Precision calculation with FRG
the 2D Bose gas as an example

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Benchmarking the FRG

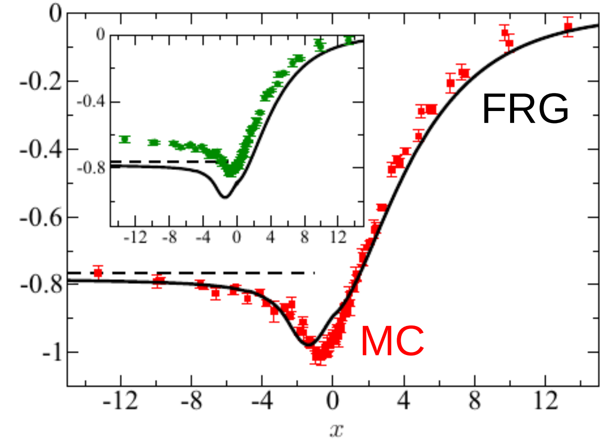
- comparison with (nearly exact) numerical simulations

3D Ising universality class

DE	ν	η
$s = 0$	0.651 03	0
$s = 2$	0.627 52	0.045 51
$s = 4$	0.630 57	0.033 57
$s = 6$	0.630 07	0.036 48
Conformal bootstrap	0.629 971(4)	0.036 297 8(20)
Six loop	0.6304(13)	0.0335(25)
High T	0.630 12(16)	0.036 39(15)
MC	0.630 02(10)	0.036 27(10)

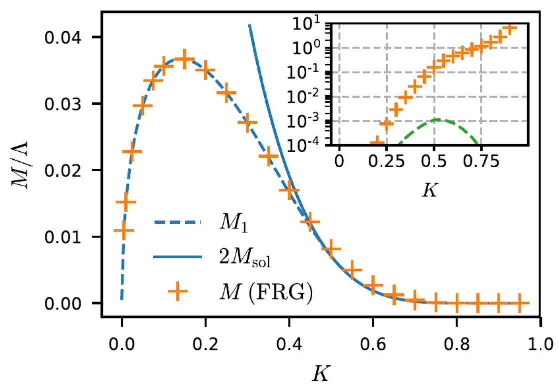
Balog *et al.* PRL 2019
De Polsi *et al.* PRE 2020

3D Heisenberg universality class



Rançon *et al.* PRB 2016

- comparison with integrable model



soliton and anti-soliton masses in the sine-Gordon model

R. Daviet and ND, PRL 2019

- comparison with experiments: 2D Bose gas

Outline

- Universal thermodynamics of a 2D Bose gas

  experiment *vs* classical field simulations *vs* FRG

- Tan's two-body contact in a 2D Bose gas

  experiment *vs* classical field simulations *vs* FRG

Dilute Bose gas

- 3D

$$\hat{H} = \int d^3r\, \hat{\psi}^\dagger \left(-\frac{\nabla^2}{2m}\right) \hat{\psi} + \frac{1}{2} \int d^3r\, d^3r'\, V(r-r') \hat{\psi}^\dagger(r) \hat{\psi}^\dagger(r') \hat{\psi}(r') \hat{\psi}(r)$$

At low energies, the potential *V(r)* is fully characterized by the *s*-wave scattering length a_3 (scattering in the *s*-wave channel only)

- quasi-2D

Harmonic trap: $V_H(z) = \frac{1}{2} m \omega_z^2 z^2 \qquad l_z = \frac{1}{\sqrt{m\omega_z}}$

$$T \ll \omega_z : \qquad \hat{H}_{2D} = \int d^2r \left\{ -\hat{\psi}^\dagger \frac{\nabla^2}{2m} \hat{\psi} + \frac{g}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right\}$$

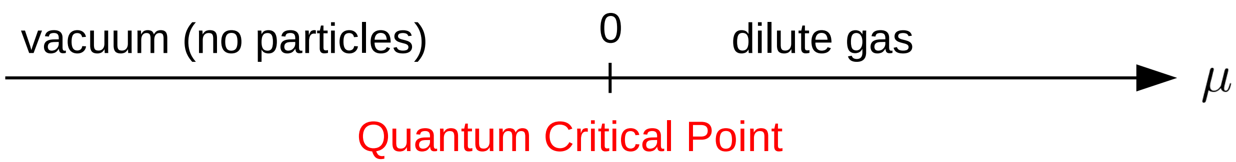
$$\text{with} \left\{ \begin{array}{ll} \Lambda \sim 1/l_z & \text{UV momentum cutoff} \\ mg = \sqrt{8\pi} \frac{a_3}{l_z} & \text{interaction constant} \end{array} \right.$$

[Petrov & Shlyapnikov, PRA 2001]

scattering length: $a_2 \simeq \frac{2}{\Lambda} e^{-\frac{2\pi}{mg} - \gamma}$

- universal thermodynamics

$$P(\mu, T) = \frac{T}{\lambda^2}\mathcal{F}\left(\frac{\mu}{T}, \tilde{g}(T)\right) \quad \text{with} \quad \begin{cases} \lambda = \sqrt{\frac{2\pi}{mT}} \\ \tilde{g}(T) = -\dfrac{4\pi}{\ln \frac{1}{2}\sqrt{2ma_2^2 T} + \gamma} \\ a_2 = \frac{2}{\Lambda} e^{-\frac{2\pi}{mg} - \gamma} \end{cases}$$



- weak-coupling limit

$$\tilde{g}(T) \simeq mg = \sqrt{8\pi}\frac{a_3}{l_z} \ll 1$$

$$P(\mu, T) = \frac{T}{\lambda^2}\mathcal{F}\left(\frac{\mu}{T}, mg\right)$$

PRL **107,** 130401 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Exploring the Thermodynamics of a Two-Dimensional Bose Gas

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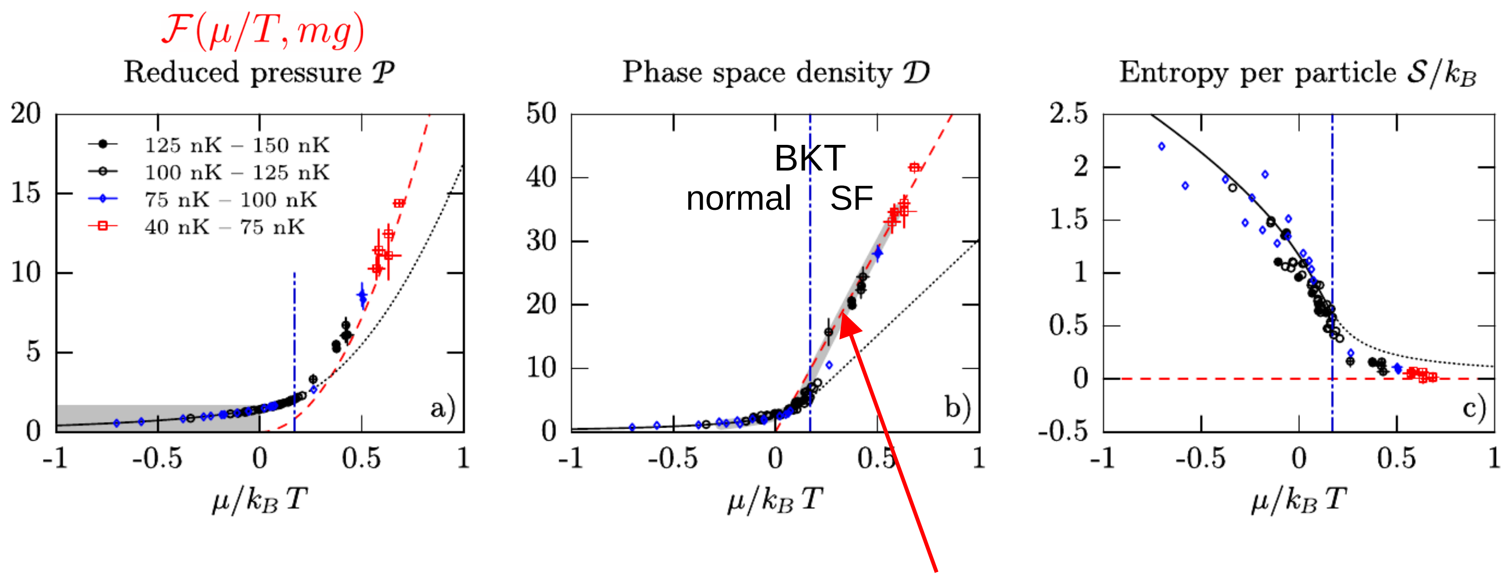
(Received 30 May 2011; revised manuscript received 11 July 2011; published 19 September 2011)

only parameter: $mg = \sqrt{8\pi}\frac{a_3}{l_z} = 0.109(5)$



PHYSICAL REVIEW A **66**, 043608 (2002)

Two-dimensional weakly interacting Bose gas in the fluctuation region

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(Received 14 June 2002; published 11 October 2002)

Monte Carlo simulations of a (classical) 2D φ^4 theory

PHYSICAL REVIEW A **85**, 063607 (2012)

Universal thermodynamics of a two-dimensional Bose gas

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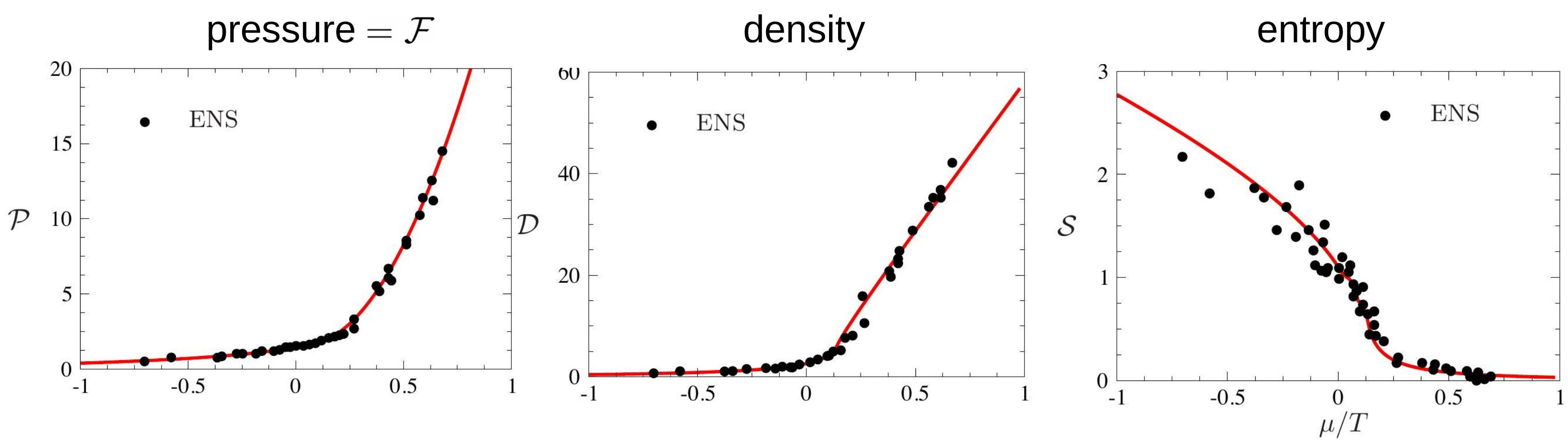
(Received 13 March 2012; published 5 June 2012)



$$\Gamma_k[\phi^*,\phi] = \int_0^\beta d\tau \int d^2r \left\{ \phi^* \left(-Z_{A,k}\frac{\nabla^2}{2m} - V_{A,k}\partial_\tau^2 + Z_{C,k}\partial_\tau \right) \phi + U_k(|\phi|^2) \right\}$$

LPA’ or DE2

Two-body contact [S. Tan, Ann. Phys. 2008]

- thermodynamic definition:

$$P(\mu, T, a_2) = \frac{T}{\lambda^2}\mathcal{F}\left(\frac{\mu}{T}, \tilde{g}(T)\right) \quad \text{with} \quad \tilde{g}(T) \equiv \tilde{g}(m a_2^2 T)$$

$$d\Omega = -\mathcal{A}dP - TdS - \mu dN + \frac{C}{4\pi m} d(\ln a_2)$$

$$\frac{C}{4\pi m} = \frac{\partial \Omega}{\partial(\ln a_2)}\bigg|_{\mathcal{A},\mu,T} = -\mathcal{A}\frac{\partial P}{\partial(\ln a_2)}\bigg|_{\mu,T} \qquad \left(= \frac{\partial E}{\partial(\ln a_2)}\bigg|_{\mathcal{A},N,S} \right)$$

- short-distance physics

two-body density matrix

$$\rho_2(\mathbf{r}, \mathbf{r}', \mathbf{r}, \mathbf{r}') = \langle \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r})\rangle$$

$$\simeq \frac{C}{(2\pi)^2\mathcal{A}} \ln^2|\mathbf{r}-\mathbf{r}'| \quad \text{for} \quad b \ll |\mathbf{r}-\mathbf{r}'| \ll a_2, n^{-1/2}$$

momentum distribution

$$n_{\mathbf{k}} = \frac{C}{|\mathbf{k}|^4} \qquad (1/a_2, n^{1/2} \ll |\mathbf{k}| \ll 1/b)$$

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ARTICLE

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Tan's two-body contact across the superfluid transition of a planar Bose gas

Y.-Q. Zou[1], B. Bakkali-Hassani[1], C. Maury[1], É. Le Cerf[1], S. Nascimbene[1], J. Dalibard[1] & J. Beugnon[1]✉

Tan's contact is a quantity that unifies many different properties of a low-temperature gas with short-range interactions, from its momentum distribution to its spatial two-body correlation function. Here, we use a Ramsey interferometric method to realize experimentally the thermodynamic definition of the two-body contact, i.e., the change of the internal energy in a small modification of the scattering length. Our measurements are performed on a uniform two-dimensional Bose gas of ^{87}Rb atoms across the Berezinskii–Kosterlitz–Thouless superfluid transition. They connect well to the theoretical predictions in the limiting cases of a strongly degenerate fluid and of a normal gas. They also provide the variation of this key quantity in the critical region, where further theoretical efforts are needed to account for our findings.

$$\frac{\partial E}{\partial a_3} \Longrightarrow \frac{\partial E}{\partial a_2} \quad \Longrightarrow \quad \frac{C}{4\pi m} = \frac{\partial E}{\partial(\ln a_2)}$$

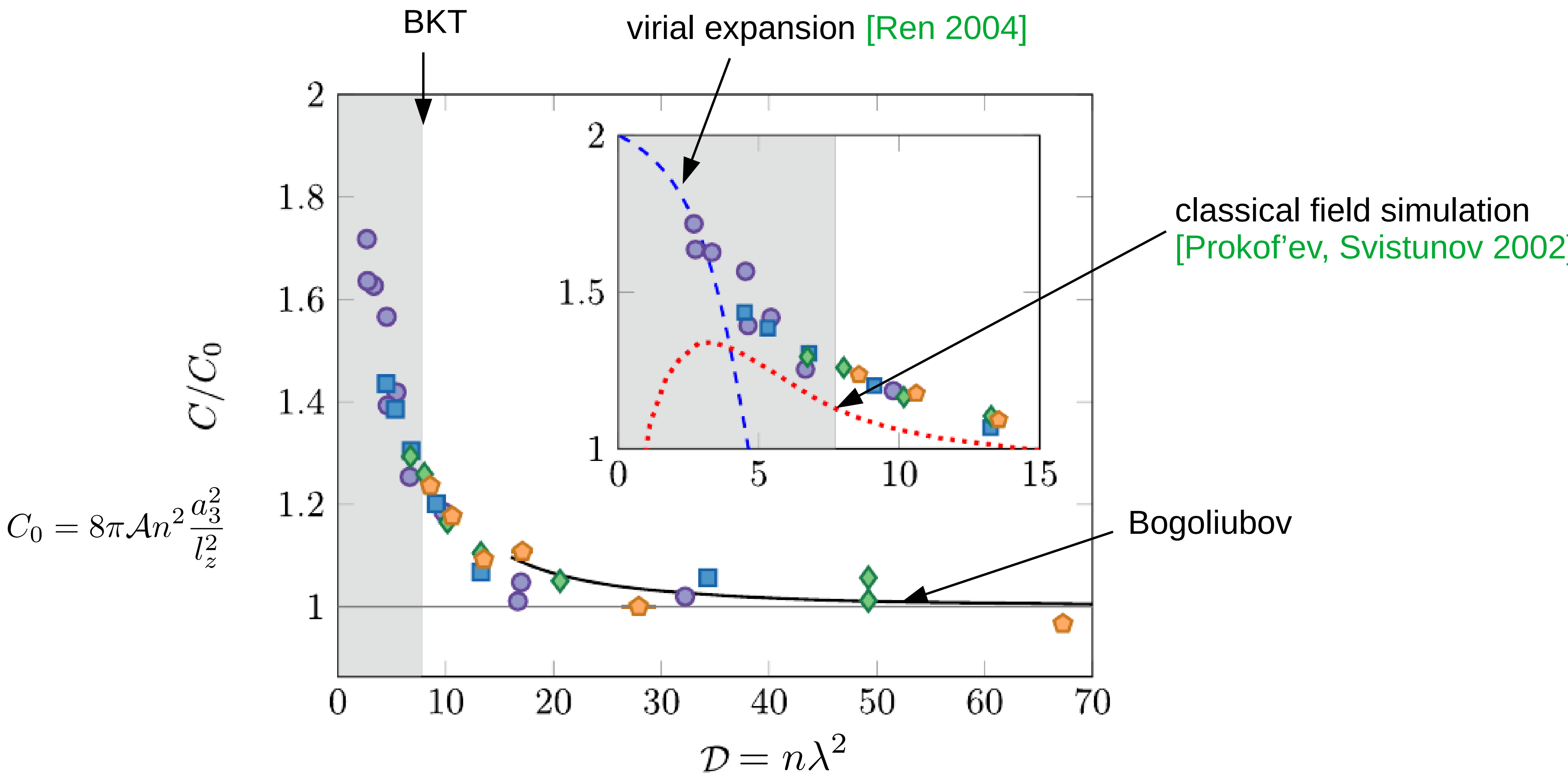
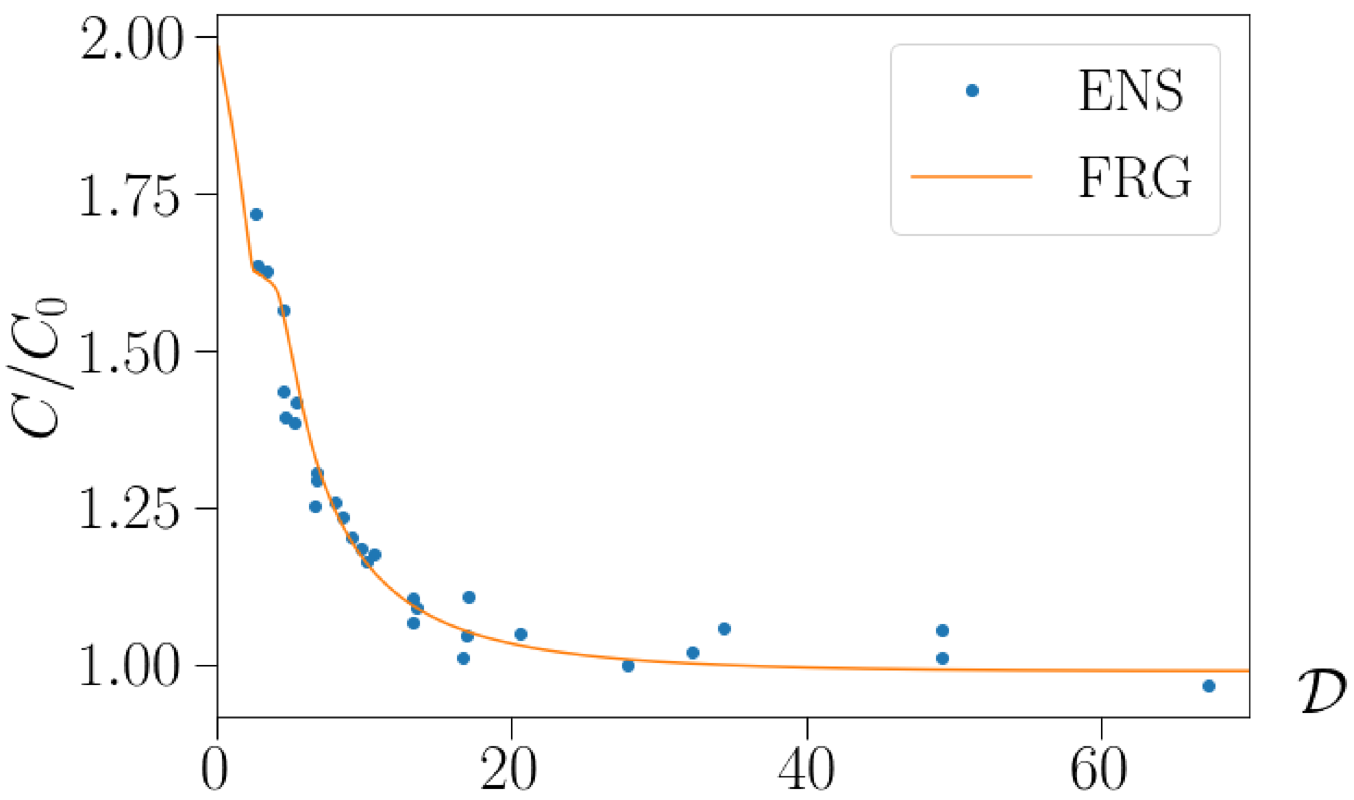


Fig. 3 Contact measurement. Variations of the normalized Tan 's contact C/C_0 with the phase-space density $\mathcal{D}$. The encoding of the experimental points is the same as in Fig. 2. The colored zone indicates the non-superfluid region, corresponding to $\mathcal{D} < \mathcal{D}_c \approx 7.7$. The continuous black line shows the prediction derived within the Bogoliubov approximation. Inset: Zoom on the critical region. The dashed blue line is the prediction from ref. [46], resulting from a virial expansion for the 2D Bose gas. The dotted red line shows the results of the classical field simulation of ref. [47].

FRG calculation of the contact

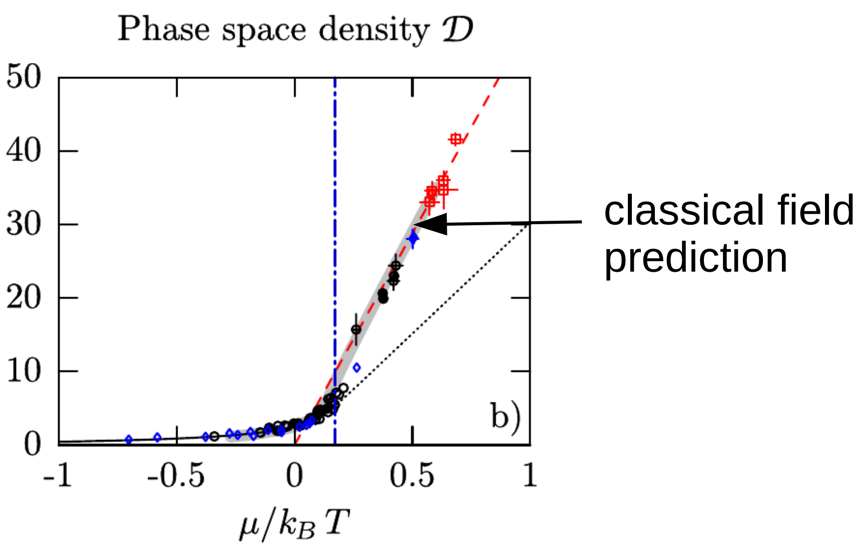


A. Rançon and ND, to be published

Classical field predictions [Prokof'ev, Svistunov 2002]

- very accurate for the equation of state

Yefsah *et al.* 2011

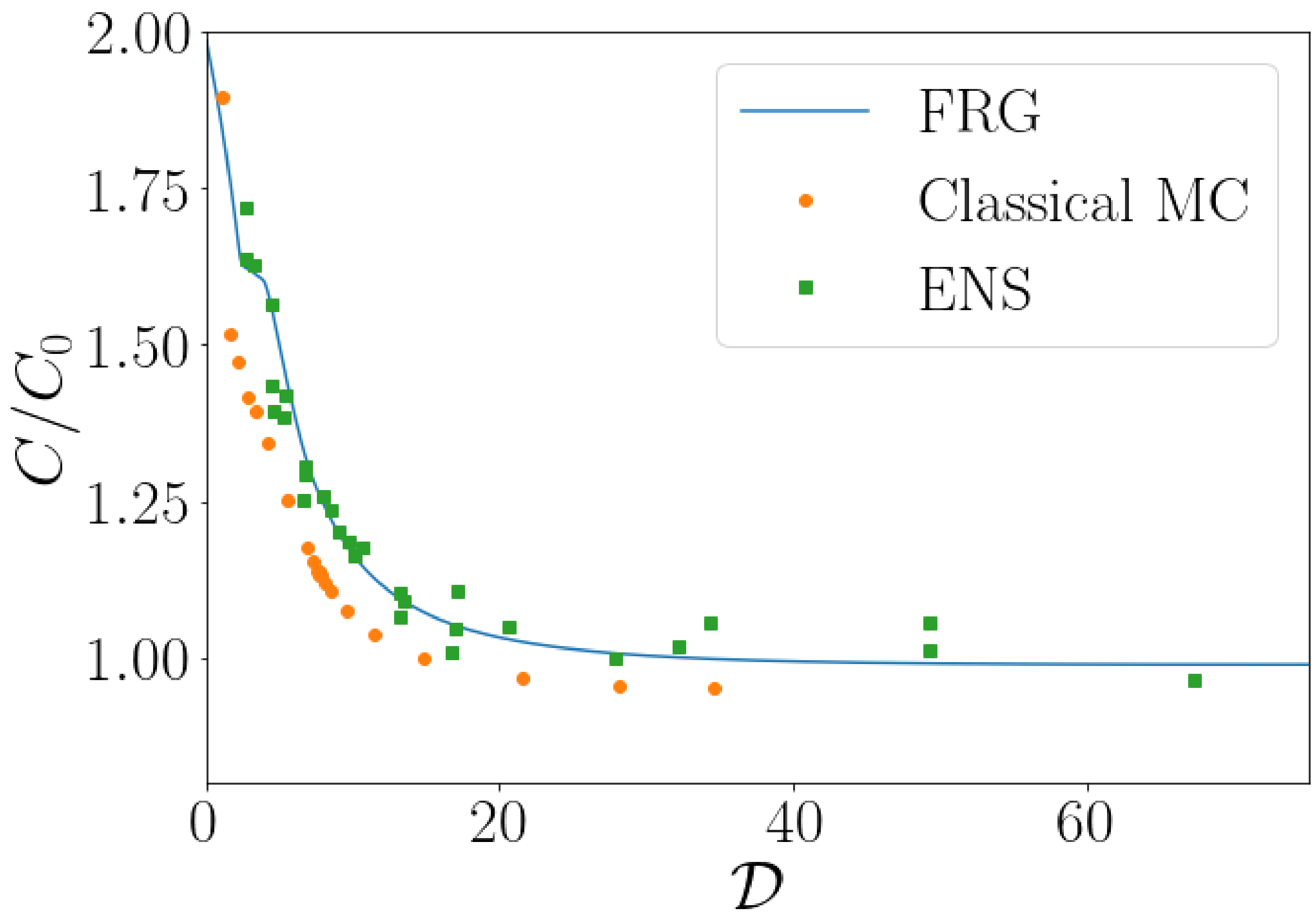


- contact – first method [Zou *et al.* 2021]

$$\frac{C}{4\pi m} = \frac{\partial \Omega}{\partial(\ln a_2)} = \frac{mg^2}{2\pi}\frac{\partial \Omega}{\partial g} = \frac{mg^2}{2\pi}\frac{L^2}{2}\langle|\psi|^4\rangle$$

not accurate for $T>T_c$

- contact – second method $$\frac{C}{4\pi m} = \frac{mg^2}{2\pi}\frac{\partial \Omega}{\partial g}$$



FRG
Classical MC
ENS
2.00
1.75
1.50
1.25
1.00
0
20
40
60
C/C_0
$\mathcal{D}$

Article

Observation of first and second sound in a BKT superfluid

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Panagiotis Christodoulou[1] ✉, Maciej Gałka[1], Nishant Dogra[1], Raphael Lopes[2], Julian Schmitt[1,3] & Zoran Hadzibabic[1]

Superfluidity in its various forms has been of interest since the observation of frictionless flow in liquid helium II[1,2]. In three spatial dimensions it is conceptually associated with the emergence of long-range order at a critical temperature. One of the hallmarks of superfluidity, as predicted by the two-fluid model[3,4] and observed in

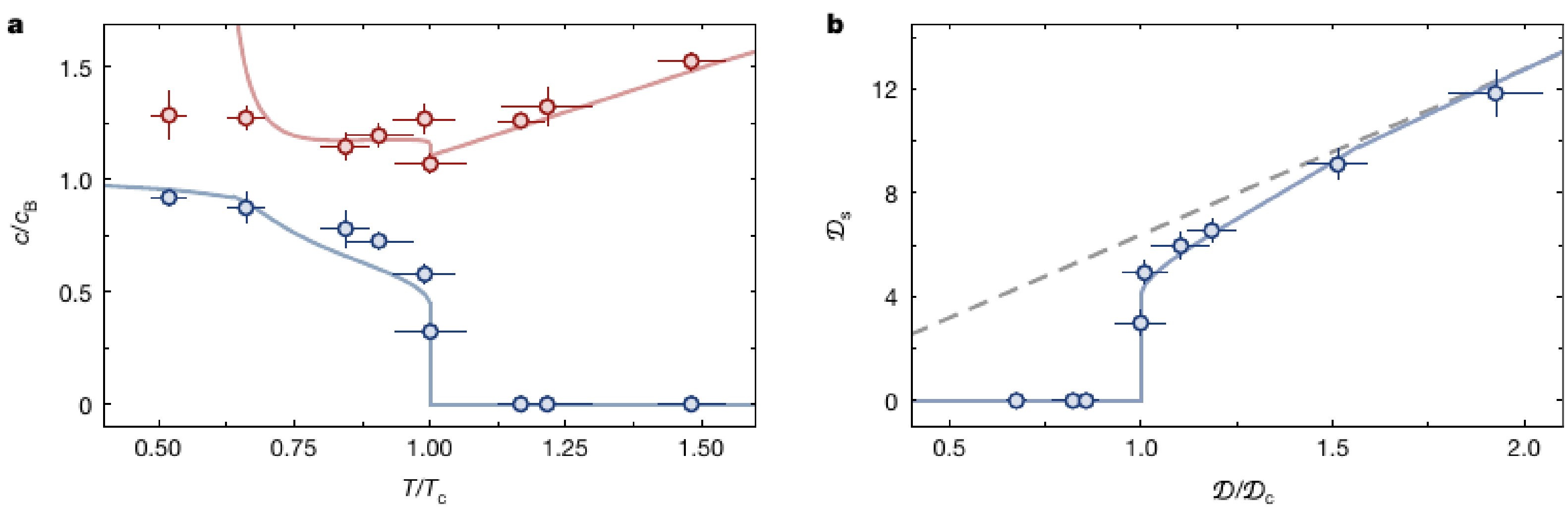

Fig. 3 | The sound speeds and the superfluid density. a, Normalized sound speeds, c_1/c_B (red) and c_2/c_B (blue), and the corresponding theoretical predictions without any free parameters. Owing to scale invariance in two dimensions, the predicted $c_{1,2}/c_B$ are functions of just T/T_c and $\tilde{g}$. Their discontinuities at T_c correspond to the infinite-system jump in superfluid density. **b**, The superfluid phase-space density, $\mathcal{D}_s$, deduced using the measured sound speeds. The solid line, showing the universal jump of $\mathcal{D}_s$ from 0 to 4 at $\mathcal{D} = \mathcal{D}_c$, is the infinite-system theoretical prediction without any free parameters. The dashed line corresponds to a 100% superfluid ($\mathcal{D}_s = \mathcal{D}$). All error bars show standard statistical errors.

Conclusion

- The planar Bose gas provides us with a good experimental platform to benchmark the FRG.
- The experimental dertermination of the equation of state is in very good agreement with both the FRG and Monte Carlo simulations of a classical field theory.
- The recent measurement of Tan’s contact is very well fitted by the FRG (but not by the classical field theory).
- Does FRG also agree with the recent measurement of the first- and second-sound velocities?

Thank you !