The long-range XY model and other examples of exotic BKT scaling

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- *G. Giachetti, ND, S. Ruffo and A. Trombettoni* (EPL 133, 57004, 2021, arXiv:2012.14896)
- *G. Giachetti, ND, S. Ruffo, and A. Trombettoni* (Phys. Rev. Lett. 127, 2021, arXiv:2104.13217)
- *G. Giachetti, A. Trombettoni, S. Ruffo, and ND (Phys. Rev. B 106, 014106, 2022)*

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Long Range $O(N)$ models

$$H = -J \sum_{\mathbf{i},\mathbf{r}} r^{-d-\sigma} \mathbf{s_i} \cdot \mathbf{s_{i+r}} \qquad \sigma > 0$$

Recursion Relations and Fixed Points for Ferromagnets with Long-Range Interactions*

J. Sak

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(Received 14 February 1973)

Sak's Criterion

$$\sigma < \sigma_* = 2 - \eta_{\mathrm{SR}}$$

$$= \; p^{-2+\eta} \qquad \text{vs} \qquad p^{-\sigma}$$

- *ND, A. Trombettoni, A. Codello, Phys. Rev. E 92, 052113 (2015)*
- *ND, A. Trombettoni, S. Ruffo, Phys. Rev. B 94, 224411 (2016)*
- *ND, A Trombettoni, S Ruffo, Phys. Rev. B 96, 104432 (2017)*

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Why $d = 2$ XY model?

$$H = -\sum_{\mathbf{i},\mathbf{r}} \frac{J}{r^{2+\sigma}} \cos\left(\theta_{\mathbf{i}} - \theta_{\mathbf{i}+\mathbf{r}}\right)$$

- In the SR: a line of fixed points
- Numerical results: hard to get
- Analytical mappings fail beyond n-n couplings (e.g. Villain, Sine-Gordon representations)

Short-range BKT transition

Nearest-neighbors case:

$$H = -J \sum_{<\mathbf{i},\mathbf{j}>} \cos\left(\theta_{\mathbf{i}} - \theta_{\mathbf{j}}\right)$$

Low-temperature approximation

$$\cos\left(\theta_{\mathbf{i}} - \theta_{\mathbf{j}}\right) \approx \frac{1}{2}|\nabla\theta|^2$$

Gaussian power-law correlations

$$\langle \mathbf{s}_0 \cdot \mathbf{s}_{\mathbf{r}} \rangle_0 = \langle \cos(\theta_0 - \theta_{\mathbf{r}}) \rangle_0 \sim r^{-\frac{1}{2\pi J}}$$

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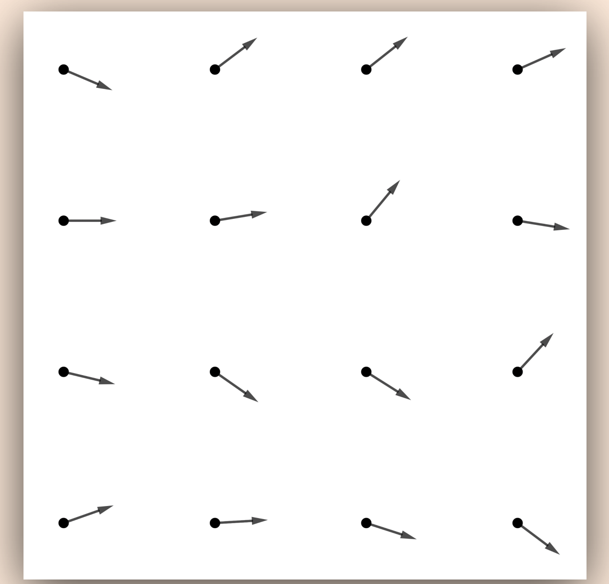
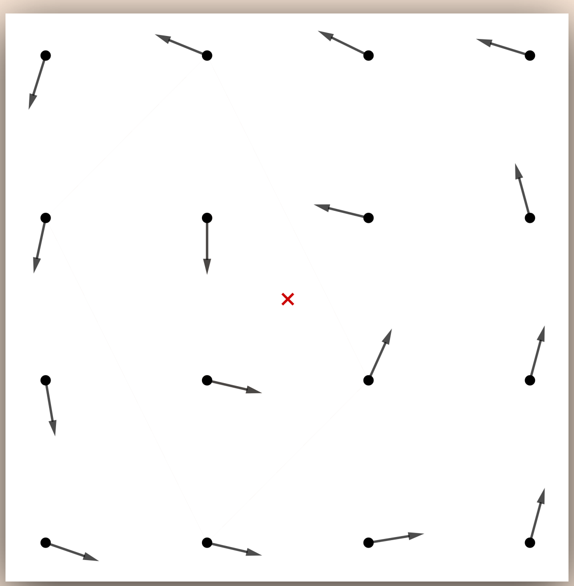
- *M. Gräter and C. Wetterich, Phys. Rev. Lett. 75, 378 (1995).*
- *P. Jakubczyk, N. Dupuis, B. Delamotte Phys. Rev. E 90, 062105 (2014)*
- *ND, A. Trombettoni, I. Nándori, T Enss, Phys. Rev. B 96, 174505 (2017)*

What about topological configurations?





Coulomb gas representation:

$$H = \pi J \sum_{i \neq j} m_i m_j \ln |\mathbf{r}_i - \mathbf{r}_j| + \epsilon_c \sum_i m_i^2$$

$$y = e^{-\epsilon_c}$$

Vortex fugacity

BKT flow equations

$$\dot{J} = -4\pi^3 J^2 y^2$$
$$\dot{y} = (2 - \pi J)\, y$$

Gaussian FPs line

Vortices relevant for

$$J < J_{BKT} = \frac{2}{\pi}$$

Low-temperature phase

$$\langle \mathbf{s}_0 \cdot \mathbf{s}_\mathbf{r} \rangle \sim r^{-\eta_{sr}} \qquad \eta_{sr} < \frac{1}{4}$$

Field-theoretical approach: long-range

Long-range perturbation:

$$H = H_{\rm SR} - \sum_{\mathbf{i},\mathbf{r}} \frac{g}{r^{2+\sigma}} \cos\left(\theta_{\mathbf{i}} - \theta_{\mathbf{i}+\mathbf{r}}\right)$$

Tricky continuous limit

Non-local

$$S_g = -g \int d^2\mathbf{x} \;\left(\cos\theta \nabla^\sigma \cos\theta + \sin\theta \nabla^\sigma \sin\theta\right)$$

Interacting

Dimension of the perturbation

$$\langle \cos\theta \nabla^\sigma \cos\theta \rangle \sim L^{-\sigma} \langle \cos\theta \cos\theta \rangle \sim L^{-(\sigma + \eta_{sr}(J))}$$

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Gaussian FPs line

$\eta_{sr} > \frac{1}{4}$ → Destroyed by vortices

$\eta_{sr} < 2 - \sigma$ → Destroyed by long-range

BKT transition survives

$$\frac{7}{4} < \sigma < 2$$

First Order Phase Transition in the Plane Rotator Ferromagnetic Model in Two Dimensions

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Abstract. We show that the two-dimensional isotropic ferromagnetic rotator model exhibits a first order phase transition if the interaction decays as $r^{-\alpha}$ with $2<\alpha<4$.

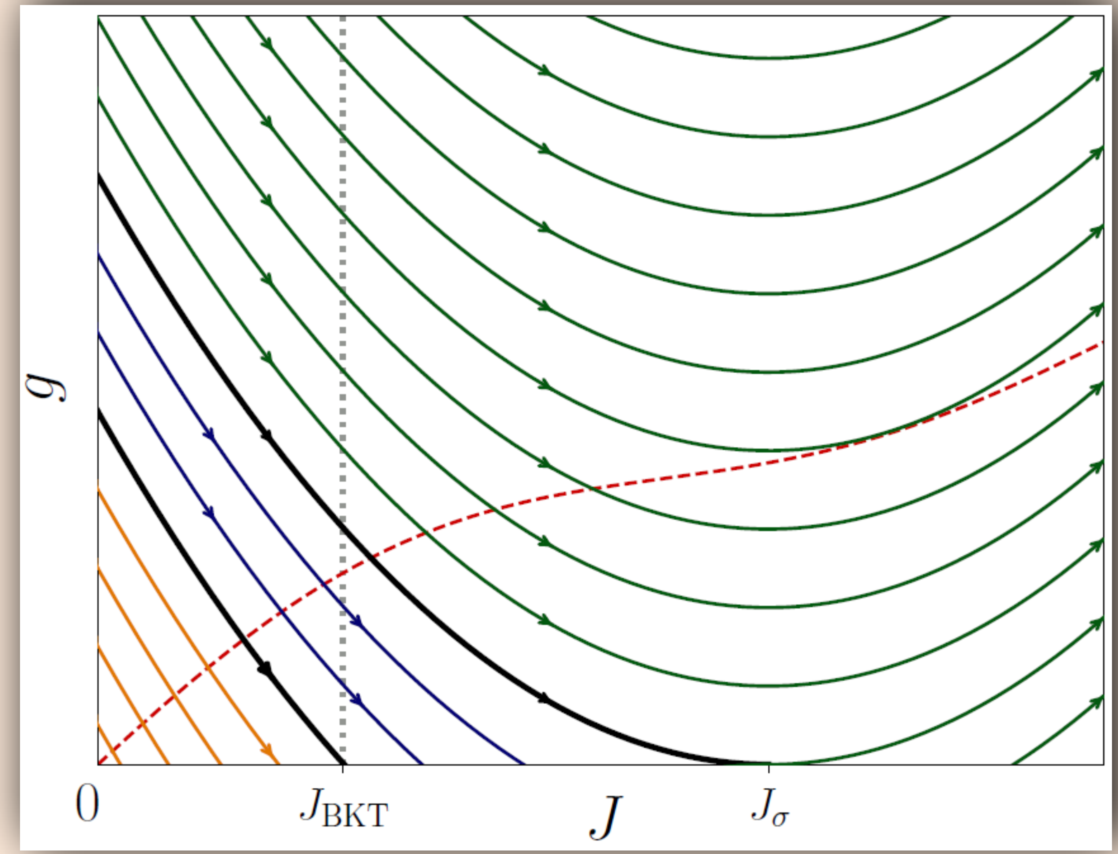
$$\dot{g} = (2 - \sigma - \eta_{sr}(J))\, g$$

$$\dot{J} = \eta_{sr}(J) g$$

$$\dot{y} = (2 - \pi J) y$$

Long-range

Vortices



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Low-Temperature phase

For low temperatures:

$$g \to \infty \qquad \eta_{sr} \to 0$$

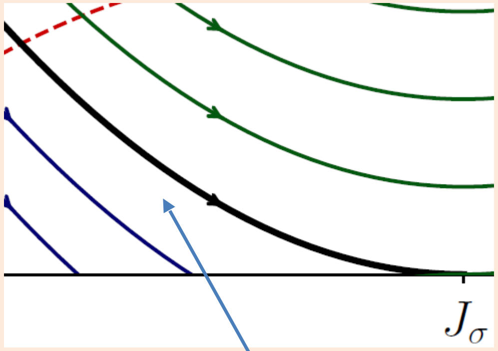
Effective Gaussian theory

$$\boxed{S_g \sim -g \int d^2\mathbf{x}\, \theta\, \nabla^\sigma \theta}$$

$$g \sim e^{\ell(2-\sigma)} = a^{2-\sigma} \qquad m = \langle \cos\theta \rangle \sim e^{-a^{2-\sigma}/g}$$

Universal scaling

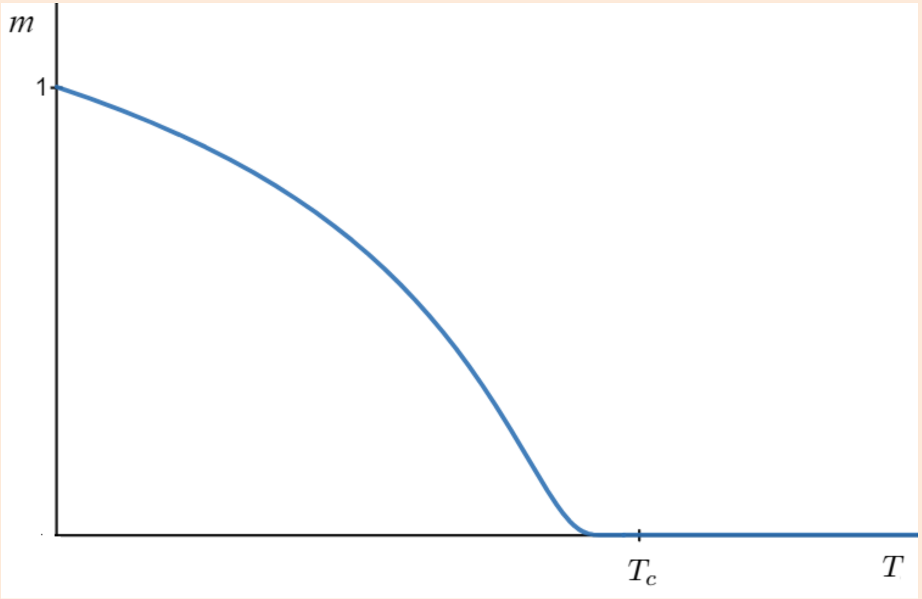
$$g \sim e^{-(T_c - T)^{-1/2}} a^{2-\sigma}$$

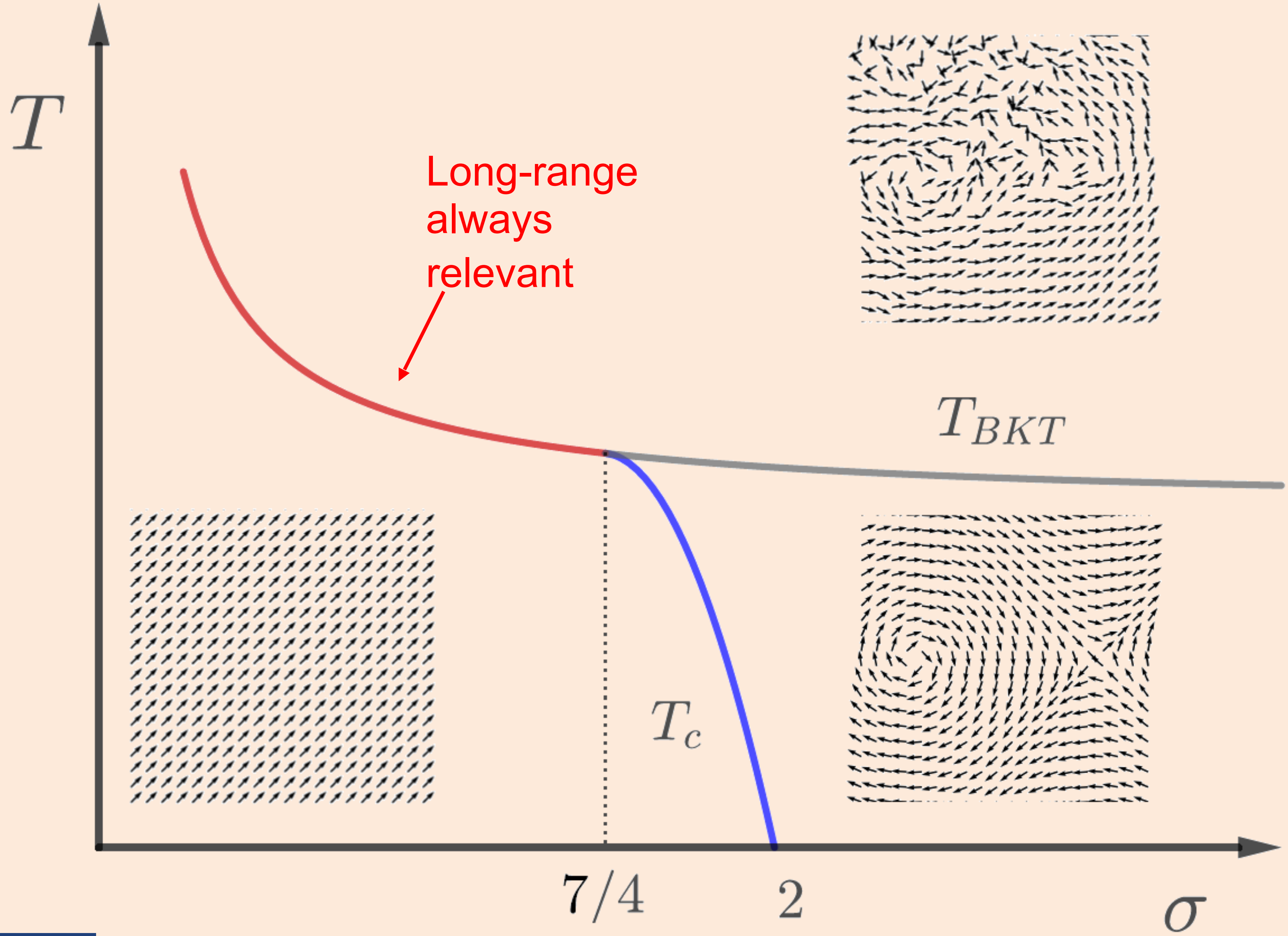


Slowing down

Infinite order phase transition

$$\ln m \sim -e^{(T_c - T)^{-1/2}}$$




T
Long-range
always
relevant
T_{BKT}
T_c
$7/4$
2
σ

Villain model

Long-range version of the Villain model

Discrete link variables

$$H = J \sum_{\mathbf{i},\mathbf{r}} r^{-2-\sigma} \left(\theta_{\mathbf{i}+\mathbf{r}} - \theta_{\mathbf{i}} - 2\pi n_{\mathbf{i},\mathbf{r}}\right)^2$$

Continuous

Always a relevant perturbation

$$S_{sr} \sim \int d^2x \; \theta \nabla^2 \theta \qquad S_g \sim \int d^2x \; \theta \nabla^\sigma \theta$$

Not in the same universality class of *XY*!

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Vortices gas Hamiltonian

$$H = \pi J \sum_{i \neq j} m_i m_j U(|\mathbf{r}_i - \mathbf{r}_j|) + \epsilon_c \sum_i m_i^2$$

Vortex-vortex potential

$$U(r) \sim r^{2-\sigma} - 1$$

$1 < \sigma < 2$

$\sigma < 1$

$$U(r) \sim \ln r$$

Always irrelevant

Puzzling results!

Conclusions

- SCHA: suggests BKT survives
- QFT: coexistence of BKT and SSB for $\frac{7}{4} < \sigma < 2$
- Infinite order phase transition
- What about $\sigma < \frac{7}{4}$?

Non-Hermitian many-body phases of matter

Adiabatic Elimination

$$\Gamma \gg \delta, \Omega$$

$$\hat{\mathscr{H}}_{\text{eff}} = \int dx \hat{\Psi}^\dagger(x) \left(-\frac{\hbar^2 \nabla^2}{2m} + V(x) \right) \hat{\Psi}(x) + \frac{g}{2} \int dx \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \hat{\Psi}(x)$$

Cold atoms in imaginary potential

$$V(x) = V_{\text{r}} \cos\left(\frac{2\pi x}{d}\right) - iV_{\text{i}} \sin\left(\frac{2\pi x}{d}\right)$$

$$V_{\text{i}} = \frac{|d|^2 \mathscr{E}_0^2}{\hbar \Gamma}$$

Tomonaga Luttinger Liquid Theory

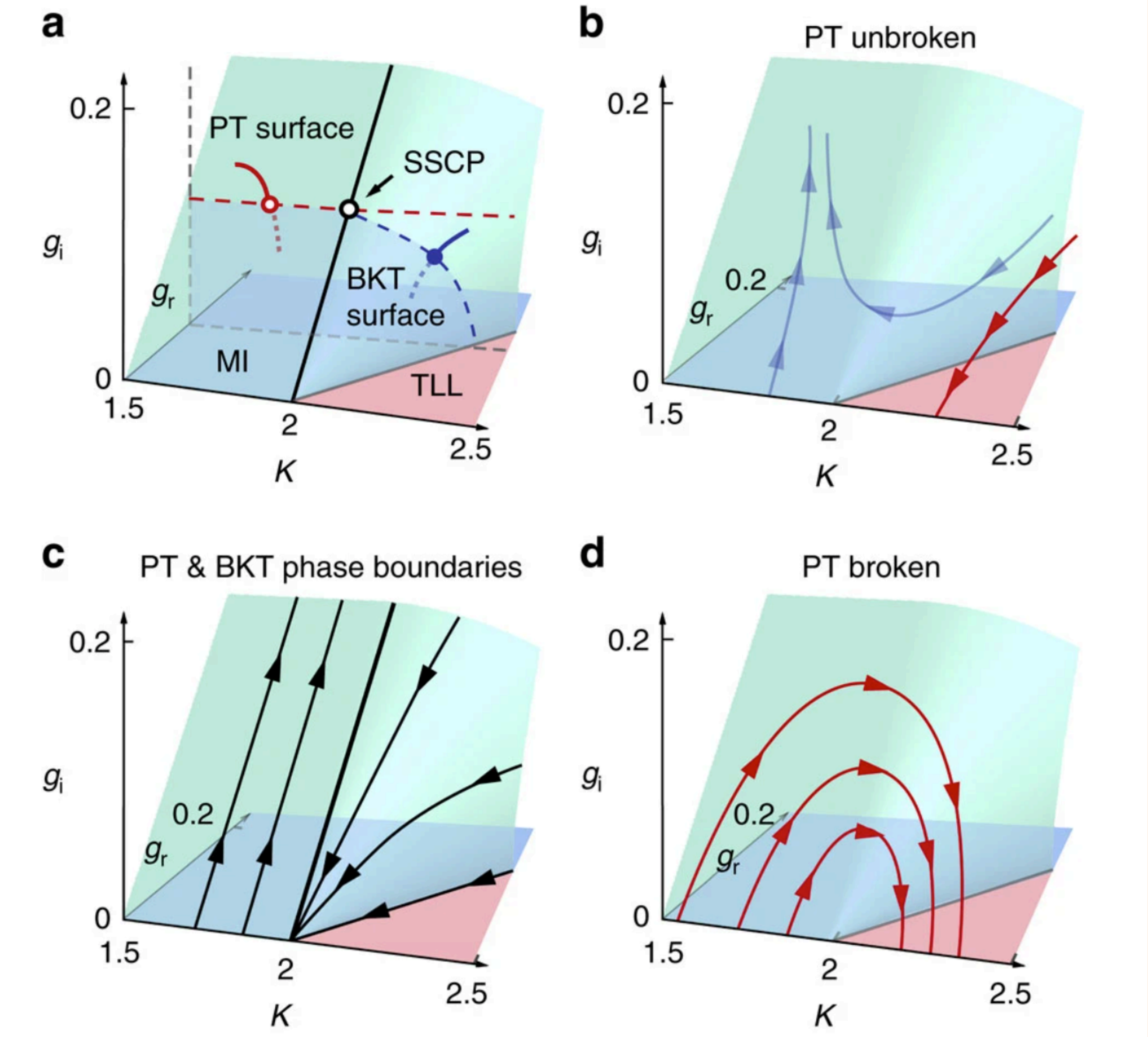
$$\hat{H}_{\mathrm{TLL}} = \int dx \frac{\hbar v}{2\pi} \left[K \left(\partial_x \hat{\theta} \right)^2 + \frac{1}{K} \left(\partial_x \hat{\phi} \right)^2 \right]$$

Bosonic field: $\hat{\Psi}^\dagger(x) = \sqrt{\hat{\rho}(x)}\, e^{-i\hat{\theta}(x)}$

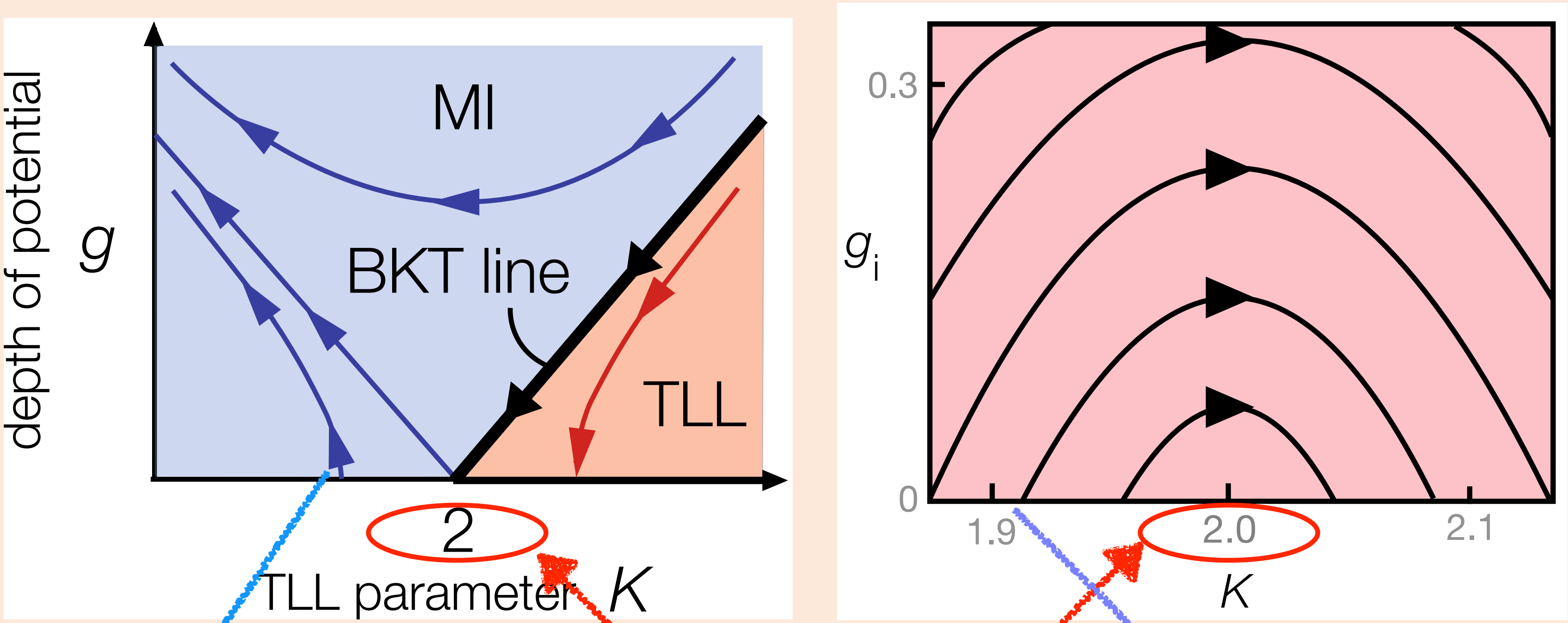
$$\hat{H} = \int \mathrm{d}x \left\{ \frac{\hbar v}{2\pi} \left[K \left(\partial_x \hat{\theta} \right)^2 + \frac{1}{K} \left(\partial_x \hat{\phi} \right)^2 \right] + \frac{g_{\mathrm{r}}}{\pi} \cos(2\hat{\phi}) - \frac{i g_{\mathrm{i}}}{\pi} \sin(2\hat{\phi}) \right\}$$

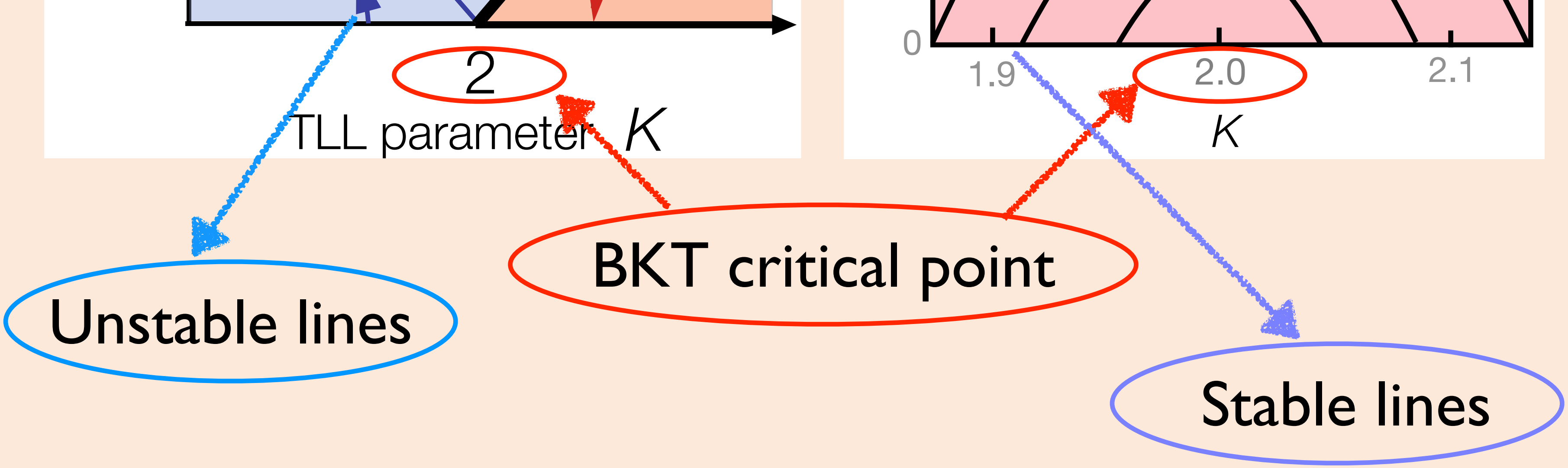
Y. Ashida, S. Furukawa & M. Ueda Nat. Comm. 8, 15791 (2017)

PT-Symmetric sine-Gordon model

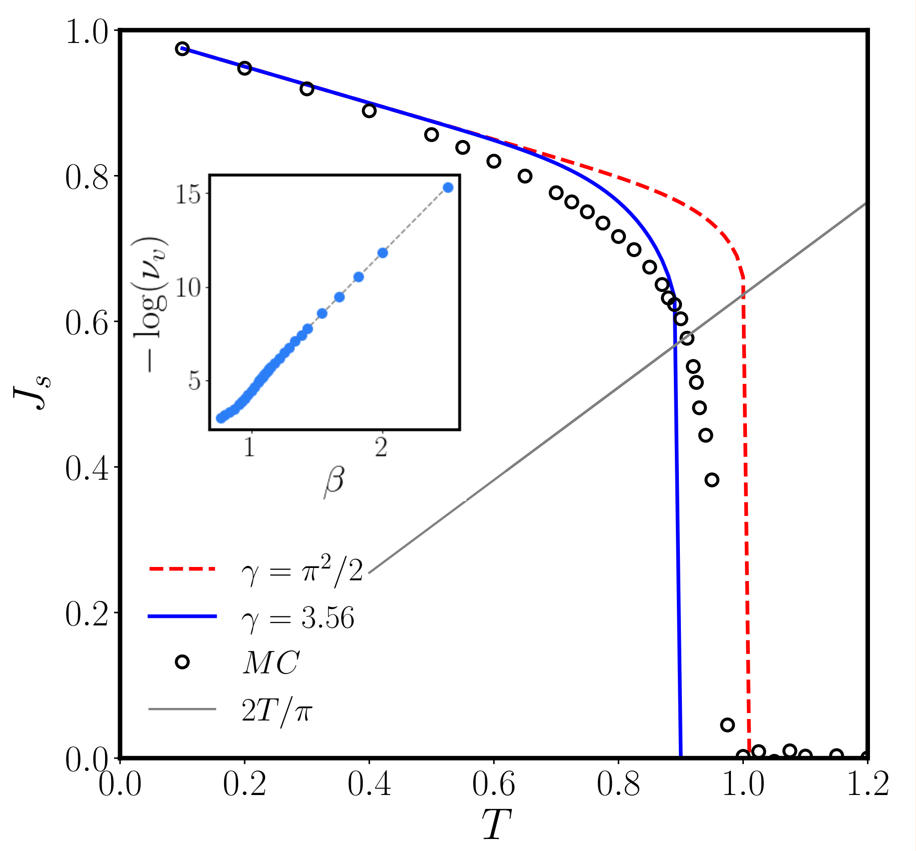


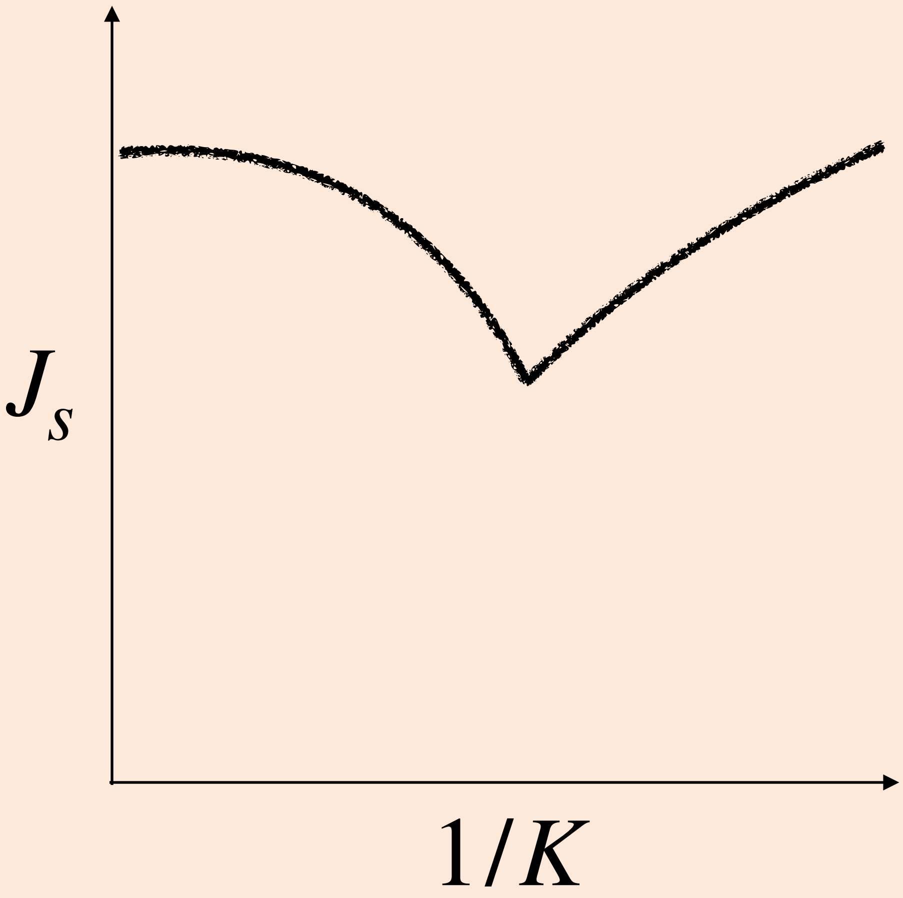
BKT Flows compared





Superfluid Stiffness compared





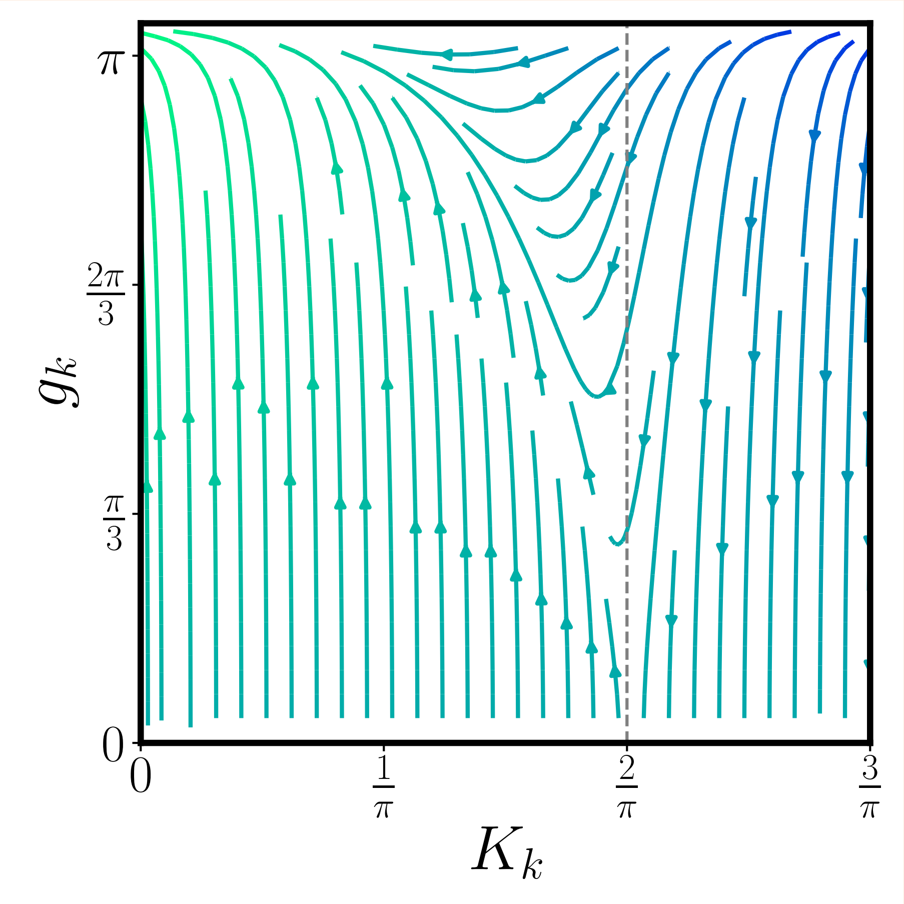
[1] I. Maccari, N. Defenu, L. Benfatto, C. Castellani, and T. Enss, Phys. Rev. B 102, 104505 (2020)

FRG description of the sine-Gordon model

$$(2+k\partial_k)\tilde{u}_k = \frac{a}{2\pi z_k \tilde{u}_k}\left[a-\sqrt{a^2-\tilde{u}_k^2}\right]$$

$$R_k = ak^2$$

$$k\partial_k z_k = -\frac{a}{24\pi}\frac{\tilde{u}_k^2}{[a^2-\tilde{u}_k^2]^{\frac{3}{2}}}$$



BKT at a d=4 Lifshitz point?

ND, A. Trombettoni, D. Zappalà

arXiv:2003.04909

non-Hermitian RG vs FRG

$$g_k = \frac{u_k}{\pi} \qquad K_k = \frac{1}{4\pi^2 z_k}$$

$$\frac{\mathrm{d}K}{\mathrm{d}l} = -\left(g_\mathrm{r}^2 - g_\mathrm{i}^2\right) K^2,$$

$$\frac{\mathrm{d}g_\mathrm{r}}{\mathrm{d}l} = (2-K)g_\mathrm{r} + 5g_\mathrm{r}^3 - 5g_\mathrm{i}^2 g_\mathrm{r}$$

$$\frac{\mathrm{d}g_\mathrm{i}}{\mathrm{d}l} = (2-K)g_\mathrm{i} - 5g_\mathrm{i}^3 + 5g_\mathrm{r}^2 g_\mathrm{i}\,.$$

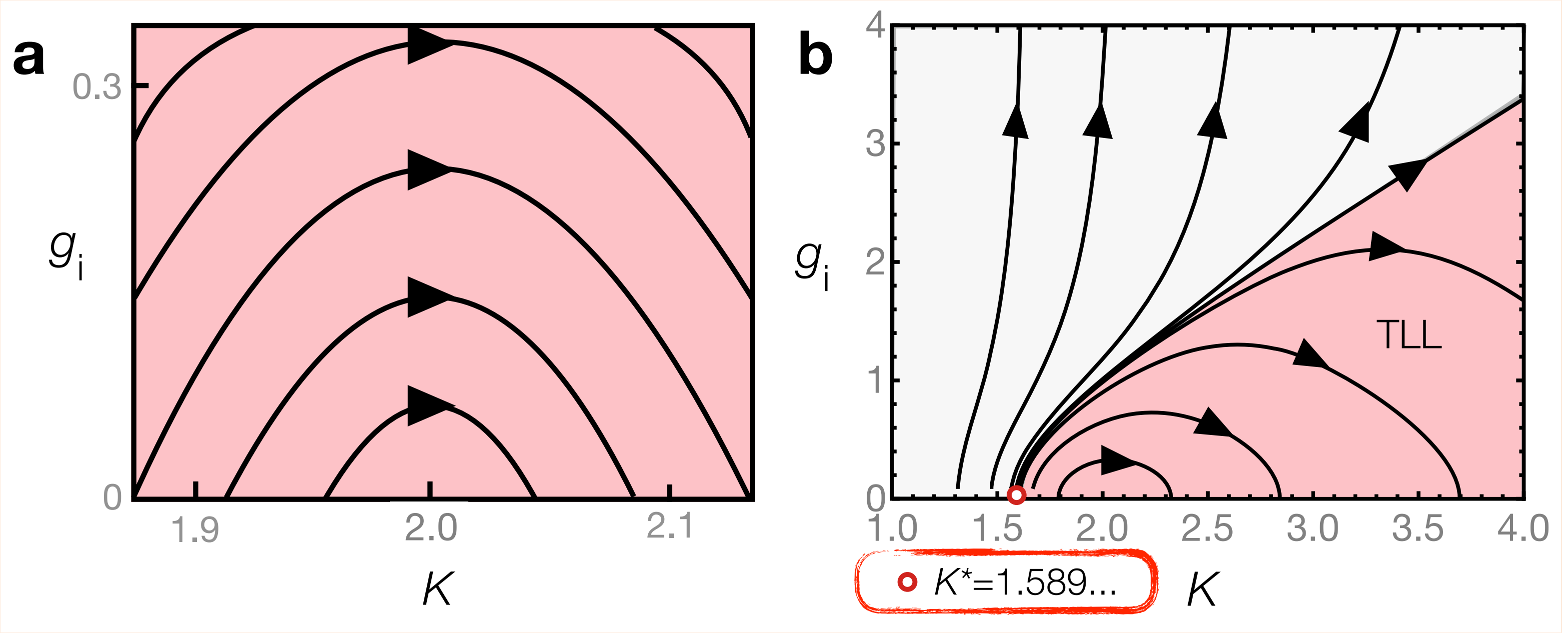
$$\left(2 + k\partial_k\right)\bar{u}_{1,k} = \frac{1}{2\pi z_k}\frac{\bar{u}_{1,k}}{\left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)}\left[1 - \sqrt{1 - \left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)}\right]$$

$$\left(2 + k\partial_k\right)\bar{u}_{2,k} = \frac{1}{2\pi z_k}\frac{\bar{u}_{2,k}}{\left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)}\left[1 - \sqrt{1 - \left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)}\right]$$

$$k\partial_k z_k = -\frac{1}{24\pi}\frac{\left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)}{\left(1 - \left(\bar{u}_{1,k}^2 - \bar{u}_{2,k}^2\right)\right)^{3/2}}$$

[1] I. Maccari, N. Defenu, L. Benfatto, C. Castellani, and T. Enss, Phys. Rev. B 102, 104505 (2020)

non-Hermitian RG vs FRG

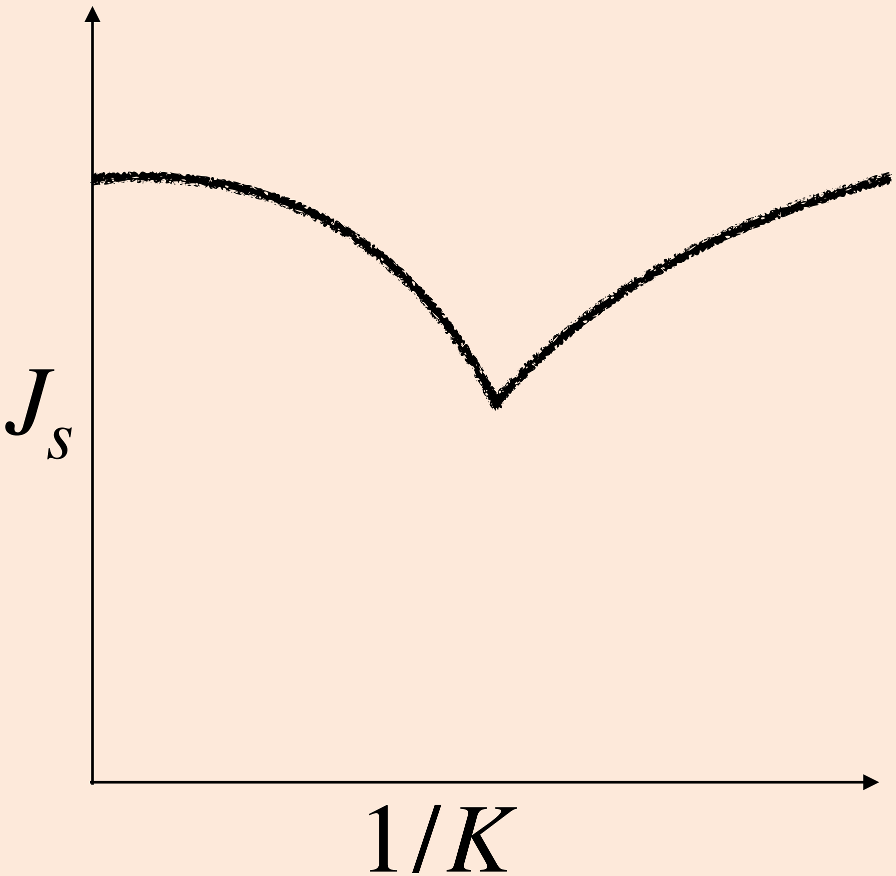


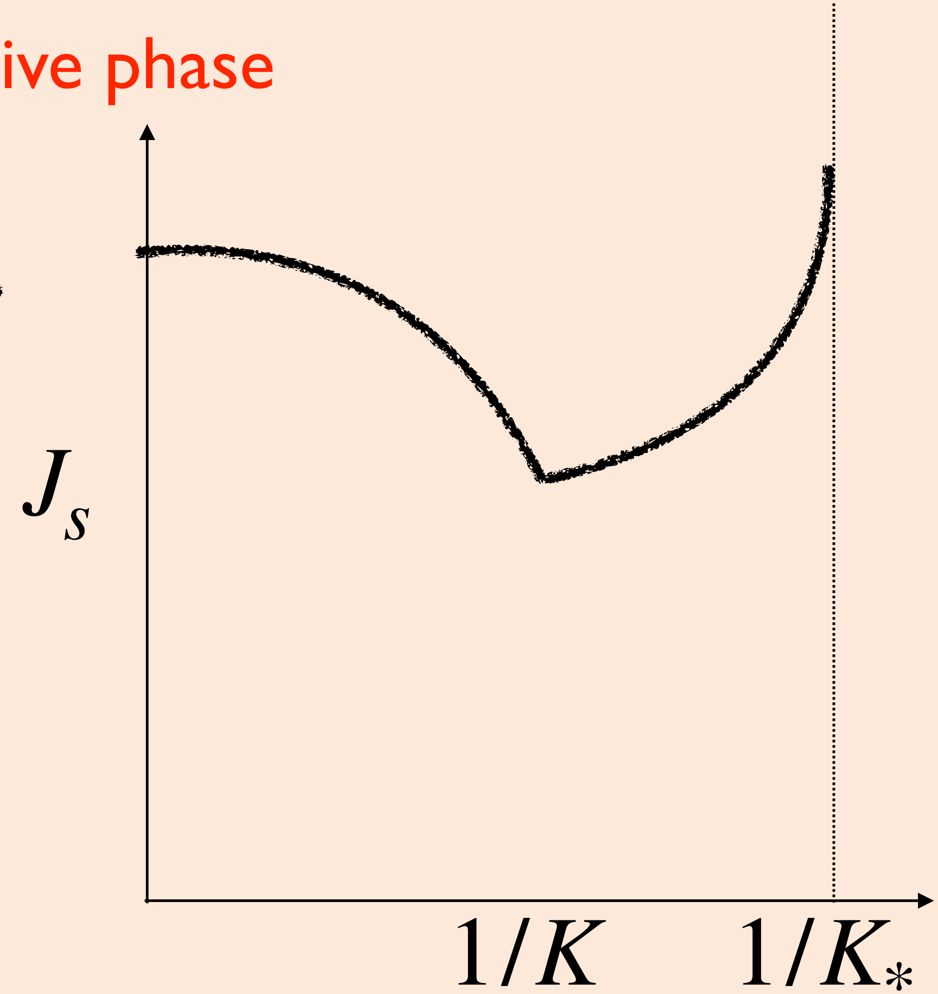
Breakdown of the TTL phase also in the PT broken regime

New massive phase

non-Hermitian RG vs FRG

New massive phase





Breakdown of the TTL phase also in the PT broken regime

Thank you for your attention!

I acknowledge Guido Giachetti for his support in the preparation of the slides

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