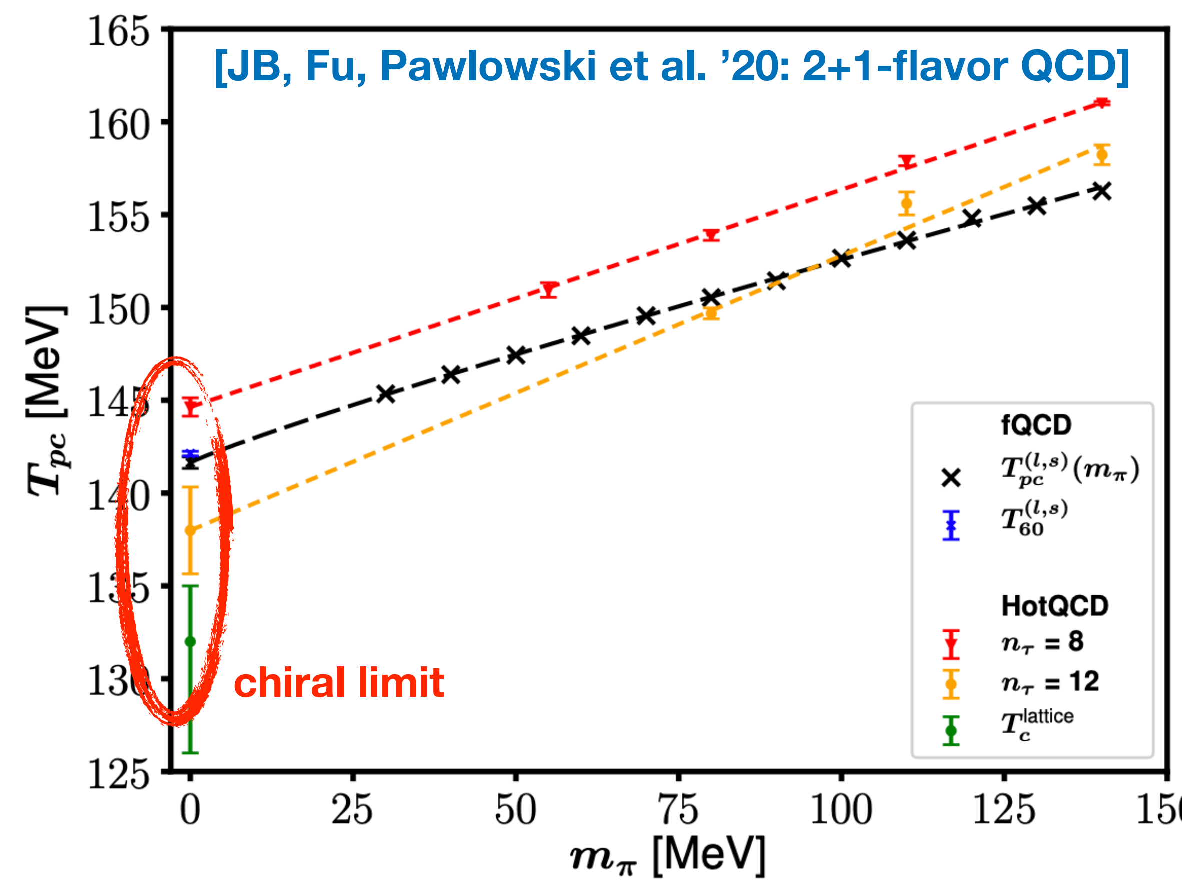
- **Impressive progress** on description of scattering processes [Eser & Blaizot '22] & spectral functions [Horak et al. '22]

Phase structure



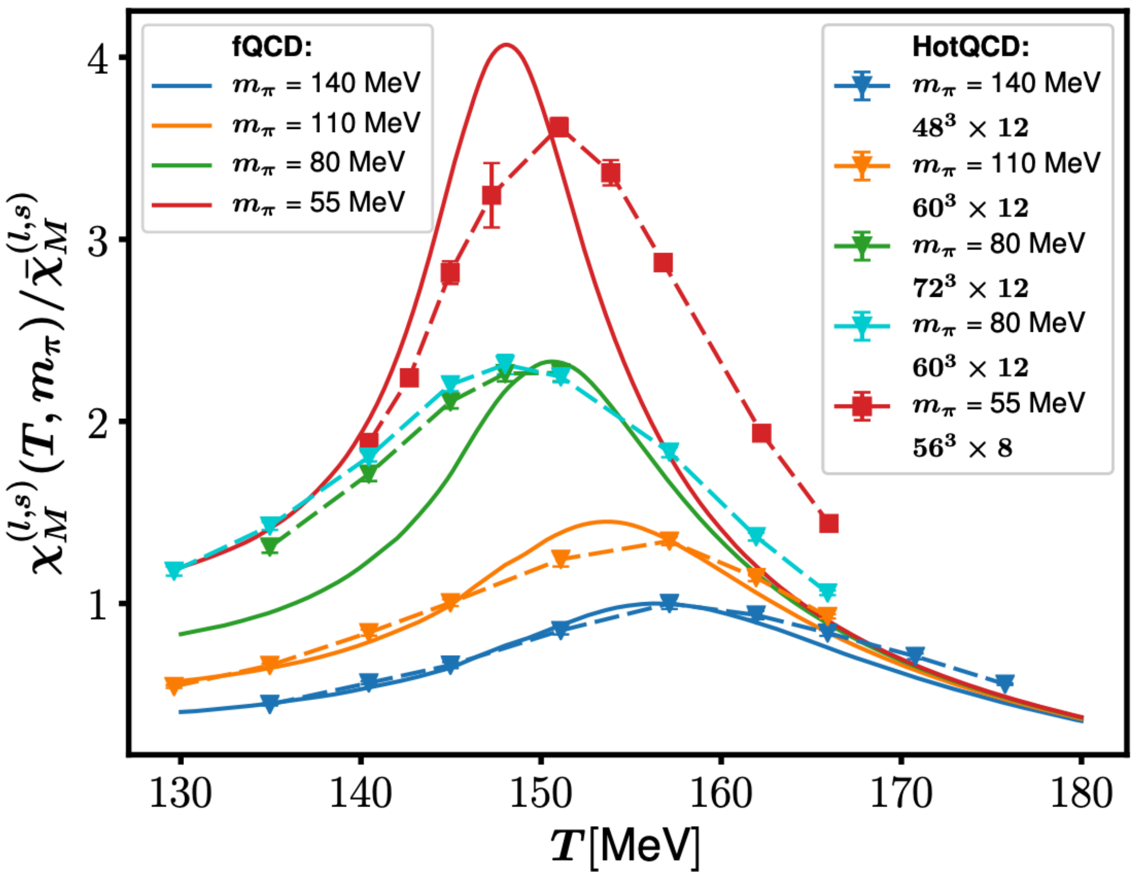
Nature of the chiral transition

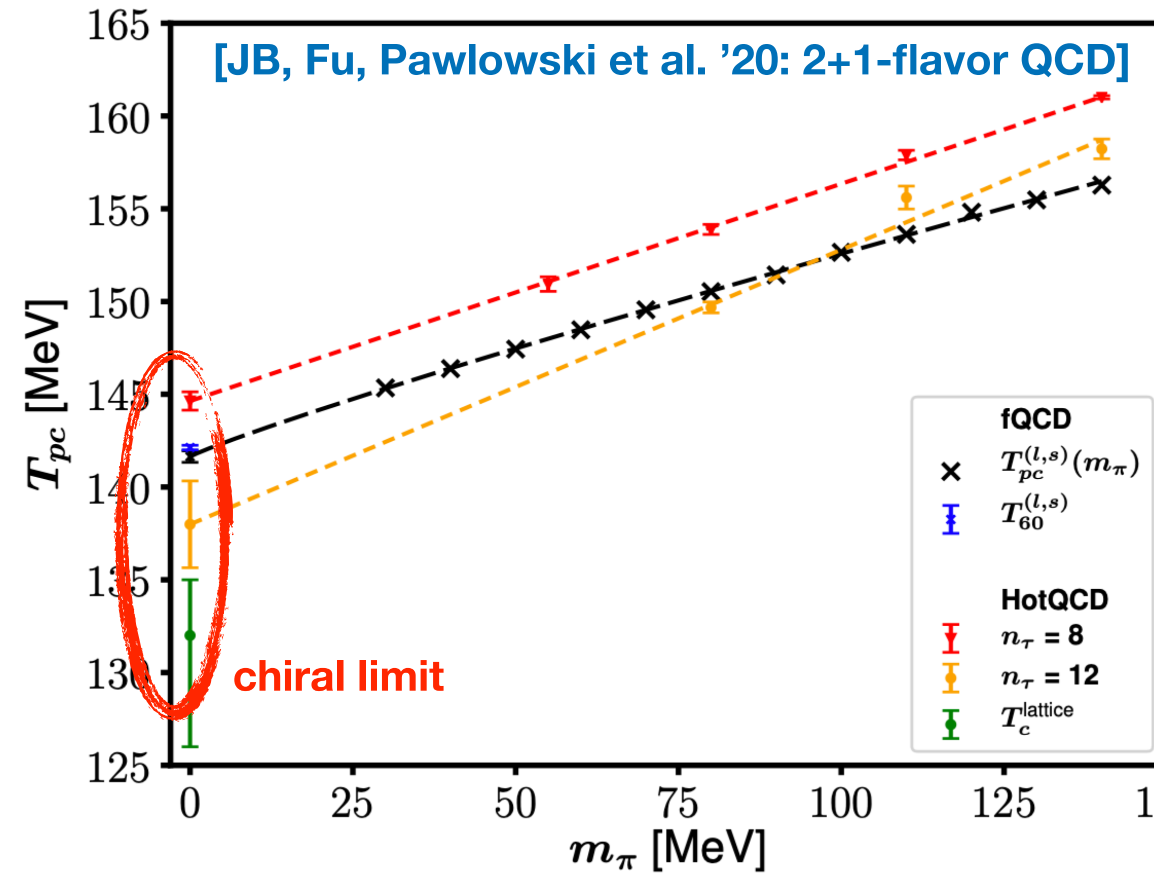




- Good agreement with lattice QCD: $T_c \sim 130\ldots140$ MeV
  [Ding et al. ’19; JB, Fu, Pawlowski et al. ’20; Kotov, Lombardo, Trunin ’21]
- Phase transition appears to fall into the O(4) universality class [JB, Fu, Pawlowski et al. ’20; JB, Pawlowski, Rosenblüh ’21]

Nature of the chiral transition





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- Scenarios: Nature of the transition in 3-flavor QCD?
  [Fejos '22]

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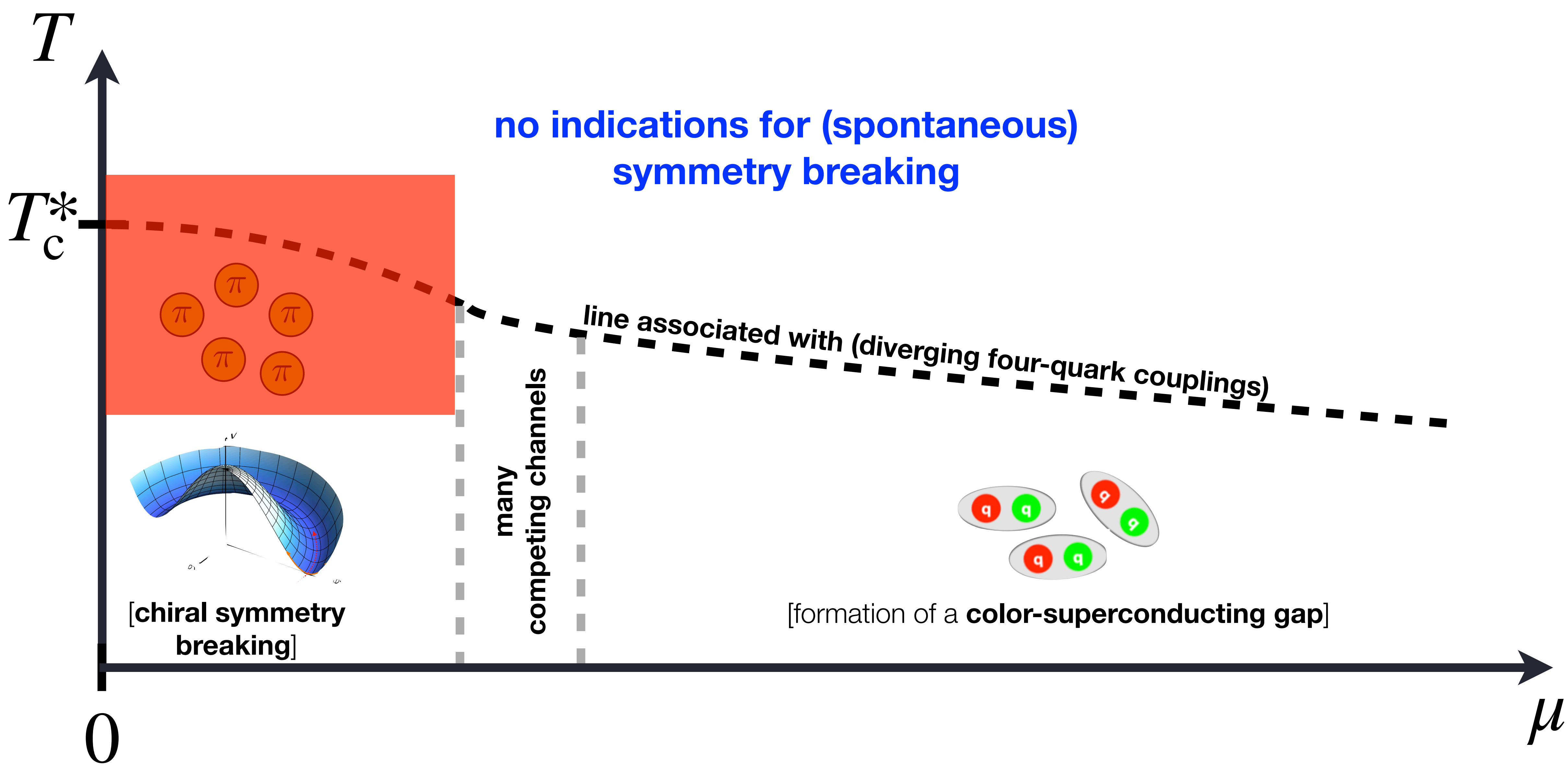
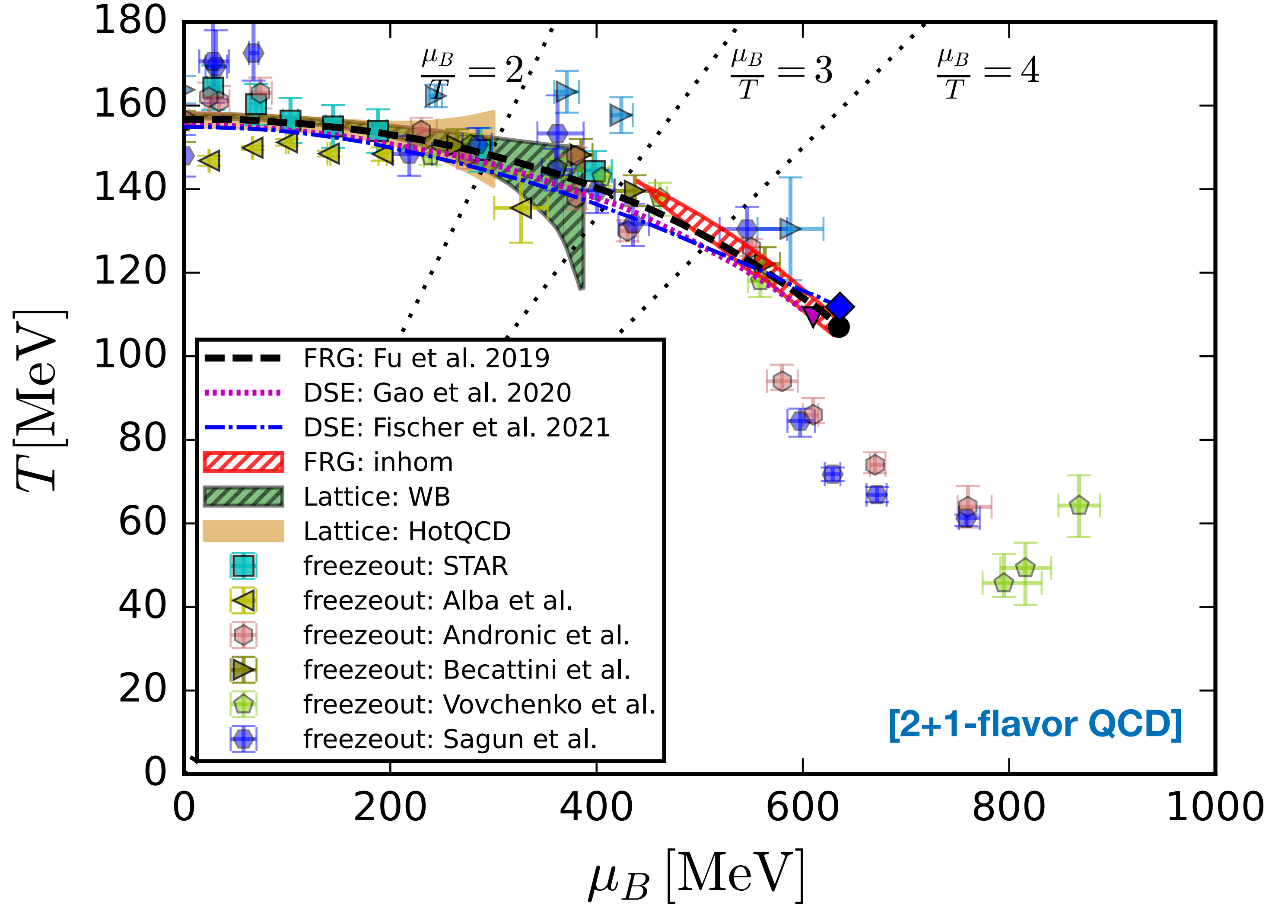
Phase structure

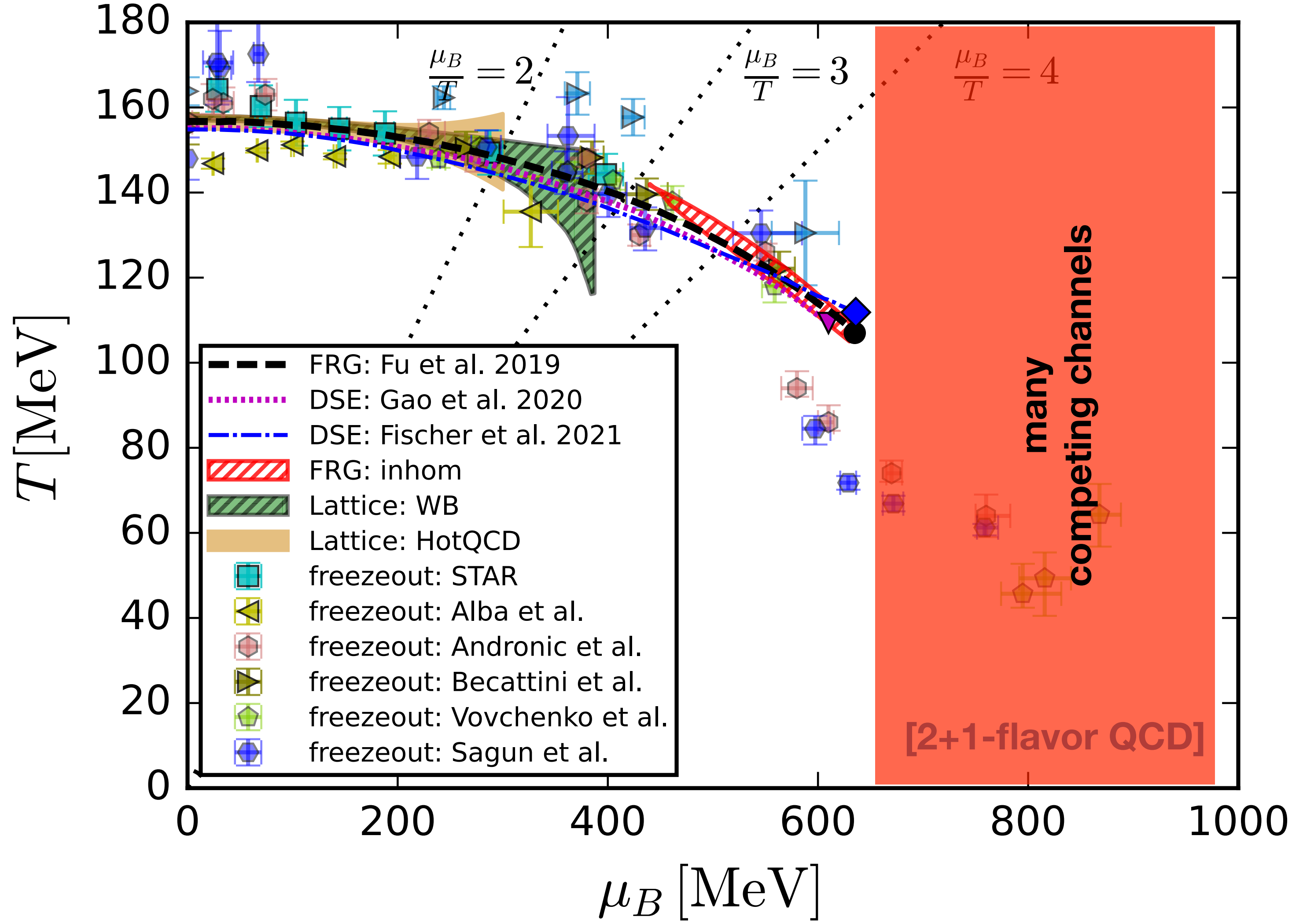


Phase structure at small chemical potential



- In very good agreement with lattice QCD results at small chemical potential [Fu, Pawlowski, Rennecke '20]
- **Prediction** for the critical point [Fu, Pawlowski, Rennecke '20]

Phase structure at small chemical potential



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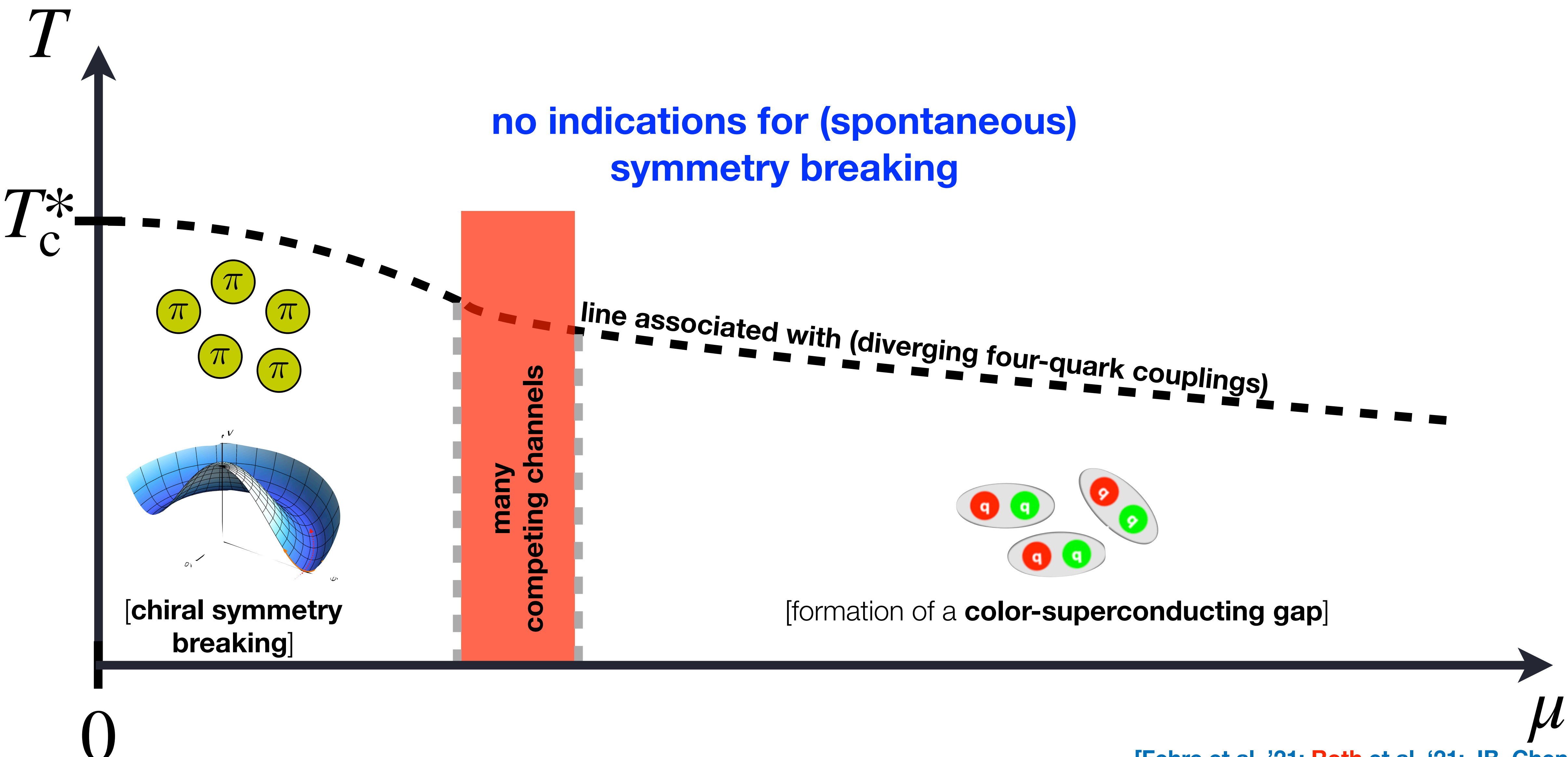
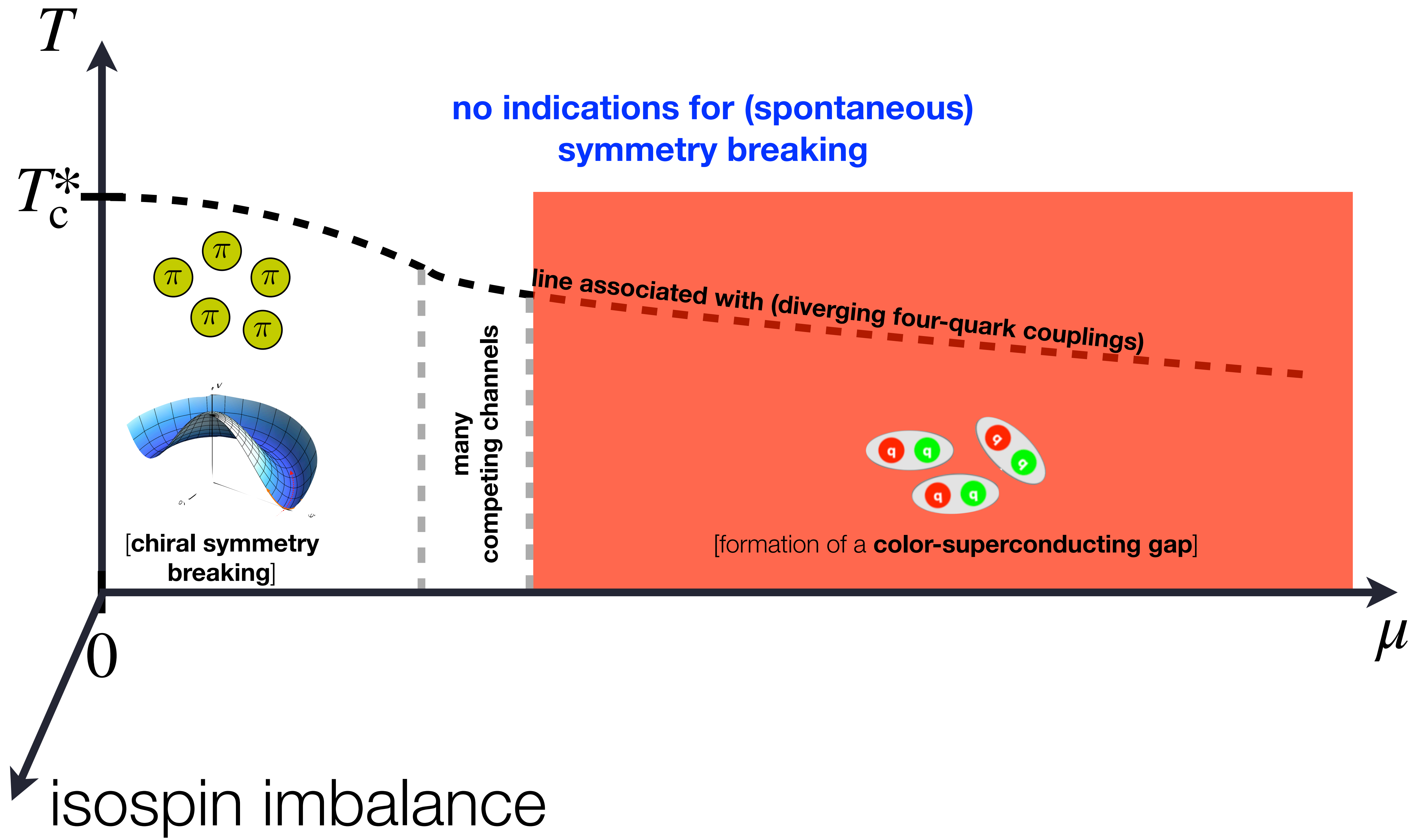
Phase structure



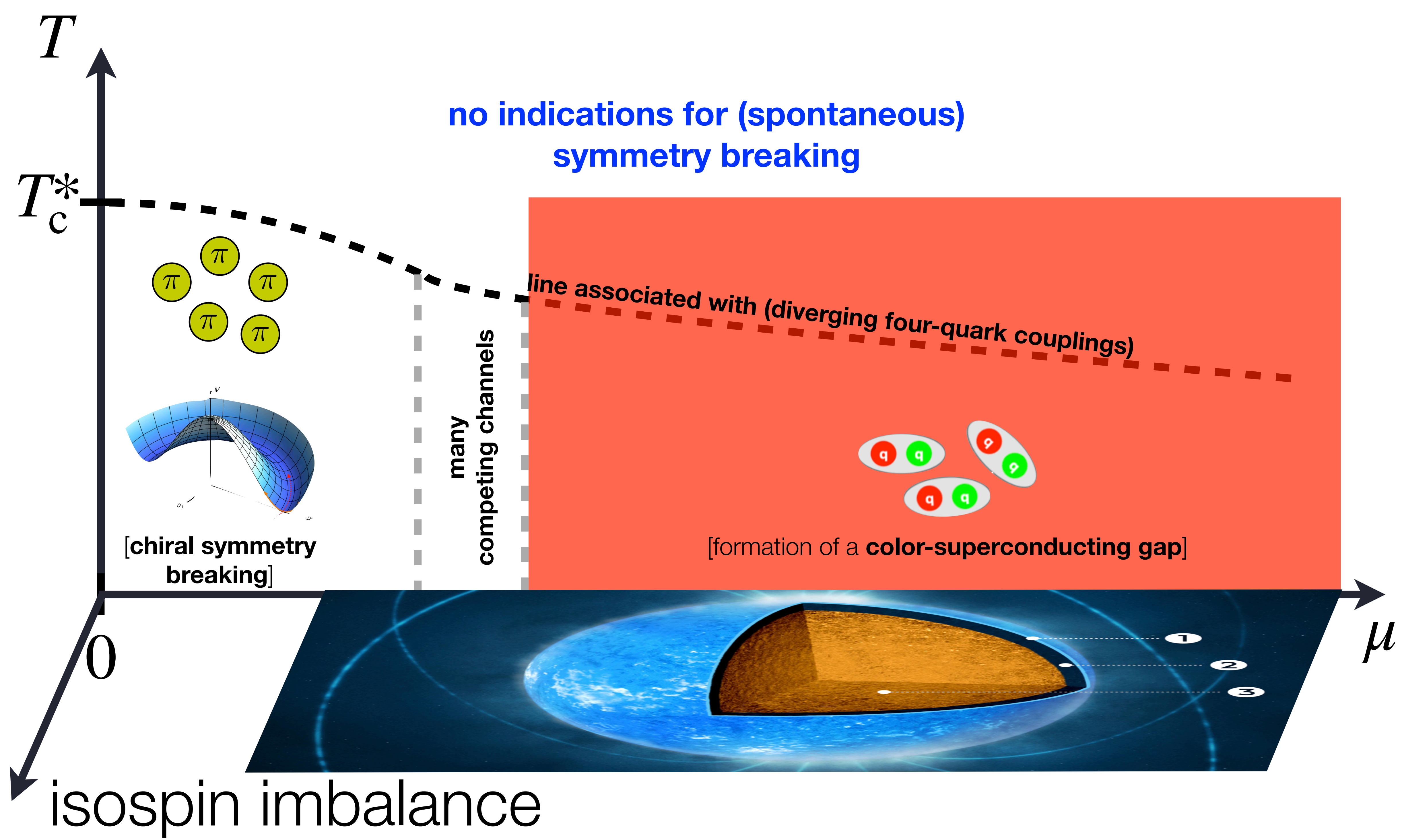
[Fehre et al. '21; Roth et al. '21; JB, Chen, Fu, Geissel, Horak, Huang, Ihssen, Pawłowski, Reichert, Rennecke, Tan, Toepfel, Wessely, Wink '22; Otto et al. '22]

- **Promising** developments of regulators and numerical techniques [Wink & Grossi '19; Koenigstein et al. '21; Stoll, Zorbach, Koenigstein et al. '21; Grossi, Ihssen et al. '22]

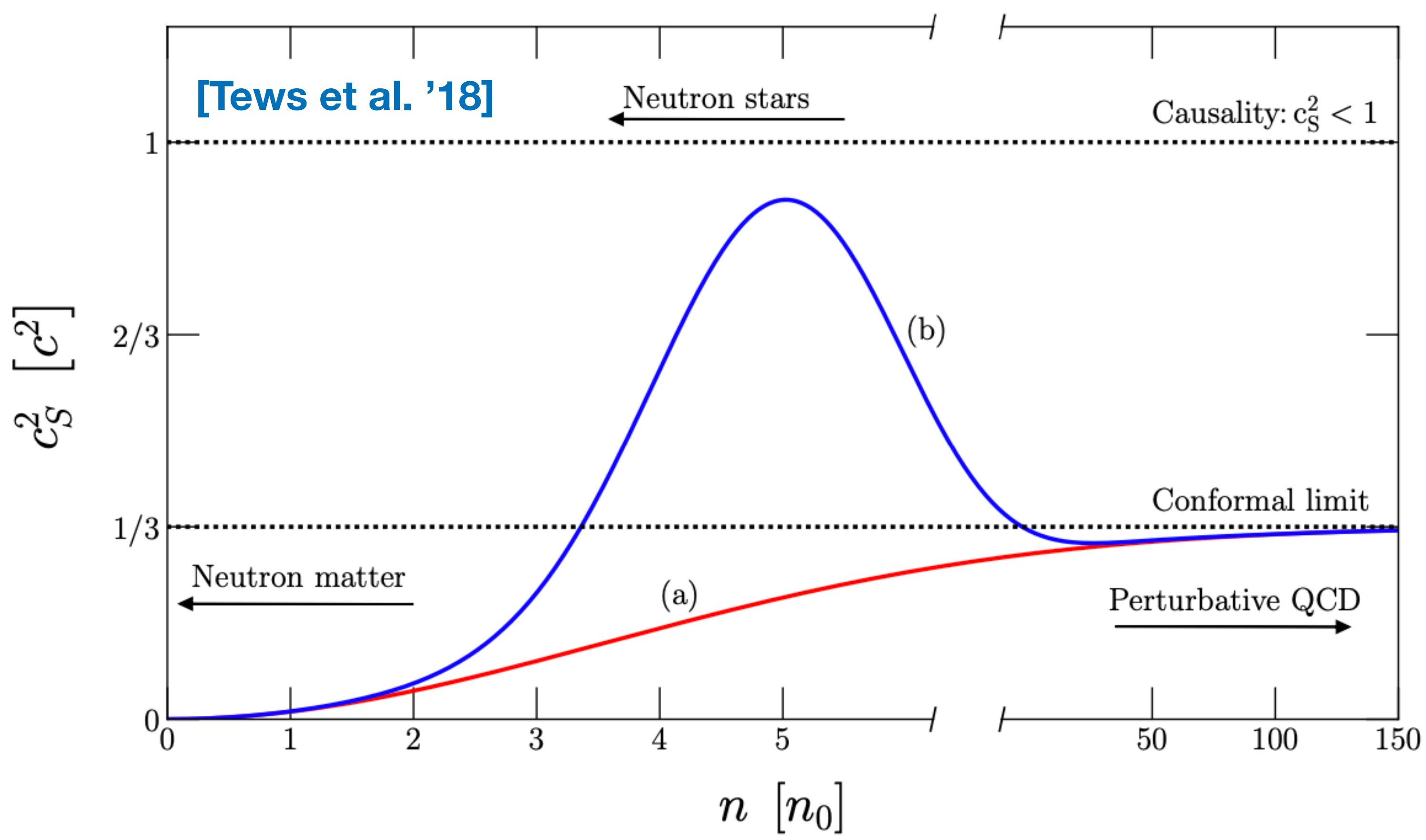
From the phase structure to **thermodynamics**



Thermodynamics of dense matter: why?



Constraints on dense matter: speed of sound



- **Speed of sound:** $c_s^2 = \partial P / \partial \epsilon$
- Constraints from neutron-star observations and nuclear physics suggest the existence of a global **maximum of the speed of sound** [Bedaque, Steiner '15; Tews et al. '18; Greif et al. '18; Raaijmakers '19]

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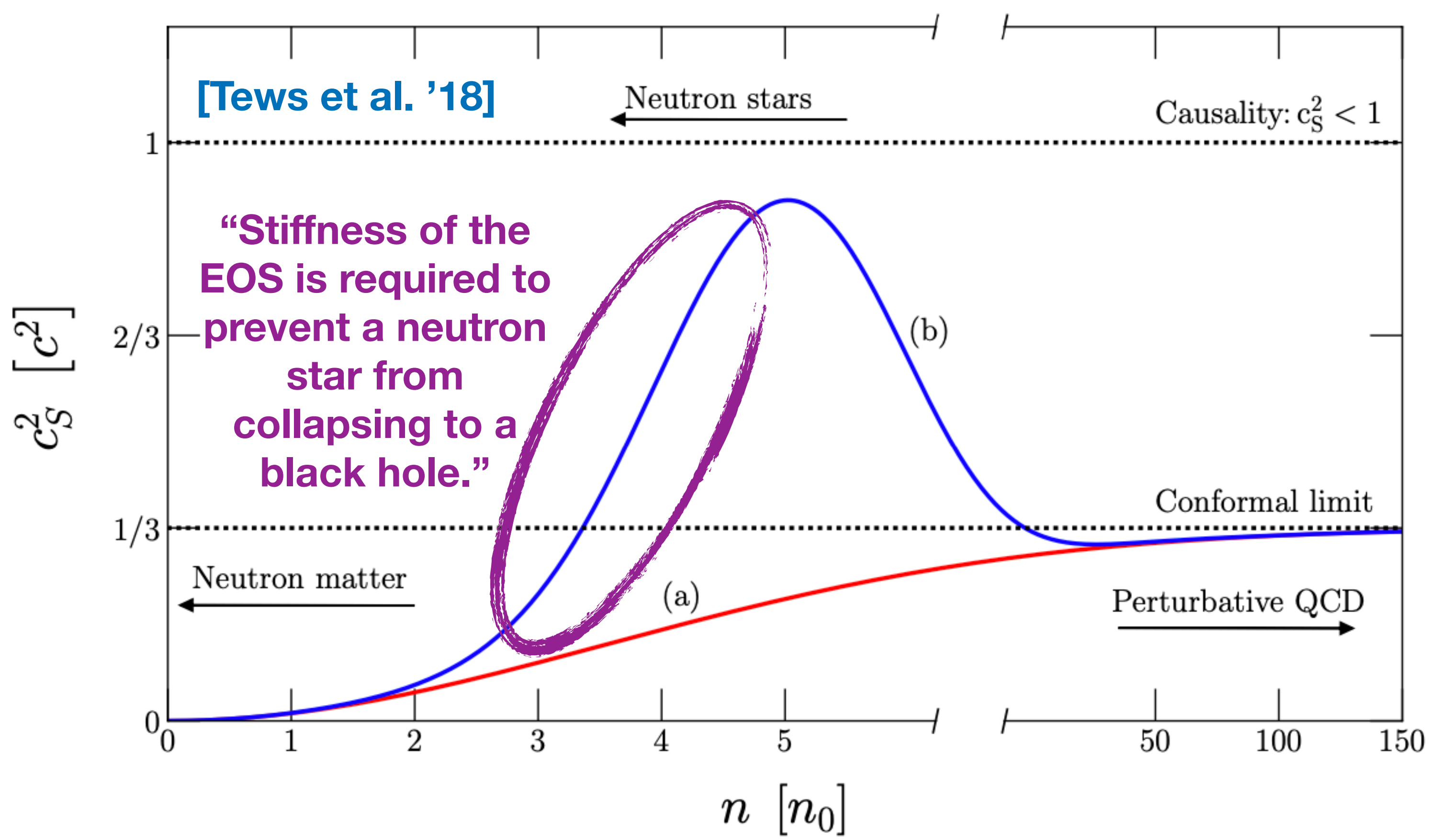

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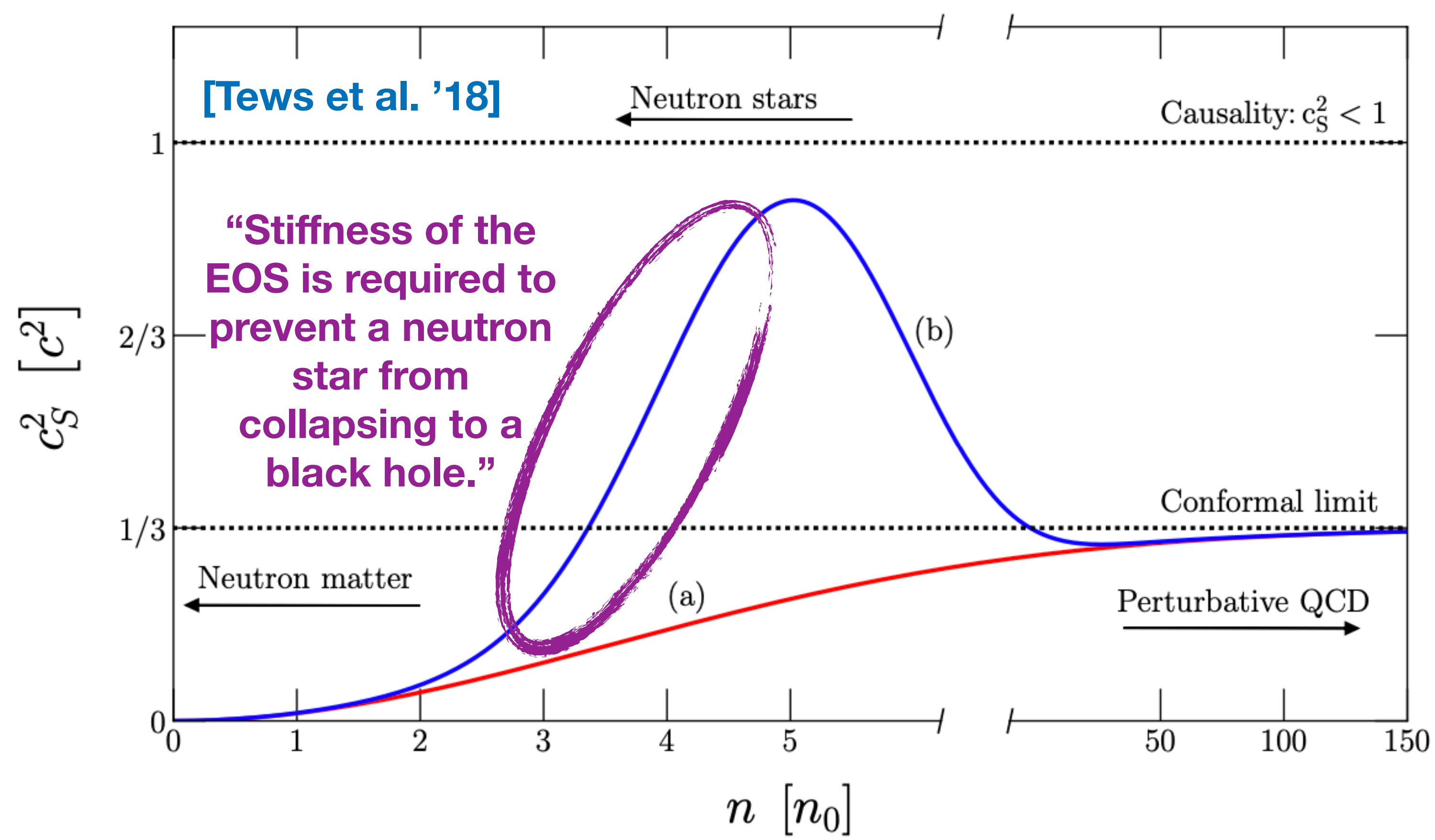

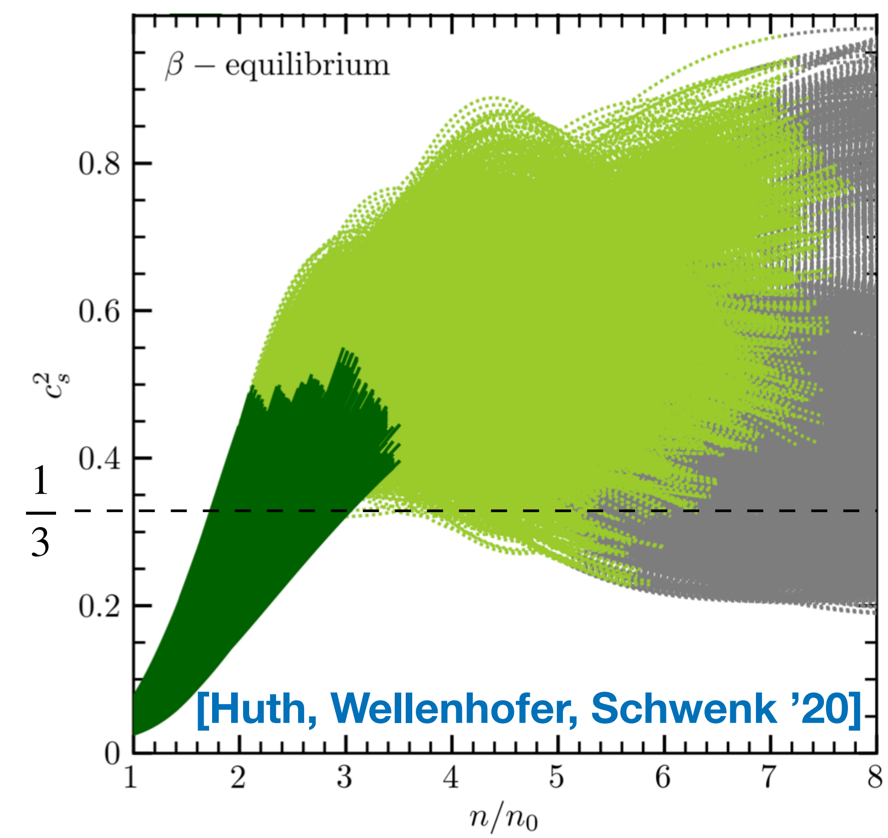

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Speed of sound of dense symmetric matter

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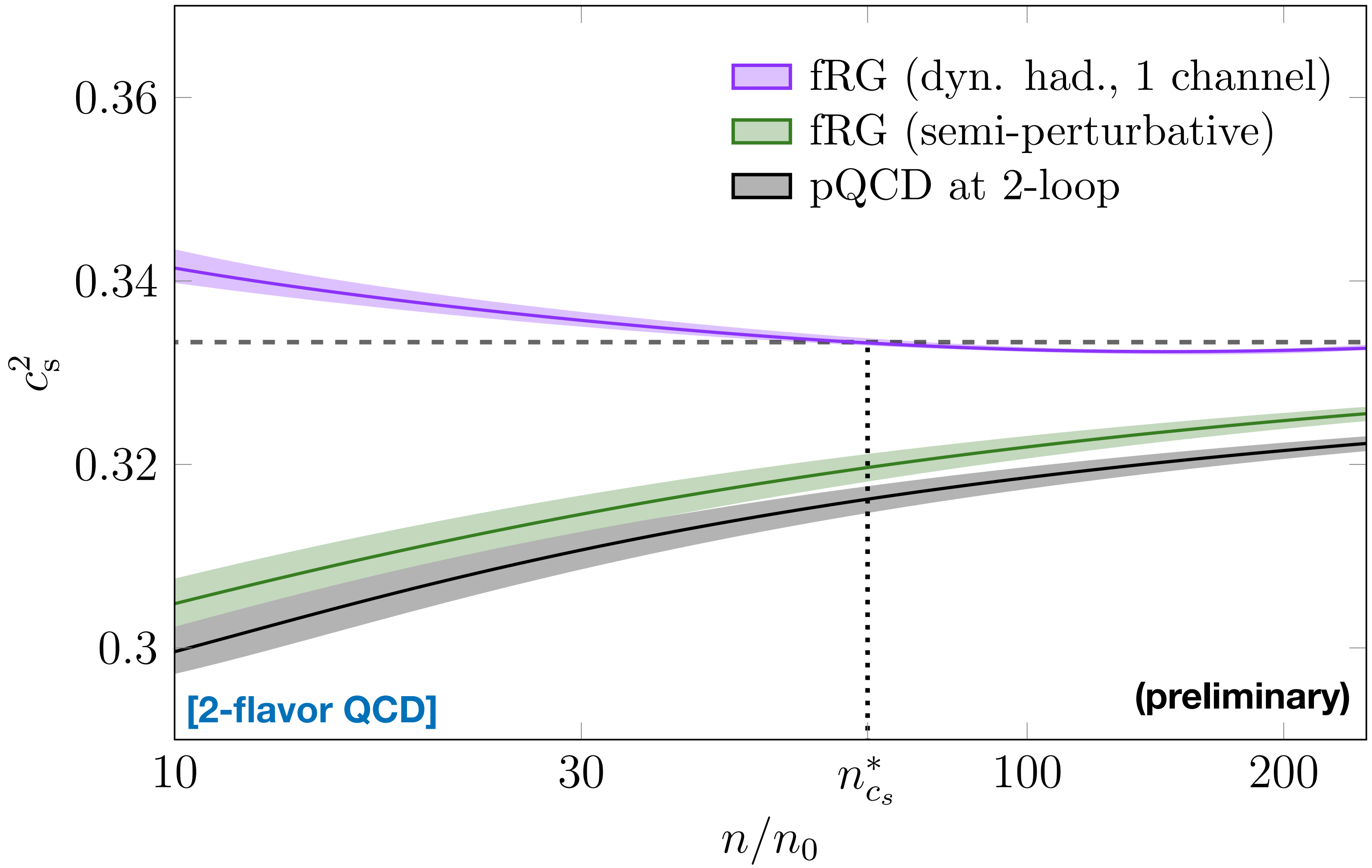
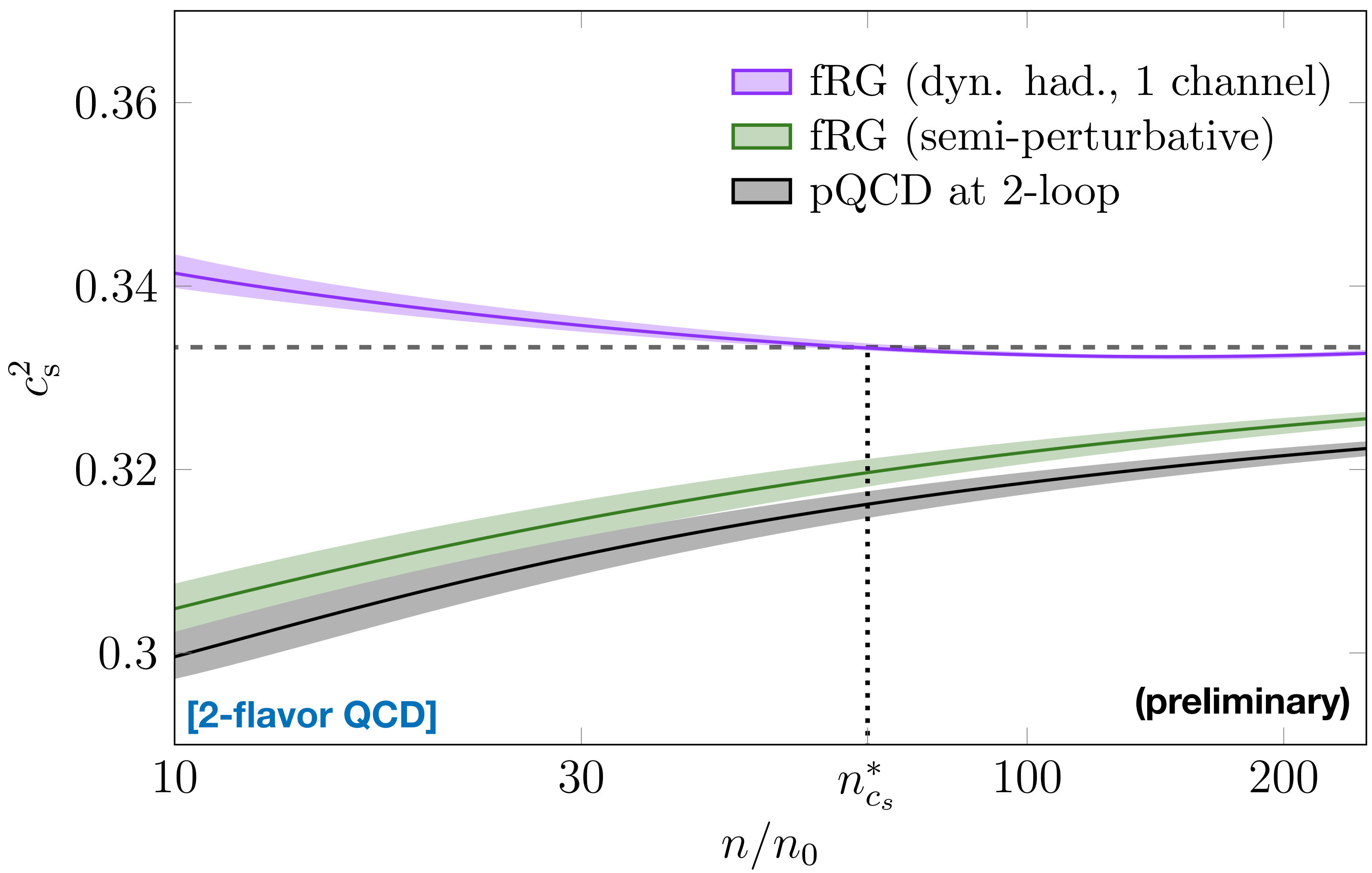
[JB & Schallmo ’21; JB, Geissel, Schallmo ’22; JB, Geissel, Schallmo (in prep.)]



- Consistent with **perturbative QCD at high densities**
- Deviations from pQCD because of non-perturbative form of the **gap**: $\Delta_0 \sim \exp(-c/(g^4\mu^2))$

Speed of sound of dense symmetric matter

[JB & Schallmo '21; JB, Geissel, Schallmo '22; JB, Geissel, Schallmo (in prep.)]



- **Scaling law** for the speed of sound: [JB, Geissel, Schallmo '22]

$$c_s^2 = \frac{1}{3} + \bar{c}_0(1+\sigma)n^{\frac{2(\sigma-1)}{3}} - \bar{c}_1/\ln(\bar{c}_2 n^{\frac{1}{3}})$$

Speed of sound of dense symmetric matter

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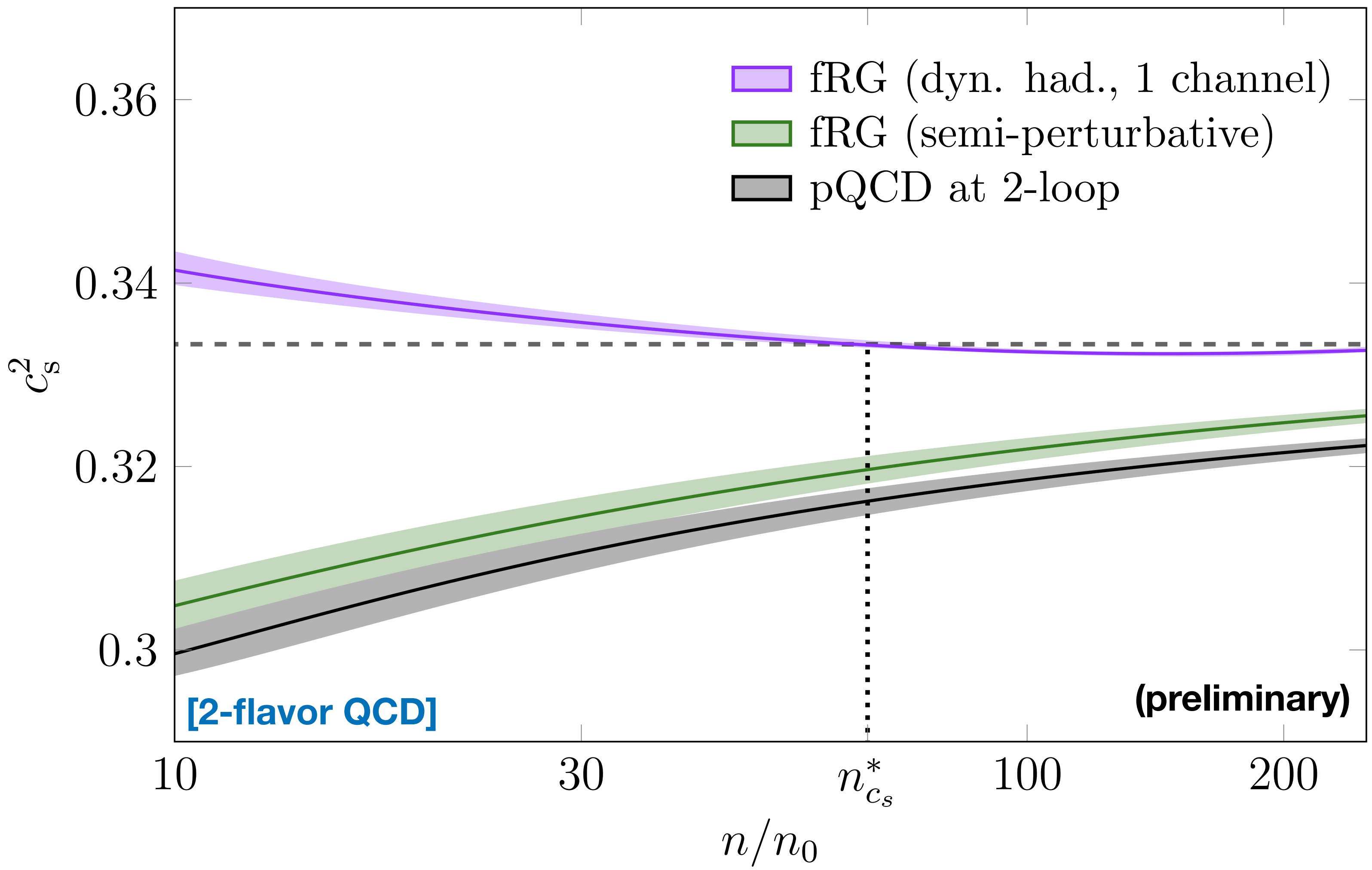

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free gas

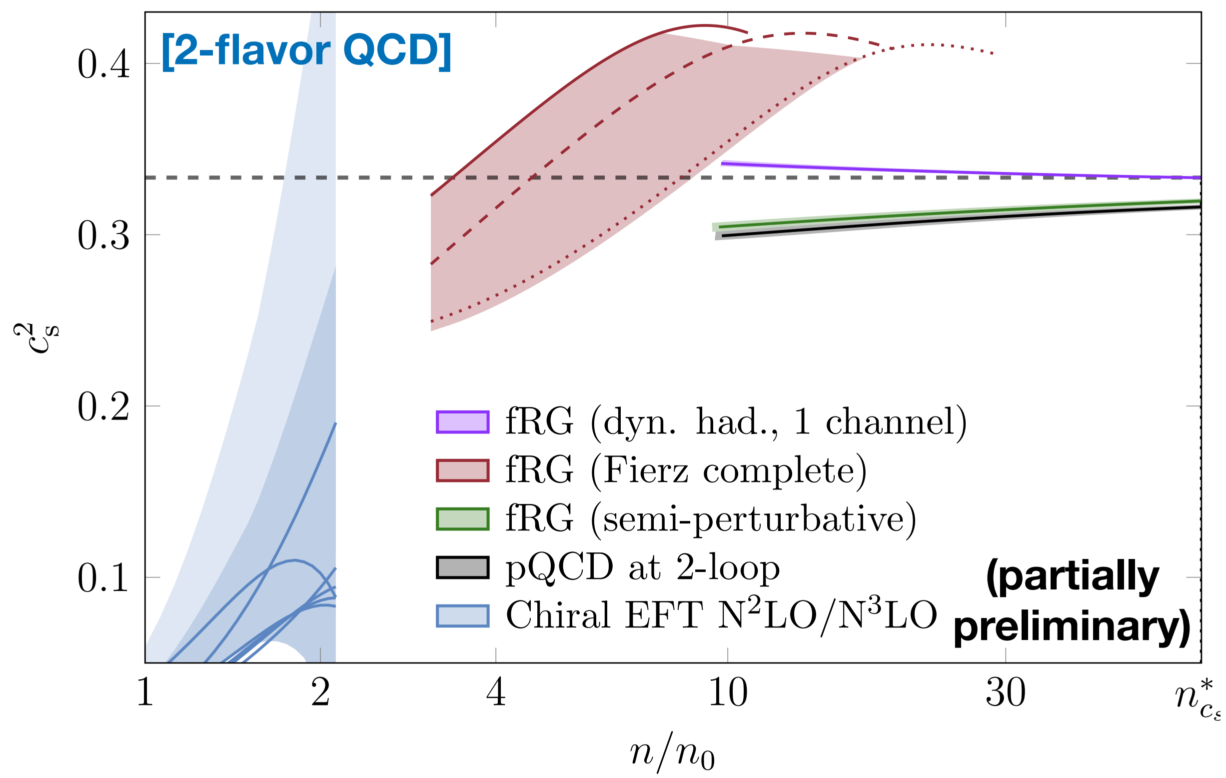
perturbative QCD

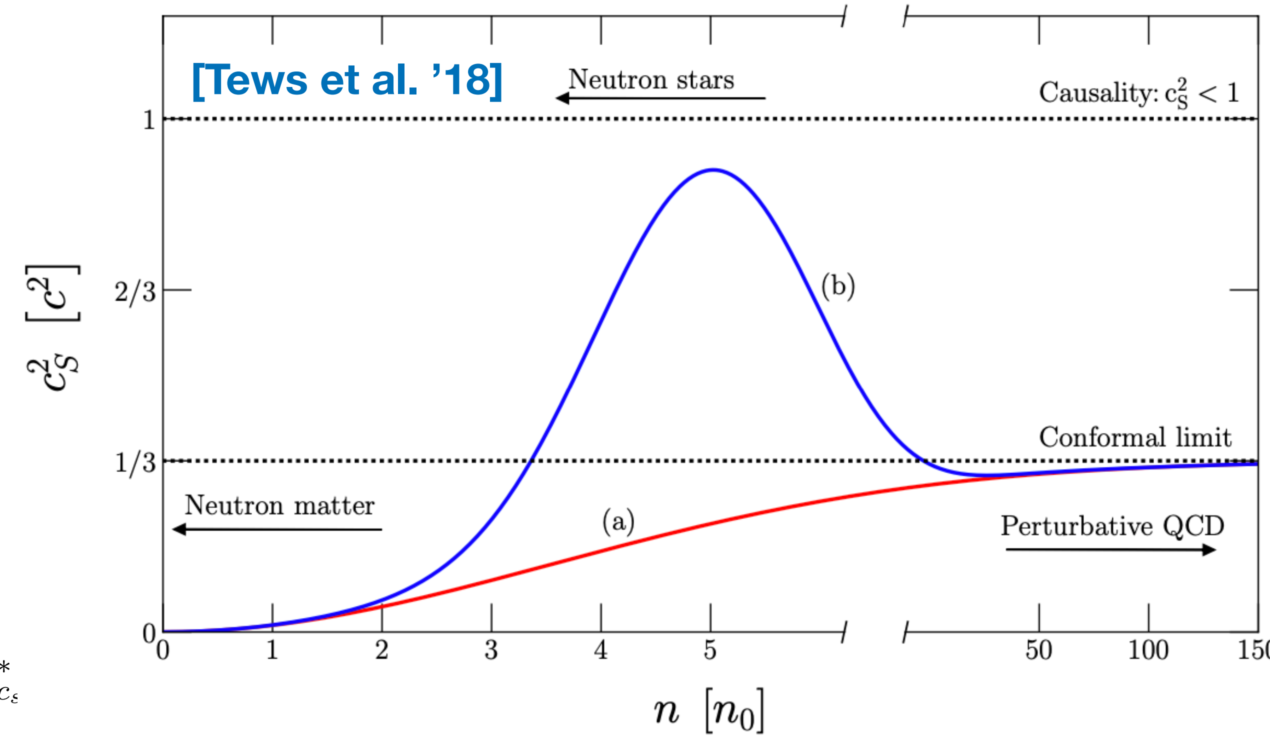
$$c_s^2 = \frac{1}{3} + \bar{c}_0(1+\sigma)n^{\frac{2(\sigma-1)}{3}} - \bar{c}_1/\ln(\bar{c}_2 n^{\frac{1}{3}})$$

gap-induced correction

Cautious comparison to astrophysical constraints

[Leonhardt et al. '19; JB & Schallmo '21; JB, Geissel, Schallmo '22; JB, Geissel, Schallmo (in prep.)]





- Existence of a maximum is in accordance with neutron-star observations and nuclear physics [Bedaque, Steiner '15; Tews et al. '18; Greif et al. '18; Raaijmaakers '19; Huth et al. '20; Huth et al. '22; Altiparmak, Ecker, Rezzolla '22; Ecker & Rezzolla '22]

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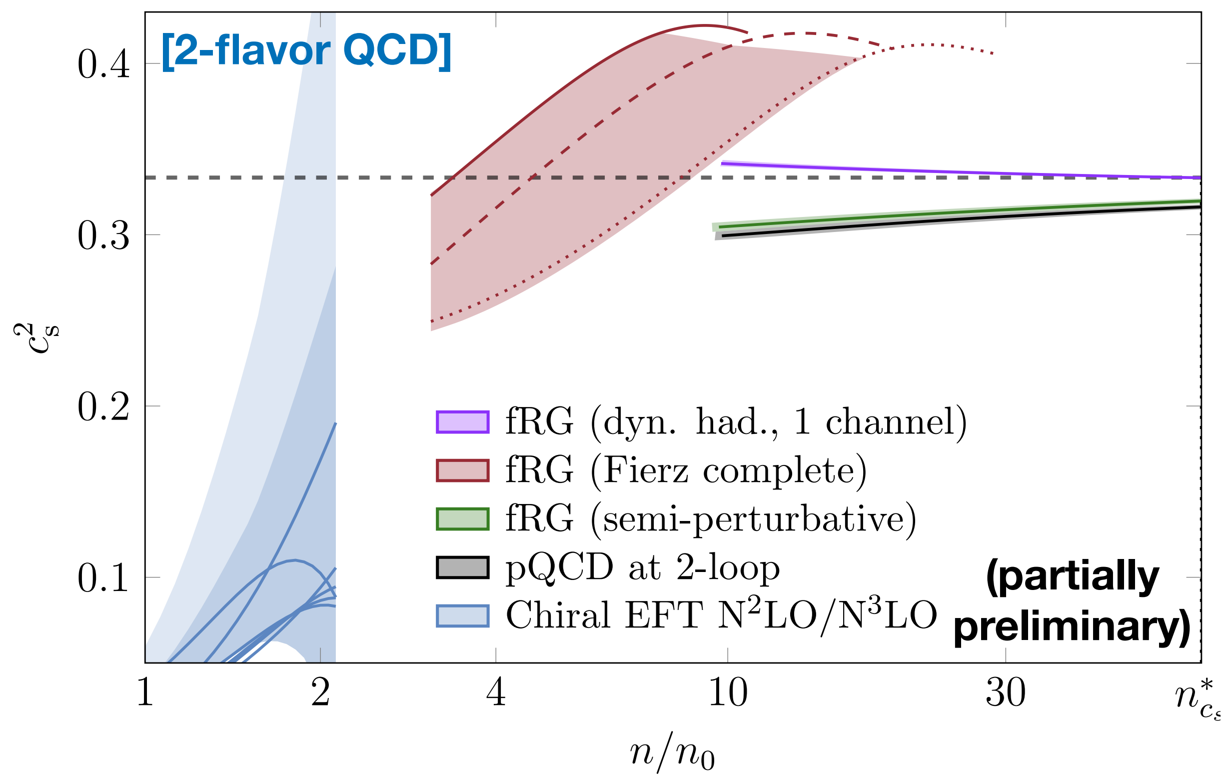

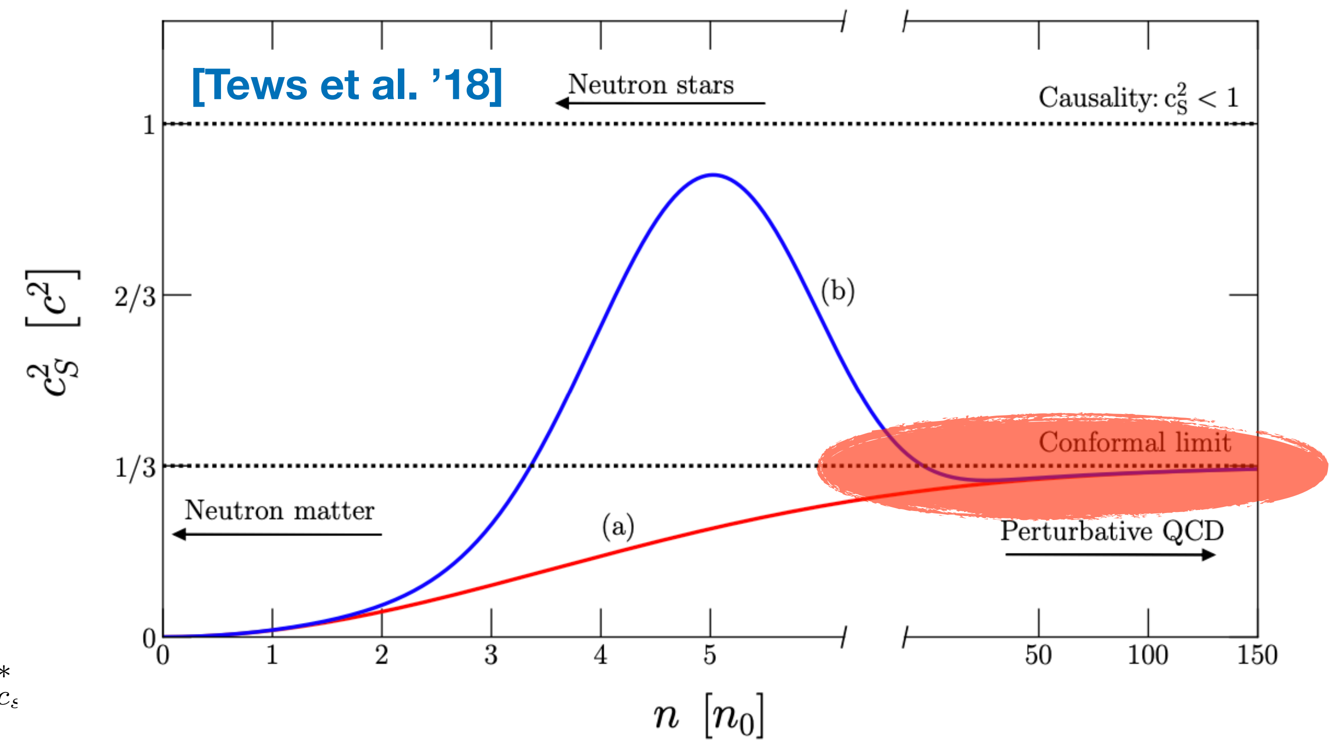

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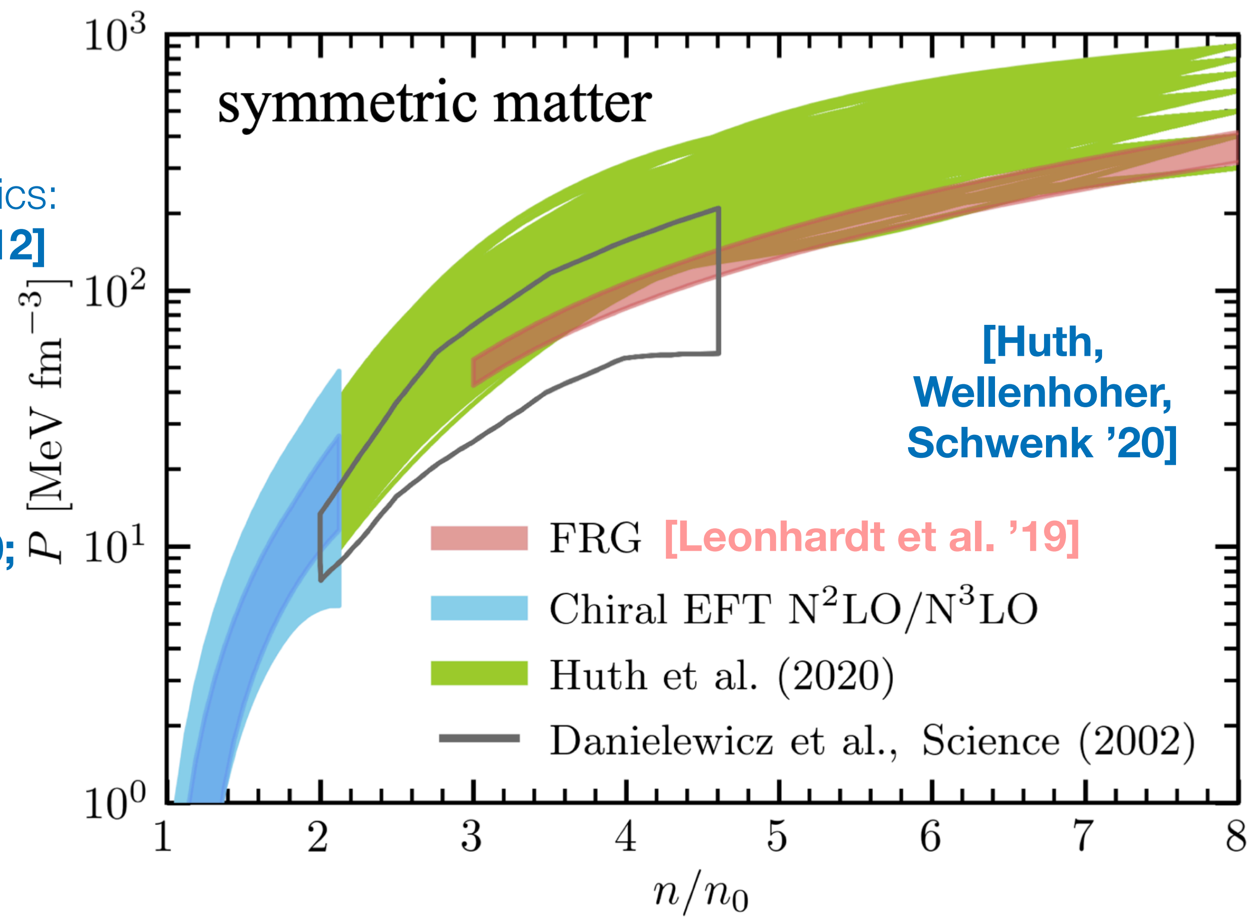
- **"High-density crossing point"** appears to be directly related to the **color-superconducting gap**

Constraints on the EOS of dense symmetric matter

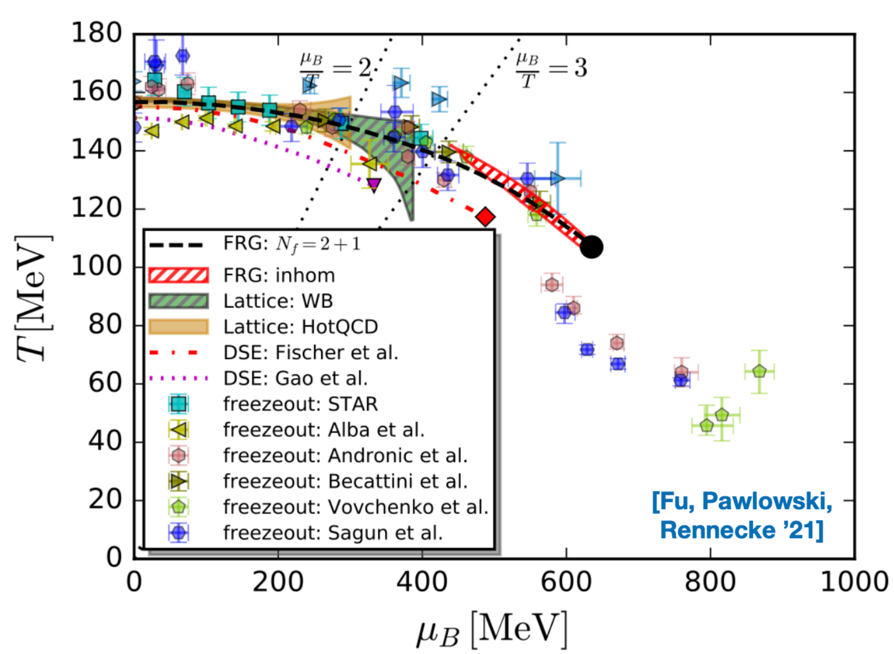
[incomprehensive list of studies of **low-density** thermodynamics:
Floerchinger & Wetterich '12]
Wellenhofer et al. '14;
Drews, Weise '15;
Hu et al. 16;
Drischler, Hebeler, Schwenk '17,
Otto, Oertel, Schaefer '20;
Tripolt et al. '21;
Lu et al. (NLEFT collaboration) '20;]



- FRG results can be used to constrain **new density functionals** together with constraints coming from neutron-star observations & chiral EFT **[Huth, Wellenhofer, Schwenk '20]**
- Consistent with constraints from **heavy-ion collision experiments**

Conclusions

- **Exciting times**: Constraints from heavy-ion collision experiments and neutron-star observations



- Good agreement of FRG results & lattice QCD data at low densities

- **FRG predictions** for the finite-temperature phase structure of QCD at finite density

- **EOS results** are consistent with **results from studies based on chiral EFT** at low densities and with **neutron-star observations**