Functional RG for zero- and one-dimensional Fermi systems

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goal: probe the limits of functional RG for many-particle Fermi systems

here: vertex expansion and U being the small parameter

physics challenges:

- phase transitions at finite U?
- many-body effects in (topological) insulators?
- many-body effects in non-equilibrium?
- non-Hermitian systems?

technical challenges:

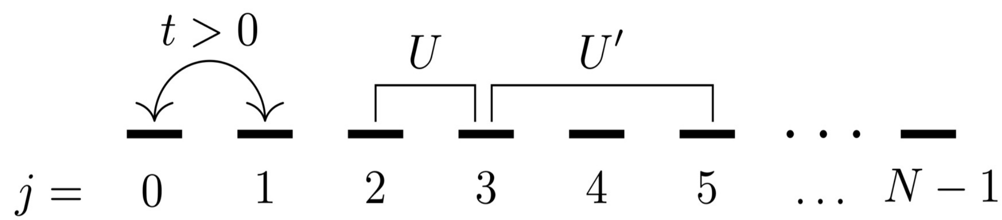
- feedback of two-particle vertex and self-energy?
- position/momentum and frequency dependence?
- number of equations and expensive computations?
- sharp structures for real frequencies (Keldysh)?
- violation of conservation laws due to truncation?
- consider non-Hermitian (gain/loss) systems?

why 0d and 1d:

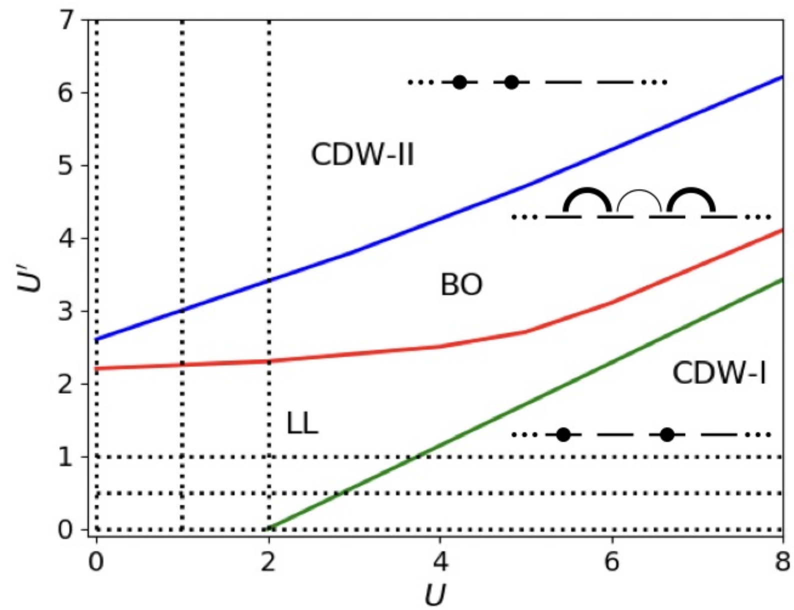
- large number of emerging many-body phenomena
- exact analytical and numerical results to compare
- less expensive numerics as in 2d and 3d

Phase transitions: the „classic“ application of functional RG for Fermi systems

the model:



the phase diagram at half-filling



t = 1

(adopted from Mishra et al. 2011)

note: mean-field gives transition for infinitesimal U, perturbation theory none

(for 1d extended Hubbard model see Tam, Tsai, Campbell 2006)

the approximations:

(real-space eCLA adopted from Weidinger et al. 2017)

1. neglect 3-particle vertex
2. keep one frequency in each vertex channel (p,x,d)
3. keep dynamical intra-channel feedback, static feedback of other channels, feedback of bare vertex
4. restrict feedback length *L* of vertex
5. keep full feedback of self-energy
6. sharp Matsubara frequency cutoff

vertex and self-energy at least to second order in U, U´

numerical challenges:

1. vertex has order $N_f * N^2 * L^2$ variables
2. convergence in N_f, L and N, is $L << N$?

how does this differ from common 2d studies?

full feedback of dynamical self-energy
order parameter can be computed from propagator

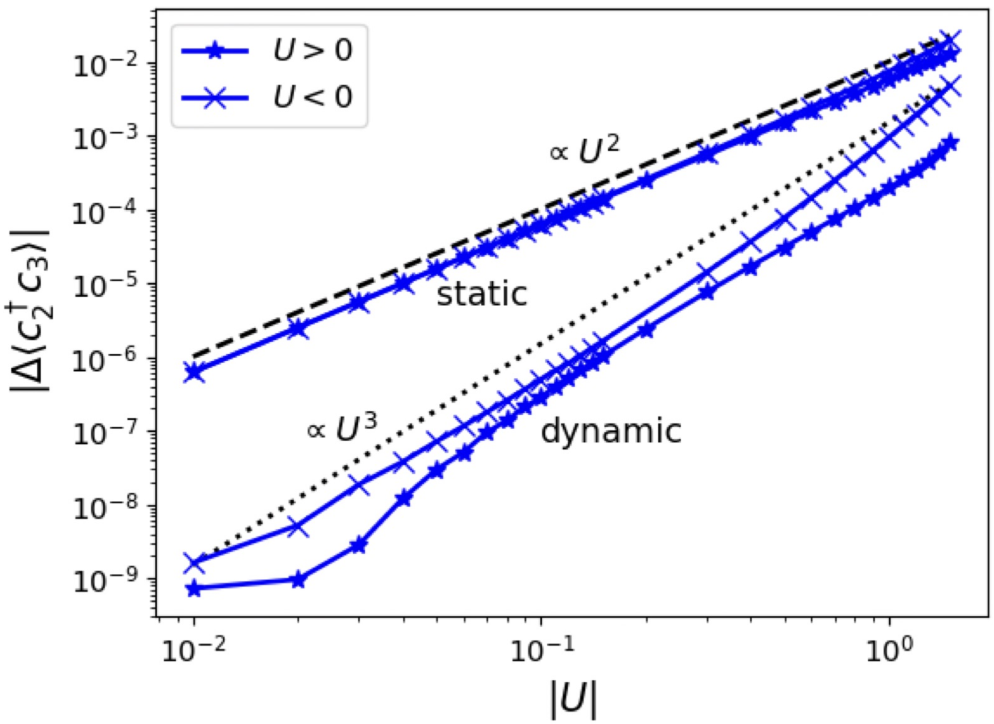
$$\langle c_i^\dagger c_j \rangle = \frac{1}{2\pi} \left[\int_{-\infty}^{\infty} d\omega \, \mathrm{e}^{\mathrm{i}\omega 0^+} \, \tilde{\mathcal{G}}^{\Lambda_\mathrm{f}}(\omega) \right]_{i,j}$$

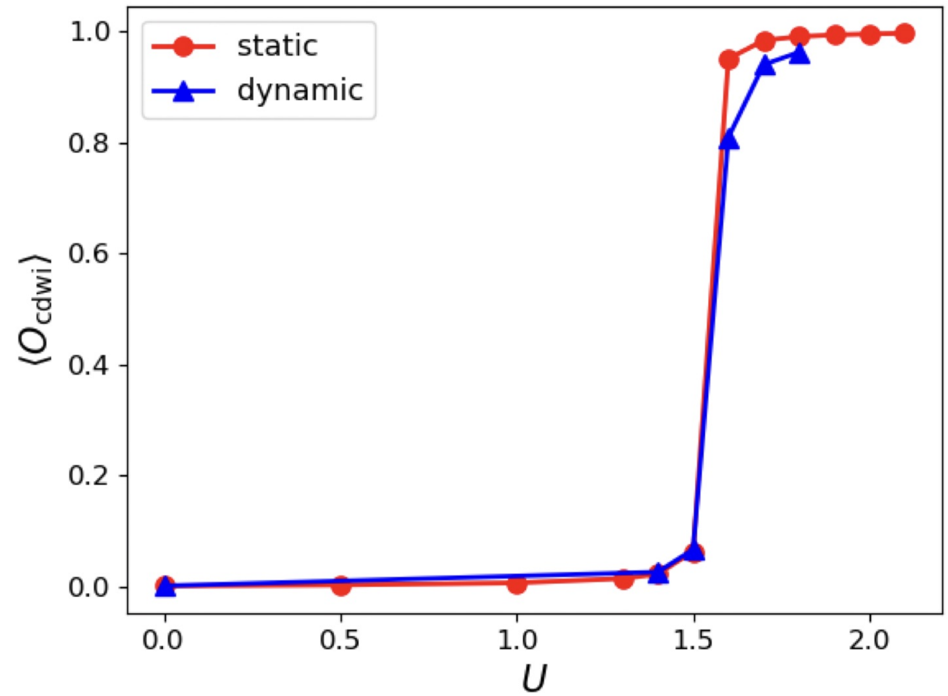
cutoff to zero if infinitesimal nudge (convergence, bias?)

$$\Sigma_{j,j}^{\Lambda_\mathrm{i}}(\omega_n) = (-1)^j \, S$$

Is the frequency dependence crucial (U'=0)?

$$\langle O_{\rm cdwi}\rangle = \frac{2}{N}\sum_{j=0}^{N-1}(-1)^{j+1}\langle n_j\rangle$$

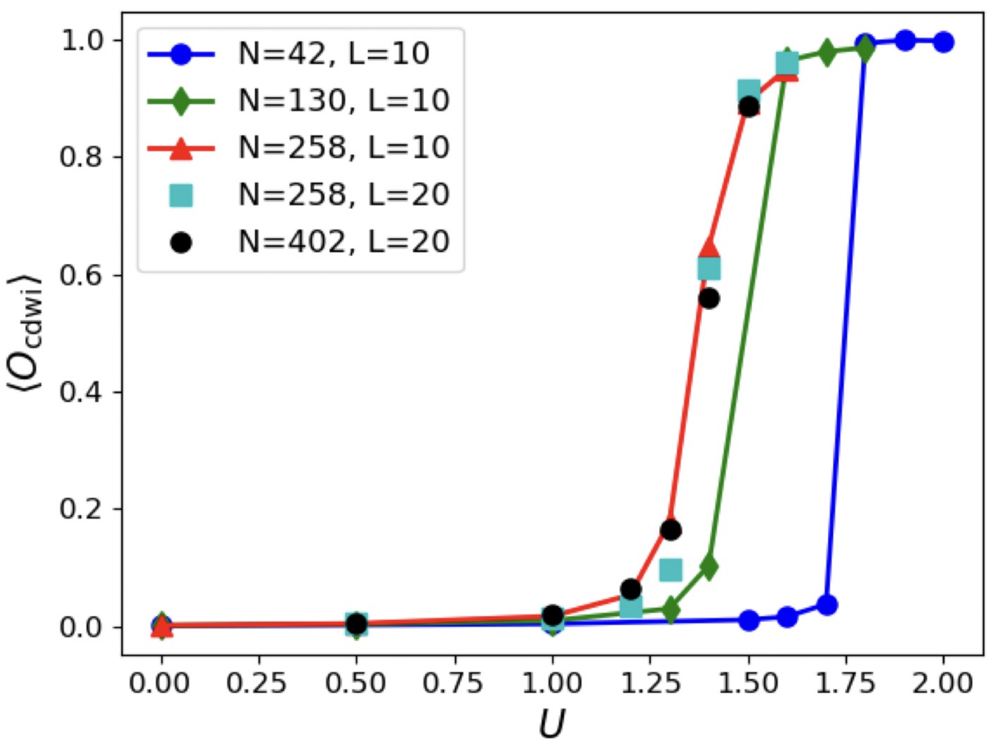




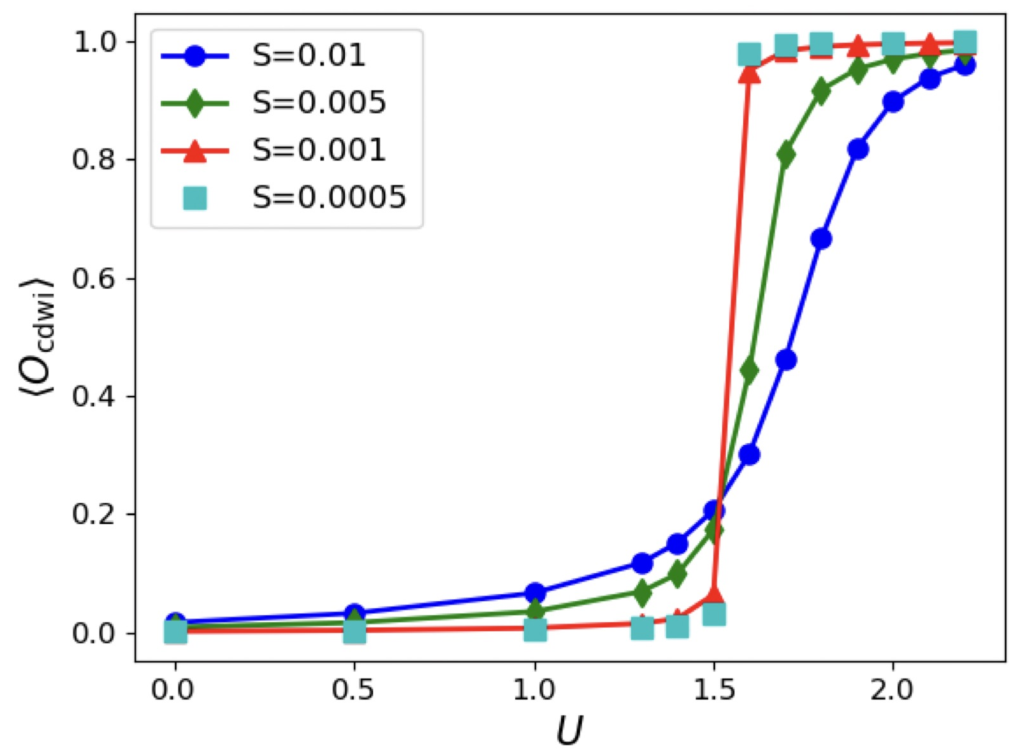
N=10, *L*=2

N=82, *L*=*10*, *S*=0.001

Convergence issues



S=0.001



N=82, L=10

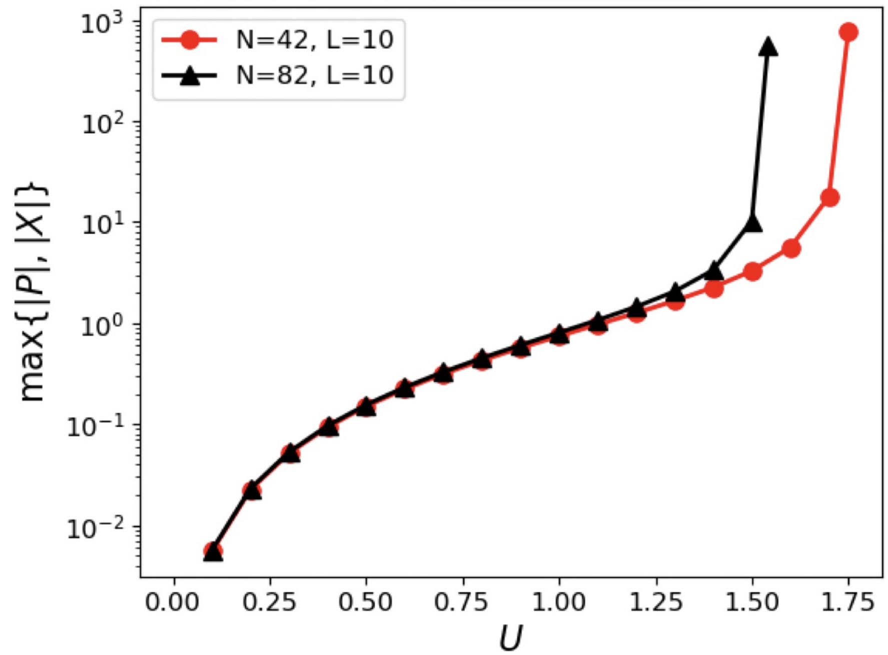
note: transition is Kosterlitz-Thouless

order parameter increase from 0 to 1 is much slower

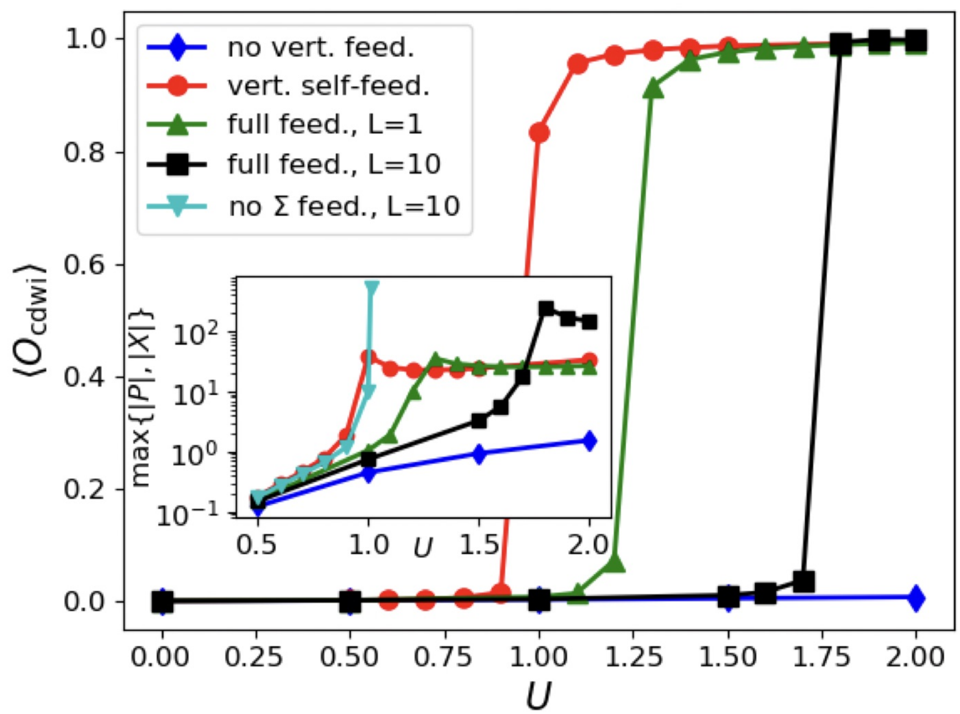
exact transition at U=2

Details

absence of initial nudge:
vertex diverges
also true for „wrong“ nudge

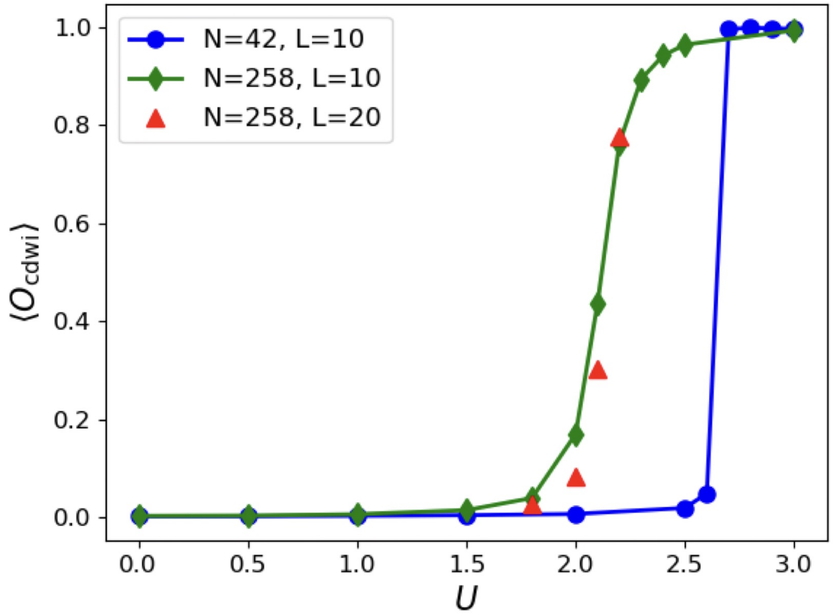


role of feedback

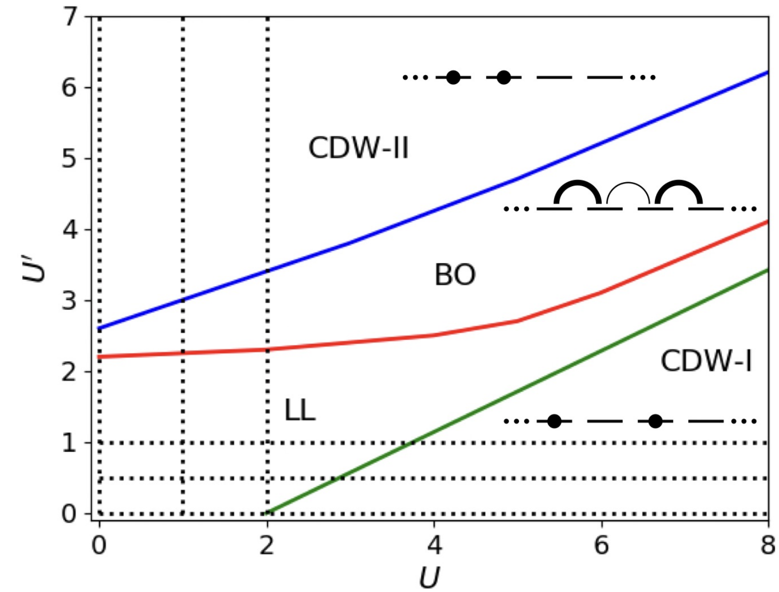


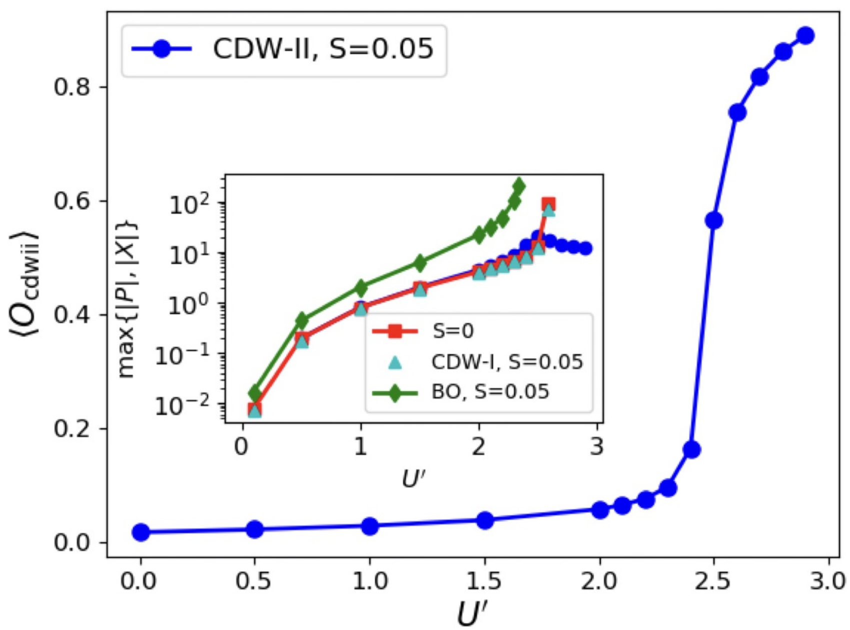
N=42, *S*=0.001

Other *U′* and the other phases?

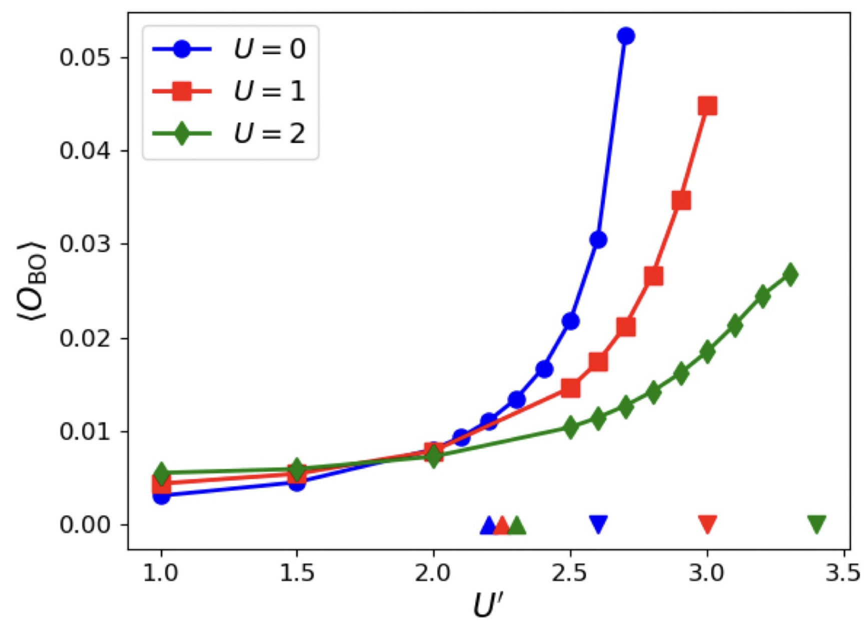


U'=0.5, S=0.001





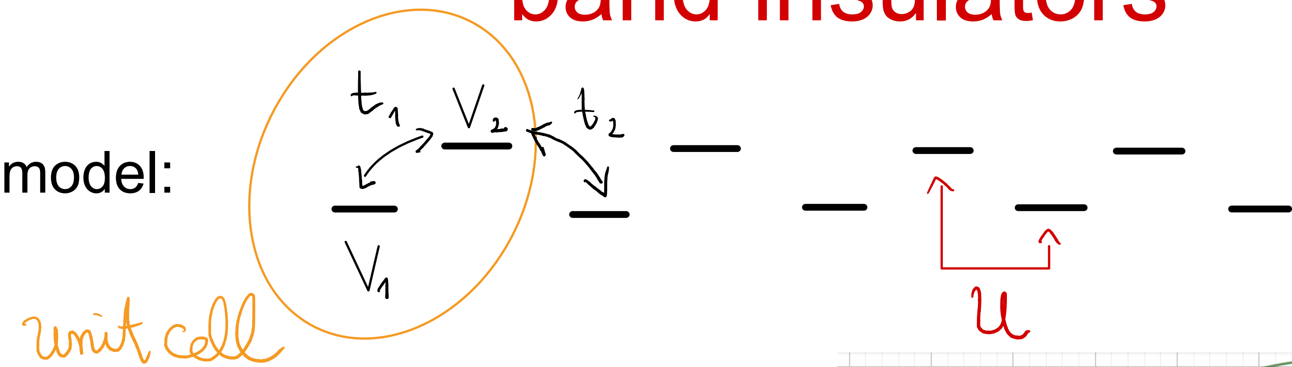
U=0, *N*=40, *L*=10



N=42, L=10, S=0.001

Functional RG for (topological) band insulators

the model:



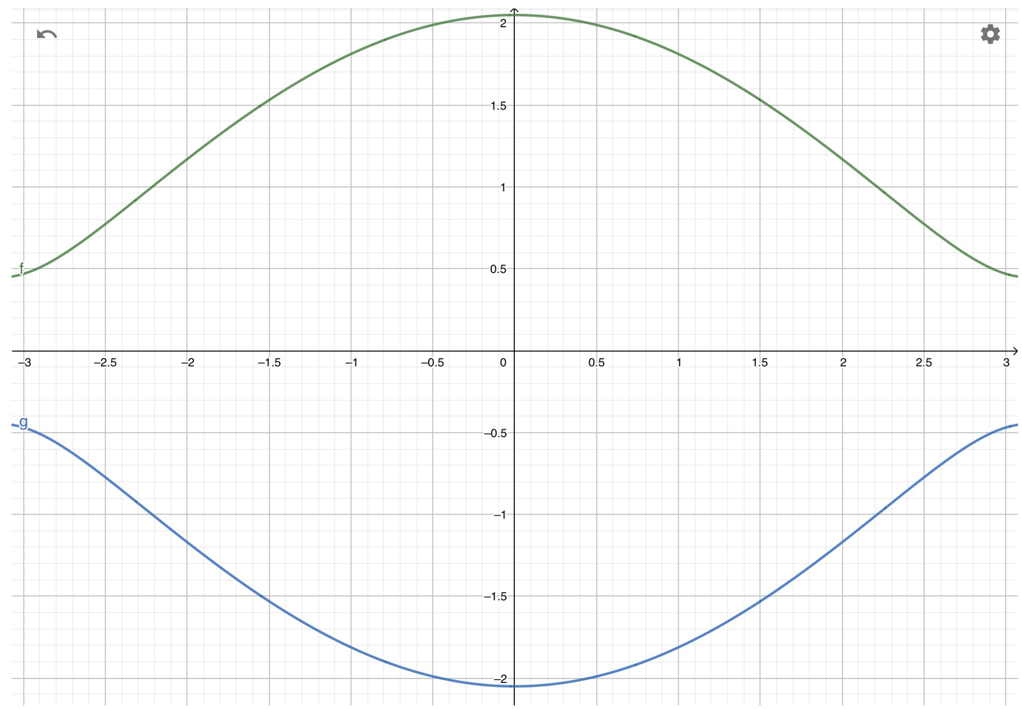
$$V_1 = -V_2 = V$$

$$t_{1/2} = t \pm \delta t$$

the single-particle dispersion:

$$2\Delta = 2\sqrt{V^2 + 4\delta t^2}$$

$$W = 2t$$



consider bulk (e.g. gap) and boundary („Friedel“ osc., edge states) physics

the approximations:

1. neglect the flow of the two-particle vertex
2. keep the self-energy feedback on its own flow
3. effective single-particle picture with renormalized parameters
4. if OBC: spatial structure beyond unit cell one
5. density from its own flow equation
6. sharp Matsubara frequency cutoff

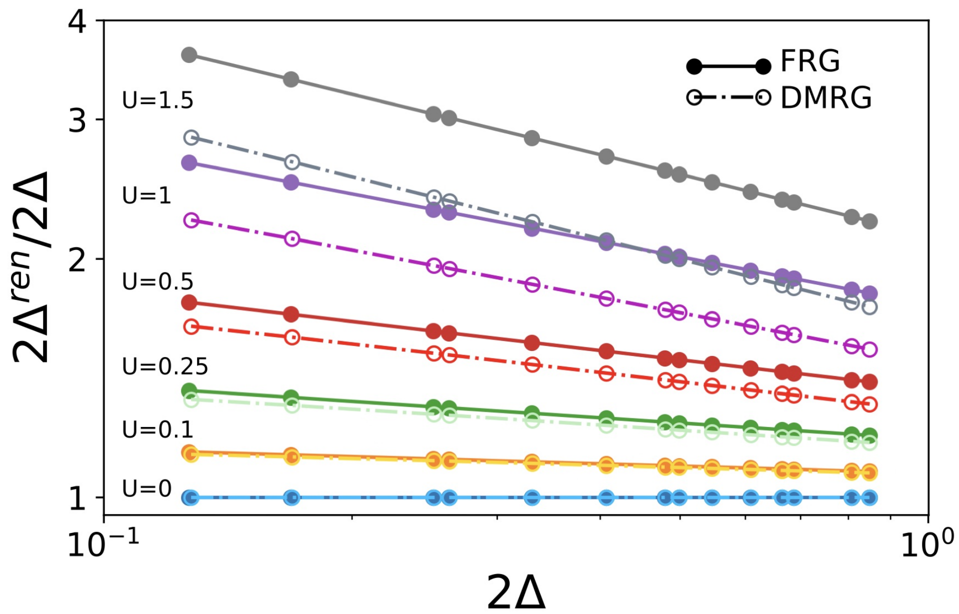
the numerical challenge:

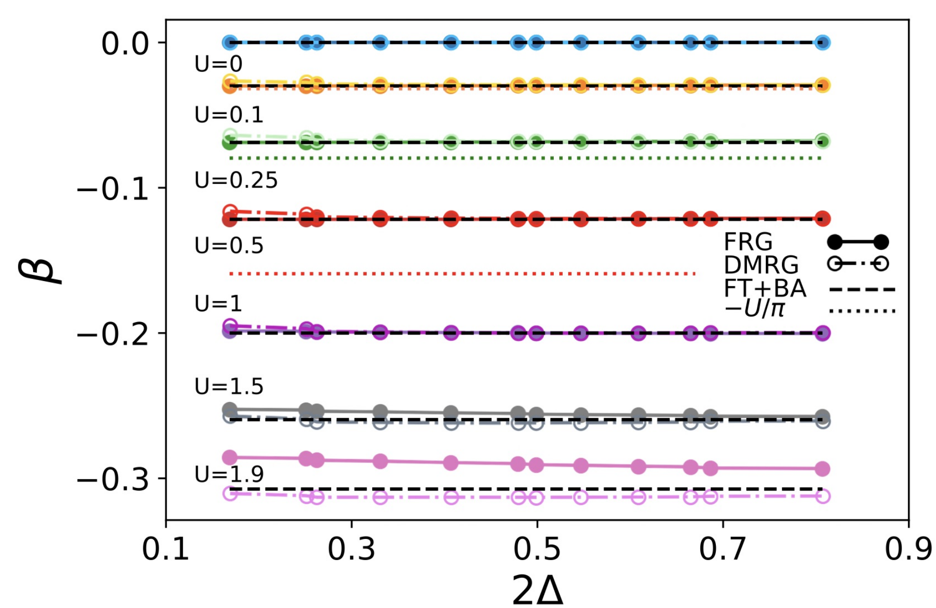
to access low-energy physics: as large systems as possible

The gap

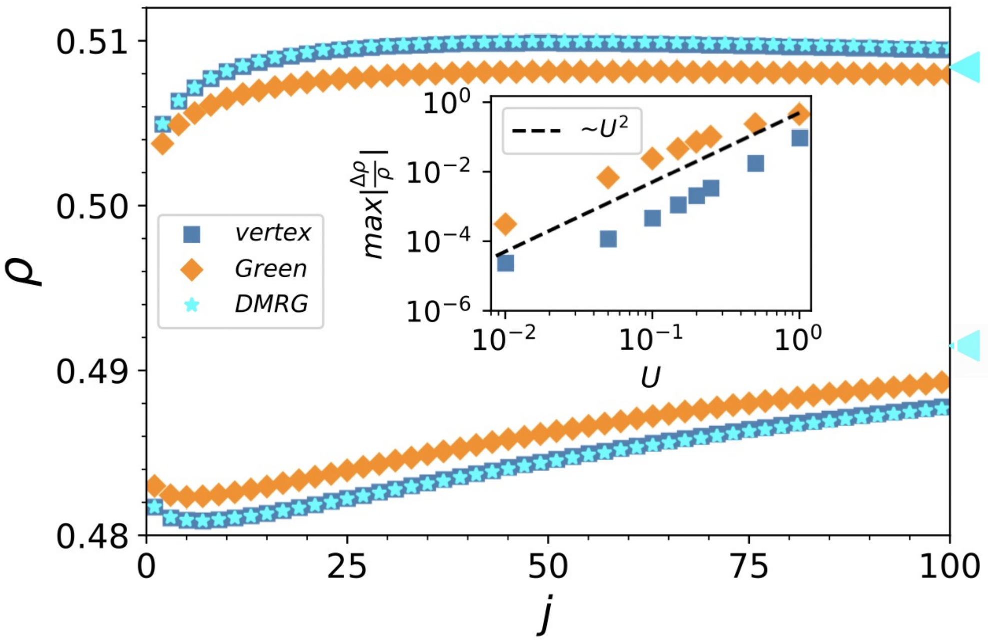
perturbation theory in *U*: $\Delta^{\mathrm{ren}} = \Delta\left(1 - \frac{U}{\pi}\ln\frac{\Delta}{W}\right)$

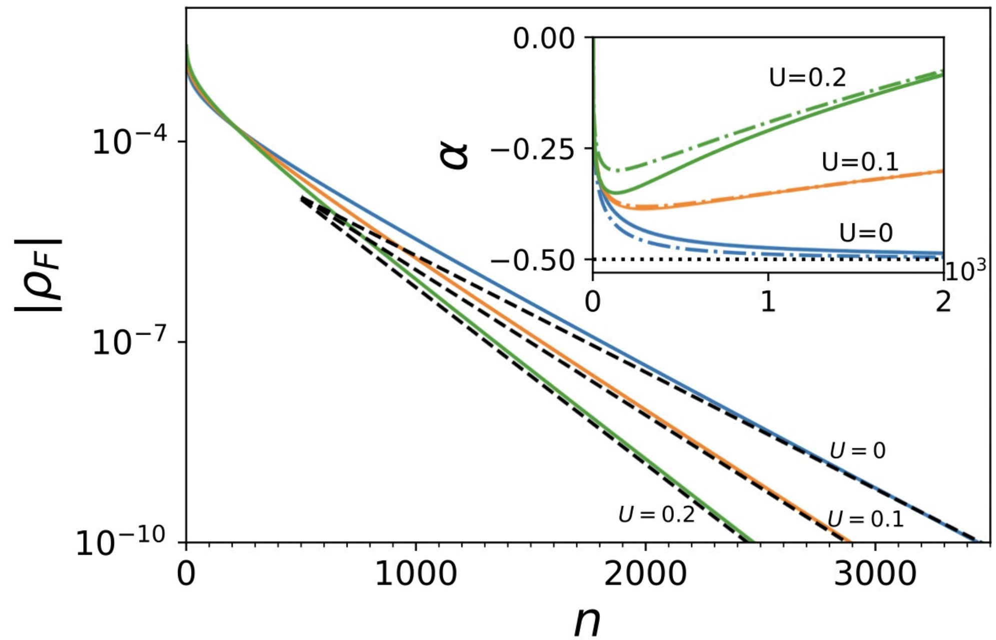
functional RG from effective parameters: $\frac{\Delta^{\mathrm{ren}}}{\Delta} = \left(\frac{\Delta}{W}\right)^{-U/\pi}$ for $\Delta \ll W$





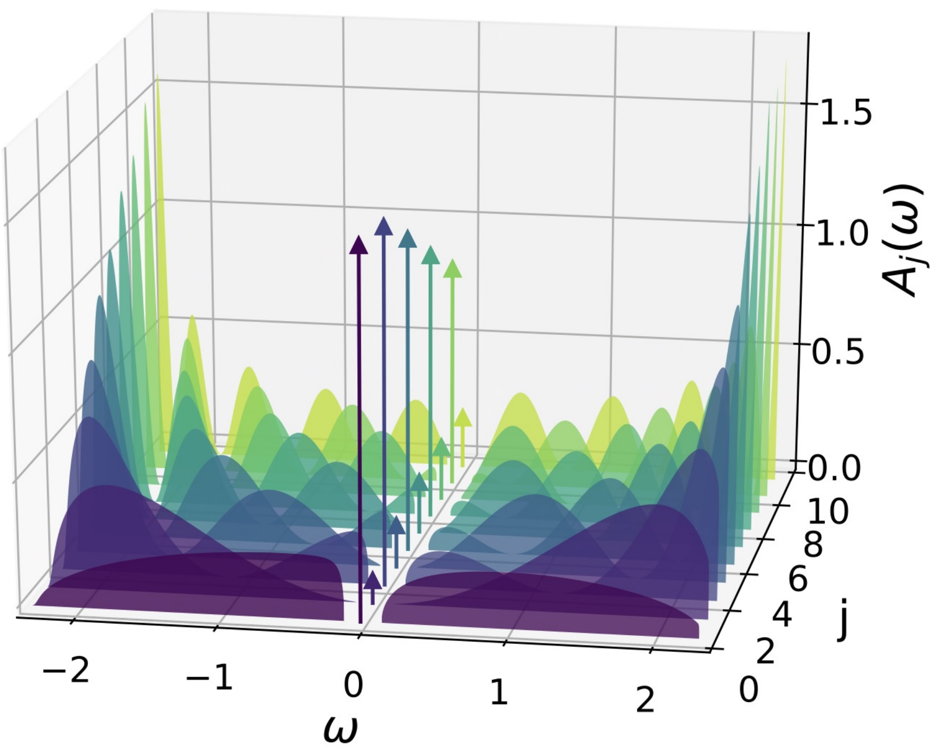
The „Friedel“ oscillations

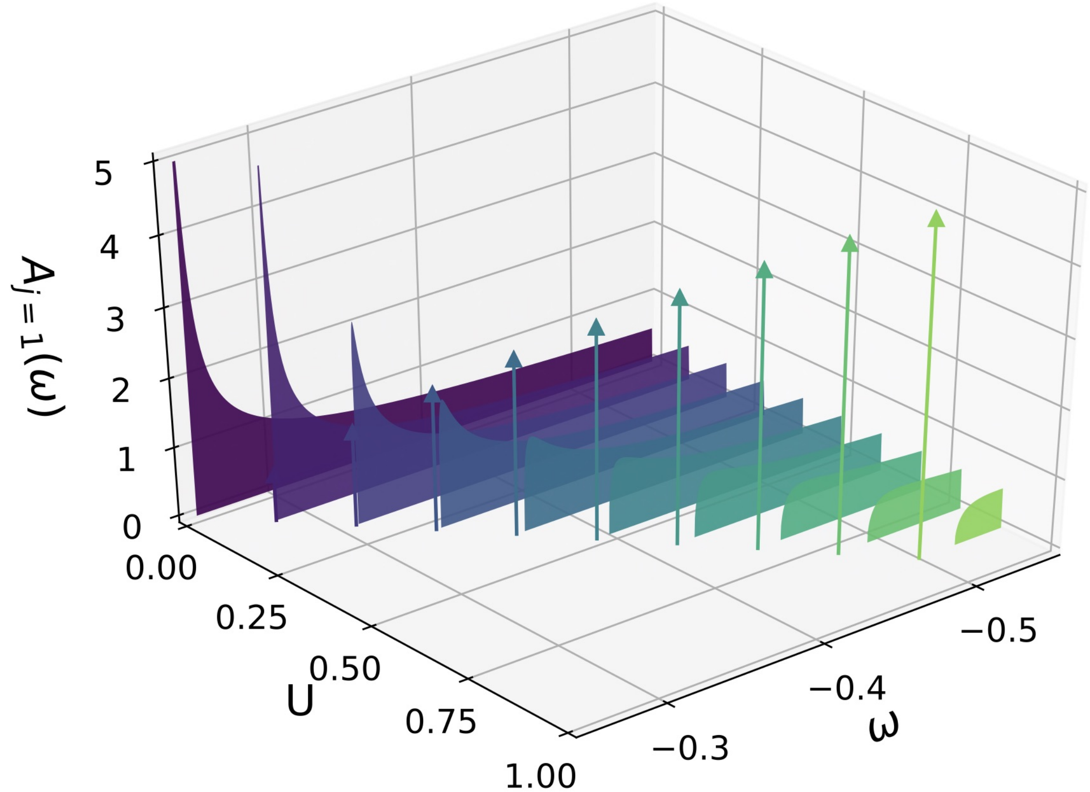




exponential decay and pre-exponential function

Edge states



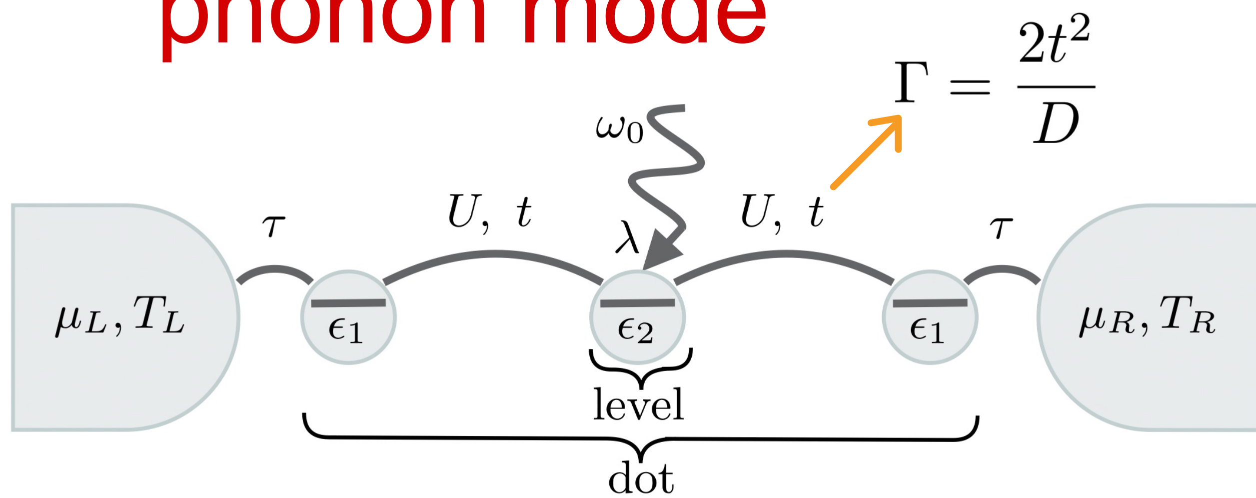


bulk-boundary correspondence at finite *U*?

→ consider the boundary charge instead

A quantum dot coupled to a phonon mode

the model:



the question: signatures of the Coulomb interaction and the phonons in finite bias voltage and temperature gradient particle and thermal steady-state current

the challenge: simple example in which the Keldysh functional RG even in the lowest truncation leads to a frequency dependent self-energy and violation of current conservation, is this a generic problem?

the approximations:

1. neglect flow of the two-particle vertex
2. after integrating out the phonon mode: retarded (frequency dependent) two-particle interaction; self-energy is frequency dependent
3. keep the self-energy feedback on its own flow
4. auxiliary reservoir cutoff

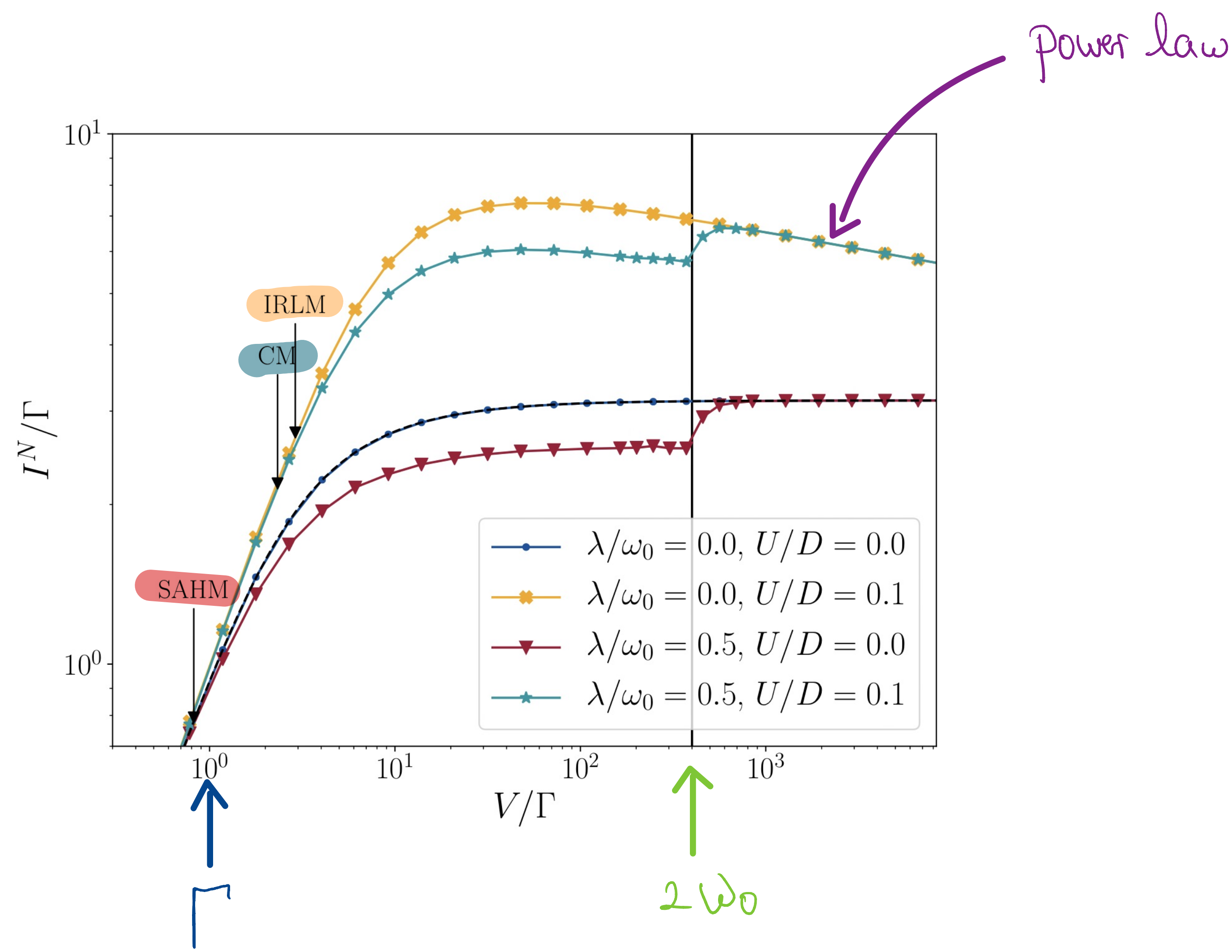
the numerical challenges:

1. real frequency (Keldysh) sharp structures to be integrated over on rhs of flow equations
2. high-resolution frequency grid around characteristic energies

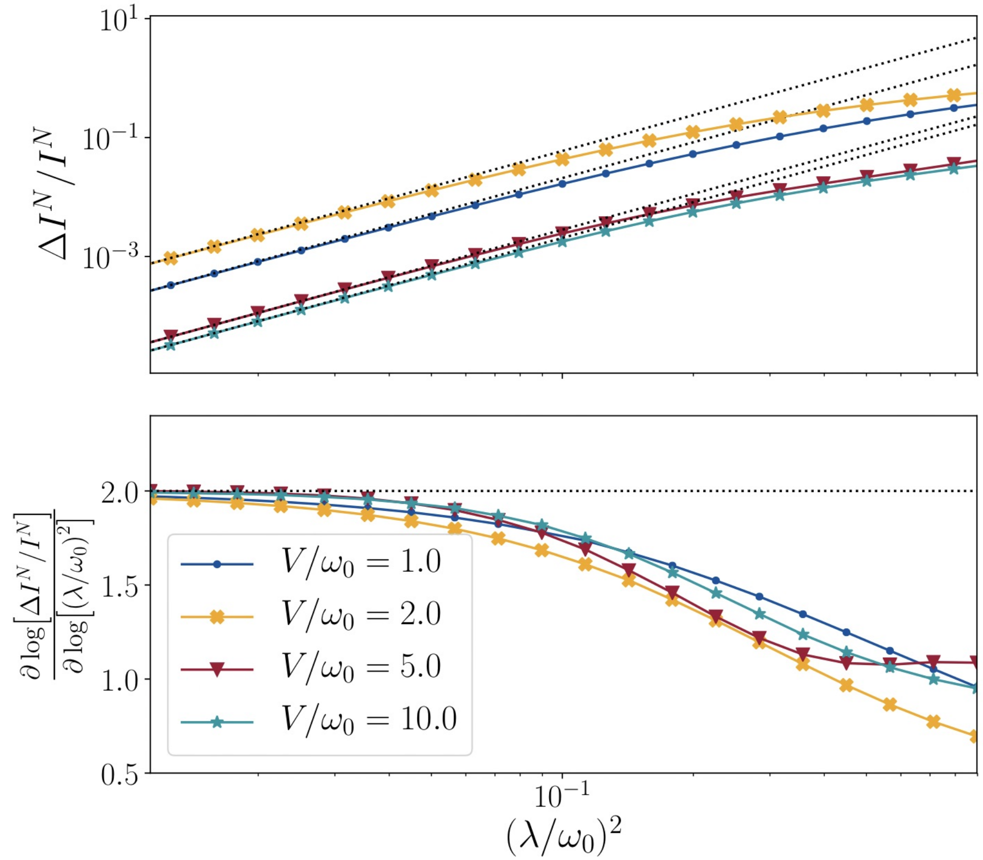
more interesting: anti-adiabatic regime with „fast" phonon $\Gamma \ll \omega_0$

perturbation theory: terms $\lambda^2 \ln\left(\frac{\Gamma}{\omega_0}\right)$, $U \ln\left(\frac{\Gamma}{D}\right)$

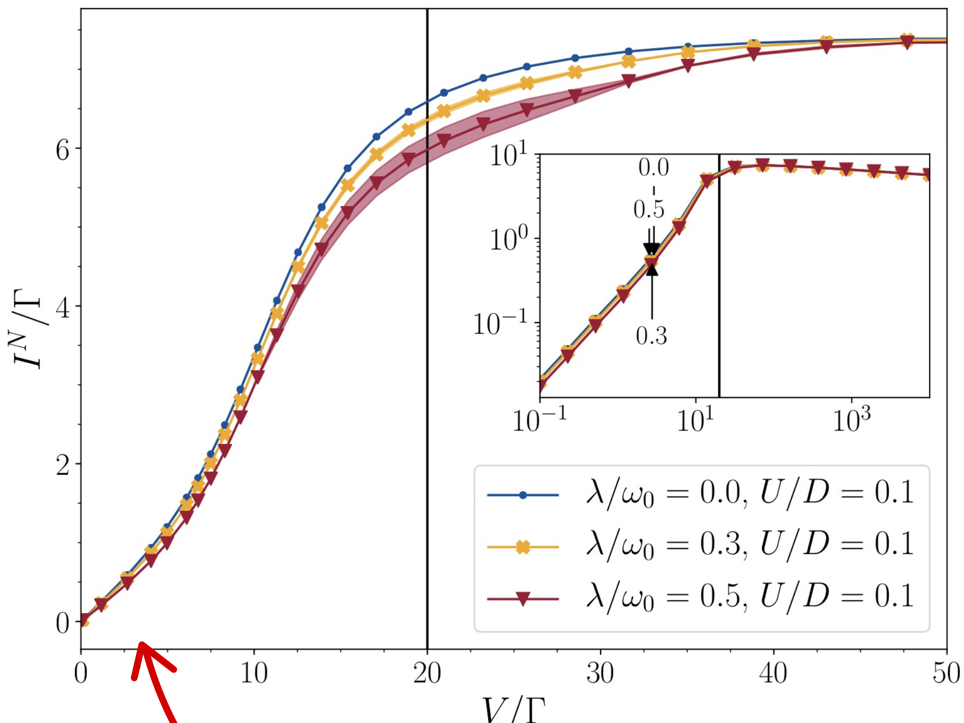
The particle current (at p-h-symmetry)

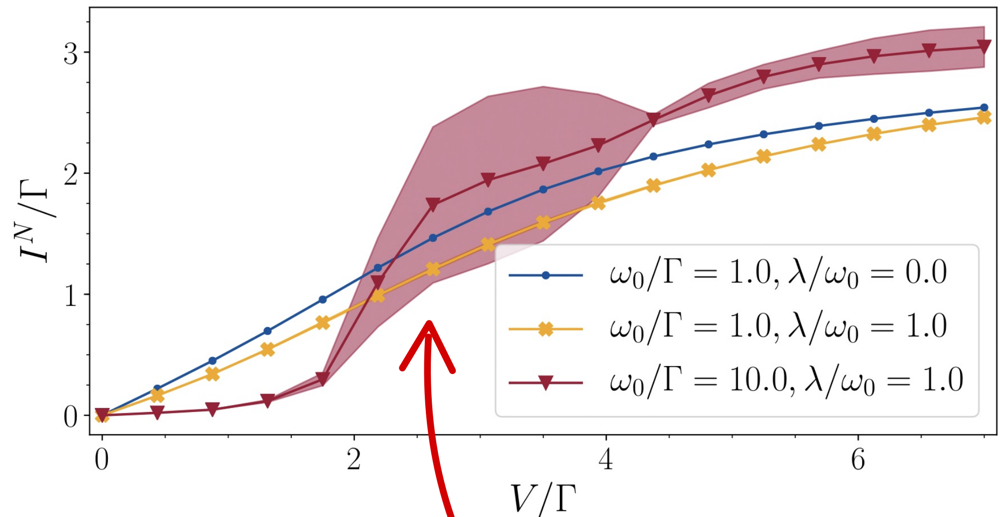


Is the implementation correct?



The current conservation issue (away from p-h-sym.)

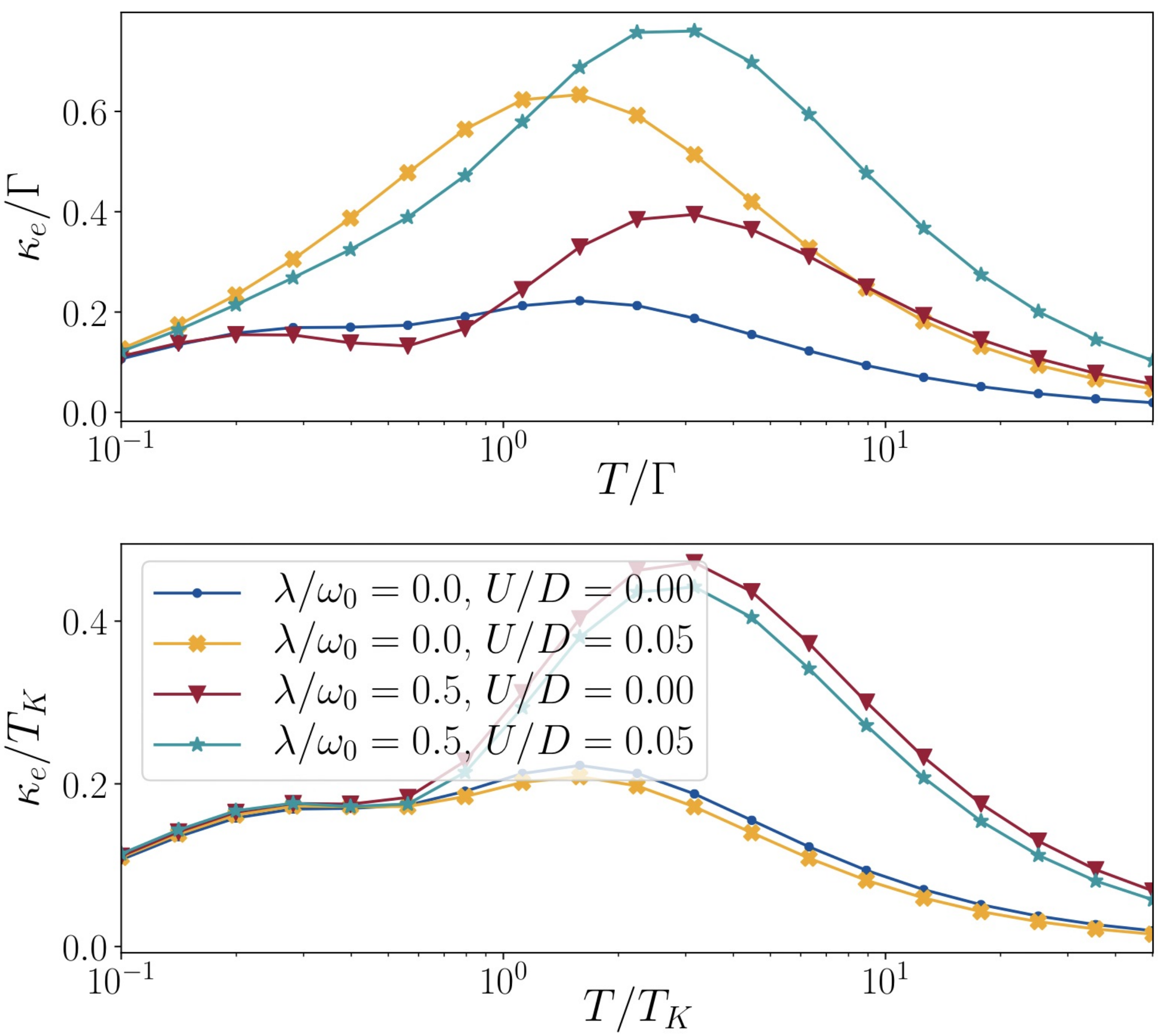




lowest order useless

Frank-Condon blockade

A little bit more on the physics (thermal conductance at finite voltage)



Towards new applications of functional RG: non-Hermitian and PT-symmetric systems

is this something fundamentally new? (Bender et al., Mostafazadeh, Brody,…, late 90ies)

are these merely effective theories for open systems with gain and loss?

(Ashida, Ueda,…, more recent)

starting point: consider interacting toy problem with real spectrum
(no PT-symmetry breaking)

$$H = \frac{p^2}{2} + \frac{x^2}{2} + \frac{ik}{3!} x^3 \qquad k \in \mathbb{R}^+$$

A glimpse at the formalism

(complete) biorthonormal eigenbasis

$$i\partial_t |R_\alpha\rangle = H|R_\alpha\rangle = E_\alpha |R_\alpha\rangle$$

$$i\partial_t |L_\alpha\rangle = H^\dagger |L_\alpha\rangle = E_\alpha^* |L_\alpha\rangle$$

$$\langle L_\alpha|R_\beta\rangle = \delta_{\alpha\beta}$$

$$\sum_\alpha |R_\alpha\rangle\langle L_\alpha| = \sum_\alpha |L_\alpha\rangle\langle R_\alpha| = 1$$

generating functional as usual

$$W(j) = \log\big(Z(j)\big) = \log\left(\frac{1}{Z_0}\int \mathcal{D}x\, e^{-S_0(x)-S_{\text{int}}(x)+(j,x)}\right)$$

$$S(x) = \frac{1}{2}\sum_{\omega_n} x(-\omega_n)\left[\mathcal{G}_0(\omega_n)\right]^{-1} x(\omega_n) + \frac{ik}{3!\sqrt{\beta}}\sum_{\omega_1,\ldots,\omega_4} \delta_{\sum\omega_i,0}\, x(\omega_1)x(\omega_2)x(\omega_3)x(\omega_4)$$

vertex expansion around:

$$\phi^\star = \langle x\rangle\,\big|_{j=0}$$

(Schütz and Kopietz 2006)

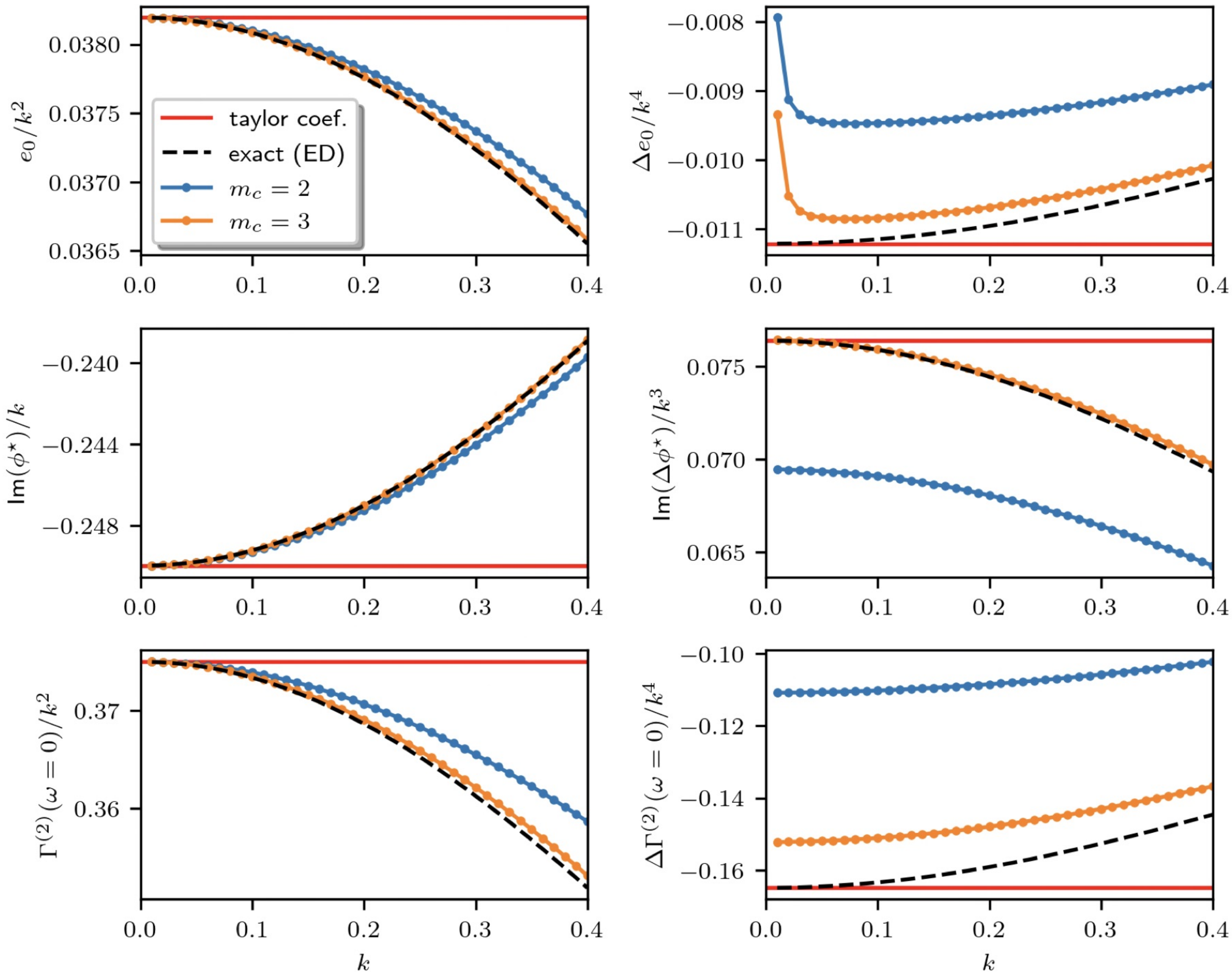
observables?

$$A = \textstyle\sum_{\alpha\beta} A_{\alpha\beta}\, |R_\alpha\rangle\langle L_\beta|$$

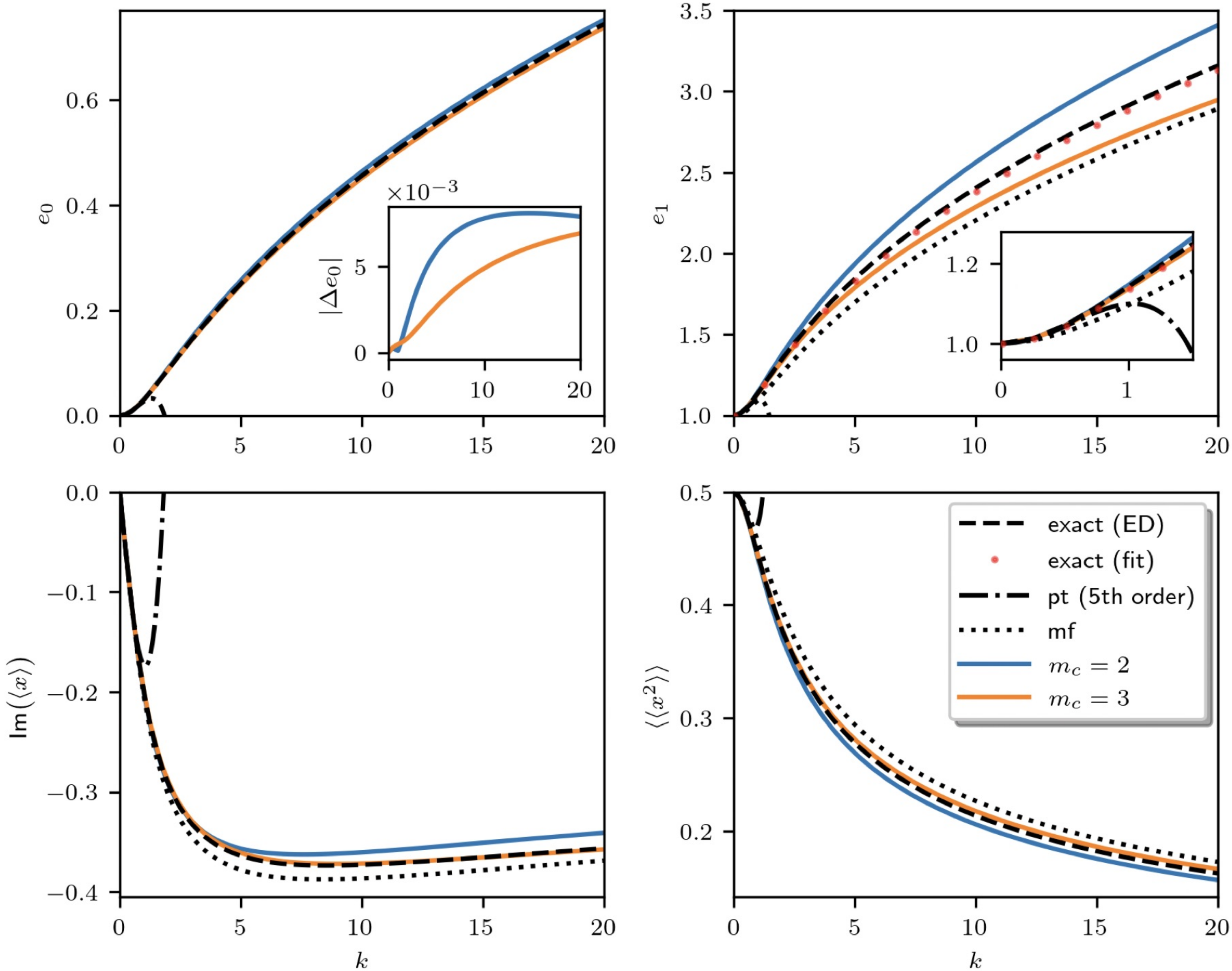
compare to ED, mean-field and perturbation theory

(Bender et al. 2006)

Is the implementation correct?



A few results



What can we learn from all this?

1. phase transitions at finite *U* are accessible (vertex feedback!)

2. meaningful results for 1d band insulators

3. parameter regimes in which the results appear meaningful despite violated conservation laws, internal consistency checks useful

4. application to non-Hermitian many-body systems is promising

5. when ever possible: check implementation of the „many“ equations, discretization in frequency or momentum, sharp structures in integrals on rhs captured,…

thanks to:
Lisa Markhof, Björn Sbierski, Christoph Karrasch
Yen-Ting Lin, Clara Weber, Mikhail Pletyukhov, Dante Kennes, Herbert Schoeller
Mara Caltapanides
Lukas Grunwald