Gravity-mediated matter scattering amplitudes from the effective action

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How to extract physics from the ERG?

- ERG: gives flow of $\Gamma_k[g, \phi]$
- Read off observables from $k \to 0$ limit:
  - $\lim_{k \to 0} \Gamma_k[g, \phi] \equiv \Gamma[g, \phi]$: effective action (EA)
  - Functional derivatives: (nonperturbatively) renormalized n-point functions
  - 1PI diagrams: renormalized scattering amplitudes

$\to$ Physics is read off from (tree level) Feynman diagrams of the EA

The effective action

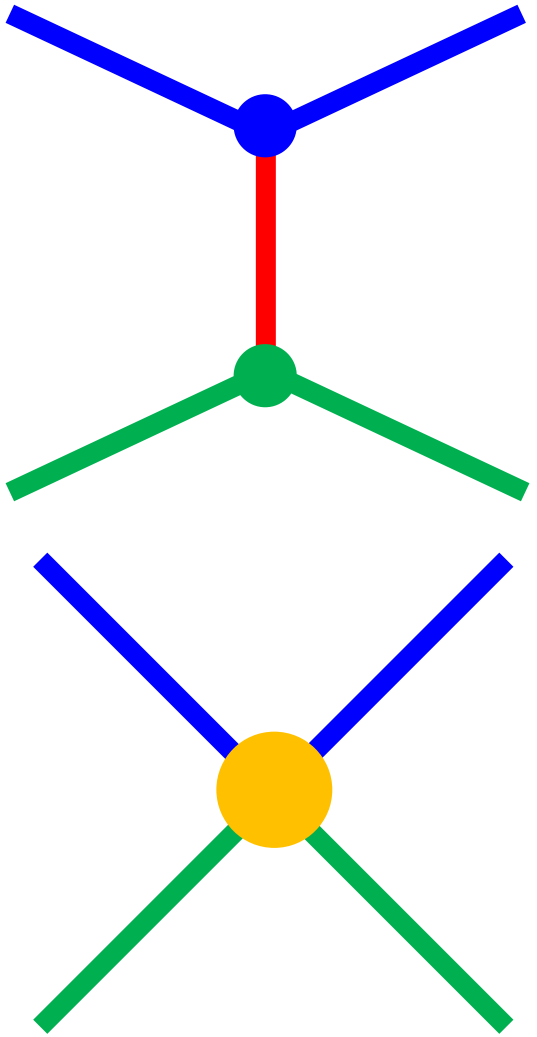
- ERG equation: generates all possible terms in the EA
    - Computation: (very) hard
        - see also talk by B. Knorr
    - Solution (almost) impossible

- Workaround: focus on process of interest
    - e.g 2-2 scattering only
    - Only compute relevant terms of the EA

- Central question in this talk:
  how to classify the relevant terms?
    - Scalars: easy
    - Photons: medium
    - Gravitons: hard

Ingredients for a scattering amplitude

- What do we want?
  - 2-2 scattering
  - Scalars ϕ, photons A
  - Flat background
  - Gravity-mediated
- What do we need:
  - Tree-level Feynman diagrams
  - Full propagators
  - Full interaction vertices



Classification of the EA

- Classify only relevant contributions in the EA
  - Flat background: up to $\mathcal{O}(\mathcal{R}^2)$
  - 4-point vertices: up to $\mathcal{O}(\phi^4, A^4)$
  - Gauge invariance: only D_μ, $R_{\mu\nu\rho\sigma}$, $F_{\mu\nu}$, ϕ
  - Additional symmetries:
    - $\mathbb{Z}_2$ symmetric scalar
    - Bianchi identities for $R_{\mu\nu\rho\sigma}$, $F_{\mu\nu}$
  - On-shell relations: $p^2 = 0$ (massless limit)
- Vertices: nth functional derivative $\Gamma^{(n)}$
- Propagators: $\left(\Gamma^{(2)}\right)^{-1}$

Classify scalars

- Gravity + self-interactions:

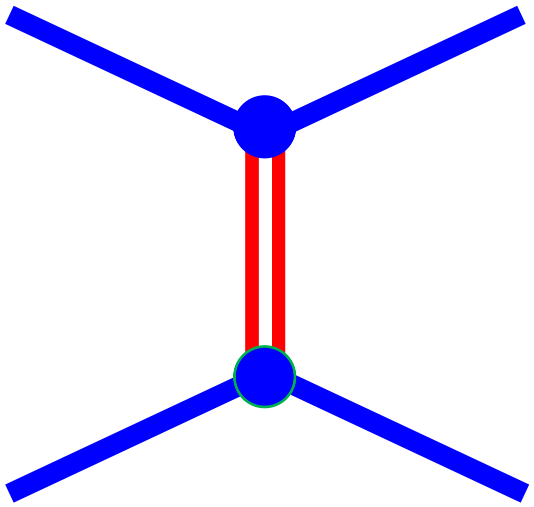
$$\Gamma_{\phi\phi} = \frac{1}{2}\int \phi\, f_{\phi\phi}(\Box)\phi$$

$$\Gamma_{h\phi^2} = \int \left(f_{R\phi\phi}(\Box_1, \Box_2, \Box_3) R\phi\phi + f_{\mathrm{Ric}\phi\phi}(\Box_1, \Box_2, \Box_3) R^{\mu\nu} \left(D_\mu \phi\right)\left(D_\nu \phi\right) \right)$$

$$\Gamma_{\phi^4} = \int f_{\phi^4}\left(\left\{ D_i \cdot D_j \right\}_{i<j} \right) \phi\phi\phi\phi$$

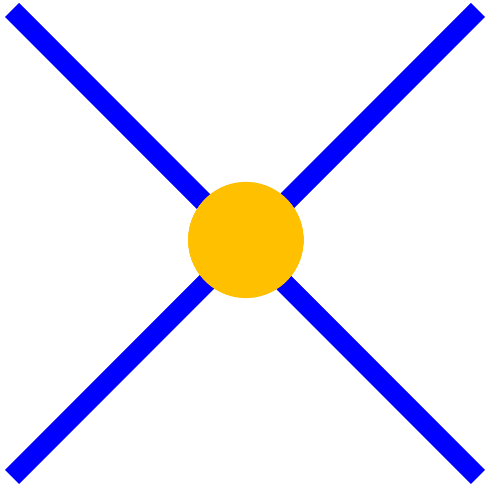
$$\Gamma_{h^2} = \frac{1}{16\pi G_{\mathrm{N}}} \int \left(-R - \frac{1}{6} R f_R(\Box) R + \frac{1}{2} C_{\mu\nu\rho\sigma} f_C(\Box) C^{\mu\nu\rho\sigma} \right)$$

Scalar scattering amplitudes



Scattering amplitudes ($m_\phi = 0$):

$$\mathcal{A}_t^\phi(s,t,u) = \frac{4\pi G_\mathrm{N}}{3}\left[\frac{\left(t\left(1+t\,f_{\mathrm{Ric}\phi\phi}\right)-12tf_{R\phi\phi}\right)^2}{t\left(1+t\,f_R(t)\right)} - (s^2-4su+u^2)\frac{\left(t\left(1+t\,f_{\mathrm{Ric}\phi\phi}\right)\right)^2}{t\left(1+t\,f_C(t)\right)}\right]$$



$$\mathcal{A}_4^\phi = f_{\phi^4}(s,t,u)$$

More matter: photons

- Classification strategy:

1. Generate all contractions of $R_{\mu\nu\rho\sigma}$, $F_{\mu\nu}$ with derivatives

   xAct: `AllContractions`

2. Reduce: avoid overcounting
   - $D^\mu D_\mu \to \Box$ (into form factor)
   - Integration by parts
   - Bianchi identities
   - Relabeling: $D_1 D_2 F^1_{\mu\nu} F^2_{\rho\sigma} = D_2 D_1 F^1_{\rho\sigma} F^2_{\mu\nu}$

Photon effective action

$$\Gamma_{A^2} = \int F_{\mu\nu} f_{FF}(\Box) F^{\mu\nu}$$

$$\Gamma_{hA^2} = \int \Big(f_{RFF}\, R\, F_{\alpha\beta}F^{\alpha\beta} + f_{\mathrm{Ric}FF} R^{\alpha\beta} F_{\alpha}{}^{\gamma} F_{\beta\gamma} + f_{\mathrm{Rm}FF} R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$
$$+ f_{D^2RFF} \left(D^{\alpha}D^{\beta}R\right) F_{\alpha}{}^{\gamma} F_{\beta\gamma} + f_{D^2\mathrm{Ric}FF} \left(D^{\alpha}D^{\beta}R^{\gamma\delta}\right) F_{\alpha\gamma}F_{\beta\delta}$$
$$+ f_{\mathrm{Ric}D^2FF} R^{\gamma\delta} \left(D^{\alpha}D^{\beta}F_{\alpha\gamma}\right) F_{\beta\delta} + f_{\mathrm{Ric}DFDF} R^{\gamma\delta} \left(D^{\alpha}F_{\alpha\gamma}\right)\left(D^{\beta}F_{\beta\delta}\right)\Big)$$

- Photon-scalar, photon 3-vertices: don't contribute
- Photon 4-vertex: 7 form factors
- Photon-scalar 4-vertex: 4 form factors

Photon scattering amplitudes

- Polarisations +/−
- Crossing symmetry

$$\mathcal{A}_t^{+--+} = 2\pi G_\mathrm{N} \frac{(2 + t^2 f_{D^2\mathrm{Ric}FF} - 2t f_{\mathrm{Ric}FF} - 4t f_{\mathrm{Rm}FF})^2}{t(1 + t\, f_C(t))}$$

$$\mathcal{A}_t^{++++} = \frac{\pi}{3} G_\mathrm{N} t^2 (s^2 - 4su + u^2) \frac{\left(-t f_{D^2\mathrm{Ric}FF} + 4 f_{\mathrm{Rm}FF} + f'_{FF}(0)\right)^2}{t(1 + t\, f_C(t))}$$

$$- \frac{\pi}{3} G_\mathrm{N} t^4 \frac{(6t f_{D^2RFF} + t f_{D^2\mathrm{Ric}FF} - 24 f_{RFF} - 6 f_{\mathrm{Ric}FF} - 4 f_{\mathrm{Rm}FF} + 2 f'_{FF}(0))^2}{t(1 + t\, f_C(t))}$$

- + 2 more classes of polarisations

- 4-point diagrams: too long for this page.
- Photon-scalar amplitudes

Conclusions and outlook

- This talk:
  - Classification of photon effective action
  - 2-2 scattering amplitudes of photons and scalars
- Challenge: graviton 2-2 scattering
  - $h_{\mu\nu}$ has 2 indices
  - 4-point vertex: classify up to 4 curvature tensors
  - Many Bianchi identities
- Future goal:
  - Compute form factors using ERG
  - Plug into scattering amplitude formula
  - Find fully renormalized cross-sections
  - Generalization to curved spacetime