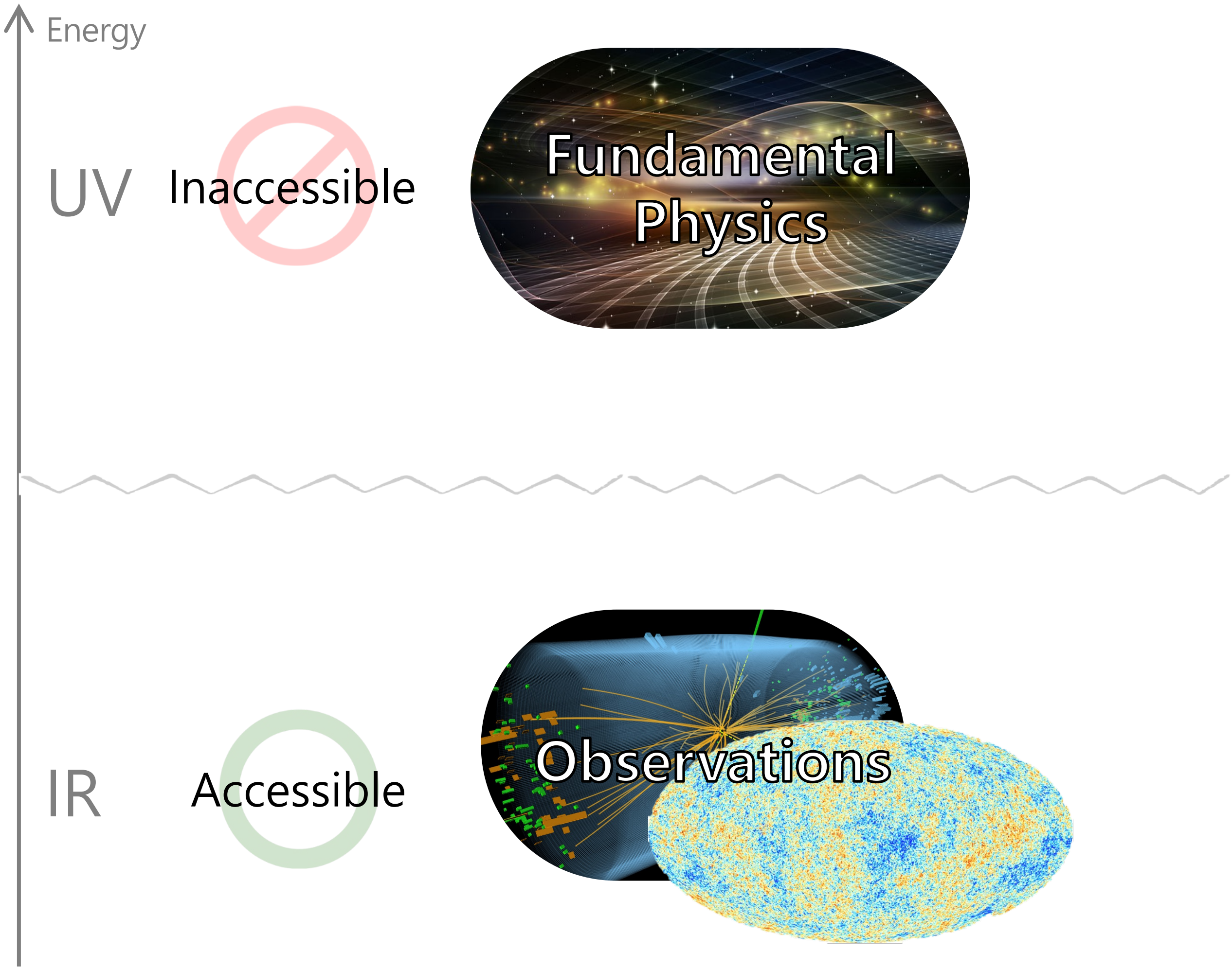
Positivity bounds in Effective Field Theories

Scott Melville

25 Jul 2022

Big Picture

Big Picture



Energy

UV

Inaccessible

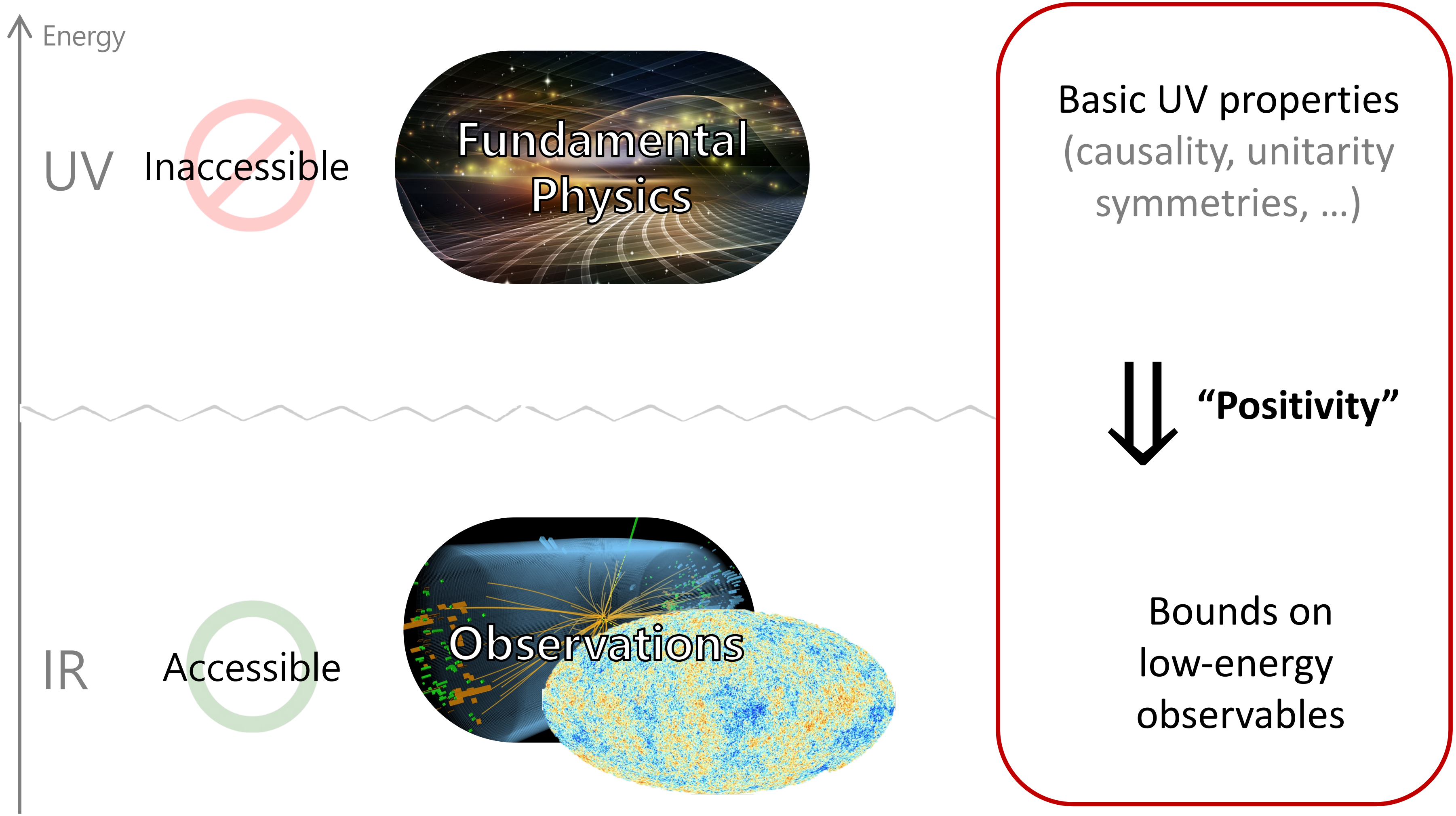
Fundamental Physics

IR

Accessible

Observations

Big Picture



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Basic UV properties
(causality, unitarity
symmetries, …)

$\Downarrow$ **“Positivity”**

Bounds on
low-energy
observables

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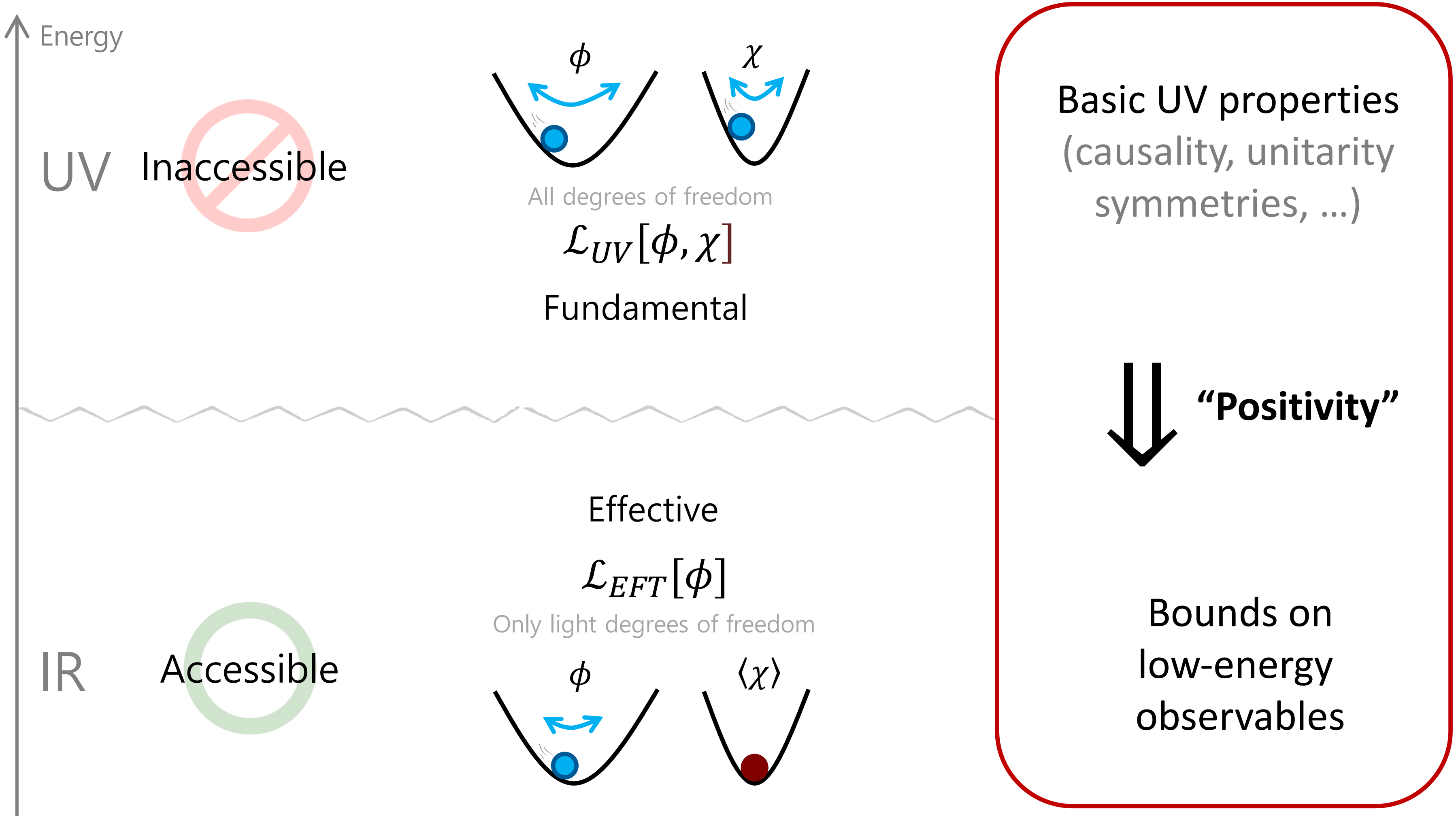

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All degrees of freedom

$\mathcal{L}_{UV}[\phi,\chi]$

Fundamental

Effective

$\mathcal{L}_{EFT}[\phi]$

Only light degrees of freedom

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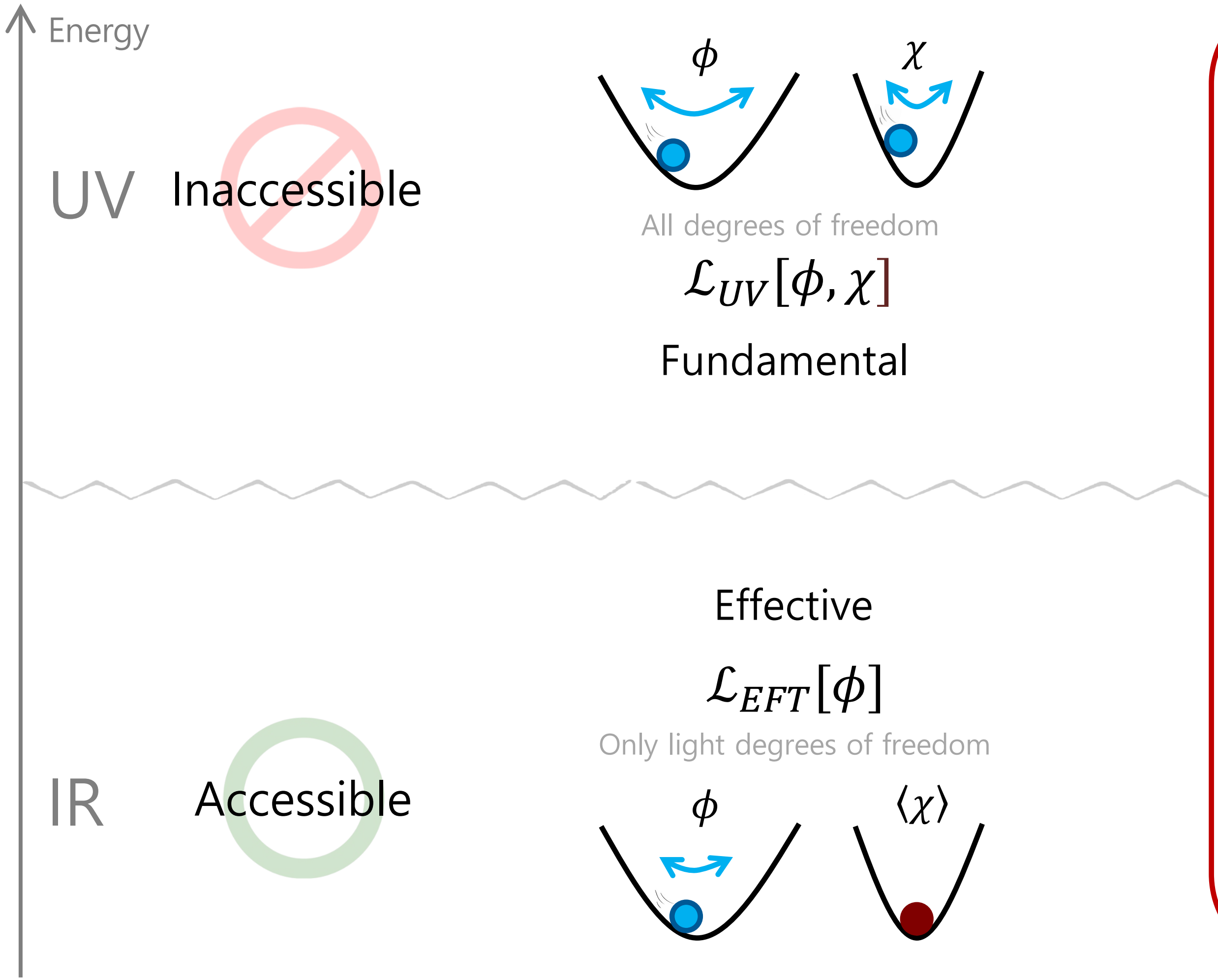

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Constraints on the RG flow

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$$\mathcal{L}_{\text{UV}}[\phi, \chi, \dots]$$

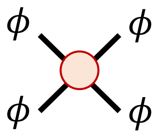
$$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_{\text{renorm.}} + c_2(\Lambda)(\partial\phi)^4 + \cdots$$

Heavy physics is captured by non-renormalizable interactions

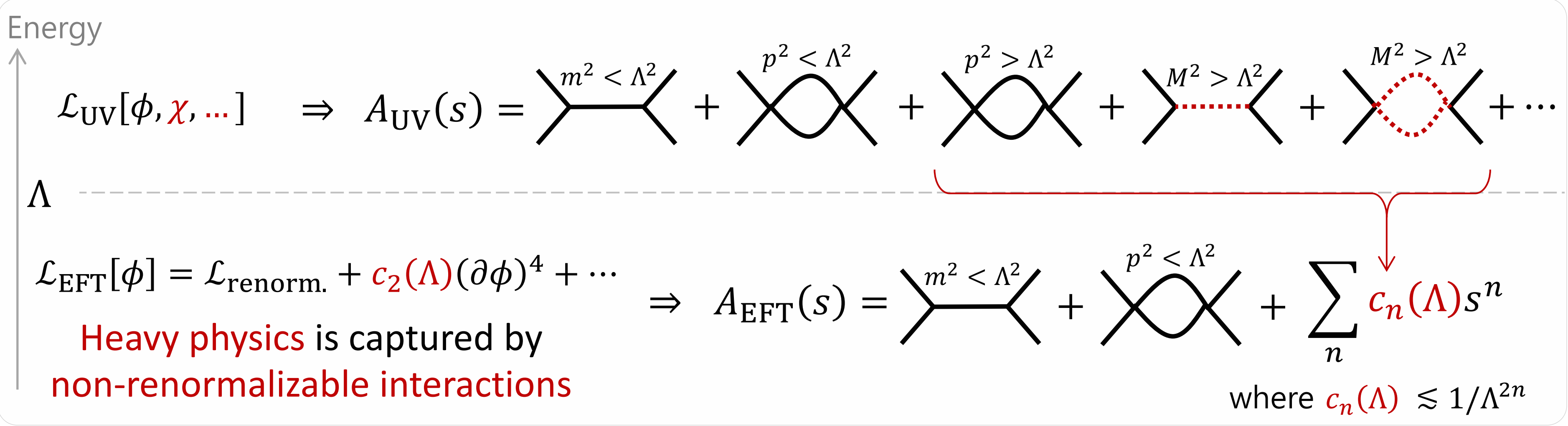
Big Picture

2 → 2 scattering amplitude

$A(s) =$



Total energy,
$s = (p_1 + p_2)^2$



$\mathcal{L}_{\mathrm{UV}}[\phi, \chi, \ldots] \quad \Rightarrow \quad A_{\mathrm{UV}}(s) = \; + \; + \; + \; + \; + \cdots$

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$\Rightarrow \quad A_{\mathrm{EFT}}(s) = \; + \; + \sum_n c_n(\Lambda) s^n$

where $c_n(\Lambda) \lesssim 1/\Lambda^{2n}$

Big Picture

2 → 2 scattering
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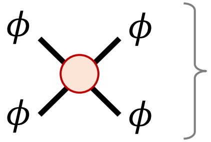

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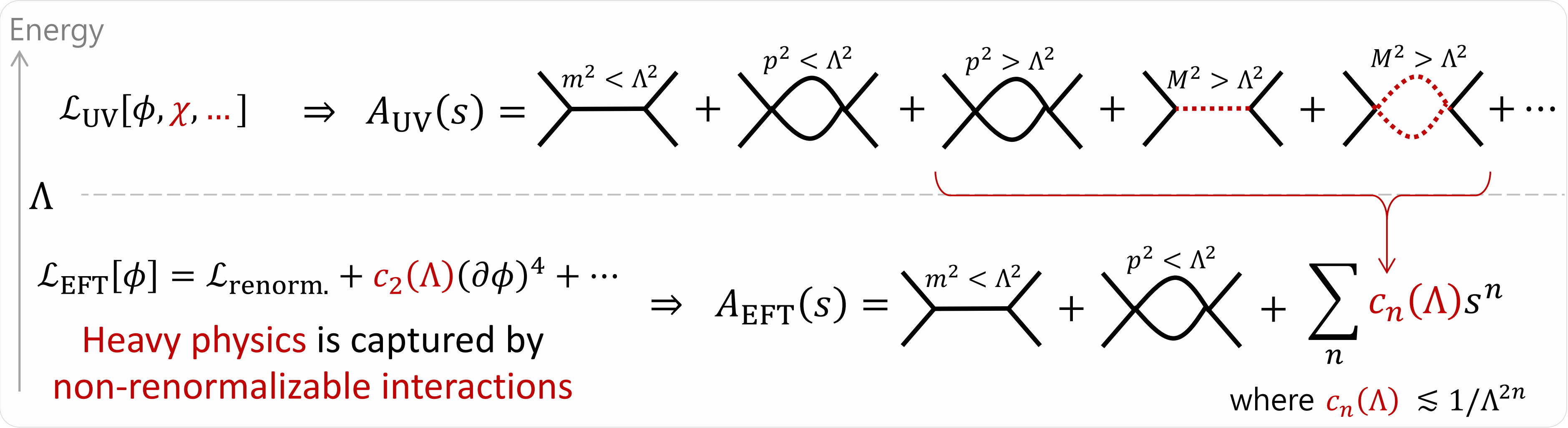

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Λ

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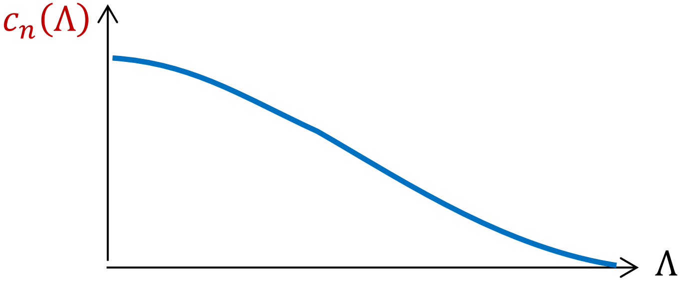
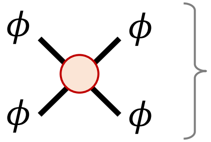
where $c_n(\Lambda) \lesssim 1/\Lambda^{2n}$

Main result: Causality+unitarity of heavy physics ⇒ Constraints on RG flow e.g. $\dfrac{\partial c_n(\Lambda)}{\partial \Lambda} \leq 0, \ldots$



Big Picture

2 → 2 scattering amplitude $A(s) =$



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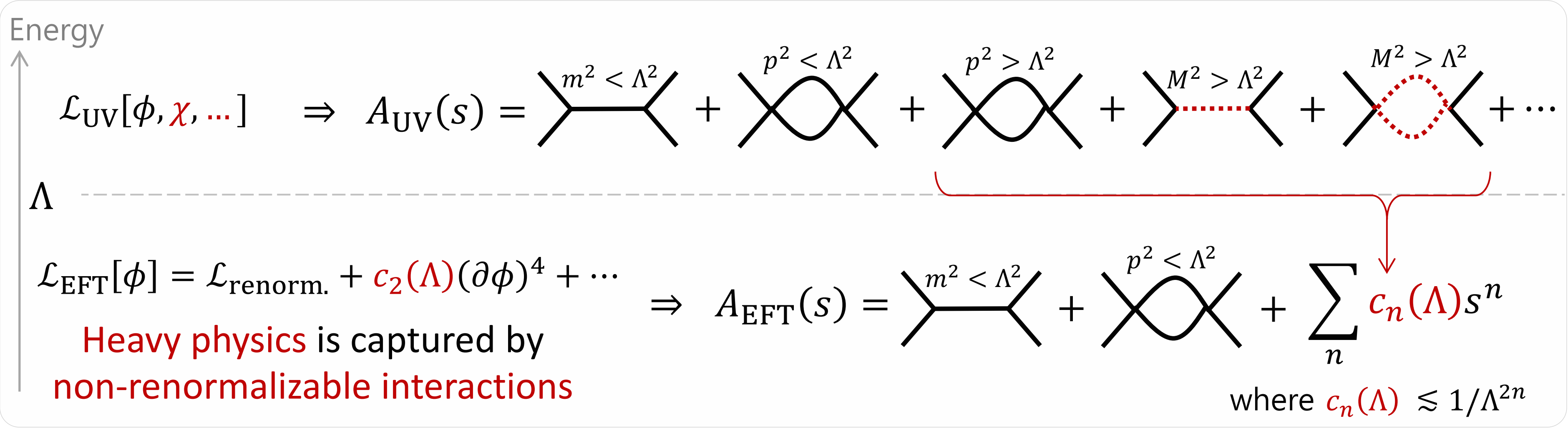

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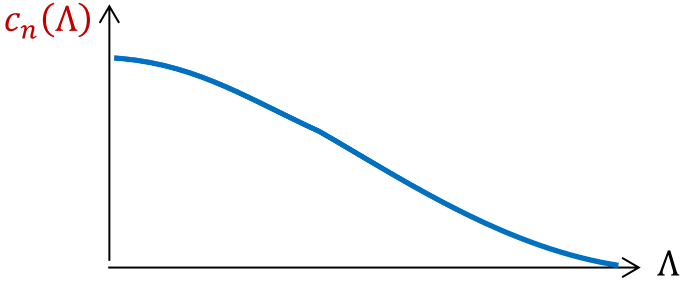
Main result: Causality+unitarity of heavy physics ⇒ Constraints on RG flow e.g. $\dfrac{\partial c_n(\Lambda)}{\partial \Lambda} \leq 0, \dots$

Useful because: Renormalizable UV fixed point $c_n(\infty) = 0$ ⇒ Bounded EFT coefficients e.g. $c_n(\Lambda) \geq 0$



Outline

Positivity bounds on $\partial_\Lambda c_n(\Lambda)$

GR as an EFT

Dark Energy as an EFT

Positivity bounds

Integrating out heavy physics at the scale Λ produces terms like s^n/Λ^{2n} in $A(s)$.

So expect $\Lambda\partial_\Lambda c_n(\Lambda) \sim \frac{1}{\Lambda^{2n}}$

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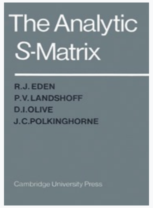
Sketch of proof:

Causality dictates the
analytic structure of amplitude

Modern version of the
S-Matrix Program

[Bremermann, Bros, Epstein, Froissart, Glaser, Gribov, Hepp, Jin, Kallen, Lehmann, Mandelstam, Martin, Taylor, ..., 1960s]



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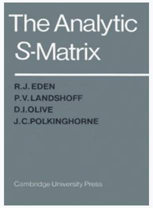
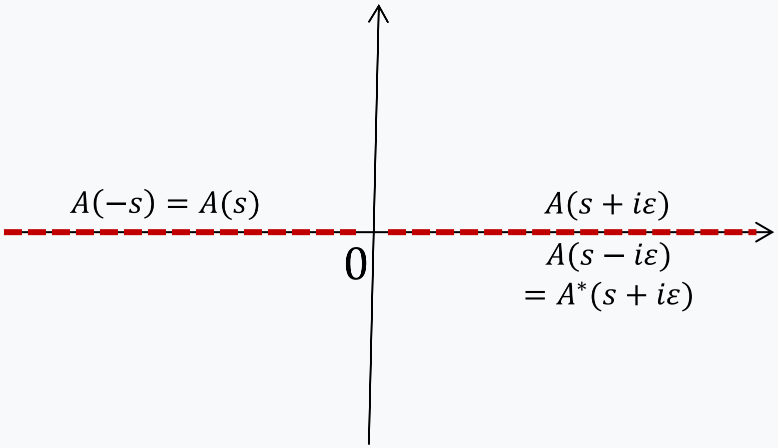


The Analytic
S-Matrix

R.J. EDEN
P.V. LANDSHOFF
D.I. OLIVE
J.C.POLKINGHORNE

Cambridge University Press

$A_{UV}(s)$ is analytic except for branch cuts
on the real axis with discontinuity $2\pi i\, \rho(\sqrt{s})$



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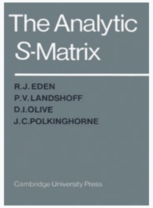

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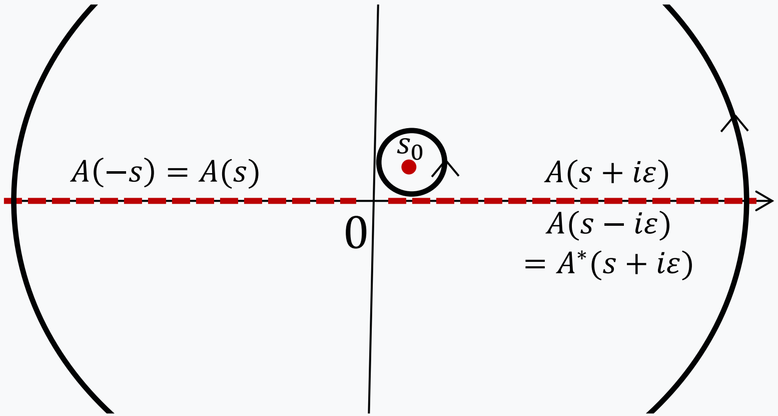

Cauchy's theorem then relates the low- and high-energy amplitudes

$$A_{UV}(s_0) = \oint_{s_0} \frac{ds}{2\pi i}\,\frac{A_{UV}(s)}{s-s_0} = A_{UV}(\infty) + \int_0^\infty ds\, \frac{\rho(\sqrt{s})}{s-s_0}$$

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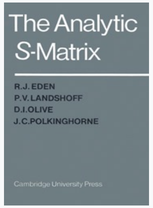

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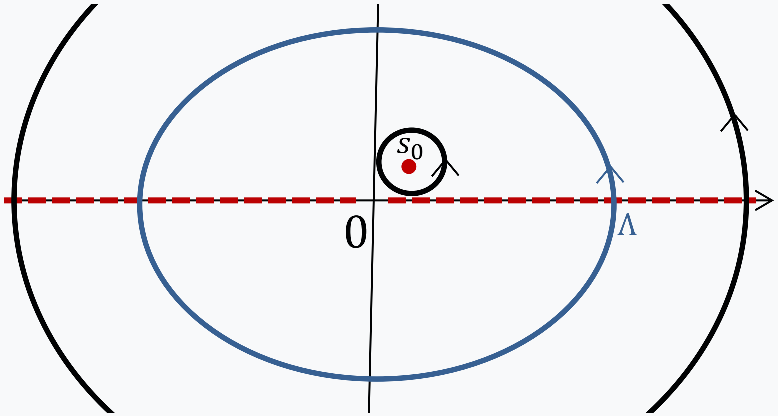

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EFT expansion corresponds to an intermediate contour

$$A_{EFT}(s_0) = \sum_n c_n(\Lambda)\, s_0^n + \int_0^{\Lambda^2} ds \, \frac{\rho(\sqrt{s})}{s - s_0}$$

where $c_n(\Lambda) = c_n(\infty) + \int_{\Lambda^2}^\infty \frac{ds}{s} \frac{\rho(\sqrt{s})}{s^{2n}}$

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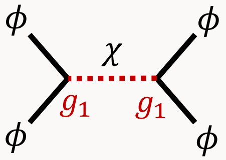
Simple example: $\mathcal{L}_{\text{UV}} \supset g_1 \chi \phi^2 + g_2 \chi^2 \phi^2$

$\Rightarrow A_{\text{UV}}(s) \supset$



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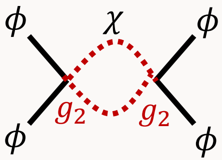

$$= \frac{g_1^2}{M_\chi^2 - s - i\varepsilon} + g_2^2 \sin^{-1}\left(\frac{\sqrt{s}}{2M_\chi}\right) \sqrt{\frac{4M_\chi^2}{s} - 1}$$

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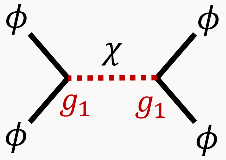

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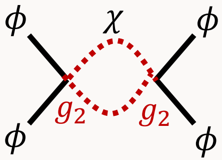

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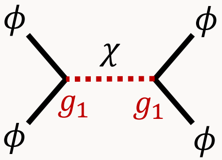

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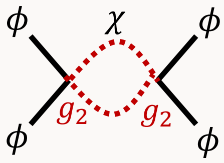

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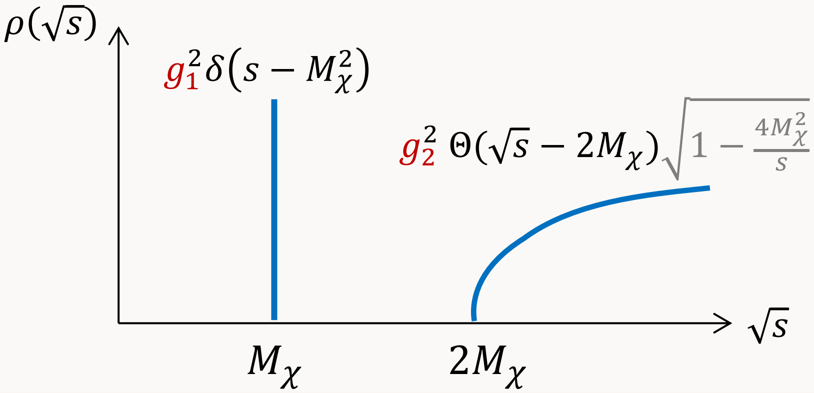
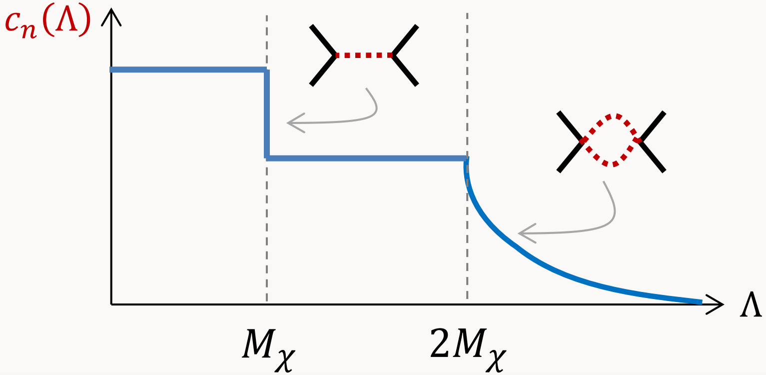


$\Longleftrightarrow$



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The positivity of $\rho(\Lambda)$ is a general consequence of unitarity (probabilities add up to 1),

since the probability $\langle p_1 p_2 | p_1 p_2 \rangle = |1 + i\, A(s)|^2 \leq 1 \;\; \Rightarrow \;\; 2 \operatorname{Im} A_{\text{UV}}(s) \geq |A_{\text{UV}}(s)|^2 \geq 0$

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This monotonicity leads to a variety of bounds on the EFT coefficients $c_n(\Lambda)$

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e.g.
assuming UV
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$$c_2(\infty) = 0,\quad c_4(\infty) = 0,\quad c_6(\infty) = 0$$

Causality
$\Rightarrow$
Unitarity

$c_2(\Lambda) \geq 0, \qquad c_4(\Lambda) \geq 0, \qquad c_2(\Lambda) \geq 0$ [Adams++, hep-th/0602178]

$\Lambda^8 c_6(\Lambda) \leq \Lambda^6 c_4(\Lambda) \leq \Lambda^4 c_2(\Lambda)$ [de Rham+SM+Tolley+Zhou 1702.06134]

$c_6(\Lambda) c_2(\Lambda) \geq c_4(\Lambda) c_4(\Lambda)$ [Bellazzini++, 2011.00037] [Tolley+Wang+Zhou, 2011.02400]

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e.g. assuming UV fixed point with

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$\overset{\text{Causality}}{\underset{\text{Unitarity}}{\Longrightarrow}}$

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We can directly compare these with low-energy measurements of the $c_n(\Lambda)$

Positivity bounds

These bounds have been developed for every EFT operator that contributes to the 2 → 2 amplitude

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General
2 → 2 scattering
amplitude

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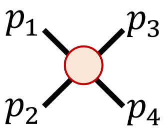


depends on

Ingoing energy, $s = (p_1 + p_2)^2$
Transferred energy, $t = (p_1 - p_3)^2$
(note that $(p_1 - p_4)^2 = -s - t$)

$$\Rightarrow \quad A_{\mathrm{EFT}}(s,t) = \sum_{a,b} c_{2a,b}(\Lambda)(s + t/2)^{2a} t^b$$

Causality of
heavy physics

$$\Rightarrow \quad \Lambda \partial_\Lambda c_{2a,b}(\Lambda) = \partial_t^b \left[\frac{-\rho(\Lambda,t)}{(\Lambda^2 + t/2)^{2a}} \right]_{t=0}$$

where $\rho(\Lambda,t) = \frac{2}{\pi} \operatorname{Im} A_{UV}(s = \Lambda^2, t)$

Unitarity of
heavy physics

$$\Rightarrow \quad \partial_t^b \rho(\Lambda,t) \Big|_{t=0} \geq 0$$

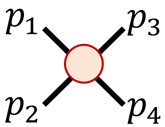
see e.g. [Nicolis+Rattazzi+Trincherini, 0912.4258]

Positivity bounds

These bounds have been developed for every EFT operator that contributes to the 2 → 2 amplitude

General
2 → 2 scattering
amplitude

$A_{UV}(s,t) =$



depends on

Ingoing energy, $s = (p_1 + p_2)^2$
Transferred energy, $t = (p_1 - p_3)^2$
(note that $(p_1 - p_4)^2 = -s - t$)

$$\Rightarrow \quad A_{\text{EFT}}(s,t) = \sum_{a,b} c_{2a,b}(\Lambda)(s + t/2)^{2a} t^b$$

Causality of heavy physics $\Rightarrow$

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Causal + Unitary $\Rightarrow$ Every $\Lambda \partial_\Lambda c_{2a,b}(\Lambda)$ is bounded

e.g. $\Lambda^6 \partial_\Lambda c_{2a,1} + \frac{2a+1}{2} \Lambda^4 \partial_\Lambda c_{2a,0} \geq 0$

[de Rham+SM+Tolley+Zhou, 1702.06134]

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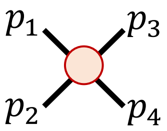

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[de Rham+SM+Tolley+Zhou, 1702.06134]

Also implies non-linear bounds on the $c_{2a,b}$ [Bellazzini++, 2011.00037]

Using Bose symmetry ($s \leftrightarrow t \leftrightarrow u$ crossing) gives further bounds [Tolley+Wang+Zhou, 2011.02400]

Bounds get even stronger for spinning particles [Davighi+SM+You, 2108.06334]

Positivity bounds on $\partial_\Lambda c_n(\Lambda)$

Heavy physics is captured by the EFT expansion,

$$A_{\text{EFT}}(s,t) = \sum_{a,b} c_{2a,b}(\Lambda)(s+t/2)^{2a} t^b$$

Causality
+
Unitarity
⇒

$$\Lambda^{4a}\partial_\Lambda c_{2a,0} = \rho(\Lambda,0) \geq 0$$

$$\Lambda^{4a+2}\partial_\Lambda c_{2a,1} + \tfrac{2a+1}{2}\Lambda^{4a}\partial_\Lambda c_{2a,0} = \partial_t \rho(\Lambda,0) \geq 0$$

$$\vdots$$

A UV fixed point (i.e. $c_{2a,b}(\infty) = 0$) can only exist if the $c_{2a,b}(\Lambda)$ satisfy an infinite number of coupled constraints,

e.g. $c_{4,0} \geq 0$ and $\Lambda^6\partial_\Lambda c_{2,1} + \frac{3}{2}\Lambda^4\partial_\Lambda c_{2,0} \geq 0$

GR as an EFT

Dark Energy as an EFT

GR as an EFT

Pure gravity in 4 spacetime dimensions is described at low energies by, (see e.g. [Donoghue, gr-qc/9512024])

$$\mathcal{L}_{\mathrm{EFT}}[g_{\mu\nu}] = M_P^2 R + c_4(\Lambda)\left(R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}\right)^2 + \tilde{c}_4(\Lambda)\left(R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma}\right)^2 + \cdots$$

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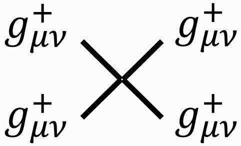
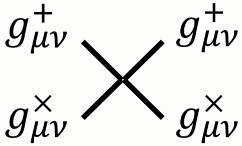
Two distinct scattering amplitudes:





$$A_{++++}(s) = s^4\ c_4(\Lambda) \qquad A_{+\times+\times}(s) = s^4\ \tilde{c}_4(\Lambda)$$

Renormalizable UV fixed point
+ causal, unitary RG flow ⇒

$$c_4(\Lambda) \geq 0,$$
$$\tilde{c}_4(\Lambda) \geq 0$$

[Bellazzini+Cheung+Remmen, 1509.00851]

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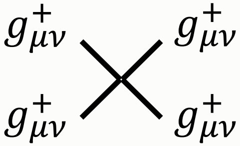

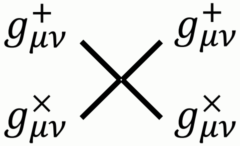

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$$c_4(\Lambda) \geq 0,$$
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[Bellazzini+Cheung+Remmen, 1509.00851]

Many recent developments:
- including matter fields
- stronger bounds (more UV input)
- different spacetime dimensions
- connections with string swampland

e.g. [Tokuda+Aoki+Hirano, 2007.15009]
[Alberte+de Rham+Jaitily+Tolley,, 2007.12667]
[Caron-Huot+Li+Parra-Martinez+Simmons-Duffins, 2201.06602]
[Bern+Herrmann+Kosmopoulos+Roiban, 2205.01655]
[Herrero-Valea+Koshelev+Tokareva, 2205.13332], ...

Dark Energy as an EFT

Scalar-tensor EFT $\mathcal{L}_{EFT}[\overset{\text{Metric}}{g_{\mu\nu}}, \overset{\text{Light scalar field}}{\phi}]$ with $\Lambda^3 \sim M_P H_0^2 \ll M_P^3$

Many possible interactions. Two ways to proceed:

Dark Energy as an EFT

Metric

Light scalar field

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(1) Assume more symmetries, e.g. $\phi \to \phi + c + c_\mu x^\mu$

$$\mathcal{L}_{EFT} = c_2 X + c_3 \mathcal{L}_3 + c_4 \mathcal{L}_4 + c_5 \mathcal{L}_5$$

with

$$\mathcal{L}_3 = 2X\left(\nabla_\mu \nabla^\mu \phi\right), \qquad \mathcal{L}_4 = 2X\left(\left(\nabla_\mu \nabla^\mu \phi\right)^2 - \left(\nabla_\mu \nabla_\nu \phi\right)^2\right)$$

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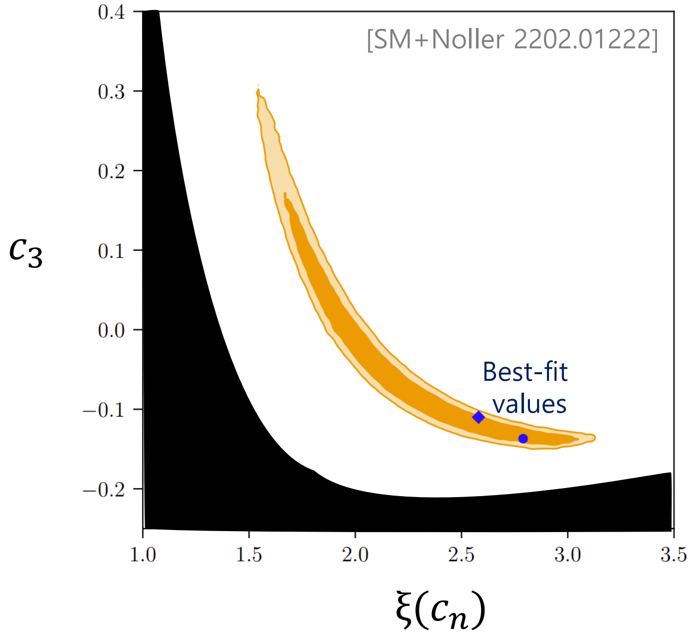
This "covariant Galileon" EFT has a ΛCDM-like background,

$$\frac{\nabla\nabla\phi}{\Lambda^3} = -\xi(c_n)\left(\delta_\mu^\nu + q\,\delta_\mu^0 \delta_0^\nu\right)$$

[Renk et al, 1707.02263]

Fixed by eom

Deceleration parameter



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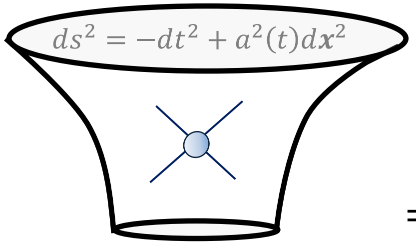

Fluctuations with energy $H \ll \omega \ll \Lambda$ do not feel background curvature.

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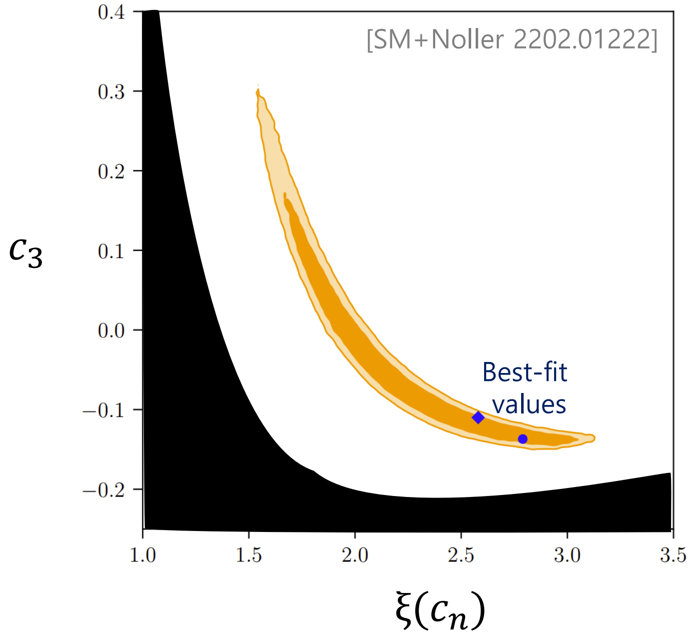

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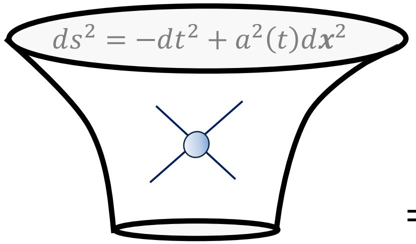

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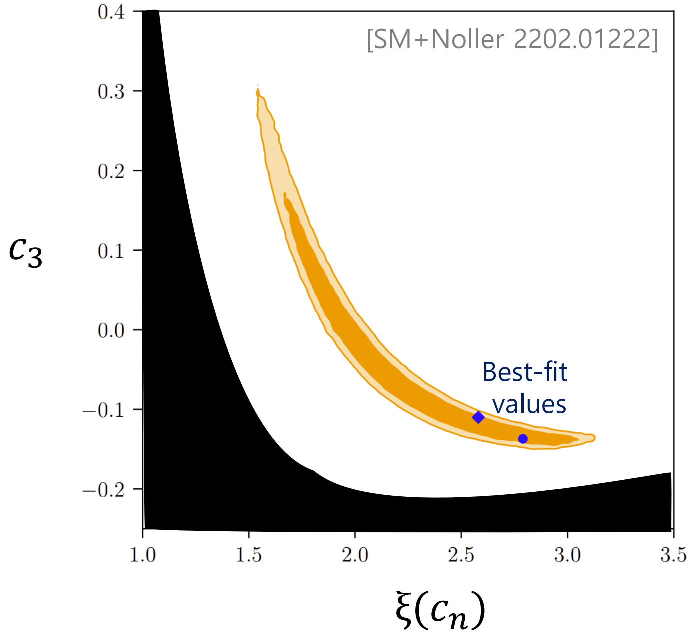

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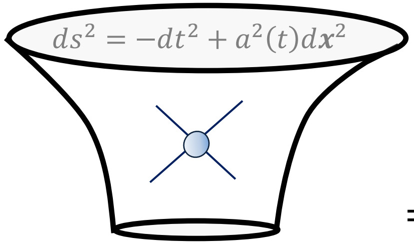

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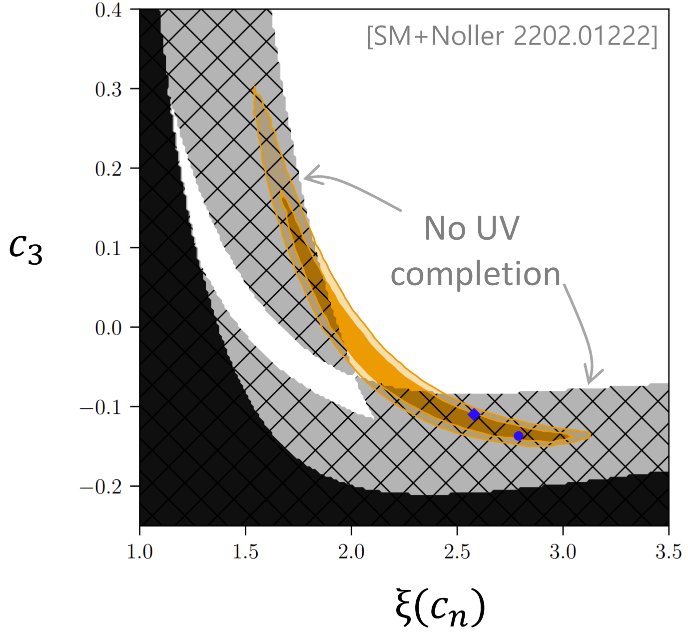
This amplitude violates the positivity bounds required for a unitary UV completion in large regions of parameter space.



[SM+Noller 2202.01222]

No UV completion

Dark Energy as an EFT

Metric

Light scalar field

Scalar-tensor EFT $\mathcal{L}_{EFT}[g_{\mu\nu}, \phi]$ with $\Lambda^3 \sim M_P H_0^2 \ll M_P^3$

Many possible interactions. Two ways to proceed:

(2) Work with general parametrisations

$$\mathcal{L}_{EFT} = M^2(X)R + P(X)$$

with $X = (\partial\phi)^2$

e.g. [SM+Noller, 1904.05874]
[Herrero-Valea+Timiryasov+Tokareva, 1905.08816]
[SM+Grall, 2102.05683]
[de Rham+SM+Noller, 2103.06855]
[Noumi+Tokuda, 2105.01436], …

Some general lessons:

(i) Subluminal GWs are difficult* to UV complete

[de Rham+Tolley, 1909.00881]
[de Rham+SM+Noller, 2103.06855]

(ii) Screening mechanisms are difficult* to UV complete

[SM+Davis, 2107.00010]

* difficult = any operator that contributes to 2 → 2 scattering only satisfies these positivity bounds if they lead to superluminal GWs and no screening

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Causality
+
Unitarity

$$\Rightarrow \quad \begin{aligned} \Lambda^{4a}\partial_\Lambda c_{2a,0} &= \rho(\Lambda,0) \geq 0 \\ \Lambda^{4a+2}\partial_\Lambda c_{2a,1} + \tfrac{2a+1}{2}\Lambda^{4a}\partial_\Lambda c_{2a,0} &= \partial_t \rho(\Lambda,0) \geq 0 \\ &\vdots \end{aligned}$$

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has $c_4(\Lambda) > 0$ and $\tilde{c}_4(\Lambda) > 0$

Dark Energy as an EFT

(i) Subluminal GWs are difficult to UV complete

(ii) Screening mechanisms are difficult to UV complete